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CONSTRUCTION OF A PHYSICAL-MATHEMATICAL MODEL OF OSCILLATIONS OF THE UNBALANCED VIBRATOR OF THE PNEUMATIC SORTING TABLE

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A physical-mathematical model of oscillations of unbalanced vibrators of a pneumatic sorting table as non-stationary oscillations of an impulse-loaded round plate with various options for fixing its contour has been built. The axisymmetric non-stationary oscillations of a round plate supported by a one-sided round base were considered in two ways of fixing its contour, namely, when it is tightly clamped and freely supported. It was assumed that the linearly elastic base resists only compression and does not perceive stretching. It is shown that for certain durations of the transverse force pulse in time, the amplitude of the deflection of the middle of the plate in the direction of action of the external pulse can be smaller than the amplitude of the deflection in the opposite direction. At the same time, in the second case, there is no contact of the plate with the base. It has been proven that this dynamic effect of the asymmetry of the elastic characteristic of the system also applies to bending moments and is more clearly manifested when the contour is freely supported than when it is tightly clamped. For rectangular and sinusoidal pulses, closed-loop solutions of the equations of motion of the plate during its contact with the base and after separation from the base were constructed. Compact formulas were derived for calculating the amplitudes of positive and negative deflections in both directions from the zero position of static equilibrium. Formulas have been obtained for calculating the time it takes for the plate to obtain extreme deflection values, which is achieved due to the selection of a special axisymmetric distribution of dynamic pressure on the plate. Under such a load, the plate simultaneously detaches from the base at all points, except for the contour points, which reduces the nonlinear boundary value problem to a sequence of two linear problems. Numerical integration of the differential equation was carried out to check the reliability of the constructed analytical solutions. Adequacy of the model was proven for the following values of initial parameters: modulus of elasticity, 2.1·10^{11} Pa; the Poisson ratio of the plate material, 0.25; plate thickness, 7...10 mm; the maximum pressure on the plate, 4·10^{3} Pa; the bending stiffness of the plate, 6402.6667 N·m

Keywords: unbalanced vibrator, pneumatic sorting table, oscillation of a round plate, dynamic effect of asymmetry

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1. Introduction

Reducing the cost of production, provided it is of high quality, is one of the main factors in the functioning of enterprises, which affects their competitiveness. This is especially important under conditions of high competition of enterprises in the agro-industrial complex. The solution to the problem is related to the development of new technologies and devices and the improvement of existing ones [1].

It should be noted that reducing the cost of production is especially important for technological processes that are used for large volumes of raw materials. One of these processes is the separation of grain mixtures. During the collection, storage, and processing of grain mixtures, hundreds of thousands of tons of raw materials are subject to separation, which leads to significant effects in energy and resource efficiency, provided that technologies and equipment for this process are improved [2].

Separation of grain mixtures in a vibro-pneumatic fluidized bed is one of the main techniques of separating impurities from the seeds of the main crop or separating different crops. According to this technique, the mixture is divided into grain fractions according to a set of physical and mechanical properties: geometric dimensions, shape, density, aerodynamic parameters, and particle elasticity [3]. At the same time, scientifically based management of the separation of grain mixtures by applying physical-mathematical modeling of the process under various factors that affect its energy and resource efficiency remains relevant.

2. Literature review and problem statement

There are a number of technical solutions for ensuring the process of separation of grain mixtures in a vibro-pneumofluidized bed, which differ in the shape of the working surfaces, the scheme of movement of the grain mixture, the technique of creating the fluidity of the raw material layer, etc. [4].

Among the technical implementations of equipment for the separation of grain mixtures, pneumatic sorting tables should be singled out, on which the separation of grain mixtures containing seed material is carried out. The separation of seed material from the grain mixture imposes certain requirements on the functioning of the separation equipment. During the selection of this fraction, its particles are subjected to repeated mechanical impact from the working surfaces, which leads to injury of the particles. As a result, damage to seed particles can reach critical values, which significantly reduces the biological activity of the final product [5]. Ways to solve this problem should obviously be sought in the justified management of mechanical impact on raw material particles. Such control can be carried out by developing and applying physical-mathematical models of oscillation of the working bodies of the separation equipment.

The process of stratification of the grain mixture into fractions and their further transportation along the working surface of the pneumatic sorting table to the unloading devices takes place under the influence of vibrations of the working surface and the flow of air through the layer of raw materials. At the same time, the frequency of oscillations is determined by the operation of frequency converters, and the amplitude is varied by unbalanced vibrators [6]. It should be noted that the stability of vibrations of the working surface and, as a result, the quality of separation of the grain mixture, are determined by the stability of the devices that create these vibrations.

Unbalanced vibrators are mechanical devices with fixed positions. Their form changes according to the set requirements from circle to sector [7]. The research examines one of the limit fixed positions of unbalanced vibrators – a round shape. At the same time, for the construction of a mathematical model, an assumption was made: the unbalanced vibrator is considered as a circular plate.

Round-shaped plates are common structural elements. These are the bottoms of liquid storage tanks, foundation slabs of silos and other buildings with a round shape in plan, hatch covers, glazing of portholes, etc. In applied mechanics, considerable attention is paid to calculations of their strength, both under the action of static and dynamic loads [8].

Unsteady oscillations of a circular plate on an elastic base are considered in [9], where the classical equation of the plate theory is not used, but its refined version of the first approximation. The problem of impulse deformation is solved by numerical methods, using the method of dipping or expanding the shape of the area to the canonical one. The circular area expands to a square one. The results obtained in [9] concern only the continuous movement of the plate from the base, which is provided for in the statement of the problem. Article [10] considered the possibility of using the theory of plates with continuous contact from the base to calculate the strength of hard asphalt concrete road surfaces. Oscillations of rectangular plates under the action of moving loads arising during the movement of transport were studied. In them, in addition to direct problems of mechanics, several inverse problems of identification of external dynamic loads based on the results of experimental measurement of deflections or deformations of the hard surface are solved. The problems are reduced to the numerical solution of integral equations, with separate regularization, but study [10] does not concern round plates.

The statics of layered plates of a symmetrical rectangular structure is described in [11]. In the work, only the static deformation of plates on an elastic base is considered, without analyzing their dynamics. The authors of paper [12] suggest using an elastic base as a means of passively influencing plate oscillations, since the base significantly changes the first natural frequencies at which resonances are possible. The work does not provide for the calculation of plates for the action of impulse loads. Harmonic oscillations of round three-layer plates, without separation from the base, are discussed in article [13]. An analytical solution of the boundary value problem was constructed in cylindrical functions, but it, as in [12], is not suitable for calculating the impulse deformation of the plate. Such deformation is not foreseen in articles [14, 15], where only static deformation of layered orthotropic plates is considered. Static bending of a three-layer plate on a two-parametric basis is considered in [16]. Analytical calculation formulas were obtained, but the study is less general compared to [14, 15], where the plates are multilayered.

It is necessary to single out papers [17, 18], where static bending of plates on a two-parameter basis is also performed. They consider special variants of external load that arise in certain environments, as well as non-traditional theories of plates, which take into account the porosity and heterogeneity of the material, the presence of piezoelectric elements, etc. However, there are no dynamics problems in the works.

The analysis of the latest publications reveals that they assume a two-sided connection of the plates with the base. Such an assumption simplifies the statement of the problem and its solution but does not correspond to the research task, where the connections are one-sided [19, 20]. Dynamic problems of deformation of a plate with a one-sided bond with the base belong to nonlinear problems since in the general case they have unknown areas of separation and contact that change during movement. These problems are of scientific and practical interest because they correspond to more adequate theoretical models of plate deformation in practice. Under such conditions, effects are observed that cannot be described by traditional models of plates on a two-sided elastic base. This predetermines the purpose of the current research.

3. The aim and objectives of the study

The purpose of this study is to build a physical-mathematical model of oscillations of unbalanced vibrators as non-stationary oscillations of an impulse-loaded round plate with various options for fixing its contour in one-sided contact with an elastic base. The application of the model built will make it possible to obtain a reasonable approach to managing the resource efficiency of the process of separating grain mixtures using a pneumatic sorting table, provided it is used in the general physical-mathematical model of this process.

To achieve the goal, the following tasks were set:

– to obtain an analytical solution in cylindrical functions with further calculations of the boundary value problem of plate dynamics with rigid clamping of the contour;

– to obtain an analytical solution in cylindrical functions with further calculations of the boundary value problem of plate dynamics with a hinged contour;

– to check the adequacy of the physical and mathematical model of the dynamics of a circular plate, both under the condition of rigid clamping of the contour, and under the condition of hinged contour.

4. The study materials and methods

The object of research is the oscillations of unbalanced vibrators, a physical-mathematical model of non-stationary oscillations of a plate separated from the base.

The schematic view of the unbalanced vibrator, which is used to vary the amplitude of oscillations of the pneumatic sorting table and for which a physical-mathematical model is constructed, is shown in Fig. 1.

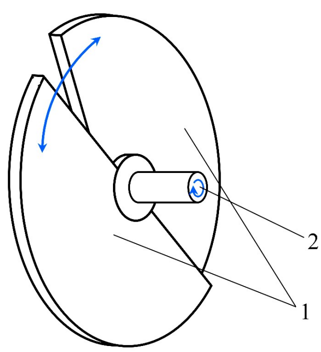

Fig. 1. Schematic view of an unbalanced vibrator: 1 – balanced mass; 2 – axis

In order to obtain accurate analytical solutions to the problems posed in the study for two options for changing the force pulse over time (step and sinusoidal), a special distribution of the external load along the radial coordinate is adopted. This makes it possible to move from a continuous nonlinear system to a system with one degree of freedom.

5. Building a physical-mathematical model of the dynamics of a circular plate on a one-sided elastic base caused by a force impulse

5. 1. Physical-mathematical model of the dynamics of a plate with rigid contour clamping

The oscillatory motion of the plate upon contact with its base can be described by the differential equation:

$$D\nabla^2\nabla^2 z + cz + \rho_* \frac{\partial^2 z}{\partial t^2} = q f(r)\varphi(t)\left[H(t) - H(t-t_1)\right]. \quad (1)$$

It contains a differential operator $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$; $D = \frac{Eh^3}{12(1-v^2)}$ – bending stiffness of the plate with thickness h; $z=z(r, t)$ is the transverse deflection of the plate as a function of the radial coordinate r and time t; c is the coefficient of the elastic base; E, ν – modulus of elasticity and Poisson's ratio of the plate material; ρ_* is the total mass of a unit area of the system, taking into account the attached mass of the base; q – maximum pressure on the plate; $f(r)$ is the pressure distribution function along the radius; $\varphi(t)$ is a function of the change in momentum over time; t_1 – pulse duration; $H(t)$, $H(t-t_1)$ are unit step functions.

It is assumed that the plate contour $r=R$ is tightly clamped, i.e.:

$$z(R, t) = 0; \left.\frac{\partial z(r, t)}{\partial r}\right|_{r=R} = 0. \quad (2)$$

The solution of equation (1) can be represented by the product:

$$z = A(t)\cdot f(r), \quad (3)$$

in which $A(t)$ is an unknown time function, and the second factor $f(r)$ is:

$$f(r) = C_1 J_0\left(\beta\frac{r}{R}\right) + C_2 I_0\left(\beta\frac{r}{R}\right), \quad (4)$$

where C_1, C_2, β – unknown constants; $J_0(x)$, $I_0(x)$ – cylindrical zero index functions.

Considering the derivatives [21]:

$$\frac{dJ_0(x)}{dx} = -J_1(x), \quad \frac{dI_0(x)}{dx} = -I_1(x),$$

and expressions (2)–(4), one can obtain a system of equations:

$$C_1 J_0(\beta) + C_2 I_0(\beta) = 0;$$

$$C_1 J_1(\beta) + C_2 I_1(\beta) = 0, \quad (5)$$

in which $J_1(x)$, $I_1(x)$ – cylindrical functions of index one.

The equality to zero of the determinant of this system gives the transcendental equation:

$$J_0(\beta)I_1(\beta) + J_1(\beta)I_0(\beta) = 0, \quad (6)$$

to determine the constant β.

The smallest positive root of equation (6) is:

$$\beta \approx 3.1962206.$$

To find unknown C_1 and C_2, we introduce restrictions:

$$f(0) = C_1 + C_2 = 1.$$

Then, according to (5):

$$C_1 = \frac{I_0(\beta)}{I_0(\beta) - J_0(\beta)} \approx 0.9472274;$$

$$C_2 = -\frac{J_0(\beta)}{I_0(\beta) - J_0(\beta)} \approx 0.0527726. \quad (7)$$

The chart of function $f(r)$ is plotted as a solid line in Fig. 2.

The greatest pressure is present in the center of the plate, and on its contour it is zero.

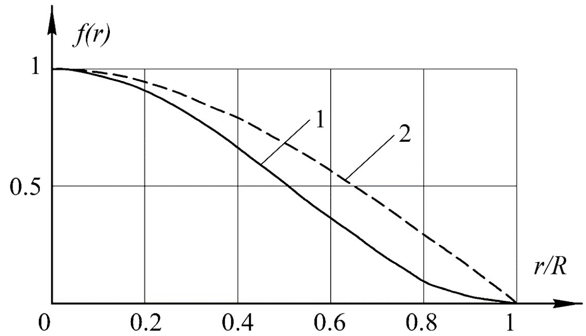

Fig. 2. Chart of function $f(r)$:
1 — rigid clamping, 2 — hinged support

By substituting (3) in (1), we can obtain the equation:

$$\frac{d^2A}{dt^2}+\Omega^2A=\frac{q}{\rho_*}\varphi(t)\left[H(t)-H(t-t_1)\right], \tag{8}$$

in which $\Omega=\sqrt{\frac{\left(\left(D\beta^4/R^4\right)+c\right)}{\rho_*}}$ is the frequency of free oscillations of axisymmetric oscillations of a plate supported by a two-sided elastic base.

The solution of equation (8) is constructed under zero initial conditions:

$$A(0)=0;$$

$$\left.\frac{dA}{dt}\right|_{t=0}=0.$$

It was believed that before the impulse was applied, the plate was at rest and had zero deflection. Under the following initial conditions:

$$A(t)=\frac{q}{\rho\Omega}\int_0^{u_*}\varphi(u)\sin\Omega(t-u)\,\mathrm{d}u, \tag{9}$$

and $u_*=\begin{cases}t \text{ with } t\le t_1,\\ t_1 \text{ with } t>t_1.\end{cases}$

Expression (9) makes it possible to consider different momentum scans in time.

First you need to consider the action of a rectangular pulse when $\varphi(t)$=1=const. In this case, calculating integral (9) yields:

$$A(t)=\begin{cases}z_c\left(1-\cos(\Omega t)\right) \text{ with } t\le t_1,\\ 2z_c\sin\left(\frac{\Omega t_1}{2}\right)\sin\left(\Omega\left(t-\frac{t_1}{2}\right)\right) \text{ with } t>t_1.\end{cases} \tag{10}$$

Here, $z_c=\frac{q}{\left(D\beta^4/R^4\right)+c}$ – static deflection of the center of the plate, on an elastic base, caused by pressure $q(r)=q\cdot f(r)$.

Here, we limited ourselves to considering the action of a pulse of short duration when $t_1<2\pi/\Omega$. With such a variant of the load, the solution (10) will be integral on the interval $t\in(0;t_*)$, where:

$$t_*=\frac{\pi}{\Omega}+\frac{t_1}{2}.$$

At $t_*=t_1$, according to (10):

$$A(t_*)=0,\ \frac{dA}{dt}=-\upsilon_0=-2z_c\Omega\sin\left(\frac{\Omega t_1}{2}\right). \tag{11}$$

The amplitude of the deflection of the center of the plate in the direction of action of the pulse, in accordance with (3), (4) and (10), is:

$$a_0(t_1)=\begin{cases}2z_c\sin\left(\frac{\Omega t_1}{2}\right) \text{ with } t_1\in(0;\pi/\Omega),\\ 2z_c \text{ with } t_1\in[\pi/\Omega;2\pi/\Omega].\end{cases} \tag{12}$$

It takes into account that $J_0(x)=I_0(x)=1$.

This extreme occurs when:

$$t=t_c=\begin{cases}\frac{\pi}{2\Omega}+\frac{t_1}{2} \text{ with } t_1\in(0;\pi/\Omega),\\ \frac{\pi}{\Omega} \text{ with } t_1\in[\pi/\Omega;2\pi/\Omega].\end{cases}$$

From the moment of time $t=t_*>t_1$, the movement of the plate starts opposite to the direction of the pulse. The unloaded plate loses contact with the base, so its bending is described by the differential equation:

$$D\nabla^2\nabla^2z+\rho\frac{\partial^2z}{\partial t^2}=0. \tag{13}$$

Here ρ is the mass per unit area of the plate.

Substituting the expression (3) into (13), we obtained the differential equation:

$$\frac{d^2A}{dt^2}+\omega^2A=0. \tag{14}$$

Here, $\omega=\frac{\beta^2}{R^2}\sqrt{\frac{D}{\rho}}.$

The solution to equation (14), under initial conditions (11), is:

$$A(t)=-2z_c\frac{\Omega}{\omega}\sin\left(\frac{\Omega t_1}{2}\right)\sin\left(\omega(t-t_*)\right).$$

It follows from it that the amplitude deviation of the center of the plate from the base is:

$$a_1(t_1)=2z_c\frac{\Omega}{\omega}\sin\left(\frac{\Omega t_1}{2}\right). \tag{15}$$

And the time of its achievement is determined by the expression:

$$t=t_m=\frac{\pi}{2\omega}+t_*.$$

Thus, the ratio of the amplitudes of the deflections of the center of the plate according to (12), (15), is equal to:

$$\frac{a_1(t_1)}{a_0(t_1)}=\begin{cases}\frac{\Omega}{\omega} \text{ with } t_1\in(0;\pi/\Omega),\\ \frac{\Omega}{\omega}\sin\left(\frac{\Omega t_1}{2}\right) \text{ with } t_1\in[\pi/\Omega;2\pi/\Omega].\end{cases}$$

If $\Omega>\omega$, then $a_1(t_1)>a_0(t_1)$, that is, there is a dynamic effect of asymmetry of the elastic characteristic of the system, which was discussed in [22]. When $t_1>\pi/\Omega$, it appears only for pulse durations $t_1\in(\Omega>\omega;\ t_G)$, where:

$$t_G=\frac{2}{\Omega}\left(\pi-\arcsin\frac{\omega}{\Omega}\right).$$

At the next stage of the research, it is necessary to consider the action of a sinusoidal pulse on the plate, when $\varphi(t)=\sin(\lambda t)$, $\lambda\geq\Omega$, $t_1=\pi/\lambda$.

In this case, after calculating the integral in (9), we obtained, for $\lambda\geq\Omega$:

$$A(t)=\frac{q}{\rho_*}\cdot\frac{\lambda}{\Omega\left(\lambda^2-\Omega^2\right)}\times$$
$$\times\begin{cases}\sin(\Omega t)-\dfrac{\Omega}{\lambda}(\lambda t)\text{ with }t\leq t_1,\\ \sin(\Omega t)+\dfrac{\Omega}{\lambda}\sin\left(\Omega\left(t-t_1\right)\right)\text{ with }t>t_1.\end{cases}\qquad(16)$$

If $\lambda=\Omega$, then the disclosure of uncertainty in (16) gives:

$$A(t)=\frac{z_c}{2}\begin{cases}\sin(\Omega t)-\Omega t\cdot\cos(\Omega t)\text{ with }t\leq t_1=\pi/\Omega,\\ -\pi\cos(\Omega t)\text{ with }t>t_1=\pi/\Omega.\end{cases}\qquad(17)$$

Taking the derivative t from expression (16), at $t>t_1$, we obtained:

$$\frac{dA(t)}{dt}=\frac{q}{\rho_*}\cdot\frac{\lambda}{\lambda^2-\Omega^2}\left[\cos(\Omega t)+\cos\left(\Omega\left(t-t_1\right)\right)\right].\qquad(18)$$

The time when this derivative is zero corresponds to the stop of the plate upon contact with the main one and is determined by the transcendental equation:

$$\cos(\Omega t)+\cos\left(\Omega\left(t-t_1\right)\right)=0.$$

It has the root:

$$t=t_e=\frac{\pi}{2\Omega}+\frac{t_1}{2}.\qquad(19)$$

The amplitude of the positive deflection of the center of the plate in the direction of the acting pulse was determined by substituting (19) into (16) and (17). As a result, you can get:

$$a_0\left(t_1\right)=A\left(t_e\right)=$$
$$=\begin{cases}\dfrac{2q}{\rho_*}\cdot\dfrac{\lambda}{\Omega\left(\lambda^2-\Omega^2\right)}\cos\left(\dfrac{\Omega t_1}{2}\right)\text{ with }\lambda>\Omega,\\ 0.5\pi z_c\sin\left(\dfrac{\Omega t_1}{2}\right)\text{ with }\lambda=\Omega.\end{cases}\qquad(20)$$

According to (16), the unloaded plate will return to position $z(r,t)=0$ when:

$$\sin(\Omega t)+\sin\left(\Omega\left(t-t_1\right)\right)=0.$$

This equation has a solution:

$$t=t_*=\frac{\pi}{\Omega}+\frac{t_1}{2}.\qquad(21)$$

By substituting (21) in (18), you can get the formula for the speed of movement of the center of the plate when $z(r,t)=0$. It takes the form:

$$\left.\frac{dA}{dt}\right|_{t=t_*}=\upsilon_0=-\frac{2q}{\rho_*}\begin{cases}\dfrac{\lambda}{\lambda^2-\Omega^2}\cos\left(\dfrac{\Omega t_1}{2}\right)\text{ with }\lambda>\Omega,\\ \dfrac{\pi}{4\Omega}\text{ with }\lambda=\Omega.\end{cases}$$

To obtain negative deflections of the plates when it is detached from the base, equation (14) was solved under initial conditions:

$$A\left(t_*\right)=0;\quad\left.\frac{dA}{dt}\right|_{t=t_*}=\upsilon_0.$$

The resulting solution is:

$$A(t)=\frac{\upsilon_0}{\omega}\sin\left(\omega\left(t-t_*\right)\right).$$

It follows from it that the amplitude of the negative deflection of the center of the plate is achieved when:

$$t=t_m=\frac{\pi}{2\omega}+t_*,$$

and is equal to:

$$a_1\left(t_1\right)=A\left(t_m\right)=\frac{\left|\upsilon_0\right|}{\omega}.\qquad(22)$$

So, in accordance with (20) and (22), the following ratio of the amplitudes of the negative and positive deflections of the center of the plate is obtained:

$$\frac{a_1\left(t_1\right)}{a_0\left(t_1\right)}=\frac{\Omega}{\omega}=\text{const}.$$

In the considered case, it does not depend on the duration of the pulse and is equal to the ratio of the frequencies of free oscillations of the plate supported and not supported by an elastic base, when $t_1\in(0;\pi/\lambda)$, $\lambda\geq\Omega$. When $\Omega>\omega$, the dynamic effect of the asymmetry of the elastic characteristics of the system, which also had the effect of a rectangular pulse, is manifested.

At the next stage of the research, it was necessary to find out whether the dynamic effect of asymmetry applies only to deflections or to other characteristics. In particular, this applies to the bending moments that determine the stressed state of the plate. To this end, in the expression of the bending moment:

$$M(r,t)=$$
$$=-D\left(\frac{\partial^2 z(r,t)}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial z(r,t)}{\partial r}-\frac{1-\nu}{r}\cdot\frac{\partial z(r,t)}{\partial r}\right),$$

we substituted the solution (3). Considering that [23]:

$$\lim_{r\to0}\frac{J_1\left(\dfrac{\beta}{R}r\right)}{r}=\lim_{r\to0}\frac{I_1\left(\dfrac{\beta}{R}r\right)}{r}=\frac{\beta}{2R},$$

and expressions derived from cylindrical functions, in the center of the plate we obtained:

$$M(0,t)=\frac{D(1+\nu)}{2}\cdot\frac{\beta^2}{R^2}(C_1-C_2)A(t).$$

So, the ratio:

$$\frac{M(0,t)}{z(0,t)}=\frac{D(1+\nu)}{2}\cdot\frac{\beta^2}{R^2}(C_1-C_2)=\text{const},$$

does not depend on time. Therefore, the mentioned dynamic effect concerns not only deflections, but also bending moments, which in the center of the plate can be greater when the plate is detached from the base at moments when the plate comes into contact with the base.

5. 2. Physical-mathematical model of plate dynamics with hinged contour

When constructing a physical and mathematical model of the dynamics of a plate with a hinged loop, differential equation (1) was solved under boundary conditions:

$$z(R,t)=0,\quad M(R,t)=0. \tag{23}$$

As above, the solution was sought in the form (3), and the function $f(r)$ was given by expression (4). By substituting (3) into (23), we obtained the system of equations:

$$C_1J_0(\beta)+C_2I_0(\beta)=0,$$

$$C_1\left[\frac{1-\nu}{\beta}J_1(\beta)-J_0(\beta)\right]+C_2\left[I_0(\beta)-\frac{1-\nu}{\beta}I_1(\beta)\right]=0.$$

The equality of the determinant of this system to zero leads to the transcendental equation:

$$\frac{I_1(\beta)}{I_0(\beta)}+\frac{J_1(\beta)}{J_0(\beta)}=\frac{2\beta}{1-\nu}. \tag{24}$$

In contrast to the previous case of fixing the contour of the plate, the roots of equation (24) depend on the values of the coefficient of transverse deformations v.

After determining β, constants C_1 and C_2 were found using formulas (7).

In order to simplify the calculations, Table 1 gives the values of constants β, C_1 for different values.

As can be seen from Table 1, the value of v affects the constants β, C_1, C_2 by no more than 5 %.

Since the form of the solution of the dynamic boundary value problem does not change when the boundary conditions are changed, the formulas obtained above remain valid for the case of hinged abutment of the plate edges. For their application, it is only necessary to use the corresponding values of the constants β, C_1, C_2.

Table 1

Values of constants β, C_1, C_2 at different v

v	β	C_1	C_2
0.20	2.1869110	1.0473143	–0.0473143
0.25	2.2045701	1.0426163	–0.0426163
0.30	2.2215195	1.0382659	–0.0382659
0.35	2.2378079	1.0342274	–0.0342274
0.40	2.2534790	1.0304698	–0.0304698
0.45	2.2685723	1.0269661	–0.0269661

5. 3. Checking the adequacy of the physical and mathematical model of round plate dynamics

To check the adequacy of the developed physical-mathematical model using the derived formulas, three cases can be considered. The first case corresponds to non-stationary oscillations of a plate with a clamped contour when it is loaded with a rectangular pulse. The second case corresponds to the dynamic response of a plate with a closed contour to the action of a sinusoidal pulse. The third is the dynamic deflection of a plate with a hinged contour caused by a rectangular pulse.

In the first case, which corresponds to the non-stationary oscillations of a plate with a clamped contour when it is loaded with a rectangular pulse, by using formulas (12), (15), the calculation of the oscillation parameters was carried out at: E_1=2.1·10^{11} Pa; ν=0.25, h=0.007 m; ρ=54.6 kg/m^2; ρ$_*$=100 kg/m^2; c=10^6 N/m^3; q=4·10^3 Pa; R=1.5 m. For these initial data: D=6402.6667 N·m; z_c=3.5336·10^{-3} m; Ω=106.3950 s^{-1}; ω=49.1672 s^{-1}; t_G=0.0500 s. The calculated plots for $a_0(t_1)$ and $a_1(t_1)$ are shown in Fig. 3.

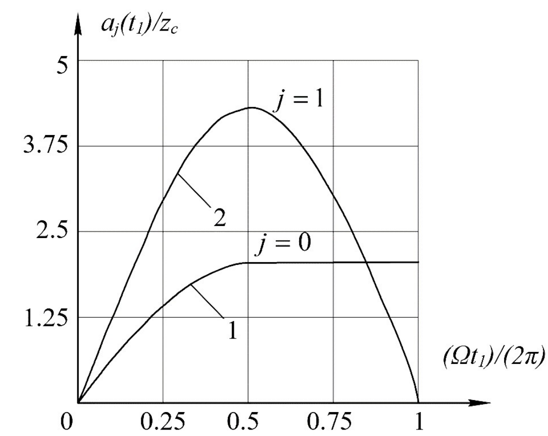

Fig. 3. Plots $a_j(t_1)$ for: 1 – j=0; 2 – j=1

For the second case, which corresponds to the dynamic response of a plate with a closed contour to the action of a sinusoidal pulse, it is possible to calculate the extrema of the function $A(t)$ for the initial data from the first case. In this case, the values of the parameter λ will be equal to λ=Ω, λ=2Ω, λ=3Ω. The results of calculations according to formulas (20), (22) are given in Table 2. The extreme deflections of the plate and the time of their achievement when t_1=π/λ are given.

Table 2

Deflection amplitudes and time to achieve them at λ=jΩ, j=1, 2, 3

j	$a_0(t_1)/z_c$	$a_1(t_1)/z_c$	t_e, s	t_m, s
1	1.5708	3.3991	0.0295	0.0762
2	0.9428	2.0402	0.0221	0.0689
3	0.6495	1.4055	0.0197	0.0664

For the third case, which corresponds to the dynamic deflections of the plate with a hinged contour caused by a rectangular pulse, the initial parameters are chosen the same as in the first case. These initial parameters according to Table 1 correspond to the constants: β=2.2045701; C_1=1.0426163; C_2=–0.0426163. The plot of the function $f(r)$ is shown by a dotted line in Fig. 2. From the performed calculation, the following was obtained: z_c=3.884·10^{-3} m; Ω=101.4827 s^{-1}; ω=23.3910 s^{-1}; t_G=0.05733 s. The calculated amplitudes of deflections of the center of the plate according to formulas (12), (15) and the time of their achievement for three pulse durations are given in Table 3.

In order to check the plausibility of the constructed analytical solutions, for the numerical data of the third case, equations (8) and (14) were integrated. The resulting plots of plate center movements at t_1=0.03; 0.05733; 0.06 s are shown in Fig. 4.

Table 3

Deflection amplitudes and time to achieve them at $\lambda = j\Omega$, j=1, 2, 3

t_1, s	$a_0(t_1)/z_c$	$a_1(t_1)/z_c$	t_e, s	t_m, s
0.0300	1.9976	8.6668	0.0305	0.1131
0.05733	2.0000	2.0000	0.0310	0.1268
0.0600	2.0000	0.8413	0.0310	0.1281

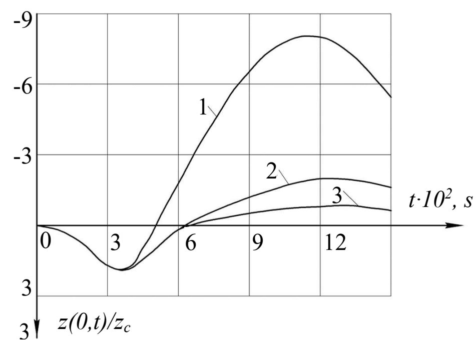

Fig. 4. Plots of movement of the center of the plate: 1 – t_1=0.03 s; 2 – t_1=0.05733 s; 3 – t_1=0.06 s

Extreme values in Table 3 are found on these plots, that is, there is convergence of the calculation results by two techniques.

It should be noted that the discrepancy between the obtained theoretical results and the results of experimental studies does not exceed 3...5 %. This proves the possibility of using a physical-mathematical model of the dynamics of a round plate to calculate the amplitude of its oscillations under the action of rectangular or sinusoidal pulses under the conditions of experimental studies.

6. Discussion of results of modeling the dynamics of a circular plate on a one-sided elastic base caused by a force impulse

During the verification of the adequacy of the developed physical-mathematical model using the derived formulas, three cases were considered. At the first stage, the case of non-stationary oscillations of a plate with a clamped contour when it is loaded with a rectangular pulse is considered. At the second stage, there is a dynamic response of the plate with a closed contour to the action of a sinusoidal pulse. At the third stage, the dynamic deflection of a plate with a hinged contour caused by a rectangular pulse is considered.

As for the first case, the calculations prove that at $t_1 \in (0;t_G)$ the inequality $a_1(t_1) > a_0(t_1)$ holds, that is, the dynamic effect of the asymmetry of the force characteristic of the system is manifested (Fig. 3).

In the second case, the numerical analysis shows that when the parameter λ increases, the ratios $a_0(t_1)/z_c$ and $a_1(t_1)/z_c$ decrease, but $a_1(t_1)/a_0(t_1)$ remains constant, and $a_1(t_1) > a_0(t_1)$ (Table 2). That is, the dynamic effect of the asymmetry of the elastic characteristics of the system is also manifested.

In the third case, for the first pulse duration $a_1(t_1) > a_0(t_1)$, for the second – $a_1(t_1) = a_0(t_1)$, and for the third – $a_1(t_1) < a_0(t_1)$. Only in the first version of the load is the dynamic effect of the asymmetry of the elastic characteristics of the system observed, while the ratio of the amplitudes of the deflections is 4.3386 (Table 3). Its calculation for deflections of the plate clamped along the contour at t_1=0.03 s gives: $a_1(t_1) = a_0(t_1)$=2.1633. Therefore, the mentioned dynamic effect is manifested to a greater extent when the contour of the plate is hinged against, than when it is clamped.

Listed in Tables 2, 3, the numerical results confirm the suitability of derived formulas for calculations. They show a fundamental difference between the dynamic behavior of a circular plate having one-sided contact with the base and the behavior of a plate with two-sided contact with an elastic base when $a_1(t_1) \leq a_0(t_1)$.

Thus, thanks to the use of the fitting method and the selection of a special distribution of the load along the radius of the plate, accurate analytical solutions of dynamic boundary value problems for two options for fixing the contours of the plate have been constructed. This distinguishes our results from the results obtained in works [17, 18], where only a static problem is considered, which cannot be applied to the dynamics of oscillations of unbalanced vibrators. The solutions obtained in the study are expressed in terms of cylindrical functions of real and imaginary arguments. The nonlinear boundary value problem is reduced to a sequence of two linear problems, as a result of which compact formulas are derived for calculating the amplitudes of deflections and bending moments and the time of their achievement. It was established that for certain durations of the external pulse, the dynamic effect of the asymmetry of the elastic characteristic of the system can be manifested. At the same time, the amplitude of the deflection of the plate, after its separation from the base, is greater than the amplitude of the deflection of the supported plate in the direction of its impulse. This is the fundamental difference between the dynamic behavior of a plate with a one-sided bond and the behavior of a plate on a two-sided basis [19, 20]. In the case of two-sided contact of the plate with the elastic base, which is traditionally considered in the scientific literature, the mentioned effect cannot be manifested. The reliability of the obtained analytical solutions is confirmed by the consistency of the numerical results they lead to with the results of the numerical solution of the differential equation.

It should be noted that the physical-mathematical model of the dynamics of a round plate can be used to calculate the strength of common structural elements of a round shape, both under static and dynamic loads. At the same time, the adequacy of the model was proven for the following values of the initial parameters: modulus of elasticity, 2.1·10^{11} Pa; the Poisson ratio of the plate material, 0.25; plate thickness, 7...10 mm; the maximum pressure on the plate, 4·10^{3} Pa; the bending stiffness of the plate, i6402.6667 N·m. At the same time, it is possible to simulate cases: oscillation of a plate with a clamped contour when it is loaded with a rectangular or sinusoidal pulse; oscillations of a plate with a hinged contour under the action of a rectangular pulse. The possibility of theoretical modeling of the amplitude of oscillations of a round plate for these cases is proven by the low discrepancy between the theoretical results and the results obtained experimentally, which is no more than 5 %.

The shortcoming of the study is the lack of examples of connection with the physical-mathematical model of the process of separation of grain mixtures on a pneumatic sorting table. This is explained, firstly, by the possibility of applying the obtained model in processes other than the separation process. Secondly, the results reported in the study are intermediate and, accordingly, represent a part of the general physical and mathematical model of the process of separation of grain mixtures. Obviously, from this point of view, establishing a correlation between the vibration parameters of unbalanced vibrators and the quality of the obtained products will not be correct.

The future study is to extend the developed physical-mathematical model to a more general case, when the plate is not solid, but ring-shaped, that is, it has a central concentric hole.

7. Conclusions

1. Unsteady oscillations of a uniform round plate supported by a one-sided elastic base under impulse loading were theoretically investigated. For two variants of the boundary conditions and two cases of change of momentum in time, closed analytical solutions of the equations of motion of the plate when it is in contact with the base and when it is separated from the base were constructed. Solutions are expressed in cylindrical functions of real and imaginary arguments. It was established that as a result of separation of the plate from the base, with pulse durations from 0 to 0.05 s, there is a dynamic effect of asymmetry of the elastic characteristics of the system, which is absent when the plate is in continuous contact with the two-sided base.

2. An analytical solution in cylindrical functions of the boundary value problem of the dynamics of a plate with a hinged contour has been obtained. It is noted that the obtained analytical solutions for non-stationary oscillations of a uniform circular plate supported by a one-sided elastic base remain valid for the case of hinged abutment of the plate edges. For their application, it is necessary to use the corresponding values of the constants β, C_1, C_2. It was noted that the effect of asymmetry is manifested to a greater extent when the contour of the plate is hinged against, than when it is rigidly clamped under equivalent other conditions. It was established that this applies not only to deflections of the plate but also to bending moments that are proportional to the deflections for the accepted distribution of the external load along the radius of the plate.

3. The adequacy of the developed physical-mathematical model of the dynamics of a circular plate has been proven by obtaining numerical solutions that confirm the suitability of the derived formulas for calculations. The fundamental difference between the dynamic behavior of a round plate having one-sided contact with the base and the behavior of a plate with two-sided contact with an elastic base is shown. The possibility of theoretical modeling of the amplitude of oscillations of a round plate with a clamped contour when it is loaded with a rectangular or sinusoidal pulse, as well as oscillations of a plate with a hinged contour under the action of a rectangular pulse, has been proven. In this case, the values of the initial parameters are equal to: modulus of elasticity, $2.1 \cdot 10^{11}$ Pa; the Poisson ratio of the plate material, 0.25; plate thickness, 7...10 mm; the maximum pressure on the plate, $4 \cdot 10^3$ Pa; the bending stiffness of the plate, 6402.6667 N·m.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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