The object of this study is the technology of ensuring magnetic cleanliness when applied to small spacecraft (SC) with overall dimensions of ~1 m in terms of minimizing the effect of magnetic interference on the on-board magnetometer. As part of solving the general problem of increasing the reliability of spacecraft, the task of improving the technology of magnetic cleanliness is considered. It is shown that the use of the multi-dipole model for the calculation of ~50 nT magnetic interference is limited in the design of small space vehicles that have an increased installation density. The expediency of using a model of its spherical harmonics instead of a multi-dipole model to represent the magnetic field of satellite components was theoretically justified. For the practical application of the model, the budgeting of the projections of the magnetic induction of hindrance to the onboard magnetometer according to the experimentally determined coefficients of the dipole, quadrupole and octupole spherical harmonics is proposed. An algorithm for calculating the coordinates of the location and projections of the magnetic moment of the dipole source inside the satellite based on the results of measuring the coefficients of spherical harmonics of its components is proposed. The possibility of representing the coefficients of the spherical harmonics of the magnetic field of the entire satellite in the form of the sum of the listed corresponding coefficients obtained during field measurements of the components is theoretically justified. It is proposed to use the difference between the calculated and measured values of the corresponding coefficients as a criterion for the quality of the work to ensure magnetic cleanliness. If the proposed procedure is used in the space industry, the quality of ensuring the magnetic cleanliness of small space vehicles can be increased, thereby improving the reliability of their operation in orbit

Keywords: magnetic cleanliness technology, spacecraft, spherical harmonics of the magnetic field

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IMPROVING THE TECHNOLOGY FOR ENSURING THE MAGNETIC CLEANLINESS OF SMALL SPACECRAFT

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1. Introduction

In the space industry, special attention is paid to the reliability of designed spacecraft (SC). This is due to the practical impossibility or economic impracticability of repairing the satellite in orbit. A number of special technologies were devised in order to ensure the reliable functioning of the spacecraft in orbit. One of such technologies, which should create conditions for the regular functioning of onboard equipment, is the technology of ensuring the magnetic cleanliness of spacecraft. The results of the application of this technology are aimed at protection against the negative influence of the magnetic field, which is induced by the components and assemblies of the spacecraft.

In particular, it is very important to reduce the magnitude of the natural magnetic moment of the equipment units and spacecraft as a whole, as this leads to a decrease in the mechanical moment that occurs during magnetic interaction with the Earth's planetary magnetic field. This is explained by the fact that the occurrence of a significant mechanical moment leads not only to the distortion of the standard orientation of the satellite in orbit. In the case of a significant mechanical moment, its emergence will cause a chaotic rotation of the satellite in orbit, which in turn will lead to incorrect orientation of the spacecraft antennas and loss of communication with the satellite. Thus, even a successfully launched spacecraft may be lost from the point of view of the possibility of its proper functioning and communication with the control center on Earth.

To reduce the value of the total magnetic moment of the spacecraft, which is the vector sum of the magnetic moments of individual units and nodes of the satellite, additional restrictions are imposed during their construction. Usually, the magnitude of the module of the magnetic moment of the components of the spacecraft being created is subject to limitation. According to the order of application of the technology for ensuring magnetic cleanliness, restrictions on the maximum magnitude of the magnetic moment module are introduced, starting with the spacecraft as a whole. Based on the technical capabilities of the active satellite orientation control system, the module of the magnetic moment of the entire spacecraft is limited. Next, the permissible values of the magnetic moments of the systems and functional units of the satellite are substantiated, and only then the requirements for their component subsystems are drawn up. The last to devise are requirements for electronic elements and materials of individual components of the spacecraft.

The technology of ensuring magnetic cleanliness is based on the experimental control of the magnetic moment of all components and elements of the spacecraft on specialized magnetic measuring benches. The order of controlling the magnitude of the magnetic moment is the opposite of the order of making the corresponding demands. First, the magnetic moment of individual elements and nodes is subject to control, and lastly, the magnetic moment of the spacecraft as a whole is to be measured.

In addition, the technology for ensuring magnetic cleanliness is aimed at leveling the negative interaction of separate systems and units of the spacecraft with each other indirectly through the magnetic field. In particular, to ensure the normal operation of the on-board magnetometer, the module of the magnetic induction of interference should not be more than 50 nT. Providing such a small magnetic interference is an urgent and technically difficult task of creating magnetically clean spacecraft. This task has become particularly acute for modern small spacecraft. Reducing the distance between units and systems, which are sources of magnetism, leads to an increase in the magnetic interference created by them. A significant value of the amount of magnetic interference to the on-board magnetometer, in turn, leads to incorrect functioning of the orientation control system.

Thus, the urgent task is the constant improvement of the technology of ensuring magnetic cleanliness, which must correspond to the modern trends in the design and construction of spacecraft.

2. Literature review and problem statement

Already the first launches of artificial satellites of the Earth showed the need to devise technology to reduce the negative magnetic interaction between them and the geomagnetic field. According to the technical reports of the National Aeronautics and Space Administration, it is possible to trace how the technology of ensuring magnetic cleanliness emerged and was filled with content. It is interesting that initially in this technology all restrictions were made on the magnitude of the induced magnetic field at several typical distances [1]. The magnetic moment was used only to estimate the external magnetic field at the extremities of the outer rods of the spacecraft, which made it difficult to analyze the spatial distribution of the field.

Modern ideas about the technology of ensuring magnetic cleanliness are recorded in the ESA standards [2, 3], which contain a general algorithm and recommended methods of its implementation. It should be noted that these standards do not contain universal (for each spacecraft mission) values of limited magnetic characteristics. For each specific mission of the spacecraft, studies should be conducted to determine them. The calculation of the values of the limited magnetic characteristics is carried out in the direction from the entire satellite to its components, and from them to the restrictions on the elements and materials used in the construction of the satellite.

In the ESA standards, the basic concept for modeling the magnetic field both inside and outside the spacecraft is the multi-dipole model [4, 5]. In a series of his works, the author improves and expands the method of multi-dipole magnetic modeling, in particular, adds the possibility of using measurements of the field gradient. A number of improvements relate to the control and prediction of the magnetic characteristics of the spacecraft, covering all phases of the project, including the final magnetic tests of the satellite. Examples of several successful missions are given in [4, 5], which demonstrate the power of the approach to solving the problem of ensuring magnetic cleanliness using the multi-dipole model. However, the author himself recognizes the weaknesses of the developed approach, such as the reverse search for the distribution of dipole sources, which leads to the problem of ambiguity of possible solutions. As a possible way to solve the problem, software for an efficient combination of deterministic and stochastic solver strategies was proposed.

A number of works on the application of magnetic cleanliness methods refer to the most stringent requirements for the interference magnetic field, which are imposed by the features of the spacecraft mission [6]. These are first of all spacecraft that have the mission of studying other planets of the solar system [7, 8] or the mission of determining the geomagnetic field [9]. For such missions, the limiting level of magnetic interference created by the spacecraft is so low that to ensure magnetic cleanliness in the installation area of the scientific magnetometer, the latter is moved away from the satellite by means of a remote rod. It should be noted that even with a length of 10 m of the extension rod, the need for work to reduce the magnetic field created by the spacecraft does not disappear. But the use of an outrigger in the case of a small spacecraft is usually not a possible technical solution, so such missions are performed by multi-meter-sized satellites with a weight of about 1000 kg. In addition, the reduced overall dimensions of small spacecraft weighing 50–200 kg lead to an increase in the density of installation of satellite components and, as a result, an increase in the generated magnetic interference.

It should be noted that the problem of ensuring magnetic cleanliness is even more acute, especially in the case of very small spacecraft (nanosatellites). Paper [10] presents a model of the time-dependent magnetic moment of electric currents flowing inside a nanosatellite, which negatively affects the dynamics of its orientation. And a compensation strategy is proposed using an autonomous control algorithm of several compensating dipole magnetic sources. The residual magnetic moment in [11] is proposed to be minimized by optimizing the layout of the nanosatellite using the accelerated particle swarm algorithm developed to find the optimal layout solution. In addition, the relatively small values of the magnetic moments of such spacecraft and the reduction in the dimensions of the measurement system negatively affect the reliability of the obtained results. Article [12] presents a method for estimating the magnetic dipole moment of a nanosatellite based on near-field magnetic induction measurements using methods developed for machine learning. With the help of complex neural network machine learning algorithms, it is possible to improve the probability of magnetic moment measurement results, but not to solve the problem completely. However, the methods proposed to ensure the magnetic cleanliness of ~10 kg nanosatellites are generally limited for application to smaller spacecraft of larger size and weight. For example, when building small spacecraft, it is not envisaged to use the free rotation of the satellite along all three axes during tests. Therefore, it is interesting for small spacecraft to use such test methods that would expand the set of experimentally determined magnetic characteristics under limited dynamic and static loads. An example of such a method is the technique for determining the magnetic moment, which uses the results of measuring the magnetic flux in cylindrical windings when the spacecraft is linearly moved through them [13]. However, there are currently no known practical examples of the application of such an approach to spacecraft, so it is not possible to confidently speak about the feasibility and advantages of the method. On the other hand, this approach was successfully applied in [14] to determine the amplitude coefficients of the spherical harmonics of the magnetic field components of the Sich-2 spacecraft. Since the method does not involve a change in the orientation of the investigated component when measuring the magnetic flux, both the permanent and the induced components of the magnetic field are determined experimentally. This feature is especially useful for devices designed with shielding. In this case, the magnitude of the induced magnetic moment exceeds the constant one and must therefore be determined experimentally. However, it should be noted that perhaps this is the only example of the use of such a method in the space industry.

The above review gives reasons to claim that improving the technology of ensuring magnetic cleanliness is a permanent task that must be solved during the construction of a spacecraft. Therefore, it is advisable to devise procedures that expand the capabilities of the technology for ensuring magnetic cleanliness for small spacecraft, in particular, by using the model of spherical harmonics of the magnetic field of its components.

3. The aim and objectives of the study

The aim of this work is to develop a theoretical basis for the experimental determination of magnetic interference inside the spacecraft based on three spatial harmonics of the magnetic field of its components (separate nodes and electronic units). This is aimed at reducing the negative impact of the magnetic field on the functioning of the satellite and increasing the reliability of the operation of spacecraft in orbit due to the improvement of the technology for ensuring magnetic cleanliness.

To accomplish the aim, the following tasks have been set:

– to analyze the existing models of magnetic interference inside the satellite, taking into account the increase in the compactness of the installation of components and the construction of separate nodes and units with a compensated magnetic moment;

– to adapt the model of magnetic interference, which is created by individual nodes and electronic units inside the spacecraft, applying for this the concept of spherical harmonics of magnetic field sources;

– to justify the possibility of calculating the magnitude and coordinates of the location of the unknown source of magnetic moment inside the spacecraft based on the results of experimental determination of the coefficients of spherical harmonics of all components and the satellite itself as a whole.

4. The study materials and methods

The technology of ensuring magnetic cleanliness in that part of it, which is aimed at minimizing the negative magnetic interaction between component parts and components of small spacecraft, is subject to research. The main method of research is the simulation of a stationary magnetic field, as sources of which all components of the satellite are considered.

The models based on the dipole magnetic moment of the spacecraft and its components are the most common for the study of magnetic interference created by the components of the satellite on-board magnetometer. In addition, the magnetic moment is the main magnetic characteristic used in the magnetic cleanliness technology to limit the magnetic field created by the spacecraft components. One of the harmful factors of the disturbance that changes the standard orientation of the spacecraft is the torque that arises as a result of the interaction of the satellite's own magnetic moment with the Earth's magnetic field. Therefore, during the construction of spacecraft intended for work in near-Earth orbits, a set of measures is taken to reduce the magnetic moment of the satellite, as a rule, to the level of 0.1 A·m^2 for each cubic meter of the device's volume. Since the magnetic moment of the entire satellite is the vector sum of the magnetic moments of the components, its reduction can be achieved by reducing the contributions from the spacecraft components. Therefore, to limit the magnitude of the satellite's magnetic moment in the space industry, requirements for the level of magnetic moments of all component parts of the spacecraft are developed separately. According to the recommendations of the European Space Agency, to ensure the magnetic cleanliness of the spacecraft, a budget is used based on the projections of the magnetic moment vector of each of the instruments and components of the satellite [2, 3]. Then the values of the projections of the expected (predicted) magnetic moment of the entire spacecraft are obtained by arithmetically adding up the corresponding contributions from the components. Thus, the budget for the magnetic moment of the spacecraft is the basis for drawing up the initial restrictions and further control of the magnetic moments of the satellite components. Therefore, constant monitoring of the magnitude of the magnetic moment is an integral part of the technology of creating objects of space technology. Control of the magnetic moment is carried out on magnetometric benches, which are equipped with square Helmholtz coils for compensation of the geomagnetic field (Fig. 1).

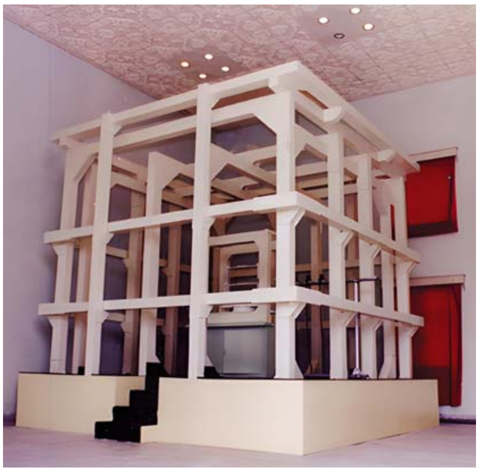

Fig. 1. A system for measuring the magnetic moment of spacecraft components at the Beijing Institute of Space Research (BISEE)

In addition to the minimization of the magnetic moment for small spacecraft with a system for controlling the orientation and position in the orbit according to the magnetic field, it is necessary to minimize the magnetic interference created by the spacecraft to the on-board magnetometer (Fig. 2). A similar task arises to ensure regular functioning of scientific magnetometers of geophysical satellites [9].

To calculate the magnetic field using the dipole model of a generalized complex source, its geometric center is associated with a point dipole magnetic moment $\vec{M}$. Then the simulation can be considered correct if the distance R from the center of the source to the point of calculation of the magnetic field is more than three times its maximum overall size $3 \cdot l$.

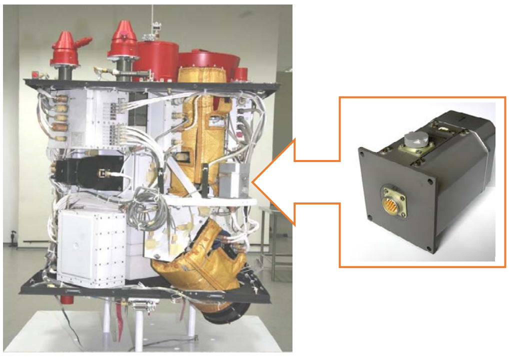

Fig. 2. The location of the onboard magnetometer inside the spacecraft Sich-2

Therefore, a multi-dipole model is usually used to model the distribution of magnetic induction inside the spacecraft $\vec{B}$ (for example, at point P of the magnetometer installation) (Fig. 3).

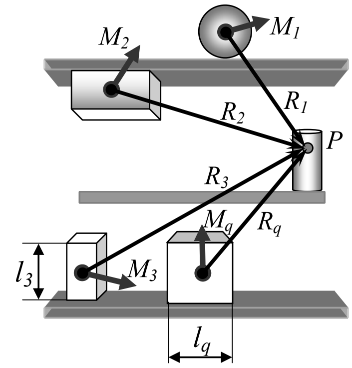

Fig. 3. Multipole mathematical model of the spacecraft's magnetic field

It is based on the definition of the total magnetic field, which is calculated from the values of the magnetic moments $\vec{M}_q$ of all components, provided that the inequality $R_q>3\cdot l_q$ is fulfilled for the overall size l_q of each of the components of the spacecraft:

$$\vec{B}(P)=-\frac{\mu_0}{4\pi}\sum_q \operatorname{grad}\frac{\left(\vec{M}_q,\vec{R}_q\right)}{\left(R_q\right)^2}. \qquad (1)$$

That is, the model uses the assumption that the spatial distribution of the magnetic field created by each component spacecraft is exclusively dipole in nature. However, for nodes and units, in the construction of which methods of reducing the value of their magnetic moment were used, such an assumption is incorrect and, accordingly, expression (1) cannot be used in the calculation of magnetic interference. Thus, existing restrictions on the application of the multi-dipole model necessitate the use of other approaches to modeling the magnetic field inside small spacecraft.

5. Research results of the application of the model of spherical harmonics of the magnetic field in the technology of ensuring magnetic cleanliness

5. 1. Modeling of magnetic field interference by spherical harmonics of spacecraft components

The current trend in the construction of spacecraft for remote sensing of the Earth is the use of small spacecraft. Minimizing the dimensions of such devices leads to the need to use on-board magnetometers and electromagnets to solve satellite navigation and orientation tasks. The quality of control in the magnetic version of the feedback circuit of the spacecraft orientation control system depends on the error of the information provided by the on-board magnetometer. This error will, first of all, be determined by the amount of magnetic interference in the volume of sensors of the on-board magnetometer, which is created by the satellite itself. Modeling the interference magnetic field at the place of installation of the on-board magnetometer makes it possible to take into account and minimize errors when solving problems of control and navigation of small spacecraft.

In addition, it is additionally necessary to ensure the minimum values of the magnetic interference created by the satellite itself in the locations of the payload devices (primarily scientific magnetometers) at significantly reduced distances to the sources of the magnetic field. The last requirement significantly limits the permissible value of magnetic interference and increases the complexity of ensuring the magnetic cleanliness of the spacecraft.

In this case, the methods of technology for ensuring magnetic cleanliness, which are based on the multi-dipole model, may not always be correct for describing the interference magnetic field inside the satellite. With reduced distances between the components of spacecraft, it is actually necessary to conduct simulation of the magnetic field directly near their surface. In this case, the condition of the inequality $R_q>3\cdot l_q$ is not fulfilled, and the use of the many-dipole representation (1) is not correct.

An alternative model of the magnetic field interference created by the spacecraft to the on-board magnetometer is a model based on the spherical harmonics of the magnetic field, the sources of which are the components of the on-board equipment. With its help, the calculation of magnetic interference at the installation point of the on-board magnetometer can be carried out on the basis of predetermined amplitude coefficients g_n^m and h_n^m of spherical harmonics of the field of each of the components and nodes. At the same time, the values of the amplitude coefficients should be determined experimentally in reference to the local coordinate system, the center of which coincides with the geometric center of the component of a spacecraft (Fig. 4).

Fig. 4. System for measuring the coefficients of spherical harmonics of the magnetic field of the component, which is rotated around the vertical axis (BISEE)

Then the magnetic interference induction is the vector sum of contributions from spherical harmonics [15]:

$$\vec{B} = -\frac{\mu_0}{4\pi} \times$$
$$\times \mathrm{grad} \sum_{n=1}^{\infty} \frac{1}{r^{n+1}} \sum_{m=0}^{n} \left(g_n^m \cos m\phi + h_n^m \sin m\phi \right) P_n^m \left(\cos\theta \right), \qquad (2)$$

where r, θ, φ are the spherical coordinates of the location of the magnetometer in the local system related to the geometric center of the source (spacecraft component).

The rationality of using the spherical harmonics model for such purposes is explained by the complex spatial distribution of the magnetic field in the near zone of the spacecraft components and the extremely limited distance from them to the magnetometer. It is the combination of these two factors that makes it incorrect to use the multi-dipole model of the satellite's magnetic field at the location of the on-board magnetometer.

It should be noted that the use of a more complex model of the magnetic field based on spherical harmonics is a necessity for those components that were designed to minimize their magnetic moment using compensating sources and (or) shielding. This is explained by the essentially non-dipole nature of the spatial distribution of the magnetic field of such nodes and units, with an artificially minimized contribution of the dipole magnetic moment to the magnetic induction created by the device.

From the point of view of the steps of application of magnetic cleanliness technology, practical determination of the amplitude coefficients of spherical harmonics of the magnetic field of components and devices is easily incorporated into its algorithm. For example, the determination of the amplitude coefficients of the spherical harmonics of the magnetic field can be combined with tests on the experimental determination of the projections of the magnetic moment of each of the spacecraft components.

Usually, in practice, the number of spherical harmonics of the component magnetic field involved in the model is limited (for example, to octupole harmonics). Then, in the image of the magnetic induction of interference in the form of a series of spherical harmonics, in addition to the coefficients of the dipole harmonic g_1^m, h_1^m (n=1), the coefficients of the quadrupole g_2^m, h_2^m and octupole g_3^m, h_3^m harmonics are subject to experimental determination.

An example of the experimental determination of the coefficients of the spherical harmonics of the magnetic field of the component satellite is the results of magnetic tests of the transmitter of the neutral plasma component (NPC). These tests were carried out as part of work to ensure the magnetic cleanliness of the Sich-2 spacecraft. In [14], the method and system of measuring the magnetic flux applied during the tests were described when the component is moved through the cylindrical windings (Fig. 5).

For the convenience of further analysis of the magnetic induction created by dipole, quadrupole, and octupole harmonics according to (2), it is appropriate to represent the numerical values of the full set of coefficients in the form of Table 1.

As follows from Table 1, not all amplitude coefficients have a significant contribution to the magnetic field created by the spacecraft component. This is explained by the presence of symmetry of the magnetically active part of the device with respect to some of the proper axes of the spatial harmonics. Therefore, when calculating the magnetic induction of interference, the contributions from the harmonic coefficients determined at the level of experimental error should be excluded, and the value of the corresponding coefficient should be assigned zero.

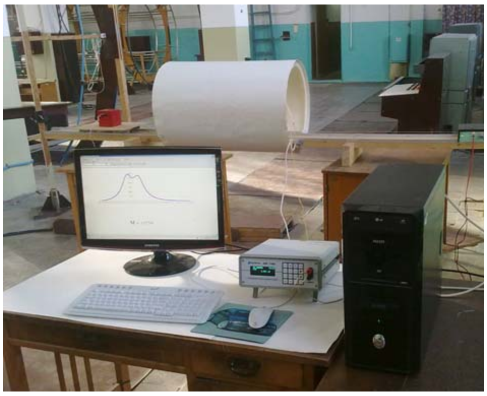

Fig. 5. System for experimental determination of coefficients of spherical harmonics at the National Technical Center of the National Academy of Sciences of Ukraine

Table 1

Spherical harmonics of the NPC sensor magnetic field

Harmonic type	Coefficient designation	Coefficient value
Dipole	g_1^0	+0.042 (A·m^2)
	g_1^1	+0.087 (A·m^2)
	h_1^1	–0.084 (A·m^2)
Quadrupole	g_2^0	+0.0177 (A·m^3)
	g_2^1	<10^{-4} (A·m^3)
	g_2^2	<10^{-4} (A·m^3)
	h_2^1	<10^{-4} (A·m^3)
	h_2^2	<10^{-4} (A·m^3)
Octupole	g_3^0	+0.00314 (A·m^4)
	g_3^1	<10^{-5} (A·m^4)
	g_3^2	<10^{-5} (A·m^4)
	g_3^3	<10^{-5} (A·m^4)
	h_3^1	<10^{-5} (A·m^4)
	h_3^2	<10^{-5} (A·m^4)
	h_3^3	<10^{-5} (A·m^4)

5. 2. A model of magnetic interference to an on-board magnetometer based on spherical harmonics of the magnetic field

The algorithm for calculating the magnetic interference created by one of the components of the on-board magnetometer is considered. For this purpose, I used the general idea of magnetic induction of spherical harmonics (2), in which the series is limited to power n=3:

$$B_r = \frac{\mu_0}{4\pi} \times$$
$$\times \sum_{n=1}^{3} \frac{n+1}{r^{n+2}} \sum_{m=0}^{n} \left(g_n^m \cos m\phi + h_n^m \sin m\phi \right) P_n^m \left(\cos\theta \right), \qquad (3)$$

$$B_\theta = -\frac{\mu_0}{4\pi}\times$$
$$\times\sum_{n=1}^{3}\frac{1}{r^{n+2}}\sum_{m=0}^{n}\left(g_n^m\cos m\phi + h_n^m\sin m\phi\right)\frac{\partial}{\partial\theta}\left(P_n^m(\cos\theta)\right), \tag{4}$$

$$B_\phi = \frac{\mu_0}{4\pi}\times$$
$$\times\sum_{n=1}^{3}\frac{1}{r^{n+2}}\sum_{m=1}^{n}m\left(g_n^m\sin m\phi - h_n^m\cos m\phi\right)P_n^m(\cos\theta). \tag{5}$$

In (3) to (5), the spherical coordinates r, θ, φ of the location of the sensors of the on-board magnetometer in the local system of the spacecraft component are used, which can be defined as:

$$r = \sqrt{(x_0-x)^2+(y_0-y)^2+(z_0-z)^2}, \tag{6}$$

$$\cos\theta = \frac{z_0-z}{r},$$

$$\sin\theta = \frac{\sqrt{(x_0-x)^2+(y_0-y)^2}}{r}, \tag{7}$$

$$\phi = \operatorname{arctg}\left(\frac{y_0-y}{x_0-x}\right), \tag{8}$$

where x, y, z and x_0, y_0, z_0 are the coordinates of the geometric center of the electronic unit and the coordinates of the location of the magnetometer transducers in the so-called design coordinate system, which is related to the center of the satellite. Hereafter, it is additionally assumed that the directions of the corresponding axes of the design and local coordinate systems of all components coincide.

Replacing in (3) to (5) the attached Legendre functions P_n^m with their expressions, the images of the three spherical projections of the magnetic interference take the form:

$$B_r = \frac{\mu_0}{4\pi}\times$$
$$\times\left(\begin{array}{l}
g_1\frac{2}{r^3}\cos\theta + \\
+\left[g_1^1\cos\phi + h_1^1\sin\phi\right]\frac{2}{r^3}\sin\theta + \\
+g_2\frac{3/2}{r^4}\left(3\cos^2\theta - 1\right) + \\
+\left[g_2^1\cos\phi + h_2^1\sin\phi\right]\frac{9}{r^4}\cos\theta\sin\theta + \\
+\left[g_2^2\cos(2\phi) + h_2^2\sin(2\phi)\right]\frac{9}{r^4}\sin^2\theta + \\
+g_3\frac{2}{r^5}\left(5\cos^3\theta - 3\cos\theta\right) + \\
+\left[g_3^1\cos\phi + h_3^1\sin\phi\right]\frac{2}{r^5}\sin\theta\left(15\cos^2\theta - 3\right) + \\
+\left[g_3^2\cos(2\phi) + h_3^2\sin(2\phi)\right]\frac{60}{r^5}\cos\theta\sin^2\theta + \\
+\left[g_3^3\cos(3\phi) + h_3^3\sin(3\phi)\right]\frac{60}{r^5}\sin^3\theta
\end{array}\right), \tag{9}$$

$$B_\theta = \frac{\mu_0}{4\pi}\times$$
$$\times\left(\begin{array}{l}
g_1\frac{1}{r^3}\sin\theta - \left[g_1^1\cos\phi + h_1^1\sin\phi\right]\frac{1}{r^3}\cos\theta + \\
+g_2\frac{3}{r^4}\cos\theta\sin\theta - \\
-\left[g_2^1\cos\phi + h_2^1\sin\phi\right]\frac{3}{r^4}\cos(2\theta) + \\
+\left[g_2^2\cos(2\phi) + h_2^2\sin(2\phi)\right]\frac{-6}{r^4}\cos\theta\sin\theta + \\
+g_3\frac{1/2}{r^5}\left(15\cos^2\theta - 3\right)\sin\theta + \\
+\left[g_3^1\cos\phi + h_3^1\sin\phi\right]\frac{1/2}{r^5}\times \\
\times\left(-15\cos^2\theta + 30\sin^2\theta + 3\right)\cos\theta + \\
+\left[g_3^2\cos(2\phi) + h_3^2\sin(2\phi)\right]\frac{15}{r^5}\times \\
\times\left(\sin^2\theta - 2\cos^2\theta\right)\sin\theta + \\
+\left[g_3^3\cos(3\phi) + h_3^3\sin(3\phi)\right]\frac{-45}{r^5}\cos\theta\sin^2\theta
\end{array}\right), \tag{10}$$

$$B_\phi = \frac{\mu_0}{4\pi}\times$$
$$\times\left(\begin{array}{l}
\left[g_1^1\sin\phi - h_1^1\cos\phi\right]\frac{1}{r^3}\sin\theta + \\
+\left[g_2^1\sin\phi - h_2^1\cos\phi\right]\frac{3}{r^4}\cos\theta + \\
+\left[g_2^2\sin(2\phi) - h_2^2\cos(2\phi)\right]\frac{6}{r^4}\sin\theta + \\
+\left[g_3^1\sin\phi - h_3^1\cos\phi\right]\frac{1/2}{r^5}\left(15\cos^2\theta - 3\right) + \\
+\left[g_3^2\sin(2\phi) - h_3^2\cos(2\phi)\right]\frac{30}{r^5}\cos\theta\sin\theta + \\
+\left[g_3^3\sin(3\phi) - h_3^3\cos(3\phi)\right]\frac{45}{r^5}\sin^2\theta
\end{array}\right). \tag{11}$$

Finally, three Cartesian projections of the magnetic induction of interference in the design coordinate system of the satellite are calculated using the formulas:

$$B_x = B_r\sin\theta_0\cos\phi_0 + B_\theta\cos\theta_0\cos\phi_0 - B_\phi\sin\phi_0, \tag{12}$$

$$B_y = B_r\sin\theta_0\sin\phi_0 + B_\theta\cos\theta_0\sin\phi_0 + B_\phi\cos\phi_0, \tag{13}$$

$$B_z = B_r\cos\theta_0 - B_\theta\sin\theta_0, \tag{14}$$

where θ_0, φ_0 are the spherical coordinates of the location of the magnetometer in the system connected to the satellite defined as:

$$\cos\theta_0 = \frac{z_0}{\sqrt{(x_0)^2+(y_0)^2+(z_0)^2}}, \tag{15}$$

$$\sin\theta_0 = \frac{\sqrt{(x_0)^2+(y_0)^2}}{\sqrt{(x_0)^2+(y_0)^2+(z_0)^2}}, \tag{16}$$

$$\phi_0 = \operatorname{arctg}\left(\frac{y_0}{x_0}\right). \tag{17}$$

The obtained images (3) to (17) make it possible to calculate the contributions of each of the components of the spacecraft to the three projections of the magnetic induction of interference with the on-board magnetometer. To obtain the value of the total interference magnetic field, it is advisable to compile a magnetic budget in the form of a table of three projections of magnetic induction created by all components (Table 2). The last line of the budget (Table 2) should contain the value of the projections of the magnetic induction of the total interference of the spacecraft components to the on-board magnetometer.

Table 2

An example of a magnetic induction interference budget with an on-board magnetometer

Component name	B_x, nT	B_y, nT	B_z, nT
Unit No. 1	tbd	tbd	tbd
Unit No. 2	tbd	tbd	tbd
Unit No. *i*	tbd	tbd	tbd
Unit No. *q*	tbd	tbd	tbd
$\sum$	tbd	tbd	tbd

In the example of the magnetic interference budget (Table 2), tbd means "to be determined".

In a complete analogy, by making the budget for three projections of magnetic induction created by each of the components of the spacecraft according to the considered algorithm, it is also possible to determine the magnetic interference at the points of the payload location.

5. 3. Application of the spherical harmonics model to calculate the position of the magnetic field source inside the spacecraft

Experimentally obtained values of coefficients $\left(g_n^m\right)_{sk}$ and $\left(h_n^m\right)_{sk}$ of spherical harmonics during magnetic measurements of the entire spacecraft can be used to control the number and location of magnetic moment sources inside the satellite.

To this end, one should obtain a representation of the magnetic field in the design coordinate system for each component, using the appropriate harmonic coefficients $\left(g_n^m\right)_q$ and $\left(h_n^m\right)_q$ for the *q*-th device. The latter should be previously experimentally obtained in local (instrument) coordinate systems related to the geometric centers of the corresponding components.

For the transition from amplitude coefficients of harmonics obtained in local (instrument) coordinate systems to the values of amplitude coefficients of harmonics associated with the design coordinate system of the entire spacecraft, one can use the mathematical apparatus of addition theorems [16] of spherical functions. According to these theorems, the values of the amplitude coefficients, which were obtained when measuring the entire spacecraft in the design coordinate system, can be theoretically calculated. At the same time, their calculation is based on the sums of the corresponding amplitude coefficients of all components and component parts of the satellite, defined in local coordinate systems. However, in practice, a case of neglecting the magnetic field of some unknown or erroneously "left" source is possible. Then, marking the amplitude coefficients of the unknown source, which must be found inside the spacecraft as $\left(g_n^m\right)_k$ and $\left(h_n^m\right)_k$, the following three equations for the dipole coefficients can be constructed as follows:

$$\left(g_1\right)_k=\left(g_1\right)_{sk}-\sum_q\left(g_1\right)_q\pm\Delta_1;$$

$$\left(g_1^1\right)_k=\left(g_1^1\right)_{sk}-\sum_q\left(g_1^1\right)_q\pm\Delta_1;$$

$$\left(h_1^1\right)_k=\left(h_1^1\right)_{sk}-\sum_q\left(h_1^1\right)_q\pm\Delta_1, \tag{18}$$

where Δ_1 is the experimental uncertainty of the dipole coefficients of the spacecraft as a whole. At the same time, the experimental uncertainty of the dipole coefficients of the spacecraft components and assemblies is neglected since it is an order of magnitude smaller than when measuring the entire satellite.

For simplification, it is assumed that the unknown source has a pronounced dipole character of the spatial distribution of the magnetic field with a point magnetic moment, the projections of which M_z, M_x, M_y are given by segments in (18). Applying the model [16] of a magnetic point dipole $M_x=\left(g_1^1\right)_k$, $M_y=\left(h_1^1\right)_k$ and $M_z=\left(g_1\right)_k$, shifted to a point with coordinates x_n, y_n, z_n, five coefficients of the quadrupole harmonic of its magnetic field in the design coordinate system are calculated. The expressions for the quadrupole coefficients obtained in this way are located in the left part of the system of five equations. Then the right-hand parts of the equations are the differences between the experimentally found coefficients of the quadrupole harmonics of the magnetic field created by the entire spacecraft and the calculated contributions to the quadrupole harmonics from all devices:

$$\begin{aligned}&2z_n\left(g_1\right)_n-x_n\left(g_1^1\right)_n-y_n\left(h_1^1\right)_n=\\&=\left(g_2\right)_{sk}-\sum_q\left(g_2\right)_q-2\sum_q z_q\left(g_1\right)_q+\\&+\sum_q x_q\left(g_1^1\right)_q+\sum_q y_q\left(h_1^1\right)_q\pm\Delta_2;\end{aligned} \tag{19}$$

$$\begin{aligned}&x_n\left(g_1^0\right)_n+z_n\left(g_1^1\right)_n=\\&=\left(g_2^1\right)_{sk}-\sum_q\left(g_2^1\right)_q-\sum_q x_q\left(g_1^0\right)_q-\sum_q z_q\left(g_1^1\right)_q\pm\Delta_2;\end{aligned} \tag{20}$$

$$\begin{aligned}&y_n\left(g_1^0\right)_n+z_n\left(h_1^1\right)_n=\\&=\left(h_2^1\right)_{sk}-\sum_q\left(h_2^1\right)_q-\sum_q y_q\left(g_1^0\right)_q-\sum_q z_q\left(h_1^1\right)_q\pm\Delta_2;\end{aligned} \tag{21}$$

$$\begin{aligned}&\frac{1}{2}x_n\left(g_1^1\right)_n-\frac{1}{2}y_n\left(h_1^1\right)_n=\\&=\left(g_2^2\right)_{sk}-\sum_q\left(g_2^2\right)_q-\frac{1}{2}\sum_q x_q\left(g_1^1\right)_q+\\&+\frac{1}{2}\sum_q y_q\left(h_1^1\right)_q\pm\Delta_2;\end{aligned} \tag{22}$$

$$\begin{aligned}&\frac{1}{2}y_n\left(g_1^1\right)_n+\frac{1}{2}x_n\left(h_1^1\right)_n=\\&=\left(h_2^2\right)_{sk}-\sum_q\left(h_2^2\right)_q-\frac{1}{2}\sum_q y_q\left(g_1^1\right)_q-\\&-\frac{1}{2}\sum_q x_q\left(h_1^1\right)_q\pm\Delta_2,\end{aligned} \tag{23}$$

where Δ_2 is the experimental uncertainty of quadrupole coefficients of the spacecraft as a whole; x_q, y_q, z_q – coordinates of the q-th source (spacecraft component) of the magnetic field relative to the geometric center of the satellite in the design coordinate system. I also neglect the experimental uncertainty of the quadrupole coefficients of the spacecraft components and assemblies as a value that is an order of magnitude smaller than the uncertainty in measurements of the entire satellite.

To calculate the system (19) to (23), the segments of the dipole coefficients of the unknown source obtained in (18) must be substituted into the left parts of the equations. Then, obtaining for its coordinates x_q, y_q, z_q in the design coordinate system segments of their possible values, the volumes of the possible location of the unknown source are actually determined. Such volumes will be smaller, the smaller the uncertainties Δ_1 and Δ_2 are.

In the case of obtaining by (18) values close to the uncertainty of Δ_1, it can be assumed that the methods of ensuring magnetic cleanliness were correctly applied both to the components and to the spacecraft as a whole. Thus, the comparison of the left parts in (18) with the uncertainty Δ_1 can be considered as one of the criteria for the quality of work to ensure magnetic cleanliness.

In the case of significantly larger average values of the segments in (18) than the uncertainty Δ_1, the proposed algorithm can be used not only to find the coordinates of a dipole source inside the spacecraft that was not taken into account for any reason. The average values of the segments obtained by (18) can be used as projections M_z, M_x, M_y of the additional compensation source of the magnetic moment, which should be installed in the spacecraft. As for the optimal position (coordinates in the design system) of the compensating source of the dipole magnetic moment, its search must be carried out taking into account the additional magnetic interference to the on-board magnetometer and the payload.

Thus, knowledge of the amplitude coefficients of the spatial harmonics of the magnetic field of all components and nodes of the space vehicle allows simulation of the total magnetic field. The use of the model of spherical harmonics of the magnetic field and the experimental determination of the harmonic coefficients is a universal approach to the analysis and purposeful change of the magnetic field of the spacecraft. In particular, the use of spherical harmonics makes it possible to simulate the magnetic field of components with an unknown magnetically active part, which is especially valuable when studying the magnetism of satellite payload equipment.

6. Discussion of results of applying the model of spherical harmonics of the magnetic field in the technology of ensuring magnetic cleanliness

The proposed methodical improvements to the technology of ensuring magnetic cleanliness do not contain complex algorithms for calculating magnetic field characteristics based on (19) to (23). The novelty and advantages of the obtained results are based on the use of the most universal model of the magnetic field based on the spherical harmonics of its scalar potential (2). However, the main condition for the practical application of the model based on (3) to (17) is the possibility of experimentally determining the amplitude coefficients of the spherical harmonics of the magnetic field of the spacecraft and its components. Therefore, it should be noted that in modern world space centers, the experimental determination of the coefficients of spatial harmonics has been carried out for a long time to reduce the methodical error of measuring the magnetic moment of the spacecraft [2], or its component (Fig. 4). The spherical harmonics model is not only more complex than the dipole model but also more accurate and versatile in modeling the magnetic field.

To measure the magnetic moment of a spacecraft and its components, the method [1], which was first applied at the NASA Goddard Space Research Center (USA), became the most widely used in the world. The method operates on the results of measurements of magnetic induction by four magnetometers when the spacecraft rotates around its vertical axis of symmetry (Fig. 4). Usually, the magnetic moment is calculated based on the experimentally determined spatial distribution of the magnetic induction around the source. However, when measuring the magnetic induction in the immediate vicinity of the spacecraft, where the spatial distribution of the magnetic field is not dipole, the application of the dipole model of the field source is incorrect. On the other hand, conducting measurements near the surface of the spacecraft is caused by the need to obtain sufficiently reliable data on the spatial distribution of the magnetic field. As the magnitude of the field created by the spacecraft rapidly decreases as it moves away from the spacecraft, the "useful signal"/noise ratio deteriorates. Therefore, when experimentally determining the magnetic moment, magnetic induction is measured not in the far zone [3], where the use of the dipole model would be justified and correct, but on imaginary circles covering the spacecraft. The smallest circle is located in the near zone and is part of the minimum spherical surface that completely covers the magnetic source. The next magnetometers are placed at distances in the direction from the source until the field becomes dipole. Usually [6], calculation based on the results of magnetic induction measurement is subject to 24 amplitude coefficients of spherical harmonics, which corresponds to the use of harmonics of power n no more than four.

The overdetermination of the obtained system of equations of magnetic induction projections allows applying additional interpolation methods to improve the probability of calculating the amplitude coefficients of spherical harmonics [5]. Therefore, the amplitude coefficients of the dipole spherical harmonic (of power n=1) found by this method are fairly precisely measured projections of the magnetic moment.

The variety of options for experimental determination of the amplitude coefficients of spherical harmonics complements the method based on the measurement of the magnetic flux through the windings, which move the investigated source [17]. When the windings are arranged on a cylindrical surface (Fig. 5), their special spatial configuration creates a selective filter of spherical harmonics of only one order m=const. Therefore, having obtained the dependence of the magnetic flux on the distance between the center of the moving source and the center of the measuring winding, its mathematical processing can be used to calculate the amplitude coefficients of the spherical harmonics of the magnetic field. The advantage of this method is the hardware separation of contributions of spherical harmonics from the total magnetic field of the source. This not only improves the "useful signal"/noise ratio of the received magnetic flux data but also allows determining the contributions of various harmonics, which create significantly different contributions to the magnetic field with the required probability.

Thus, the need for experimental determination of the spherical harmonics of the magnetic field of the spacecraft and its components cannot be considered a limiting factor for the practical application of the work results.

On the other hand, most current publications [2, 6, 10, 12, 14] tackling magnetic cleanliness are aimed at overcoming the technical problems that arise during the experimental determination of the magnetic moment of the spacecraft and its components. This is explained by the need to use both special magneto-measuring systems (Fig. 1) and appropriate procedures for treating the obtained results.

It should also be noted that the real limitations and shortcomings of the obtained results should be determined during the practical application of the proposed procedures based on the measured spherical harmonics of all components (Table 1). To date, the model of spherical harmonics of the magnetic field has been successfully applied only to a limited number of components of domestic spacecraft, so it is not possible to talk about overcoming all technical problems in the preparation of the magnetic interference budget (Table 2).

The further development of the technology for ensuring magnetic cleanliness may be the application of the spherical harmonics model for the analysis and optimization of the magnetic field of nanosatellites. The necessary increase in the accuracy of the magnetic field modeling can always be achieved by increasing the applied spherical harmonics, or harmonics with a different coverage surface of the spacecraft component. A mandatory condition for such application should be to overcome organizational and methodical difficulties in the cooperation of many enterprises that are manufacturers of components for SC.

7. Conclusions

1. The peculiarities of the application of the magnetic cleanliness technology in terms of minimizing magnetic interference to the level of 50 nT, induced by the components of the spacecraft to the on-board magnetometer, have been considered. As a result of the analysis, it was established that when building small spacecraft with overall dimensions close to 1 m and a weight of up to 200 kg, there is a tendency to increase the assembly density and decrease the distance between satellite components and nodes. It is shown that when the size of the satellite is minimized, the use of the multi-dipole model is limited in its application since the magnetic field near the surface of the components has a non-dipole spatial distribution.

2. A model of spherical harmonics of satellite components for analysis, control, and purposeful change of the magnetic field inside a small spacecraft has been adapted. For the practical use of this model, it is proposed to limit the number of used harmonics to the power $n=3$. The requirements for the maximum uncertainty of the measurement of the harmonic coefficients were defined, which should be: $<10^{-3}$ A·m^2 for dipole harmonics, $<10^{-4}$ A·m^3 for quadrupole harmonics, $<10^{-5}$ A·m^4 for octupole harmonics. It is proposed to calculate the magnetic interference budget of the on-board magnetometer using predetermined spherical harmonics of the magnetic field of all components of the satellite. On the basis of the model of spherical harmonics, direct formulas for the engineering calculation of projections of the magnetic induction of interference have been obtained.

3. The algorithm for finding a solution to the inverse problem of magnetostatics based on the results of measuring the coefficients of spherical harmonics, obtained from the entire satellite and from its components separately, has been theoretically substantiated. An algorithm for calculating the coordinates and magnetic moment of an unknown dipole source inside the satellite is proposed. The possibility of using the difference between the spherical harmonics of the entire satellite and the sums of the corresponding spherical harmonics of its components to calculate the compensating magnetic source is shown. It is proposed to use such a difference as one of the criteria for assessing the quality of the work performed to ensure magnetic cleanliness.

Conflicts of interest

The author declares that he has no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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