CONTROL PROCESSES

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DEVELOPMENT AND JUSTIFICATION OF THE SYSTEM METHODOLOGICAL APPROACH TO ASSESSING THE INVESTMENT BUSINESS PROJECT IMPLEMENTATION EFFICIENCY UNDER CONDITIONS OF THE EXTERNAL MARKET ENVIRONMENT FACTORS IMPACT

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Evaluating the effectiveness of the implementation of an investment project is a key issue when making management decisions both at the stage of setting up a startup and for expanding an existing business.

This paper reports a systematic approach to building a mathematical model to solve the task of forecasting the effectiveness of business projects, taking into account the influence of factors of the external economic environment. Proposed factors include the impact of supply and demand on the price of goods, political and industry risks, the volume of commodity supply and sales. In view of this, a method for calculating the political component of the discount coefficient using the Fourier series has been proposed. Using the theory of differential equations, correlation and regression analysis, a mathematical model for forecasting indicators of efficiency of business project implementation taking into account the influence of factors of the external economic environment has been constructed. Based on it, a generalized algorithm for applying a mathematical model to predict the effectiveness of investment projects in various business sectors has been developed.

The results from applying differential equations and variable discount coefficient showed a decrease in NPV by 14 %, and PI by 5.1 %, due to more accurate consideration of the political component in calculating the discount factor. Also, with the influence of supply and demand on the price of goods and nonlinear cash flows, it was found that the payback period does not clearly indicate the effectiveness of the implementation of an investment business project. Determining these factors provides more accurate information to the investor or business owner when forecasting the stability of a business project for making management decisions on its implementation

Keywords: mathematical model, forecasting the effectiveness of a business project, synergistic approach, net present value

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1. Introduction

The state of the international and regional economic space is characterized by a certain degree of stochasticity [1–5]. This is due to the presence of many different factors (economic shock), which are uncertain and directly affect the current state of an economic system [1]. These factors can be divided into external and internal. External factors include:

– political decisions;
– environmental disasters and pandemics;
– military-political crises;
– market fluctuations (supply-demand);
– risks arising from interaction with other economic entities;
– state of financial and credit systems;
– cross exchange rates and the state of stock markets.

In turn, internal factors include:

– internal corporate management and personnel policy;
– state of production capacities of the enterprise (for production);
– degree of modernization of the enterprise (for example, for production – innovative and technical solutions, for the service sector – improvement of customer service, transition to mass service through the introduction of electronic technologies, etc.);
– ecological state of the enterprise.

Given that private business, state-owned enterprises and corporations are subjects of economic activity, these factors directly affect them. This impact can be characterized by changes in performance indicators of the enterprise, competitiveness, changes in market positions, etc. [3]. Thus, in [3], this is explained by the possibility of the influence of macroeconomic factors on the activities of microeconomic entities, which include enterprises. Given this, the same management decisions under different influence of these factors can lead to radically different, and sometimes opposite results of financial and economic activity of the enterprise.

Given this, determining the degree of influence of external and internal factors is one of the key tasks in the analysis and planning of any business activity [1, 2]. This is especially necessary at the initial stages of doing business (starting with a startup), where the enterprise is weak and has not gained enough «turnover» of financial and economic activity.

Thus, the task of developing models for analyzing and forecasting the activities of the enterprise, taking into account the influence of these factors, is relevant.

2. Literature review and problem statement

As part of the emergence of a new Keynesian theory in macroeconomics in the 1980s, all economic processes began to be considered as dynamical systems [1]. A characteristic feature of dynamical systems is the variability of their state vector over time. Taking this into account, in addition to analyzing the past and determining the current states of micro and macroeconomic systems, the task of forecasting their future dynamics arose. In turn, this problem is quite complex and is characterized by a large degree of stochasticity (uncertainty) [1]. The degree of its solution is characterized by partial examples of forecasting the state of certain economic systems under the influence of one or more stochastic factors.

Thus, in [2], a mathematical model of macroeconomic equilibrium in a saturated stochastic environment is developed, which shows how inflation volatility affects economic growth. The model assumes that the general price level is determined by a mixed process of diffusion and jumps (the inflation model corresponds to the model of Brownian motion), and the Poisson process simulates unexpected and sudden jumps in their indices. The rate of economic growth is determined and guided by the country's domestic policy (endogenously determined) and is a function of parameters that is in equilibrium to the inflationary process. This model also makes it possible to study the robustness of the company in a particular region (in this case, the countries of Latin America were studied) in view of technological re-equipment and introduction of innovations. In this case, the approach of applying the Wiener model (Brownian motion) and the Poisson distribution was also used. However, not all stochastic processes in macroeconomic and microeconomic systems can be reduced to the application of these processes. Given this, such a model has limitations of application and is more theoretical.

One of the known approaches to the application of stochastic models are stochastic dynamic models of general economic equilibrium, dynamic stochastic general equilibrium (DSGE) [1, 3]. The main features of DSGE include:

1) supply factors are a source of cyclical fluctuations;

2) «equilibrium» is a dynamic process of equalizing supply and demand;

3) the behavior of economic agents is described using a model of intertemporal optimization and rational expectations;

4) application of forward-looking equations;

5) not the parameter itself is modeled but its deviation from equilibrium using various filters.

This model has a fairly deep theoretical economic justification and takes into account the instability of markets. However, DSGE, as noted in [4], has a fairly simplified model of the financial sector and poorly predicts economic crises and overestimates the ability of the agent to always rationally predict markets and the current economic situation. In paper [4], the expediency of taking into account the influence of factors of the external economic environment in DSGE models is shown. However, the provided mathematical models are generalized and do not take into account the specifics of a particular market environment, industry risks, and political components.

In turn, study [5] highlighted the disadvantages of using DSGE, among which are to be highlighted:

– application of the same methods of «calibration» when choosing parameters for stochastic equations;

– difficulty of parameter conversion in data;

– application of the Bayes method with ambiguously defined a priori probabilities.

Given this, it was proposed to distinguish five separate types of models, which can partially eliminate these shortcomings. These models include basic models, DSGE models, economic policy models, toy models, and forecasting models. The need to develop each of these areas according to Blanchard is quite relevant and will provide an opportunity to understand the dynamics of macroeconomic and microeconomic processes.

Thus, basic models are a general scientific and theoretical basis for understanding stochastic phenomena in economics [5]. DSGE models should be coordinated and apply proven methods developed on the basis of aggregate scientific and practical solutions of leading experts in this direction. At the same time, the purpose of economic policy models according to [5] is to analyze the effects of shock dynamics in modeling dynamic economic systems. «Toy models» are intended for quick a priori inaccurate assessment of complex economic processes, as well as illustrative educational examples for students of financial and economic training. Quite considerable attention is paid to forecasting models, where the criterion of efficiency is the accuracy of the forecast of an economic indicator [5]. However, in [5], there are no practical proposals and mathematical models for risk forecasting within DSGE.

Given the analysis of the dynamics of business processes, as well as determining the future value of the business, the most appropriate is the development of forecasting techniques [6] with elements of models of economic policy and riskology [7]. This is explained by the fact that business is influenced by both macroeconomic and microeconomic factors. Thus, in [6], on the example of studies of economic activity of US enterprises, it is shown that companies that use predictive analytics methods have significantly higher sales (USD 918,000 more) and profits than their competitors. However, it is noted that the highest efficiency of predictive analytics for an enterprise is achieved with its correct understanding and adoption of adequate management decisions. Given this, one of the conditions for improving business efficiency is the correct formation of the organizational and economic mechanism for enterprise management.

So, we can distinguish 3 basic procedures to forecast dynamic economic processes:

– application of regression analysis methods based on statistics and probabilistic models [7, 8];

– forecasting the dynamics of economic processes using artificial intelligence, machine learning [9–11] and Big Data [12];

– forecasting the dynamics of economic processes using differential equations (or difference equations) with stochastic parameters [13–15].

In [7], the features of using multifactor regression for analyzing forecasts of sales volumes under the influence of expected consumer income and advertising costs are shown. The model is represented in the form of a two-factor linear regression equation and shows the effectiveness of its application in forecasting. However, not all processes in economics can be described by linear regression equations (for example, time-periodic fluctuations in exchange rates, political risks, etc.). Given this, such models have limitations of application related to the complexity of analyzing nonlinear processes. A procedure for using piecewise linear regression is proposed in article [8]. This makes it possible to partially avoid problems in the analysis of nonlinear systems, by discretizing the analysis periods into quarters. The results of applying this procedure showed its fairly good effectiveness. In turn, the use of linear autoregressions can also give significant errors in estimates of short-period fluctuations in certain economic parameters. Also, with the possible application of these methods to analyze the activities of enterprises, it requires significant amounts of statistical data on financial and economic activity, which is not always available (especially when opening new projects, changing business activities, etc.).

Thus, with the development of computer technology, methods of using artificial intelligence and machine learning for forecasting problems are becoming more and more known [9, 11]. Among these methods are machine learning models, artificial neural networks, and deep learning methods [9]. A good example is the application of artificial intelligence and machine learning methods [10] in solving the well-known problem of income and wealth heterogeneity in macroeconomics according to Crusell and Smith [16]. With the use of deep learning in this problem with a large number of state variables, it was possible not to introduce a simplified assumption about the state space of the economy, which was previously impossible. However, these algorithms have certain disadvantages, which the authors of [10] attribute to the following:

a) training neural networks is expensive and their convergence to solutions is not guaranteed;

b) Monte Carlo simulation, which underlies the deep learning framework, has a low square root convergence rate;

c) the effectiveness of the application of stochastic optimization methods is contradictory with respect to known optimization methods.

Given this, the application of this procedure is a rather difficult task for forecasting real business processes with the influence of a significant number of factors with certain degrees of uncertainty.

As for Big Data methods [12], they are combinations of statistical methods and artificial intelligence methods. Big Data procedures have proven to be quite well in practice, however, when applying these methods in the field of econometrics, the problem of increasing the accuracy of forecasts while reducing the dimension of samples remains not fully solved.

In turn, with the evolution of computational methods, methods for applying differential equations (DE) for predicting economic processes have been further developed. The advantage of using this approach is a fairly deep study of the features of modeling various dynamic processes using DE in other fields of science and technology. Thus, in [13], the possibility of forecasting shares of the stock exchange in Brazil using DE was shown. The model demonstrated fairly good forecasting accuracy at separate short intervals. However, it remains inconvenient to determine the fundamental matrix of an DE system at each forecasting interval. Taking this into account, when dividing a large forecasting interval by a certain number of small ones, the amount of input data for the numerical solution of the DE system itself increases significantly, and hence the time for its programming increases.

However, when introducing stochastic components into differential equations [14, 15], an increase in the efficiency of forecasting in comparison with the deterministic determination of DE coefficients is shown. Thus, article [14] describes the features of constructing stochastic DE for forecasting the eBay auction. The advantages of stochastic modeling of behavior of auction participants and price fluctuations and their introduction into stochastic components of DE when forecasting prices at the end of trading are shown. The difficulty in this case is an adequate modeling of the stochastic process itself, affecting the results of the forecast. In turn, in [15], the use of calculations of stochastic correlations in time series forecasting made it possible to slightly simplify the definition of the stochastic component of DE and increase the accuracy of the forecast. Also, these approaches are applicable not only for forecasting auctions and time series. One of the actual directions of application of such approaches to the synthesis of mathematical models based on DE may be forecasting investment projects. Also, taking into account the peculiarities of the state of the world economy, and the increase in the frequency of various crises (Covid-19 pandemic, the Ukrainian-Russian war, tensions in the Middle East), it is advisable to model business processes as continuous functions in time.

Given this, it is advisable to conduct research into the development of an approach for the synthesis of a mathematical model, the use of which will allow forecasting the effectiveness of investment business projects taking into account factors of the external economic environment. In turn, the use of such models is appropriate for making balanced management decisions on the strategic development of the enterprise when implementing new business projects or expanding existing ones.

3. The aim and objectives of the study

The aim of this study is to devise a systematic approach to building a mathematical model to assess the effectiveness of the implementation of an investment business project under the influence of factors of the external market environment.

This will make it possible to more accurately assess the effectiveness of the implementation of the developed investment business project under the conditions of the analyzed enterprise by taking into account the risks caused by the influence of factors of the external economic environment.

To accomplish the aim, the following tasks have been set:

– to select and justify the necessary indicators for forecasting the effectiveness of an investment business project when making management decisions;

– to develop a method for constructing a mathematical model for forecasting the effectiveness of the implementation of an investment business project and an algorithm for its application under the influence of factors of the external market environment;

– to verify the application of the developed mathematical model for forecasting the effectiveness of the implementation of an investment business project under the influence of factors of the external market environment.

4. The study materials and methods

The object of research is the process of determining the effectiveness of investment business projects in making management decisions.

The subject of the study is the theoretical and organizational-economic aspects of developing a synergistic approach while using the methods of correlation and regression analysis, the theory of differential equations, and Fourier series to assess the effectiveness of the implementation of an investment business project.

The main hypothesis of the study assumes the influence of periodically variable discount factor, supply and demand on forecasting the effectiveness of an investment business project.

The theoretical and methodological basis of the research was the fundamental theoretical provisions of economic science in the field of analysis and evaluation of investment business projects.

To solve the problems set in the work, general scientific and special research methods were used:

– method of critical analysis, scientific abstraction and generalization to determine the directions for improving existing methods for assessing the effectiveness of forecasting the effectiveness of economic systems under the influence of factors of the external market environment;

– analytical method – when creating a representative sample of indicators characterizing the financial and economic condition of the enterprise for making management decisions on the directions and prospects of strategic development of the enterprise;

– a systematic approach for the development of a mathematical model that simultaneously uses the methods of correlation and regression analysis, the theory of differential equations, and Fourier series to assess the effectiveness of the implementation of an investment business project;

– forecasting method based on Fourier series to determine the influence of the political component in calculating the discount factor;

– forecasting method based on differential equations for calculating the main indicators for evaluating the effectiveness of an investment business project;

– regression analysis method to predict the impact of supply and demand on the unit price;

– method of correlation analysis to determine the accuracy of forecasting when choosing the type of approximating function and to calculate the matrix of correlations of the mutual influence of forecasting indicators;

– method of graphical analysis to visualize the results of calculations, forecasting the effectiveness of the implementation of an investment business project.

To study the features of forecasting the effectiveness of an investment project, a software application in C++ has been developed in Visual Studio (OS Windows). The software application represents a graphical user interface (GUI) with a developed dynamic library (dll). The GUI is designed to enter initial data and display the main indicators for forecasting the effectiveness of the implementation of an investment business project. The dynamic library (dll) consists of three submodules:

1) a statistical data processing module based on correlation and regression analysis;

2) a module for predicting the discount coefficient using Fourier series;

3) a forecasting module based on differential equations.

In turn, the GUI interface has two windows: the main and additional, which appears at the end of the calculation. The main window (Fig. 1) is intended for preliminary statistical processing of marketing research data, entering initial data for the vector of financial and economic condition of the enterprise, as well as for choosing a strategy for supplying goods to the market (production).

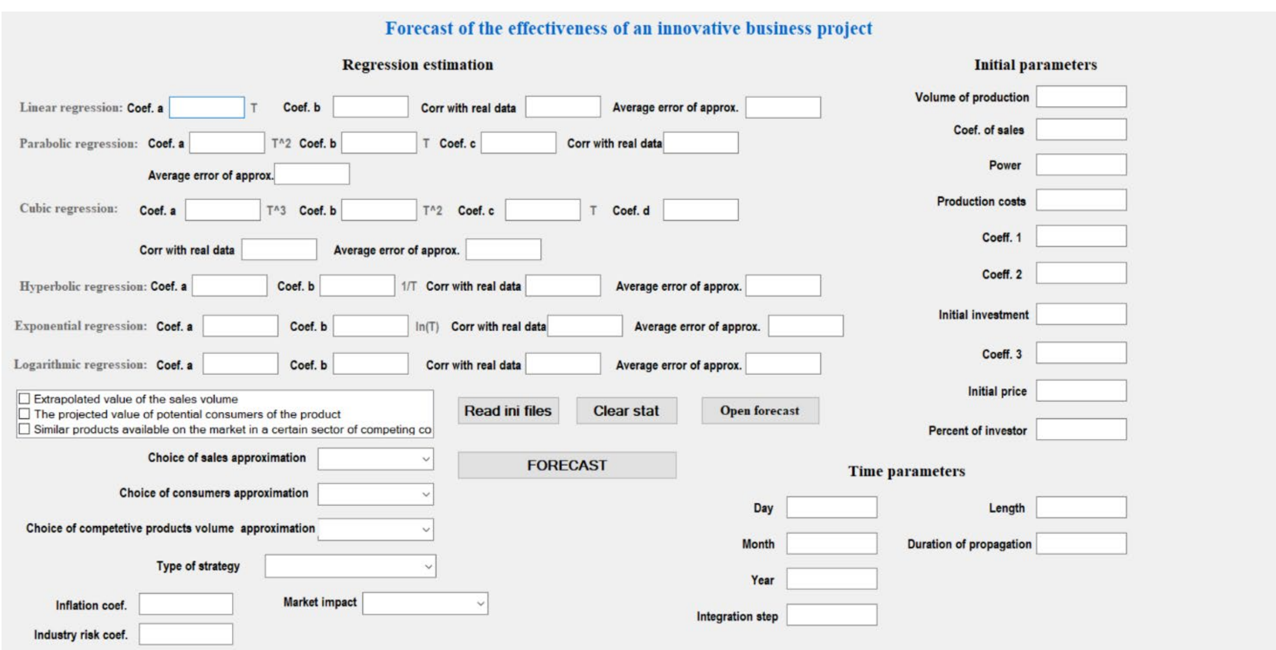


Fig. 1. Main window of the software application interface GUI

Also, to visualize indicators that are not displayed in the GUI interface, the Microsoft Excel graphical and spreadsheet apparatus was used.

To study the operability of the developed mathematical model, modeling of forecasting of an investment project related to the manufacture of electric diesel generators of low power (up to 2 kW) was carried out. In turn, the Department of Economic Cybernetics at NTU «Dnipro Polytechnic» (Ukraine) analyzed an investment project for the manufacture of generators of this type. The analysis was carried out using the classical approach of business design according to the UNIDO scheme [17] and standard methods for calculating the effectiveness of investment business projects [18].

5. Results of investigating the influence of factors of the external economic environment on forecasting the effectiveness of an investment business project

5. 1. Definition and justification of financial and economic indicators for making management decisions

Performance indicators of an investment project can be divided into three groups:

1) general financial and economic indicators that allow evaluating investment projects without taking into account the specifics of the business;

2) special financial and economic indicators that take into account the characteristics and specifics of a particular business;

3) given financial and economic indicators, combining the properties of general and special indicators.

A subgroup of general financial and economic indicators necessary for forecasting the effectiveness of implementation of an investment business project was identified:

1. Net income from sales of products (line 2000 of the Report on financial results), or *CF* (cash flow) – cash flow from sales of products.

2. The cost of sales (line 2050 of the Report on financial results).

3. Gross profit (loss) (line 2090/2095 of the Report on financial results).

4. Profit (loss) from operating activities (line 2190/2195 of the Report on financial results).

5. Net profit (loss) (line 2350/2355 of the Report on financial results).

6. Present value (*NPV*) – net present value. Traditionally, it is calculated according to the *PV* determination scheme [18–20] as follows:

$$NPV=\sum_{t=1}^{n}\frac{CF_t}{\left(1+r\right)^t}-I, \tag{1}$$

where n is the number of periods; r – discount factor; I – the value of investment, taking into account the interest of the investor.

7. Profitability index (*PI*) – return on investment. Traditionally calculated according to the scheme for determining *PV* [18, 20] as follows:

$$PI=\sum_{t=1}^{n}\frac{CF_t}{\left(1+r\right)^t}/I. \tag{2}$$

8. Discounted payback period of the project. The value of this indicator corresponds to the period in years (months) after which the value of the *NPV* indicator is positive.

9. Introduce the coefficient of autonomy of the business project – *ACB* (autonomy coefficient of business). In this case, it is proposed to calculate it as follows:

$$ACB=\frac{CF_t}{I_0\left(1+i/100\right)^t}, \tag{3}$$

where i is the investor's interest rate shown in the estimated accrual period, I_0 – the value of the initial investment.

10. Return on sales ratio (*ROS*). Traditionally, profitability of sales is calculated as the ratio of net profit to revenue. In this case, we introduce a modified profitability ratio of sales for business (*MROS*). This coefficient is proposed to be calculated as follows:

$$MROS=\frac{NPV_t}{CF_t},\text{ at } NPV_t>0, \tag{4}$$

where *NPVt* – *NPV* is calculated at time t.

Thus, 10 financial and economic indicators have been selected to forecast the effectiveness of the implementation of investment business projects and make management decisions. Thus, the first five indicators characterize the results of the enterprise. Given the peculiarities of the calculations, some indicators may be equivalent to each other. So, for example, when it comes to a general analysis of retail trade, excluding depreciation costs, logistics costs, etc., indicators 1, 3, 5 are equivalent to each other.

The last five financial and economic indicators characterize the stability of a business under the influence of factors of the external financial and economic environment in which this business operates. Thus, the selected 10 financial and economic indicators constitute a complete generalized vector of the state of business when making management decisions.

In turn, with a detailed analysis of a complex investment project (scientific and technical innovation projects, opening new production facilities, etc.), this vector of state should be expanded with special financial and economic indicators that take into account the specifics of a particular branch of management.

5. 2. Devising a method for constructing a mathematical model for forecasting the effectiveness of the implementation of an investment business project and an algorithm for its application

To determine the decision-making algorithm under conditions of instability of the external economic environment, the features of the functioning of the enterprise are considered in view of the general management theory [21]. The subject of enterprise management, depending on its size, is the executive body (board of directors), top management or one person – the director. The object of management is the enterprise itself, which conducts its activities in a market environment for profit. Given the peculiarities of the instability of the external economic space that occur at certain time intervals, it is advisable to consider the market environment as a system whose parameters are dynamically changing. In turn, changes in the parameters of the environmental market environment directly affect the financial and economic activities of the enterprise itself.

In view of this, the strategies of development, economic, industrial, and financial activities of the enterprise should be adaptive to the dynamically changing external market and economic environment. The purpose of this is to maintain the position of the enterprise in the market, its development trends, as well as increase resistance to the effects of disturbance factors.

The factors of disturbances should be understood as a variety of economic crises, market cyclical and random fluctuations, inflation/deflation, geopolitical crises affecting world trade, etc. [21]. The definition of these disturbances for the management body of the enterprise is a rather complex task that requires in-depth analytical research. In turn, the correctness of determining the factors of disturbances is key when making management decisions on the choice of a particular strategy for the financial and economic functioning of the enterprise.

Taking into account the above, when making decisions, the subject of management should perform the function of a corrective element (Fig. 2), defining plans and strategies for the enterprise under the influence of a dynamically changing market environment at certain time intervals. This corrective element is defined as the decision-making center of the enterprise (Fig. 2).

In view of this, one of the key tasks of the decision-making center is to forecast the production and economic activity of the enterprise. Taking into account the preliminary analysis of the features of forecasting financial and economic activity of different business entities, we offer a dynamic model for forecasting the production and economic activity of the enterprise (Fig. 2) with the combined use of DE, Fourier series, and correlation-regression analysis. Thus, with the use of correlation-regression analysis, marketing research is proposed. Taking this into account, three key indicators of marketing research have been identified, which should be predicted for a certain period:

– number of potential consumers N for the predefined forecasting period;

– sales volume N_1 for the predefined forecasting period;

– volume of similar products on the market from Vp competitors for the predefined forecasting period.

These indicators are formed on the basis of statistical data from the marketing research department of the decision-making center. Thus, after collecting statistical data on marketing research in a certain financial and economic environment for previous periods, three samples are formed for indicators N_1, N, and Vp. Next, using correlation-regression analysis, the most accurate approximating function is selected. The approximation in this case is proposed to be carried out using the least squares method for six types of functions: linear, quadratic (parabolic), cubic, exponential, logarithmic, and hyperbolic [22]. Given this, the types of approximating functions for these indicators can be written as follows:

$$N_1 = f_{aprox}^{N_1}(t),\ N = f_{aprox}^{N}(t), V_p = f_{aprox}^{V_p}(t),$$

$$\left.\begin{matrix} f_{aprox}^{N_1}(t) \\ f_{aprox}^{N}(t) \\ f_{aprox}^{V_p}(t) \end{matrix}\right] \Leftrightarrow \begin{cases} a_1 t + b_1, \\ a_2 t^2 + b_2 t + c_2, \\ a_3 t^3 + b_3 t^2 + c_3 t + d_3, \\ a_4 + \dfrac{b_4}{t}, \\ e^{a_5 + b_5 t}, \\ a_6 + b_6 \ln(t), \end{cases} \quad (5)$$

where $f_{aprox}^{N_1}(t)$ is the approximating function for parameter N_1; $f_{aprox}^{N}(t)$ is the approximating function for parameter N; $f_{aprox}^{V_p}(t)$ – approximating function for parameter Vp, a_1, b_1, a_2, b_2, c_2, a_3, b_3, c_3, d_3, a_4, b_4, a_5, b_5, a_6, b_6 – regression coefficients calculated by standard procedures of the least squares method [22], t – time for which statistical data are accumulated, recalculated in accordance with the real time measurement by Julian Date.

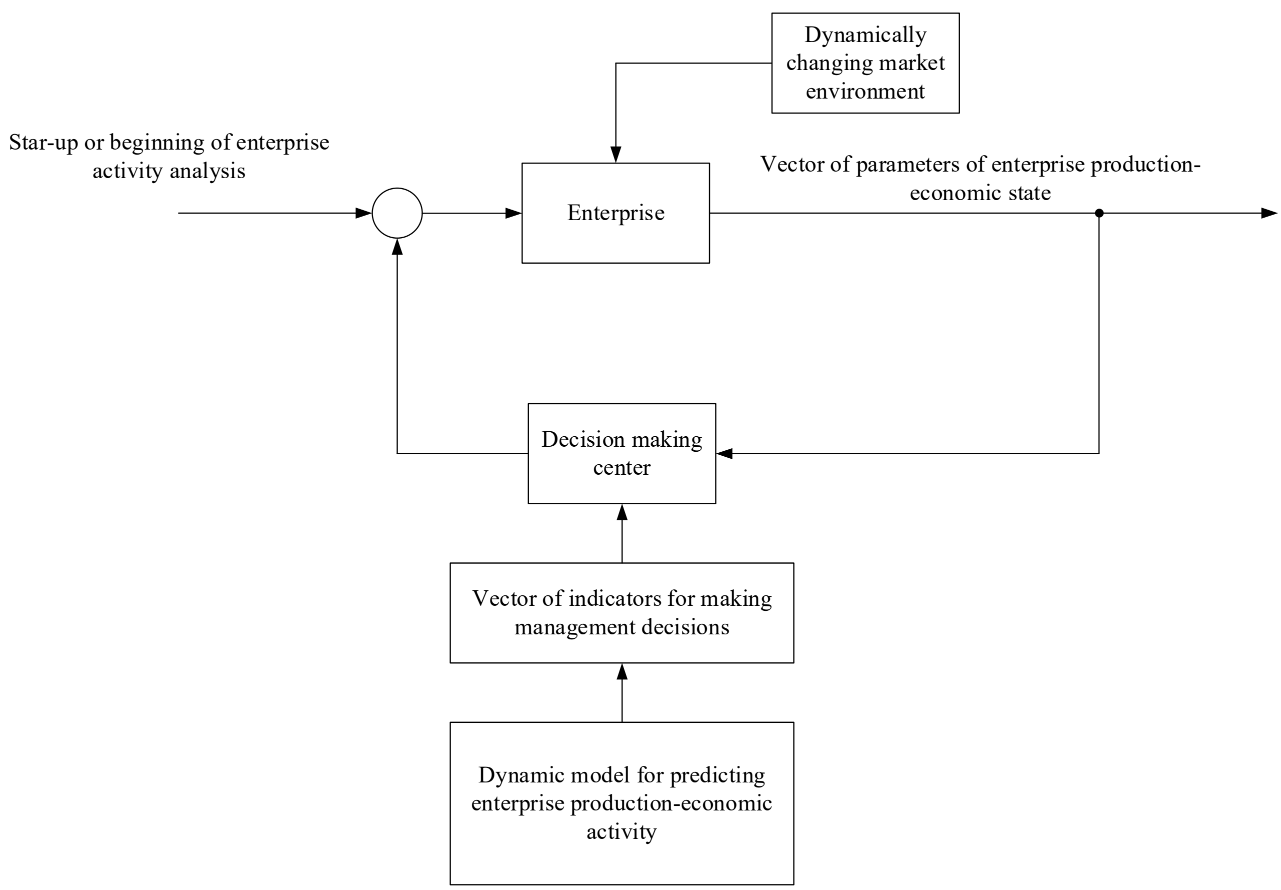


Fig. 2. Generalized scheme of the enterprise management system

In turn, the determination of the most accurate approximation function is carried out using the following conditions:

$$corr\left(f_{aprox}^{N_1}(t), Real^{N_1}\right) \to \max,$$

$$corr\left(f_{aprox}^{N}(t), Real^{N}\right) \to \max,$$

$$corr\left(f_{aprox}^{V_p}(t), Real^{V_p}\right) \to \max,$$

$$Aver\_err^{N_1} = \frac{1}{n}\left|\frac{Real^{N_1} - f_{aprox}^{N_1}(t)}{Real^{N_1}}\right| \cdot 100\,\% \to \min,$$

$$Aver\_err^{N} = \frac{1}{n}\left|\frac{Real^{N} - f_{aprox}^{N}(t)}{Real^{N}}\right| \cdot 100\,\% \to \min,$$

$$Aver\_err^{V_p} = \frac{1}{n}\left|\frac{Real^{V_p} - f_{aprox}^{V_p}(t)}{Real^{V_p}}\right| \cdot 100\,\% \to \min, \qquad (6)$$

$corr(f_{aprox}^{N_1}(t), Real^{N_1}), corr(f_{aprox}^{N}(t), Real^{N}), corr(f_{aprox}^{V_p}(t), Real^{V_p})$ are the Pearson pairwise correlation coefficients between real data samples $Real^{N_1}, Real^{V_p}$ and data samples calculated using a certain approximating function; $Aver\_err^{N_1}$, $Aver\_err^{V_p}$ – average value of approximation error in percent.

Thus, the approximating function is chosen taking into account the highest value of the positive Pearson pairwise correlation and the smallest mean value of the relative approximation error. These functions are input parameters for the prediction unit, which is built on the basis of differential equations.

The next input parameter for the prediction unit is the discount factor *r*. It is advisable to represent it as an additive quantity consisting of several components [19, 21]. Thus, according to previous studies [21], one of the main components of the discount coefficient is the political risk component. According to [21], political risk for the Ukrainian economic space increases during the presidential elections in Ukraine. This is explained by the fact that during this period certain changes in the peculiarities of the functioning of the financial and economic space of the country are possible, administrative and legal spaces and, in general, a change in the vector of development of the country. Taking into account the frequency of the choice of the President of Ukraine once every five years in [21], it was proposed to approximate the dependence of this component of the discount factor on time by empirical periodic functions. On the other hand, quite important for global business is the political risk associated with the election of the President of the United States [21]. This, on the one hand, is explained by the fact that most economies use the US dollar as the main reserve currency; on the other hand, it affects all exchange and currency processes around the world. Given this, in [21] political risk is presented as the sum of periodic functions of different frequencies.

However, the disadvantage of representing periodic functions in [21] is an empirical approach, which is rather difficult to scale to other financial and economic spaces. Taking this into account, we propose the application of the Fourier decomposition methodology of periodic components of political risk. It is known that the frequency of elections of the President of the United States 1 time in 4 years, the President of Ukraine 1 time in 5 years. For the mathematical notation of discounting, taking into account political risks between election periods, approximating functions in the form of a parabola are introduced, where the vertex is the minimum risk. Given this, the minimum political risk for the United States is 2 years after the presidential elections, respectively, for Ukraine it is 2.5 years. This can be explained by the completion of the establishment of a certain political and economic situation after the coming of a certain political force to power. In view of this, the dependence of the discount coefficient of political risks on time using parabolic approximation at three points, which are given in Table 1.

Table 1

Key points for approximation of political risks arising from the U.S. and Ukrainian presidential elections

Point of the curve	The value of the discount factor for political risk *rpol*	Timepoints for Ukraine's elections	Timepoints for the US elections
The first	$r_{pol}^{\max}$ (maximum)	–1	0
The second	$r_{pol}^{\min}$ (minimum)	1.5	2
The third	$r_{pol}^{\max}$ (maximum)	4	4

Thus, these dependences of discount coefficients related to the election of the President of the United States and Ukraine can be written as follows:

$$r_{pol}^{USA}(t_y) = \left(\frac{r_{pol.USA}^{\max}}{4} - \frac{r_{pol.USA}^{\min}}{4}\right) \cdot t_y^2 + \\ +\left(r_{pol.USA}^{\min} - r_{pol.USA}^{\max}\right) \cdot t_y + r_{pol.USA}^{\max},$$

$$r_{pol}^{UA}(t_y) = \left(\frac{4r_{pol.UA}^{\max}}{25} - \frac{4r_{pol.UA}^{\min}}{25}\right) \cdot t_y^2 + \\ +0.48\left(r_{pol.UA}^{\min} - r_{pol.UA}^{\max}\right) \cdot t_y + 0.64 r_{pol.UA}^{\min}, \qquad (7)$$

where $r_{pol.USA}^{\max}, r_{pol.UA}^{\max}$ are the maximum values of discount coefficients associated with the elections in the USA and Ukraine; $r_{pol.USA}^{\min}, r_{pol.UA}^{\min}$ – minimum values of discount coefficients related to the elections in the USA and Ukraine; *ty* – current time counted in years.

In turn, the function $r_{pol}^{USA}(t_y)$ is given by an even number, and the function $r_{pol}^{UA}(t_y)$ is in a general form.

Given the periodicity of these processes, their expansion into the Fourier series can be written as follows:

$$f_{pol}^{USA}(t_y) \sim \frac{a_0^{USA}}{2} + \sum_{n=1}^{\infty}\left(a_n^{USA}\cos\left(\frac{\pi n t_y}{2}\right)\right),$$

$$a_0^{USA} = \int_0^2 r_{pol}^{USA}(t_y)\,\mathrm{d}t_y,$$

$$a_n^{USA} = \int_0^2 r_{pol}^{USA}(t_y)\cdot\cos\left(\frac{\pi n t_y}{2}\right)\mathrm{d}t_y,$$

$$f_{pol}^{UA}(t_y) \sim \frac{a_0^{UA}}{2} + \\ +\sum_{n=1}^{\infty}\left(a_n^{UA}\cos\left(\frac{\pi n t_y}{2.5}\right) + b_n^{UA}\sin\left(\frac{\pi n t_y}{2.5}\right)\right),$$

$$a_0^{UA} = \frac{1}{2.5}\int_{-1}^{4} r_{pol}^{UA}(t_y)\,\mathrm{d}t_y,$$

$$a_n^{UA} = \frac{1}{2.5}\int_{-1}^{4} r_{pol}^{UA}(t_y)\cdot\cos\left(\frac{\pi n t_y}{2.5}\right)\mathrm{d}t_y,$$

$$b_n^{UA} = \frac{1}{2.5}\int_{-1}^{4} r_{pol}^{UA}(t_y)\cdot\sin\left(\frac{\pi n t_y}{2.5}\right)\mathrm{d}t_y, \qquad (8)$$

where $f_{pol}^{USA}(t_y)$ is the Fourier series decomposition of an even function $r_{pol}^{USA}(t_y)$; $f_{pol}^{UA}(t_y)$ – expansion into the Fourier series of the function $r_{pol}^{UA}(t_y)$; a_0^{USA}, a_n^{USA} – Fourier coefficients for the function $f_{pol}^{USA}(t_y)$; a_0^{UA}, a_n^{UA}, b_n^{UA} – Fourier coefficients for the function $f_{pol}^{UA}(t_y)$.

Thus, using the Fourier series decomposition, the values of periodic functions $f_{pol}^{USA}(t_y)$, $f_{pol}^{UA}(t_y)$ for discount coefficients when taking into account political risks associated with the elections of the USA and Ukraine are obtained. In turn, to determine the full discount ratio r, it is also proposed to take into account the component of the inflation index and the industry risk coefficient. Hence, the total discount factor can be written as follows:

$$r = f_{pol}^{USA}\left(t_y\right) + f_{pol}^{UA}\left(t_y\right) + r_{inf} + r_{bi}, \quad (9)$$

where $rinf$ – inflation index; rbi – industry risk ratio.

The coefficients of inflation and industry risk are proposed to be chosen by the method of expert assessments and set constant for a certain forecasting period.

To analyze the impact of the market on pricing and enterprise activity, we introduce the following indicators:

– q – coefficient of current demand;

– s – coefficient of current supply.

These indicators are proposed to be calculated as follows:

$$q = \frac{N}{N_1},\ 0 < q \le 0.5,$$

$$s = \frac{V_p}{N_1},\ 0 < s \le 0.5. \quad (10)$$

In turn, the impact of these indicators on pricing is proposed to be recorded as follows:

$$P = P_1 \frac{q}{s},$$

$$0.5 \cdot P_1 \le P \le 1.5 \cdot P_1, \quad (11)$$

where P_1 is the price of products calculated at the time of the start of forecasting, taking into account profits and taxes; P – current price of products taking into account the influence of the external market environment on the forecasting period.

The limitation from 0 to 0.5 in the study for q and s coefficients is explained by the introduction of the assumption that during the forecasting period, demand and forecasting cannot increase or decrease by more than 50 %. A limit is also set on the price: the price cannot be increased or decreased by more than 50 %. In turn, these ranges, if necessary, can be changed in accordance with the conditions of a particular financial and economic environment.

Thus, the required input parameters (5) to (11), describing the impact of the external economic environment on the business process, are formalized for the forecasting unit. Given that the forecasting block is proposed on the basis of differential equations, the calculations of indicators (1) and (2) will be reduced to the form of continuous discounting and interest accrual [18, 21]. With this in mind, a dynamic mathematical model based on differential equations is proposed for the forecasting unit. These equations are written as follows:

$$\frac{dV}{dt} = f\left(V_{str}\right),$$

$$\frac{dCF}{dt} = V_{cur} \cdot P \cdot k_{?.} - V_{cur} \cdot CP + CP \cdot V_{cur} \cdot ka,$$

$$\frac{dNPV}{dt} = \left(-r\right) \cdot CF \cdot e^{-rt} - I_{oi} \cdot \varphi \cdot e^{\varphi \cdot t_{inv}},$$

$$\frac{dPI}{dt} = \frac{\left(-r\right) \cdot CF \cdot e^{-rt}}{I_{oi} \cdot \varphi \cdot e^{\varphi \cdot t_{inv}}},$$

$$ACB = \frac{CF}{I_{oi} \cdot \varphi \cdot e^{\varphi \cdot t_{inv}}},$$

$$MROS = \frac{NPV}{CF}, \quad (12)$$

where V is the volume of production (number of manufactured products) or volume of services, goods (for non-production spheres and retail trade); V_{cur} – speed of production of goods, units/day; CP – cost of production (goods, services); CF – cash flow; I_{oi} – initial investment from the investor (or investment from the investor at the beginning of the forecasting period); k_n – expected sales ratio for the enterprise for the forecast period, selected by the method of expert assessments; k_a – depreciation coefficient, selected by the method of expert assessments in accordance with the characteristics of the equipment, type of activity, and method of calculating depreciation; r – discount index; φ – investor's interest rate; t_{inv} – time reduced to the periods of interest accrual to the investor in accordance with the contract (if any); t – forecasting time, integration step is set 1 day; $f(V_{str})$ is the function of the strategy of production of goods or volume of services, goods (for non-production spheres and retail). Includes 4 types of strategies:

a) line production;

b) production with an increase according to the parabolic law;

c) production with an extension according to the law of the cubic parabola.

Mathematically, this function of strategies is written as follows:

$$\frac{dV}{dt} = f\left(V_{str}\right) = \text{switch} \rightarrow \begin{array}{r|l} A\text{-str} & \left(k_1 - k_2\right)V_{cur}, \\ B\text{-str} & \left(k_1 - k_2\right)V_{cur} \cdot t, \\ C\text{-str} & \left(k_1 - k_2\right)V_{cur} \cdot t^2, \end{array} \quad (13)$$

where switch is the function of choosing a strategy for the production of goods or the volume of services, goods (for non-production areas and retail); k_1 – coefficient of arrival of funds (repair of equipment, purchase of new production units, renewal or repair of transport and logistics system, etc.); k_2 – coefficient of disposal of funds (depreciation of equipment, failures of certain production units, transport for the supply of goods, etc.); A-str, B-str, C-str – types of strategy for the production of the function $f(V_{str})$.

It should be noted that the coefficients k_1 and k_2 are selected by experts through the use of variational calculus in view of the possibilities of producing goods or providing services, goods (for non-production spheres and retail). For example, for the A-str strategy, the most optimal values of these coefficients are $k_1=2$, $k_2=1$. At such values, stable production of products (services, goods) V_{cur} per unit of time is ensured, in accordance with the linear manufacturing plan.

For nonlinear strategies B-str, C-str, these parameters are selected taking into account the following functionality:

$$\int_0^t V_{str}\mathrm{d}t \to V_{opt}, \quad (14)$$

where V_{opt} is the optimal value of products to be produced for the specified forecasting period, t.

Thus, after the completion of forecasting for the specified period, statistical correlation processing of calculation results using the differential model (12), (13) is assumed. Thus, the following matrix of correlations is proposed:

$$\begin{bmatrix} \underline{\mathrm{Matr}_{X,Y}|} & N_1 & N & V & CF & NPV & r & P & V_p & PI \\ N_1 & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ N & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ V & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ CF & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ NPV & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ r & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ P & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} \\ V_p & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 & \mathrm{r}_{X,Y} \\ PI & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & \mathrm{r}_{X,Y} & 1 \end{bmatrix}, \quad (15)$$

where $\mathrm{Matr}_{X,Y}$ is the initial matrix of correlations of statistical post-processing of forecasting results; $\mathrm{r}_{X,Y}$ – Pearson linear pairwise correlation coefficient, between X=1...9 and Y=1...9 indicators of the matrix $\mathrm{Matr}_{X,Y}$ calculated by using a standard formula [21] as follows:

$$\mathrm{r}_{X,Y} = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sqrt{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 \sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}}, \quad (16)$$

where X_i is the i-th element of the sample X; Y_i – i-th element of the sample Y; $\bar{X}$ – mathematical expectation (mean) of the sample X; $\bar{Y}$ – mathematical expectation (mean) of the sample Y; n – the number of elements in the samples X and Y.

Also, when carrying out correlation post-processing of results, all elements of X and Y indicators of the matrix $\mathrm{Matr}_{X,Y}$ are reduced to the same time dimension and sample size. Thus, a generalized mathematical model for forecasting the effectiveness of an investment project (5) to (16) is built. In view of this, its application is proposed according to the algorithm shown in Fig. 3. Thus, an algorithm and a mathematical model are proposed, the application of which allows forecasting the effectiveness of an investment project when making managerial decisions.

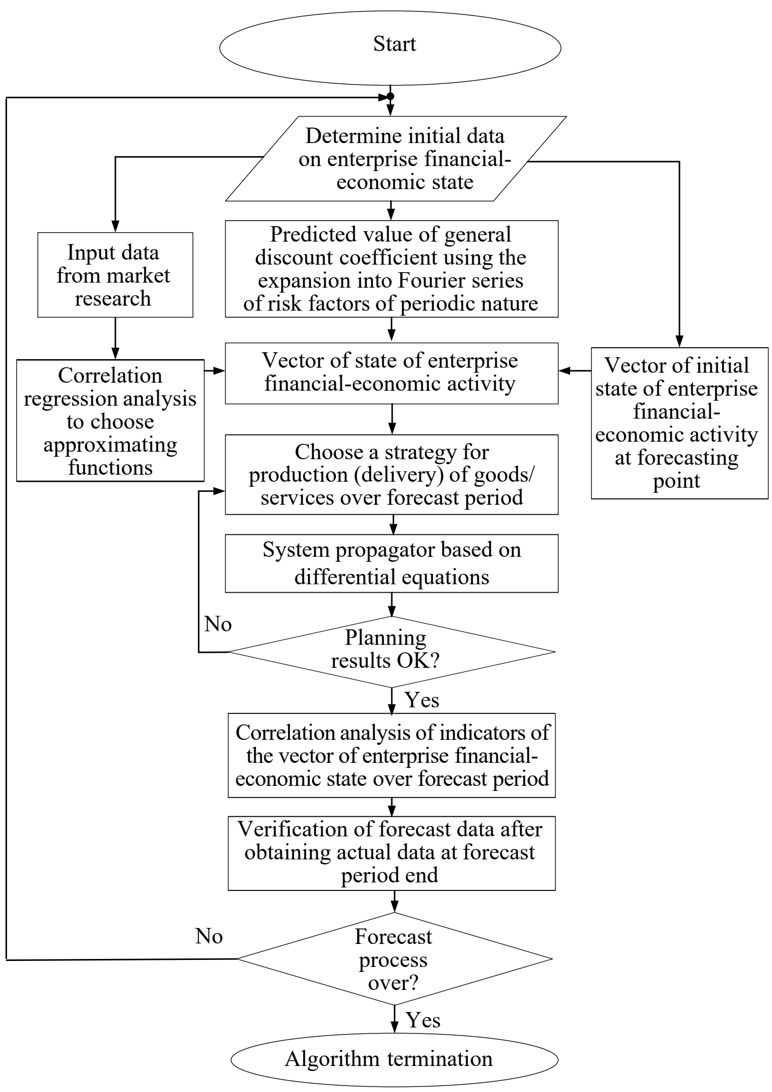


Fig. 3. Algorithm for applying a mathematical model to predict the effectiveness of an investment business project

5. 3. Verification of the application of the developed mathematical model for forecasting the effectiveness of the implementation of an investment business project

The initial data for verification of the practical application of the developed mathematical model are the financial statements from the LLC enterprise «Network of stores «Dnipro-M». Table 2 gives the determined indicators of cost and profit in the manufacture of one inverter generator.

Table 2

Determination of cost, price, and profit in the production of one inverter generator, UAH (at the rate of the National Bank of Ukraine: USD 1 – UAH 36.5686 as of 17.04.2023)

Indicator	Amount
Material costs	5505.00
Labor costs and deductions for social activities	2359.00
Depreciation of equipment	1081.00
Other expenses	885.00
Total cost	9830.00
Profitability, %	15.00
Unit price	11305.00
Profit per unit	1475.00

The business project is planned for 5 years, where in the first year the equipment is manufactured and production is launched, and in the next four years the very phase of production and sales takes place. It is estimated that to start the production of inverter generators, an initial investment of UAH 15353.3 thousand is needed (at the rate of the National Bank of Ukraine: USD 1 – UAH 36.5686 as of 17.04.2023). This project assumed that this investment would be realized at own expense, without additional interest for the investor. In this case, with a fixed total discount rate of 31.7 % for a 5-year period, the following values of *NPV* (1) and *PI* (2) were obtained: NPV=8262 UAH thousand, PI=1.54 with a payback period of 3.1 years. Project duration: from the beginning of 2022 to the end of 2026.

To compare the results of calculations using the developed model (5) to (16), with the results obtained using traditional procedures, the forecasting of the effectiveness of this project using the developed software application was carried out. Thus, for the first case, a forecast of an investment project was carried out without taking into account the influence of the market environment on the price of goods. This is necessary to analyze the influence of variable discount coefficient on forecasting results using the developed model in comparison with the use of classical approaches. Thus, for a continuous production model based on differential equations, the following input data are proposed:

1) daily output V_{cur}=13.7 units;

2) CP cost and price P of one unit of production from Table 2;

3) the expected sales ratio for the enterprise for the forecast period, selected by the method of expert assessments $k_{n.}$=1 (100 % sales);

4) depreciation coefficient k_a=0.11 (calculated from the data in Table 2);

5) investor's interest rate φ=0 (own funds);

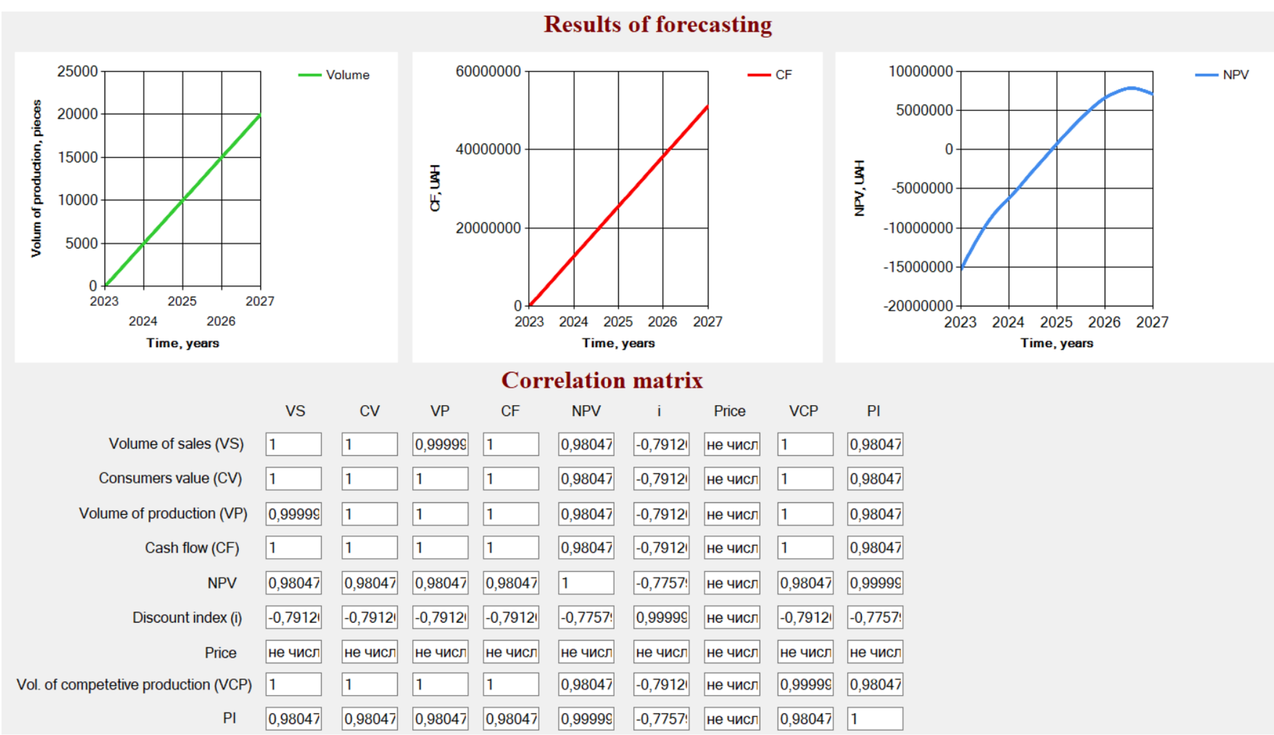
6) forecasting period from January 1, 2023, to December 31, 2026 (4 years);

8) r_{inf}=0.08 (inflation index); $r_{b\,i}$=0.05 (industry risk ratio);

7) A-str – line production strategy.

Thus, the results of calculations using the developed software application for the predefined period are shown in Fig. 4.

The calculations revealed that the net present value *NPV* of the project is positive and amounts to UAH 7105.142 thousand (at the rate of the National Bank of Ukraine: USD 1 – UAH 36.5686 as of 17.04.2023), which is 14 % less compared to calculations using traditional approaches. This difference is explained by a more accurate consideration of the discount coefficient in the developed model (Fig. 5, 6) in comparison with traditional approaches of expert assessments.



	VS	CV	VP	CF	NPV	i	Price	VCP	PI
Volume of sales (VS)	1	1	0,99999	1	0,98047	-0,7912	не числ	1	0,98047
Consumers value (CV)	1	1	1	1	0,98047	-0,7912	не числ	1	0,98047
Volume of production (VP)	0,99999	1	1	1	0,98047	-0,7912	не числ	1	0,98047
Cash flow (CF)	1	1	1	1	0,98047	-0,7912	не числ	1	0,98047
NPV	0,98047	0,98047	0,98047	0,98047	1	-0,7757	не числ	0,98047	0,99999
Discount index (i)	-0,7912	-0,7912	-0,7912	-0,7912	-0,7757	0,99999	не числ	-0,7912	-0,7757
Price	не числ	не числ	не числ	не числ	не числ	не числ	не числ	не числ	не числ
Vol. of competetive production (VCP)	1	1	1	1	0,98047	-0,7912	не числ	0,99999	0,98047
PI	0,98047	0,98047	0,98047	0,98047	0,99999	-0,7757	не числ	0,98047	1

Fig. 4. Additional interface window with calculation results

The increase in accuracy is explained by taking into account the periodicity of political risks associated with the elections of the Presidents of the United States and Ukraine. The maximum values of these risk ratios falling on the period of presidential elections $r^{max}_{pol.USA}$, $r^{max}_{pol.UA}$ were set at 0.1 for both the United States and Ukraine. The minimum values of political risk coefficients $r^{min}_{pol.USA}$, $r^{min}_{pol.UA}$ are after half the term of government of one or another president of the country (USA or Ukraine) and were determined equal to 0.026. The use of the mathematical apparatus of decomposition into the Fourier series (8) of the obtained functions (7) made it possible to simulate the periodicity of these risks.

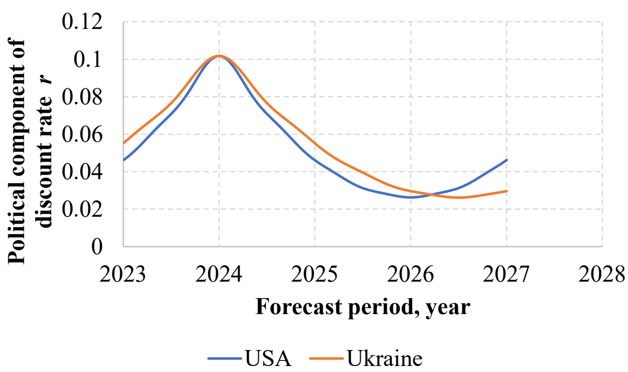


Fig. 5. Dynamics of changes in political risk components of the discount coefficient, calculated by the method of Fourier series expansion

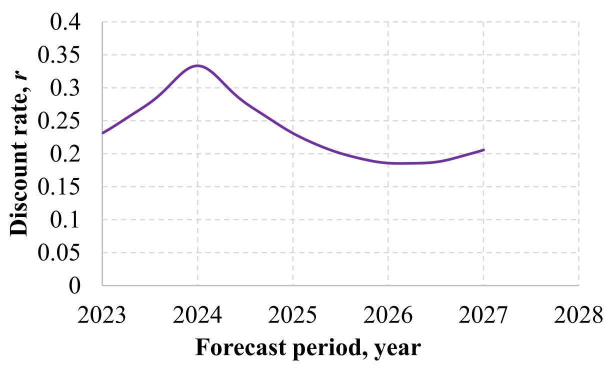


Fig. 6. Dynamics of changes in the total discount factor, calculated from formula (9)

So, Fig. 5 shows that the maximum value of political risk falls on 2024, which is true since in that year the elections of the Presidents of the United States and Ukraine occur simultaneously. Given this, during this period, the total discount rate is maximum (Fig. 6).

The dynamics of changes in the *PI* coefficient are shown in Fig. 7.

The value of the *PI* coefficient at the end of 2026 is 1.463, which is 5.1 % less compared to traditional approaches. However, despite the decrease in *PI* and *NPV* indicators, the project is profitable with a payback period of 2.9 years. The profitability index is higher than one, that is, for each hryvnia invested in the project, the company will receive UAH 1,463 of income, or UAH 0.463 of profit (at the rate of the National Bank of Ukraine: USD 1 – UAH 36.5686 on 17.04.2023). Also, the autonomy coefficients of the business project *ACB* amounted to 3.33, which indicates its stability. The dynamics of changes in the *MROS* coefficient are shown in Fig. 8.

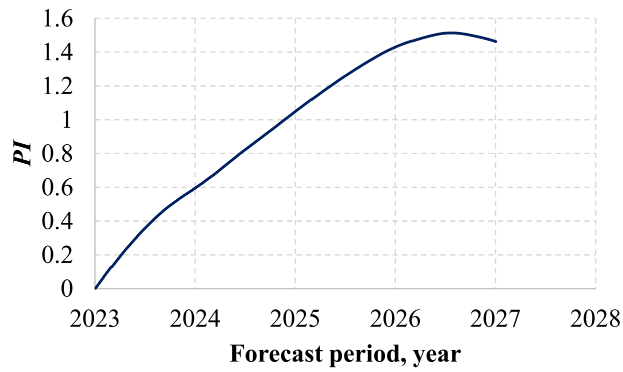


Fig. 7. Dynamics of changes in the *PI* indicator without taking into account the factors of influence of the external economic environment on pricing

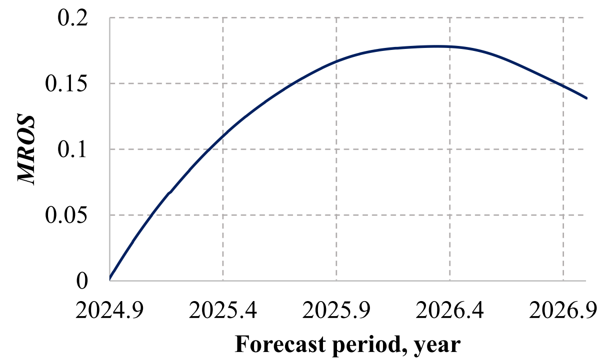


Fig. 8. Dynamics of changes in the *MROS* indicator without taking into account the factors of influence of the external economic environment on pricing

Thus, we have verified the application of the developed mathematical model for forecasting the effectiveness of the implementation of an investment business project under the influence of factors of the external market environment that showed high accuracy of forecasting calculations. This is due to a more accurate consideration of the political component when calculating the discount factor.

Taking this into account, it is proposed to forecast this investment project under the influence of factors of the external market environment on the example of solving a model problem. So, let the marketing research and business analysis department collect statistics N (number of potential consumers), N_1 (sales), and V_p (volume of similar products on the market) for 40 weeks (Fig. 9). So, the following dependences were approximated by the following functions:

– the number of potential consumers N by the cubic polynomial with $corr(f^{N}_{aprox}(t), Real^{N}) = 0.9256$ and $Aver\_err^{N} = 13.4757\,\%$;

– sales volume N_1 by the cubic polynomial with $corr(f^{N_1}_{aprox}(t), Real^{N_1}) = 0.9$ and $Aver\_err^{N_1} = 14.05\,\%$;

– the volume of similar products on the market Vp by the cubic polynomial with $corr(f^{V_p}_{aprox}(t), Real^{V_p}) = 0.8422$ and $Aver\_err^{V_p} = 19.54\,\%$.

The resulting dependences are shown in Fig. 10.

Taking into account the resulting dependences (Fig. 10), possible fluctuations in the unit price for the predefined forecasting period are obtained (Fig. 11).

Thus, taking into account fluctuations in the unit price, the dynamics of change for the coefficient *PI* for the period of forecasting the investment project are obtained (Fig. 12).

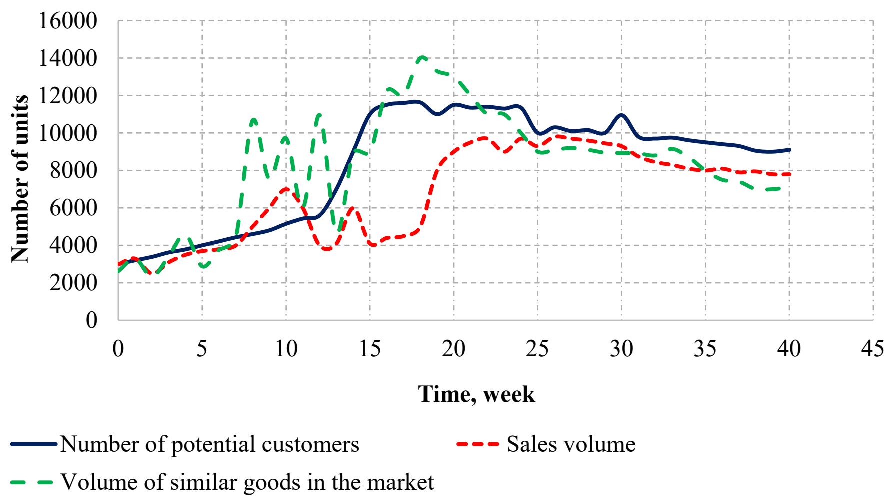


Fig. 9. Data collected by the Department of Marketing Research and Business Analysis (theoretical data for the model problem)

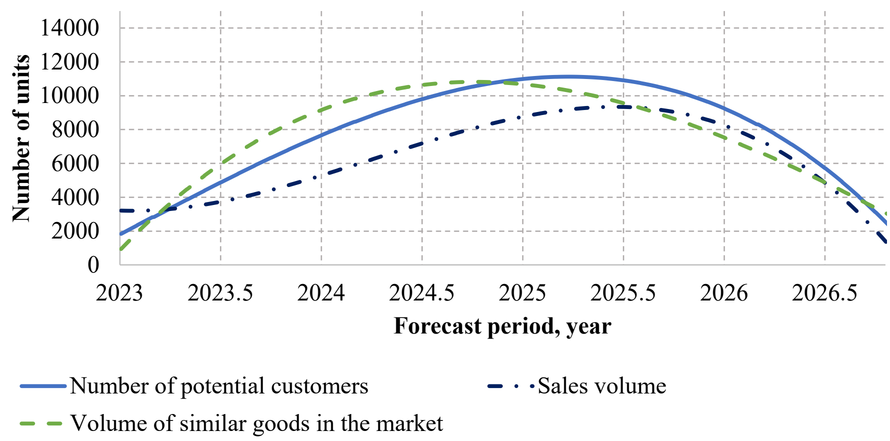


Fig. 10. The resulting approximated functional dependences for indicators N (number of potential consumers), N_1 (sales), and V_p (volume of similar products on the market)

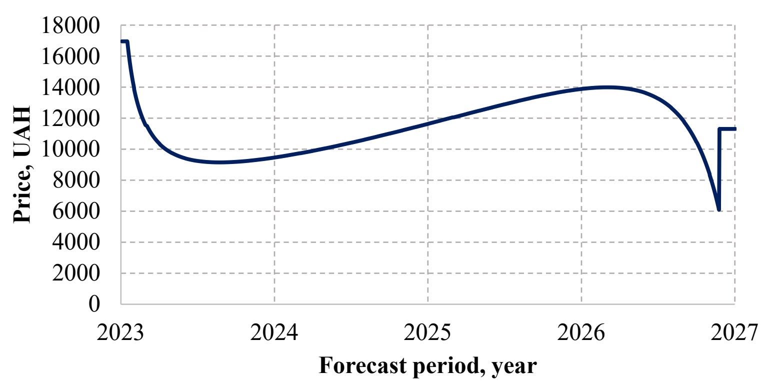


Fig. 11. Possible fluctuations in the unit price of products in the forecasting interval

Taking into account the influence of environmental factors, the *PI* coefficient at the end of 2026 was 1.5716. Given this, the investment project also remained profitable. However, the payback period of the investment business project increased from 2.9 years to 3.5 years. This is due to the nonlinearity of the dynamics of changes in cash flows *CF* (Fig. 13) with a change in unit price (Fig. 12) compared to the linear growth *CF* (Fig. 4) without taking into account factors of the external market environment.

The results of calculations of the dynamics of changes in the main indicators of efficiency of an investment business project helped construct the following correlation matrix (Fig. 14).

The results of forecasting solution on the example of a model problem demonstrate (Fig. 14) that key performance indicators *NPV* and *PI* of an investment business project are closely related to cash flows *CF* and production volume *VP* by a positive correlation.

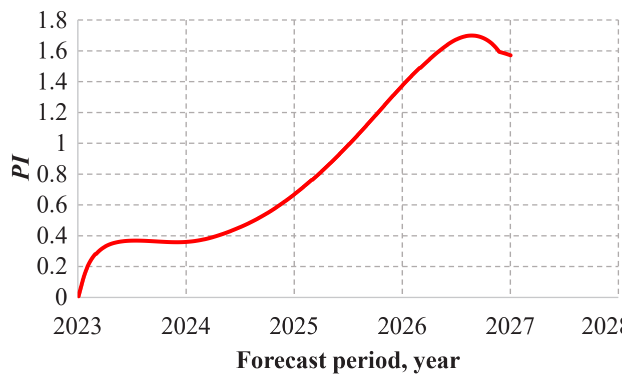


Fig. 12. Dynamics of changes in the *PI* index, taking into account the factors of influence of the external economic environment on pricing

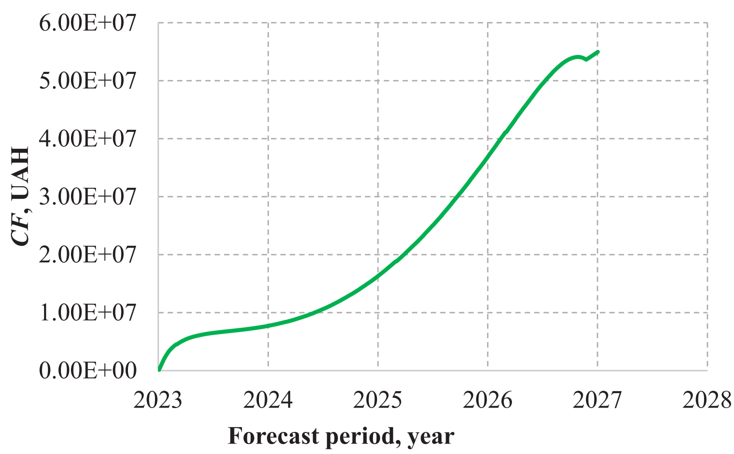
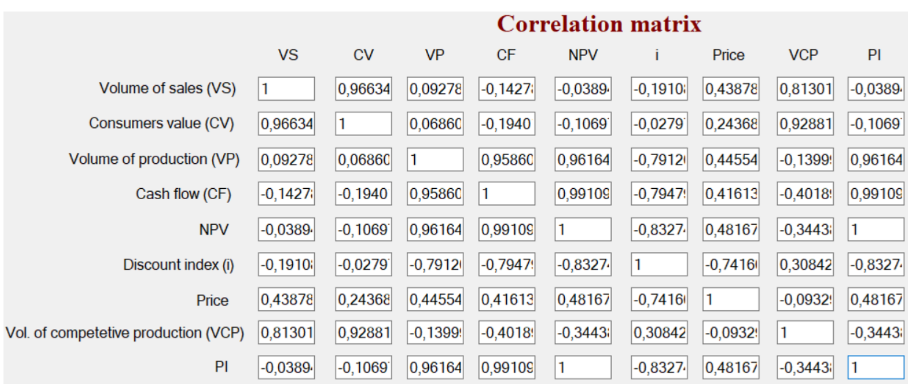


Fig. 13. Dynamics of cash flows *CF* with price changes in the market environment



Correlation matrix

	VS	CV	VP	CF	NPV	i	Price	VCP	PI
Volume of sales (VS)	1	0,96634	0,09278	-0,1427	-0,0389	-0,1910	0,43878	0,81301	-0,0389
Consumers value (CV)	0,96634	1	0,06860	-0,1940	-0,1069	-0,0279	0,24368	0,92881	-0,1069
Volume of production (VP)	0,09278	0,06860	1	0,95860	0,96164	-0,7912	0,44554	-0,1399	0,96164
Cash flow (CF)	-0,1427	-0,1940	0,95860	1	0,99109	-0,7947	0,41613	-0,4018	0,99109
NPV	-0,0389	-0,1069	0,96164	0,99109	1	-0,8327	0,48167	-0,3443	1
Discount index (i)	-0,1910	-0,0279	-0,7912	-0,7947	-0,8327	1	-0,7416	0,30842	-0,8327
Price	0,43878	0,24368	0,44554	0,41613	0,48167	-0,7416	1	-0,0932	0,48167
Vol. of competetive production (VCP)	0,81301	0,92881	-0,1399	-0,4018	-0,3443	0,30842	-0,0932	1	-0,3443
PI	-0,0389	-0,1069	0,96164	0,99109	1	-0,8327	0,48167	-0,3443	1

Fig. 14. Matrix of correlations of functions describing the dynamics of changes in the main performance indicators of an investment business project

Given this, we can say that the effectiveness of a business project directly depends on the volume of production and supply to markets.

In turn, the average positive correlation of *NPV* and *PI* is with a change in price in the market, which indicates the impact of supply and demand on this product group. The discount coefficient has a negative correlation with respect to *NPV* and *PI,* which indicates the correctness of the choice of time intervals for this business project (in the wake of a risk recession).

A comparative characteristic of *PI* and the payback period indicators of the project based on calculations using three procedures is given in Table 3.

Table 3

Comparative characteristics of *PI* and project payback period based on calculations using three procedures

Methodology of calculation	*PI*	Payback period
I. Traditional calculation method	1.54	3.1 years
II. Differential equations without taking into account the influence of supply and demand on the price of a product unit	1.463	2.9 years
III. A model problem in the presence of supply/demand statistics and regression forecasting of their impact on the price of goods	1.5716	3.5 years

From the above results in Table 3, we can draw the following conclusions:

– the results obtained when applying differential equations and variable discount coefficient (II) are close to the results with traditional calculation methods (I), which indicates the adequacy of the model (II);

– when solving a model problem and taking into account price variability from supply/demand, nonlinearities of cash flow indicators (III) are observed, affecting the dynamics of changes in *NPV* and *PI*;

– payback period with nonlinear cash flows does not clearly indicate the effectiveness of an investment business project (an increase in the payback period in III did not lead to a decrease in *PI*).

Thus, it is shown that taking into account the factors of disturbances of the external economic environment is expedient since they can influence the final performance indicators of an investment business project when making managerial decisions.

6. Discussion of research results on the application of the developed mathematical model to predict an investment business project

Known forecasting methods based on regression analysis [7, 8], artificial intelligence [10], Big Data technologies [12] have proven efficient in solving certain problems in the field of economic forecasting and decision-making. In turn, according to the analysis of the application of these models in chapter 2, they have certain limitations, which are primarily related to the insufficient amount of source data. Thus, with a shortage of initial data, the task of accurate forecasting, or training artificial intelligence (neural networks, reinforcement learning, etc.) is much more complicated, and in certain cases may be completely unsolved. One such case may be a startup of a new investment business project. In view of this, the main part of the mathematical model of forecasting was developed on the basis of differential equations (12). This is explained by the fact that this mathematical apparatus is promising and has proven efficient in other areas of economic analysis, such as stock market forecasts [13], auction forecasts [14].

Methodical approaches to building a combined mathematical model for forecasting the efficiency of investment projects on the basis of differential equations (12), with input parameters processed by statistical approximators based on the least squares method (5), (6) are proposed. However, instead of multifactor regression [7], to analyze the factors of influence of supply and demand on the price of goods, we proposed the introduction of special coefficients (10), the components of which are calculated using individual regressions based on the least squares method. This approach, on the one hand, makes it possible to increase the accuracy of taking into account one or another foreign economic factor in nonlinear data distributions. On the other hand, if there is a sufficient amount of input data, the influence of the stochastic component is reduced, which is associated with an error in regression forecasting of factors of the external market environment. The use of approximation by Fourier series (7), (8) of periodic political component of discount coefficient is proposed. These approaches significantly expand the flexibility of the process of forecasting the effectiveness of an investment business project by more accurately taking into account the influence of factors of the external economic environment compared to traditional methods [18, 19].

In turn, the increase in the accuracy of taking into account the discount coefficient is explained by the introduction into the model of the periodic component of political risks that occur during the elections of the Presidents of the United States and Ukraine. It should be noted that the introduction of these components of the discount factor is valid only for the Ukrainian economic space. The determination of political risks for other economic spaces should be carried out individually, taking into account the current geopolitical situation in a particular region.

Also, increasing the process of forecasting flexibility is to introduce the possible influence of the external market environment on the unit price of the products under consideration. Usually, in standard business planning, at the time of acceptance/rejection of a project, one price of products is calculated, taking into account profit, and it is taken constant for the entire forecasting period. However, given the dynamism and stochasticity of economic processes, this may be unfair for significant periods of business planning. In view of this, we offer methodological approaches to forecasting supply/demand and taking into account their impact on forecasting the price of products. The results of calculations based on the model problem of business planning (Fig. 9–13) showed the possible impact of product price variability on the main financial and economic indicators of the business project (Fig. 12, 13) in comparison with calculations at a constant price (Fig. 4, 7). Thus, taking into account possible price changes may be necessary for many business lines, certain groups of goods/services that significantly depend on the variability of the external market environment. Such results, firstly, are due to the introduction into the differential equations of statistical parameters of changes in supply/demand, affecting the price and calculated using regression analysis (5) (Fig. 10, 11). Secondly, by introducing into the model a variable discount coefficient (7) to (9) (Fig. 6), the component of which is calculated using the Fourier series (Fig. 5).

In turn, the use of the mathematical forecasting apparatus based on differential equations allows us to continuously assess the change in the main financial and economic indicators when making management decisions on the adoption/rejection of a certain investment business project. Also, the advantage of using this mathematical apparatus is the convenience of calculations when bringing to one time system measurement and integration with statistical approximators of indicators of the influence of the external economic environment.

Solving the problem of forecasting the effectiveness of an investment business project when making management decisions is relevant for investors, shareholders, owners, and management of a business entity.

However, there remain a number of problematic aspects and limitations in the practical use of the developed model using the following statistical approximators based on the least squares method (5), (6), among which the most significant are:

1) limitation of forecasting accuracy using linear and nonlinear regressions for significant periods (quarter or more);

2) obtaining input statistics that have «inconvenient distributions» and are rather poorly approximated by the least squares method (for example, statistics with constant significant jumps, periodic statistics with variable oscillation amplitudes, etc.);

3) insufficient statistical information to build a forecast on the basis of the least squares method.

In view of this, the problem arises of finding adaptive methods for forecasting time series for analyzing factors of the external market environment when applying the developed algorithm for forecasting the effectiveness of an investment project (Fig. 2). One of such solutions in the further development of the model may be methods of moving average, Holt-Wiener, and hybrid models [23, 24]. Another type of solution to this problem of price variability prediction in model (12) is stochastic modeling using the Monte Carlo group of numerical methods. In this case, a possible price change can be set in a certain range using random variable generators (for example, normal distribution – in the case of «white noise» models).

The disadvantage of the developed methodology is that it is intended only for the Ukrainian economic space. When using the proposed methodology to assess the effectiveness of the implementation of an investment project for business located in other countries, it is necessary to take into account the specific risks inherent in this country.

The search and selection of the most appropriate approximator for the model of differential equations (7) and the forecasting algorithm (Fig. 2) is the goal of further research.

7. Conclusions

1. When building a representative sample of indicators characterizing the financial and economic condition of the enterprise, 10 indicators for evaluating the effectiveness of the implementation of an investment business project were selected for making managerial decisions on the directions and prospects of strategic development of the enterprise. Thus, the first five indicators characterize the results of the enterprise. The last five financial and economic indicators characterize the stability of a business under the influence of factors of the external financial and economic environment in which this business operates. These indicators constitute the full vector of the state of business and allow one to make a balanced management decision on the implementation of an investment business project under the influence of factors of the external market environment.

2. Using a systematic approach, a mathematical model was developed that simultaneously uses the methods of correlation and regression analysis, the theory of differential equations and Fourier series to assess the effectiveness of the implementation of an investment business project. The use of the forecasting method based on the Fourier series made it possible to increase the accuracy of determining the political component of the discount coefficient. In turn, the use of methods of correlation and regression analysis made it possible to select the approximating function of the influence of supply and demand factors on the unit price, which most accurately reflects this dependence. Also, the use of the correlation matrix made it possible to assess the mutual influence of indicators for forecasting the effectiveness of the implementation of an investment business project. The proposed mathematical model based on differential equations made it possible to determine the dynamic dependences of the main indicators for evaluating the effectiveness of an investment business project at the predefined forecasting interval, taking into account the influence of factors of the external market environment.

This approach allows forecasting the effectiveness of an investment project starting from a startup to its completion, taking into account external and internal factors of disturbances caused by the influence of a certain financial and economic environment. This provides additional information to the investor or business owner when forecasting the stability of a business project for making management decisions on its implementation.

3. The obtained results when applying differential equations and variable discount coefficient showed a decrease in *NPV* by 14 %, and *PI* by 5.1 %, due to more accurate consideration of the political component when calculating the discount factor. The expediency of taking into account the variable-periodic component of the discount rate using the Fourier series is explained by the increase in political risks during significant events, such as presidential elections, in economic space of which the enterprise operates. This, in turn, makes it possible to predict and choose the safest time period for a startup with minimal risks.

In turn, taking into account the impact of the external market environment on the price of goods, there was an increase in the payback period of the project by almost six months. However, the increase in the payback period did not lead to a decrease in the *PI* yield index. In view of this, with nonlinear cash flows, the payback period does not clearly indicate the effectiveness of the implementation of a business project.

Thus, taking into account these factors makes it possible to more accurately assess the stability of a business project against the influence of external factors, which may be an additional argument for an investor or business owner regarding its implementation.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The manuscript contains data included as additional electronic material.

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