1. Introduction

Large-volume vertical cylindrical steel tanks are widely used in the oil refining industry to store oil or petroleum products. Considering that vertical cylindrical tanks are categorized as particularly responsible structures, their destruction can lead to environmental disasters, significant material costs, and human casualties [1–3]. Construction and operation of tanks should be based on sound scientific, technically possible, fundamentally new design and econom-
ically justified solutions. This leads to the need for additional research into the development of scientific foundations for assessing the strength, stability, and durability of vertical cylindrical tanks, taking into account actual operating conditions [4, 5]. When designing reservoirs operated in seismically hazardous areas, research is carried out in the field of structural vibrations [6–13]. As a result of seismic load, there is a possibility of structural failure of the tank in the areas of bolted joints, dents [6, 7], welds [8, 9], and in other places of stress concentration [10–13].

Therefore, studies aimed at analyzing the vibration frequencies of the wall of a steel vertical cylindrical tank for petroleum products, hardened by a prestressed winding, are relevant; the results could be in demand by research and design institutes.

2. Literature review and problem statement

To ensure long-term and trouble-free operation of large tanks, technical solutions are implemented aimed at enhancing the strength of structures. Thus, one of the options for strengthening the finished structure is to apply the winding to the outer surface of the wall when studying the free vibrations of the liquid in the tank [14]. The model of free vibrations of a tank without liquid is considered in [15]. A similar model, but with the use of composites as a winding, was built in study [16]. However, in these works, the vibration frequencies of the tank wall were investigated without taking into account prestresses. The winding is taken into account in this model as additional mass and as wall thickening. At the same time, the stressed-strained state of the tank with a winding of steel wire and composite filament with a filament winding pitch of 1:1, 1:2, and 1:3 showed the effectiveness of its use. Its application prevents the localization of stresses in the ring layers of the tank wall due to the action of liquid pressure. This model of free oscillations of the liquid in the tank. It makes it possible to determine the natural modes and natural frequencies of oil oscillations in a tank filled to a predetermined level. The authors also conducted a study [15] of displacements in the wall of the tank at the same tension of the thread for wire with a diameter of 3 mm, 4 m, and 5 mm. It should be noted that the specification of the diameter of the wire does not affect the qualitative result of studies of the laws of the influence of the pre-tension force of the winding on the vibrations of the tank. In this case, the model of free vibrations of a tank without liquid makes it possible to determine the natural frequencies and eigenmodes of the tank wall under given conditions for fixing the structure.

Article [17] reports the results of modeling the amplification of the same input movement by soil layers of two different sites. Geographically, the plots are located at a distance of 10 km from each other. Under such conditions, when designing earthquake-resistant objects, design engineers usually use the same set of estimation accelerograms to calculate the combination of emergency loads. The simulation results presented in article [17] showed that soil reinforcement during the same input movement, even in closely spaced and, at first glance, similar in geological structure, areas can differ significantly. Paper [18] considers the classification of destruction of cylindrical steel tanks from the action of seismic loads. The features of the static calculation of shells are revealed. The physical and mechanical properties of the material and the geometric parameters of the vertical tank were determined. A full-size finite-element model of a filled vertical steel tank has been developed. The calculation of the filled vertical steel tank was performed, and the values of the frequencies and periods of natural oscillations were obtained. As a reinforcement of tanks, the use of composite materials with nano-inclusions is proposed [19].

Study [4] gives a simple and effective model that takes into account the effects produced at the base of the tank (i.e., lifting and swinging), as well as the flexibility of the soil and nonlinear effects in the anchor system. A comprehensive analysis of the effects of the interaction of the soil, foundation, and structure in combination with the influence of the convective part of the liquid was carried out for three fixed cylindrical steel tanks. These structures were subject to lateral seismic loads, and three seismic vibrations of the soil and four different options for soil flexibility were also considered. The results make it possible to better understand the seismic response of anchor tanks and show that the soil-foundation-structure interaction reduces critical response parameters. These findings may serve as motivation for possible changes to existing seismic design standards. In the cited study, only the issue of forced vibrations of the structure was considered.

In [20], a parametric study was carried out using numerical simulation for a dynamic load in the form of an explosion. The study took into account the internal level of the liquid, the wall thickness, the yield strength of the material, the restrictive conditions, and the intensity of the explosion using the finite-element method. The wall thickness of the tank varies between 10 mm, 20 mm, 30 m, and 40 mm, while the filling level of the internal liquid varies between 25, 50, 75, and 100%. Similar experimental studies into the effect of liquid pressure in partially filled tanks were carried out in [21]. However, these studies looked at traditional design parameters only.

In [22], the maximum error of numerical calculations was established in comparison with the analytical solution in the consideration of problems of free oscillations of a cylindrical shell. Article [23] reports the results of modeling the operation of metal tanks filled with water. Study [24] describes a methodology for numerical analysis of a composite structure under operational loads associated with transportation. In [25], a cylindrical tank for storing petrochemical products is considered. It is noted that a typical tank design has a wall that is modeled by a thin shell. The dynamic response of the structure to the external explosive load was investigated. The structure is reinforced with anti-explosion strips. Simulation was carried out at various parameters of anti-explosion strips during an external explosion. Article [26] studies the features of deformation processes of cylindrical steel tanks with defects. The longitudinal bending of the tank wall at constant external pressure was studied. It is shown that the presence of initial defects in the design significantly reduces the strength of the tanks.

When numerically simulating vibrations of a hardened winding of the tank wall, it should be borne in mind that the construction of an accurate winding model is extremely difficult and inefficient while the use of an averaged model is effective. The possibility of using averaged mechanical characteristics for numerical studies of structures is shown in [27–29]. To take into account the change in the wall thickness of the tank in height, it is most effective to solve
the problem by the finite-element method in a three-dimen-
sional statement.

As shown in [30–32], this approach is relevant for the
study of real industrial structures. The basic laws of the
process of oscillation of engineering structures are given in
monograph [33].

Our review of publications showed that the vibration
frequencies of a steel vertical cylindrical tank for petroleum
products, hardened by a prestressed winding, are poorly
studied and require additional research related to the use of
prestress to protect against dynamic influences in the form of
horizontal vibrations, taking into account the filling level of
the structure. In the future, such studies could be positively
used as anti-seismic measures by designers and engineers in
order to improve the strength characteristics of existing cy-
lindrical structures.

3. The aim and objectives of the study

The purpose of this study is to determine the regu-
larities of the influence of the pre-tensioning force of the
winding on the vibrations of the tank wall. This issue will
make it possible to further adjust the frequencies by chang-
ing the tension force of the filament in the winding for two
cases, taking into account the influence of the tank coating
and without it.

To accomplish the aim, the following tasks have been set:
– to investigate the frequencies and vibration modes of
the winding-hardened tank wall without taking into ac-
count the influence of the tank coating;
– to investigate the frequencies and vibration modes of
the winding-hardened tank wall, taking into account the
influence of the tank coating.

4. The study materials and methods

The object of this study is the process of vibrations of the
wall of a steel vertical cylindrical tank for petroleum prod-
ucts, hardened by a prestressed winding, taking into account
and without the influence of the tank coating at different
levels of filling the structure.

During operation, the wall of the tank hardened by the
prestressed winding is exposed to the internal pressure
from the poured oil and the external pressure from the
winding. As a result of this impact, stresses arise in the
tank wall. Their magnitude affects the vibrations of the
structure. The amount of pressure on the wall of the tank
from the winding can be changed by tensioning the fil-
ament in the winding. The pressure value depends on such
parameters as the pitch of the winding turn, the thickness
of the thread, and the force of its tension [14–16]. This
pressure partially compensates for the effect of the hydro-
static pressure of the oil. At the same time, the presence of
a winding on the wall of the tank changes the frequency
spectrum of the structure [34–38]. On the one hand, the
presence of a winding increases the mass of the structure,
which leads to a decrease in the frequency spectrum. On
the other hand, the prestressed winding increases the rigid-
ity of the structure, which leads to an increase in the
frequency spectrum. Which of these effects will prevail is
determined by a number of factors related to the winding
parameters.

The spectrum of frequencies and vibration modes of the
winding-reinforced wall of a vertical cylindrical steel tank
with a volume of 3000 m³ is investigated. The problem is
solved by the finite-element method, which is implemented
in the ANSYS Workbench software suite.

The geometric characteristics of the vertical steel cylin-
drinal tank are given in Table 1.

| Table 1
<p>| Geometrical characteristics of vertical steel cylindrical tank [14–16] |</p>
<table>
<thead>
<tr>
<th>No.</th>
<th>Geometrical parameter</th>
<th>Value, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inner diameter, D</td>
<td>18.38</td>
</tr>
<tr>
<td>2</td>
<td>Tank height, h</td>
<td>11.92</td>
</tr>
<tr>
<td>3</td>
<td>The height of the bottom layer, (h_1)</td>
<td>1.49</td>
</tr>
<tr>
<td>4</td>
<td>The height of the second layer, (h_2)</td>
<td>1.49</td>
</tr>
<tr>
<td>5</td>
<td>The height of the third layer, (h_3)</td>
<td>2.98</td>
</tr>
<tr>
<td>6</td>
<td>The height of the top layer, (h_4)</td>
<td>5.96</td>
</tr>
<tr>
<td>7</td>
<td>The wall thickness of the bottom layer, (t_1)</td>
<td>0.008</td>
</tr>
<tr>
<td>8</td>
<td>The wall thickness of the second layer, (t_2)</td>
<td>0.006</td>
</tr>
<tr>
<td>9</td>
<td>The wall thickness of the third layer, (t_3)</td>
<td>0.005</td>
</tr>
<tr>
<td>10</td>
<td>Top layer wall thickness, (t_4)</td>
<td>0.004</td>
</tr>
<tr>
<td>11</td>
<td>Maximum liquid loading height in seismically hazardous areas</td>
<td>11.08</td>
</tr>
</tbody>
</table>

At the point of attachment of the wall of the tank with
the bottom, the boundary conditions for rigid fastening are
set. Boundary conditions at the upper edge are considered
of two types. The first type, the free edge, corresponds to
an uncoated tank. The second type, in the absence of move-
ments in the normal direction to the wall, corresponds to a
coated tank. The magnitude of the hydrostatic pressure of oil
on the inner surface of the tank wall depends on the height
of the oil loading. Its quantitative characteristics were deter-
mined earlier in works [14–16]. Oil splashes in the tank are
not taken into account.

The tank is made of C245 steel: modulus of elasticity,
\(E=2.1\times10^11\) Pa; Poisson ratio, \(\nu=0.3\); density, \(\rho=7850\) kg/m³;
yield strength, \(\sigma_{0.2}=245\) MPa. The appearance of zones of
plastic flow of the material is considered unacceptable. The
study examines a winding made of 65T steel with yield
strength \(\sigma_{0.2}=785\) MPa; wire diameter, 4 m; winding pitch is
close [14–16].

The oscillation of elastic shells of rotation, partially
filled with an ideal incompressible fluid, was considered. The
system of equations of motion is symbolically written in the
following form [19]:

\[
L(U) + M(\ddot{U}) = P, \tag{1}
\]

where \(U=U(x, y, z, t)\) is the displacement vector;
\(L\) is the operator of elastic forces;
\(M\) is the operator of mass forces;
\(P=P(x, y, z)\nu+P_n(x, y, z)n\) – total pressure on the tank wall;
\(n\) is the external normal to the tank wall.

Let’s consider the loading scheme of the tank wall (Fig. 1).
The outer surface of the tank wall is affected by the dis-
tributed pressure \(P_1\) from the prestressed winding. The mag-
nitude of the thread tension force is determined according to
the dependence given in [22–25].

The equilibrium equation is considered on the basis of
the moment-free theory of shells:
The integration and transformation of expression (2) leads to equality at the point \( P = N \). Based on this, it is possible to determine the contact pressure of the winding on the tank \( P_c \):

\[
P_c = \frac{N}{b^2 m R}.
\]

The distributed pressure \( P_1 \) is determined on the basis of contact pressure (3) under the assumption that the pressure is evenly distributed over the outer surface of the tank wall by the width of the wire diameter.

The hydrostatic pressure \( P_2 \) is determined by the following formula [21]:

\[
P_2(h) = -pg(h - d),
\]

where \( p \) is the density of the liquid filling the tank; 
\( g \) – acceleration due to gravity; 
\( d \) is the height to which the liquid is poured.

This study investigated crude oil with a density of 850 kg/m³. The acceleration due to gravity was assumed to average 9.81 m/s².

The oscillations of the oil tank reinforced by the winding are represented in the form of a series:

\[
U(x, y, z, t) = \sum_{k=1}^{n} c_k(t) u_k(x, y, z),
\]

where \( u_k(x, y, z) \) are the eigenmodes of the tank; \( c_k(t) \) are unknown coefficients that depend only on time.

The natural vibration modes satisfy the orthogonality condition:

\[
Lu_k = \omega_k^2 M u_k(x, y, z) = \delta_{kj},
\]

where \( \omega_k^2 \) is the natural frequency of oscillations corresponding to the \( u_k \) oscillation mode of the tank.

Thus, taking into account representation (5) and condition (6), equation (1) with loads represented in the form of (3) and (4) is reduced to the problem of eigenvalues. The problem is solved by the finite-element method in the ANSYS Workbench software package.

At the current level of development of computer technology and software, it is advisable to analyze the influence of the parameters of the prestressed winding on the vibrations of the tanks by numerical methods. Numerical studies into the frequencies and vibration modes of a tank filled to a given height with oil with a prestressed winding made of steel wire stretched with a given force are carried out in the Modal calculation module. This model is associated with the Static Structural calculation module (Fig. 2).

When determining the frequencies and vibration modes of the wall of the tank with the winding, the change in the mass of the tank due to the application of the winding was taken into account. The additional mass was modeled on the outer surface of the tank wall. The convergence check of the solution for the finite element mesh used was performed earlier [14–16].

The numerical simulation algorithm includes two stages (Fig. 3).

In the first step, the stresses in the tank wall are determined in the Static Structural design module under specified boundary conditions. The boundary conditions at the bottom end were selected as Fixed Support. And
for the upper end, two options were considered. For the first option, no restrictions were set, and for the second option, Displacement restrictions were set in the form shown in Fig. 4.

The hydrodynamic pressure is set on the inner surface of the tank wall and is determined according to dependence (6). Calculations were carried out for three cases. For the case of maximum filling of the tank with oil (Fig. 5), for the case of half filling, and for the case of an empty tank. Note that the calculation model makes it possible to simulate the hydrodynamic pressure for any arbitrary level of filling the tank with oil.

The case was considered when the winding is applied to the entire outer surface of the wall. The distributed pressure $P_1$ on the outer surface of the tank wall is given as a constant. The magnitude of this constant, depending on the tension force of the filament in the winding, can be determined by formula (3). The change in the tension force of the filament in the winding was simulated by changing the tension force coefficient of the filament $k$, $0 < k < 1$. For calculations, $P_1 = k \cdot 36.991$ kPa was taken. Calculations were carried out for four variants of the thread tension force: $k_1 = 0.2$; $k_2 = 0.4$; $k_3 = 0.6$; $k_4 = 0.8$. In this case, the distributed pressure on the outer surface of the tank wall was as follows:

$$ P_{11} = 7.398 \text{ kPa}; \quad P_{12} = 14.796 \text{ kPa}; $$

$$ P_{13} = 22.195 \text{ kPa}; \quad P_{14} = 29.593 \text{ kPa}. $$

The frequencies and modes of vibrations of pre-stressed hydrostatic pressure tanks without winding for the cases of maximum and half filling with oil were also studied.

5. Results of investigating the vibration frequencies of the wall of a steel vertical cylindrical tank strengthened by a prestressed winding

5.1. Frequencies and modes of vibrations of the winding-hardened tank wall without taking into account the influence of the tank coating

5.1.1. For a tank filled with oil as much as possible

The results of our calculations of the vibration frequencies of the tank wall, taking into account the prestressed state from the tension of the filament in the winding for the case of maximum filling with oil, showed that they are all paired. Table 2 gives the first ten significant oscillation frequencies. The vibration modes corresponding to the significant oscillation frequencies given in Table 1 were studied. It was found that the modes were the same for all coefficients $k$, i.e., they do not depend on the tension force of the wire in the winding. For the case of a tank without winding, the vibration modes take the same form. The modes corresponding to the first four frequencies are shown in Fig. 6 [15].

![Fig. 4. Boundary conditions at the upper end for the second model: a – general view of the model in axes; b – table of accepted conditions]

![Fig. 5. Scheme for calculating the hydrodynamic pressure of the maximally filled tank: a – table of accepted conditions; b – general view of the model in axes]
Vibration frequencies of the wall of the tank, reinforced by a winding of steel wire with different thread tension, taking into account the maximum hydrostatic pressure, Hz

<table>
<thead>
<tr>
<th>Significant frequency No.</th>
<th>The tension coefficient of the filament in the winding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tank without winding [15]</td>
</tr>
<tr>
<td></td>
<td>$k_1=0.2$</td>
</tr>
<tr>
<td>1</td>
<td>13.44</td>
</tr>
<tr>
<td>2</td>
<td>14.38</td>
</tr>
<tr>
<td>3</td>
<td>17.54</td>
</tr>
<tr>
<td>5</td>
<td>27.40</td>
</tr>
<tr>
<td>6</td>
<td>27.68</td>
</tr>
<tr>
<td>7</td>
<td>35.03</td>
</tr>
<tr>
<td>8</td>
<td>36.13</td>
</tr>
<tr>
<td>9</td>
<td>38.19</td>
</tr>
<tr>
<td>10</td>
<td>39.95</td>
</tr>
</tbody>
</table>

The vibration modes corresponding to the significant oscillation frequencies given in Table 3 were studied. It is found that the modes are the same for all coefficients $k$, i.e., they do not depend on the tension force of the wire in the winding. For the case of a tank without winding, the vibration modes take the same form. The vibration modes corresponding to the first four frequencies are shown in Fig. 8.

Fig. 9 shows the change in the first five vibration frequencies depending on the tension force of the wire in the winding.

5.1.2 For a tank half filled with oil

The results of our calculations of the vibration frequencies of the tank wall, taking into account the prestressed state from the tension of the filament in the winding for the case of half filling with oil, showed that they are all paired. Table 3 gives the first ten significant oscillation frequencies.
5.1.3. For a tank without oil

The results of our calculations of the vibration frequencies of the tank wall, taking into account the pre-stressed state from the tension of the filament in the winding without oil, showed that they are all paired. Table 4 gives the first ten significant oscillation frequencies.

The vibration modes corresponding to the significant oscillation frequencies given in Table 4 were studied. It is found that the vibration modes are the same for all coefficients $k_i$, i.e., they do not depend on the tension force of the wire in the winding. The modes corresponding to the first four frequencies are shown in Fig. 10. The presented vibration modes for an unfilled tank correspond to the previously obtained modes for a tank filled with oil as much as possible.

Table 4

<table>
<thead>
<tr>
<th>Significant frequency No.</th>
<th>The tension coefficient of the filament in the winding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_1=0.2$</td>
</tr>
<tr>
<td>----------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>1</td>
<td>12.75</td>
</tr>
<tr>
<td>2</td>
<td>13.367</td>
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<tr>
<td>3</td>
<td>17.253</td>
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<td>4</td>
<td>18.54</td>
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<td>26.832</td>
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<td>34.103</td>
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<td>8</td>
<td>34.781</td>
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<td>9</td>
<td>37.543</td>
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<tr>
<td>10</td>
<td>39.457</td>
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</tbody>
</table>

Fig. 11 shows the change in the first five vibration frequencies depending on the tension force of the wire in the winding.

The lower black broken line corresponds to the maximum tension coefficient $k_4=0.8$. The blue and green lines are plotted for the tension coefficients $k_3=0.6$ and $k_2=0.4$, respectively. The upper broken line of red corresponds to the tension coefficient $k_1=0.2$. At the same time, the case of a tank without winding is not shown in the plot (Table 4). Thus, for the case under consideration, a decrease in the tension force of the filament in the winding leads to an increase in the oscillation frequency.
5.2. Frequencies and modes of vibrations of the winding-hardened tank wall, taking into account the influence of the tank coating

5.2.1. For a tank filled with oil as much as possible

The results of our calculations of the vibration frequencies of the tank wall, taking into account the prestressed state from the tension of the filament in the winding for the case of maximum filling with oil, showed that they are all paired. Table 5 gives the first ten significant oscillation frequencies.

The vibration modes corresponding to the significant oscillation frequencies given in Table 5 were studied. It was found that all vibration modes corresponding to the initial frequencies have a large number of nodes around the circumference of the cylindrical shell, while there are no nodes along the generatrix. For different coefficients $k$, they may differ in the number of nodes in the circumferential direction. For the calculated case $k_1=0.2$, the modes corresponding to the first four frequencies are shown in Fig. 12.

Fig. 13 shows the change in the first five vibration frequencies depending on the tension force of the wire in the winding.

The lower black broken line corresponds to the maximum tension coefficient $k_1=0.8$. The blue and green lines are plotted for the tension coefficients $k_2=0.6$ and $k_3=0.4$, respectively. The upper broken line of red corresponds to the tension coefficient $k_1=0.2$. At the same time, the case of a tank without winding is not shown in the plot (Table 5). Thus, for the case under consideration, a decrease in the tension force of the filament in the winding leads to an increase in the oscillation frequency.

Fig. 11. Vibration frequencies of the tank at different tension forces of the wire in the winding: the case of the absence of hydrostatic pressure

Fig. 12. Vibration modes of the tank, reinforced with a winding of high-strength steel wire, taking into account the maximum hydrostatic pressure: $a$ – mode I; $b$ – mode II; $c$ – mode III; $d$ – mode IV

Fig. 13. Vibration frequencies of the tank at different tension forces of the wire in the winding: the case of maximum hydrostatic pressure
Table 5

<table>
<thead>
<tr>
<th>Significant frequency No.</th>
<th>The tension coefficient of the filament in the winding</th>
<th>Tank without winding</th>
<th>$k_1=0.2$</th>
<th>$k_2=0.4$</th>
<th>$k_3=0.6$</th>
<th>$k_4=0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>17.71</td>
<td>16.86</td>
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<tr>
<td>2</td>
<td></td>
<td>17.73</td>
<td>17.06</td>
<td>16.39</td>
<td>15.69</td>
<td>14.47</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>19.06</td>
<td>18.03</td>
<td>16.93</td>
<td>15.75</td>
<td>14.95</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>19.26</td>
<td>18.84</td>
<td>18.40</td>
<td>17.92</td>
<td>16.52</td>
</tr>
<tr>
<td>5</td>
<td></td>
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<td>20.42</td>
<td>19.21</td>
<td>17.96</td>
<td>17.50</td>
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<td>22.24</td>
<td>21.34</td>
<td>19.93</td>
</tr>
<tr>
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<td>23.90</td>
<td>22.66</td>
<td>21.99</td>
<td>21.74</td>
</tr>
<tr>
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<td>28.32</td>
<td>27.11</td>
<td>25.82</td>
<td>24.46</td>
</tr>
<tr>
<td>9</td>
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<td>28.34</td>
<td>27.61</td>
<td>26.81</td>
<td>25.98</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>29.14</td>
<td>28.38</td>
<td>27.71</td>
<td>27.06</td>
<td>26.41</td>
</tr>
</tbody>
</table>

5.2.2. For a tank half filled with oil

The results of our calculations of the vibration frequencies of the tank wall, taking into account the prestressed state from the tension of the filament in the winding for the case of half filling with oil, showed that they are all paired. Table 6 gives the first ten significant oscillation frequencies.

The vibration modes corresponding to the significant oscillation frequencies given in Table 6 were studied. It was found that all vibration modes corresponding to the initial frequencies have a large number of nodes around the circumference of the cylindrical shell, while there are no nodes along the generatrix. For different coefficients $k$, they may differ in the number of nodes in the circumferential direction. For the calculated case $k_2=0.6$, the modes corresponding to the first four frequencies are shown in Fig. 14.

Table 6

<table>
<thead>
<tr>
<th>Significant frequency No.</th>
<th>The tension coefficient of the filament in the winding</th>
<th>Tank without winding</th>
<th>$k_1=0.2$</th>
<th>$k_2=0.4$</th>
<th>$k_3=0.6$</th>
<th>$k_4=0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>17.16</td>
<td>15.03</td>
<td>14.06</td>
<td>12.95</td>
<td>11.74</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>17.19</td>
<td>15.04</td>
<td>14.11</td>
<td>13.00</td>
<td>11.79</td>
</tr>
<tr>
<td>3</td>
<td></td>
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<td>20.94</td>
<td>19.75</td>
<td>18.47</td>
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</table>

5.2.3. For a tank without oil

The results of our calculations of the vibration frequencies of the tank wall, taking into account the prestressed state from the tension of the filament in the winding without oil, showed that they are all paired. Table 7 gives the first ten significant frequencies of oscillations.

The vibration modes corresponding to the significant oscillation frequencies given in Table 7 were studied. It was found that all vibration modes corresponding to the initial frequencies have a large number of nodes around the circumference of the cylindrical shell, while there are no nodes along the generatrix. For different coefficients $k$, they may differ in the number of nodes.
of nodes in the circumferential direction. For the calculated case $k_4=0.8$, the modes corresponding to the first four frequencies are shown in Fig. 16.

Table 7

<table>
<thead>
<tr>
<th>Significant frequency No.</th>
<th>The tension coefficient of the filament in the winding</th>
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</table>

Fig. 17 shows the change in the first five oscillation frequencies depending on the tension force of the wire in the winding. A decrease in the tension force of the filament in the winding leads to an increase in the frequency of oscillations.

The lower black broken line corresponds to the maximum tension coefficient $k_4=0.8$. The blue and green lines are plotted for the tension coefficients $k_3=0.6$ and $k_2=0.4$, respectively. The upper broken line of red corresponds to the tension coefficient $k_1=0.2$. At the same time, the case of a tank without winding is not shown in the plot (Table 7). Thus, for the case at hand.

6. Discussion of results of investigating the vibration frequencies of the wall of a steel vertical cylindrical tank

The study based on finite-element modeling in the standard calculation modules of the ANSYS software package is a continuation of our work [14–16]. Previously, the model of free oscillations of a liquid in a tank and the model of free oscillations of a tank without a liquid were investigated, which is a classical modal analysis.

In this study, a variant of a typical tank with a volume of 3000 m³ with different internal loads, which is characterized by the level of filling, was also considered. Also, the outer surface of the tank wall is affected by the distributed pressure from the prestressed winding (Fig. 1). For comparison, the simulation was carried out for two variants of fixing the upper edge of the tank wall, which correspond to cases of absence or presence of a tank coating.

Prestresses $k \cdot \sigma_0$, $0 < k < 1$, caused by four options for the tension of the wire in the winding, were studied. The estimation cases for the coefficients of the tension force of the wire relative to its tensile strength were studied: at $k_1=0.2$; $k_2=0.4$; $k_3=0.6$; $k_4=0.8$.

The results showed that at maximum hydrostatic pressure without the influence of the coating, the vibration frequencies of the tank walls decrease by 22.3 % as the tension force increases to 0.8 (Table 2, Fig. 6, 7). At half hydrostatic pressure, this indicator is 12.7 % (Table 3, Fig. 8, 9). And when the tank is empty, the indicator decreases by 27.1 % (Table 4, Fig. 10, 11), in all cases only the first significant frequency was analyzed. Similar results were obtained with the influence of tank coating. For example, at maximum hydrostatic pressure, the vibration frequency of the tank wall decreased by 21.2 % (Table 5, Fig. 12, 13). And with half and empty, this indicator decreased by 31.5 % (Table 6, Fig. 14, 15) and 62.3 % (Table 7, Fig. 16, 17), respectively, where only the first significant frequency was also analyzed.

Taking into account the solved problems, the magnitude of changes in the vibration frequencies of the tank wall without the influence of the coating as a whole varies within 12–27 % and taking into account the influence of the tank coating 21–62 %, depending on the degree of filling of the tank. At the same time, for all the variable calculations carried out, the regularity of the influence of the value of the prestresses in the winding on the vibration frequencies of the tank was revealed. It relates to the fact that a decrease in the tension force of the

Fig. 16. Vibration methods of the tank, reinforced with a winding of high-strength steel wire, without hydrostatic pressure: a – mode I; b – mode II; c – mode III; d – mode IV

Fig. 17. Vibration frequencies of the tank at different tension forces of the wire in the winding: the case of maximum hydrostatic pressure
filament in the winding leads to an increase in the frequency of oscillations. Thus, it is possible to increase the frequency of oscillations of the tank by reducing the prestresses in the winding, and to lower them by increasing them. As a result of the study, a patent of the Republic of Kazakhstan for an invention [39] and a patent for a utility model [40] were obtained.

A regularity has been revealed that a decrease in the tension force of the filament in the winding leads to an increase in the frequency of oscillations. This makes it possible to adjust the vibration frequency of the tank wall by changing the prestress. By reducing it, it is possible to increase the natural vibration frequencies of the tank wall and vice versa.

Our results of studies on the effect of the pre-tensioning force of the winding on the vibrations of the tank wall could be used in the operation of the winding-hardened structure of a vertical steel tank in seismically dangerous areas. At the same time, it is possible to detune the vibration frequencies of the wall of the structure from the resonant frequencies by tensioning the winding filament by a given amount, taking into account the filling level of the tank. As a continuation of the issue under consideration, it is possible to note possible studies in the field of the impact of forced oscillations of the tank under the influence of dynamic loads, as well as the influence of the friction force between the shell and winding.

7. Conclusions

1. The frequencies and vibration modes of the winding-hardened tank wall were investigated without taking into account the influence of the tank coating with the tension force of the steel wire relative to its tensile strength: at $k_1=0.2$; $k_2=0.4$; $k_3=0.6$; $k_4=0.8$. The results showed that at maximum hydrostatic pressure, without taking into account the influence of the coating, the vibration frequencies of the tank walls decrease by 22.3% as the tension force increases to 0.8. At half hydrostatic pressure, this indicator is 12.7%, and when the tank is empty, the indicator decreases by 27.1%, in all cases only the first significant frequency was analyzed.

2. The frequencies and vibration modes of the winding-hardened tank wall were investigated, taking into account the influence of the tank coating. The tension force of the steel winding relative to its tensile strength was taken as follows: $k_1=0.2; k_2=0.4; k_3=0.6$; and $k_4=0.8$. The results showed that at maximum hydrostatic pressure, the vibration frequency of the tank wall decreased by 21.2%, and at half filling and empty tank, this indicator decreased by 31.5% and 62.3%, respectively. In all cases, only the first significant frequency was analyzed.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Data availability

All data are available in the main text of the manuscript.

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