Sustainable development has become the main focus of transport policy and planning around the world. One of the practical goals when performing multimodal transportation is the optimization of logistics costs. That is, the object of research is the process of multimodal transportation. Empirical research shows that the problem of optimization of transportation costs can be solved by different methods. But the result will be similar. In the given approach, only one component of the transportation process is subject to optimization, which is the overload time. The solutions are based on the method of mass service theory and the method based on fuzzy logic. With the help of these methods, based on practical data, time parameters were calculated that characterize overloading from railway transport to road transport. The application of the method of a weakly formalized process in relation to transport logistics was considered, taking into account not only quantitative estimates but also qualitative, vaguely defined criteria that do not lend themselves to formalization, and the relationships between them. The model was developed for further research of this process, prediction of its behavior, optimization of functioning. It is based on the technology of fuzzy sets. The results obtained using the agent model based on the mass service network and the model based on fuzzy logic differ within the permissible specified limits of no more than 5–7 %. The application of fuzzy logic in the logistics of multimodal transportation is relevant and gives the best results compared to traditional methods of the theory of mass service systems. The article includes comparisons that reflect the advantages of the proposed approach. The obtained results are of a practical nature and can be used to make a decision on choosing a route and/or when transferring from one mode of transport to another.

Keywords: multimodal transportation, freight road transportation, decision-making, fuzzy logic, supply chain, transport logistics, interaction of rail and road transport, route selection, logistics processes
Control processes

supply chain can be managed using multimodal transport networks. It will be more profitable in long-distance transportation. The decision maker must identify the transportation modes and delivery mode switching nodes to optimize the underlying distribution problem. However, one should not forget that, along with the advantages, multimodal transportation is one of the most complex types of logistics process. Thus, points of interaction of railroad and road transport, where cargo is overloaded, are an important link in the transport and logistics chain of cargo delivery from the sender to the recipient, which determines the time component of the entire transportation.

Undoubtedly, there are many factors that reduce the efficiency of the interaction of road and rail transport. These include irrational use of vehicles and loading and unloading machines, wear and tear of rolling stock, poor optimization of transportation routes, transportation management errors, and others.

Under the conditions of globalist changes taking place in the external environment, the dynamics of decision-making in the management of logistics processes are increasing. This requires prompt, almost real-time response to events, critical situations, and violations, as well as conflicts between various participants in supply chains.

The effectiveness of the financial and production activities of enterprises depends on the efficiency, economy, and reliability of management. Thus, many controversial issues arise during multimodal cargo transportation. The main of these issues is the organization of the interaction of different types of transport in transport hubs, including those that are key links in the transport and logistics systems of international multimodal transportation.

The high level of complexity of supply chains and the risks inherent in both supply and demand of resources, especially during periods of economic downturns, are recognized as major deterrents to achieving high levels of supply chain efficiency. The use of modern intelligent decision-making systems is quickly becoming an indispensable tool for designing and managing multimodal network structures of supply chains. At many transport enterprises, a powerful information environment has been created, which includes data collection systems, transmission networks, computing infrastructure, software, databases, information repositories, etc. This information environment includes transport service support and customer information service.

However, in practice, these systems are not used to their full extent because managers are faced with a huge amount of poorly structured information, which the human mind is not able to analyze in a timely manner, and therefore, difficulties arise when making optimal decisions [1].

Often there are poorly formalized tasks that cannot be solved using classical deterministic methods. We are talking about the task of analysis, planning, and economic decision-making under conditions of uncertainty, as well as incompleteness and vagueness of the source information (information that is blurred, vague, uncertain, unclear, or imprecise in nature).

An intelligent system based on agent technologies allows monitoring of critical events, as well as timely review and adjustment of plans in response to a change in the situation.

Managers of transport enterprises, who make decisions, need a powerful intelligent tool that would process data from information systems and issue an addressable analysis.

For example, options for possible management decisions and powers for various officials:

- analysis of the situation and behavior of interacting elements of the system in real time;
- provision of monitoring and diagnosis of management decisions in a dynamic mode;
- simulation of real events and processes;
- forecasting and prevention of critical situations.

Thus, managers need help in intelligent data analysis. It consists in revealing hidden rules and regularities in data arrays. It is obvious that the point of interaction of railroad and road transport, where overloading occurs, determines the time component of the entire transportation.

Optimizing vehicle idle time during multimodal transportation is a natural stage in the development of information logistics technologies. The creation and implementation of such optimization models will make it possible to increase the efficiency of transportation management, reduce non-productive expenses for the transportation of goods, accelerate the development of the national transport, territorial and information infrastructure, etc. Thus, the production task of reducing vehicle downtime is urgent.

2. Literature review and problem statement

Many countries have long since taken note of this and are actively using the devices of agent technologies and fuzzy logic to solve various tasks, including logistical ones. Thus, at the conference «Neural Network Technologies and Their Applications» in the Donbas State Machine-Building Academy, it was proposed to forecast the company’s material flows based on fuzzy logic methods [2]. As part of The 2011 New Orleans International Academic Conference, the use of fuzzy decision trees was proposed for determining the location of a logistics center [3]. Logistic problems are relevant for every enterprise. Questions about how to quickly find goods in the warehouse, how to place more materials in the smallest areas, how to plan activities in case of various supply failures, etc., are also relevant. Work [4] shows the use of methods based on fuzzy logic to solve the problem of optimization of warehouse logistics. But time constraints must be taken into account in order to satisfy customers as much as possible. These constraints correspond to cargo that must be delivered within predetermined time intervals. In addition, the requirements were represented by vague numbers, which allowed formulating the problem well closer to the real situation. The presentation of the mentioned problem with time windows and fuzzy requirements is given in [5].

The practical experience of developing systems on fuzzy sets shows that the terms and cost of their design are much lower than when using a traditional mathematical apparatus, while ensuring the required quality levels [6].

Fuzzy logic techniques are widely used in many fields and are one of the best tools for solving uncertainty problems found in datasets. In article [7], cases were considered when the method of fuzzy logic was used to measure the efficiency of the supply chain. The purpose of that study is to identify the main stages of applying fuzzy logic in the measurement of supply chain efficiency. The results are expected to be useful to supply chain performance measurement academics who are particularly interested in improving the accuracy of the results. However, the problem of low accuracy and incorrect measurements has been highlighted, as it affects incorrect
forecasts, which lead to poor supply chain management. This negatively affects the company's ability to compete.

In [8], a route selection model based on the fuzzy logic approach in a multimodal transport network is proposed. To find the most optimal route among the set of provided alternative routes. The proposed approach uses time, cost, and reliability data for each alternative route. At the same time, transport route planners can make decisions about choosing an effective route.

In [9], a modeling approach using hybrid Petri nets along with fuzzy logic is proposed for the analysis and control of a multimodal transportation system. More precisely, the study focuses on improving passenger service from the point of view of minimizing waiting time at interchange points in multimodal transportation. The paper considers the problem of combining continuous and discrete events in a single multimodal transport system. The problem is that continuous flows of passengers and a discrete number of vehicles (e.g., bus, train, taxi) are presented simultaneously. The solution to this problem is shown in the context of a modeling approach based on hybrid Petri nets and fuzzy logic.

As the review of articles [2–7] showed, most of them focus on modeling, optimization, and forecasting of production processes, and quite a large number are considering the construction of ACS, CAD, diagnostic and quality control systems. At the same time, the development of intelligent logistics systems for information and analytical support of enterprise management, technological guides, information and advisory systems, experience accumulation systems, etc., using fuzzy models, remain no less interesting and promising way of developing applied management systems.

Thus, the application of models based on fuzzy sets in the field of freight transportation is no less promising. They provide a universal way of solving many non-standard tasks, capable of capturing complex non-linear dependencies, self-improvement, learning in the process of use. The use of systems built on their basis is effective, relatively simple, and cheap. Therefore, it is not surprising that the growing interest in this technology has led to a significant expansion of the scope of its use, the appearance of many different approaches, algorithms, software products, application options, etc.

3. The aim and objectives of the study

The purpose of this study is to develop two different models that would allow optimizing the processes of multimodal transportation according to the criterion of minimizing vehicle downtime. This will make it possible to increase the economic efficiency of transportation due to the reduction of cash costs during the rental of technical means and structures of the transshipment front.

To achieve the goal, the following tasks were set:
- to obtain optimized values of the time parameters of overloading from railroad transport to road transport based on the method based on the theory of mass service;
- to obtain the optimal value of the time parameters of the overload based on the theory of fuzzy sets.

4. The study materials and methods

In case of multimodal transportation, it is necessary to solve the transport problem. At the same time, the objective function sets the parameters of the interaction of road and railroad transport in which it would be possible to reduce the time of stay of foreign wagons on the territory of the country as much as possible, and the cost of funds for implementation would be as low as possible. At the same time, the object of research is the process of multimodal transportation. The main hypothesis of the study was that the results that can be obtained when using the traditional method from the theory of mass service would be adequate to the results obtained when using the method based on fuzzy logic. In particular, the problem of synthesis (design calculations) was solved by finding the optimal values of the time parameters of the overload given the given real input data.

The agent approach is a universal tool and is based on the concept of an agent. Agent parameters are specified by variables. Agent methods are agent actions and are defined by functions. The behavior of an agent is a set of agent states connected by transitions between themselves. The mass service network was taken as the basis of the model of agent processes. When calculating the overload time parameters using the method based on the theory of mass service, which is based on Markov and semi-Markov processes, we will assume that most of the results are obtained only for exponential distribution laws. In all design calculations, it is necessary to accept some assumptions about the accuracy of the obtained results. These assumptions may vary depending upon the purpose and type of calculation.

When the number of network nodes is more than three, approximate numerical methods are used for calculations. It should be noted that it is important to analyze the utilization factors not on the entire modeling interval but on the subintervals corresponding to the load peaks. It is also necessary to classify load peaks and find solutions for each peak. Thus, the operational analysis of probabilistic mass service networks can be effectively applied when analyzing the bottlenecks of the supply chain reengineering method.

Classification of data in supply chains is a relevant area of application of fuzzy logic. Fuzzy logic models provide a universal way of solving many non-standard tasks, capable of capturing complex nonlinear dependencies, self-improvement, learning in the process of use. The use of systems built on their basis is effective, relatively simple, and cheap. The main advantage of this method lies in the ability of the algorithm to adjust the network structure to new observations (factors) and to detect complex (non-linear) relationships between the values of input and output data. It also allows for a more effective combination of formalized and informalized knowledge. Therefore, it is not surprising that the growing interest in this technology has led to a significant expansion of the scope of its use, the appearance of many different approaches, algorithms, and application options.

Models developed by different mathematical devices were compared, but at the same time, the same result was expected. Following the theory of fuzzy sets to build the model, first of all, the work performed the following:
- the factors affecting the production process, as well as their interrelationship, were determined. The full number of influencing factors is usually unknown, and not all of them are obvious. Therefore, a certain set of criteria is selected, which in the process of improving the model can be supplemented without affecting the system itself (solution method), but qualitatively improving the «portrait» of the research object;
- vague «somewhat» rules of interaction of all identified factors were determined;
Control processes

- the model development algorithm with fuzzy logic was used;
- samples were made from experimental data for a model with fuzzy logic;
- a model of the technological process was created using a computing environment – a method of identification by fuzzy knowledge bases;
- we made test samples from real data and compared the results using known mathematical methods.

So, at the input of the system there are many evaluation parameters of the transportation process, which can be divided into two groups:

- executive discipline (ratings of the \( i \)-th node of the supply chain \( j \), which are given in linguistic form: «excellent», «good» or «satisfactory», then ratings for each node of the chain) and/or in axial expert ranked assessments;
- the technological map of cargo transportation (acceptance of the application and attached documents, registration and formation of the delivery route, processing of documents, interaction during the transfer of cargo from wagons to cars, arrangement of transshipment fronts with special equipment, etc.) are also given in linguistic form: «excellent», «good» or «satisfactory».

The work assumes the use of weighted expert ranked assessments. The assumption is that the average values of point expert evaluations were used to find weighted expert evaluations. Kolmogorov and Cauchy search for average values was applied. As is known, the results of the comparison are resistant to admissible transformations of the values of the features measured on the rank scale. The study did not focus on the procedure for applying expert assessments.

On the other hand, in order to solve the specified problem using the traditional analytical method, it is necessary to study the dynamics of changes in freight transportation on the main routes of the network, to develop prospective freight flows, and to forecast the monthly unevenness of the density of transportation on the main transport lines. Then it is necessary to develop optimal schemes for loading lines of various types of transport and choose the most effective ways of increasing the capacity of these lines.

The analytical method of calculations is the most common when solving problems of the interaction of modes of transport. But it should be taken into account that the transport process is stochastic in nature. This is manifested in the unevenness and heterogeneity of traffic flows, the different duration of traffic flow maintenance, the random nature of the duration of failures of technical devices and other factors. It is because of the number of these factors that the simulation modeling method can be applied.

It is clear that the most powerful transport nodes will be those located directly next to the member states of the European Union. A vivid example is Terminal Karpaty LLC, located at the Ukrainian-Hungarian border in the Transcarpathian region [10, 11]. Cargoes arrive at the enterprise from abroad on a railroad track with a width of 1435 mm and are transshipped on road transport and vice versa. Based on the practical data of this enterprise, the dependence of financial costs on cargo transshipment operations from one transport to another was investigated. This enterprise as an object of research is characterized by a complex system with a high level of uncertainty of information and the external environment.

They focused on the fact that transshipment operations should be as mechanized and automated as possible. The dimensions of individual devices and mechanisms are determined by the volume of transshipment work, the category of cargo flows, etc. Taking into account that any phase can be implemented by several means, it is necessary to choose operations at each phase in such a way that the money costs and time costs for the implementation of the entire technological process of reloading are as small as possible.

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5. Research results on reducing the time of transshipment from rail to road transport

5.1. Obtaining the optimal value of the time parameters of transshipment from rail transport to road transport based on the method from the theory of mass service

As you know, the term of cargo delivery is determined by the sum of time characteristics of all links of the logistics chain. Fig. 1 shows a conventional diagram of the transport and logistics chain of international cargo flows alternately by rail and road transport. This logistics chain consists of the following elements: \( O \) – the origin of the cargo flow (warehouse of the consignor); \( T' \) – time for initial operations with the cargo and documents and (if necessary) transporting it from the warehouse of the consignor to the departure station; \( T_{dept} \) – downtime at the departure station; \( B_{1435} \) – cargo advancement in wagons of 1435 mm gauge; \( T_{trans} \) – downtime at the point of transshipment of cargo from railroad wagons to cars; \( B_{avt} \) – moving cargo in cars; \( T_{dist} \) – downtime at destination; \( B_{dist} \) – time for transportation from the customs office of destination to the consignee and for final operations with the cargo; \( P \) – place of payment of the wagon flow (warehouse of the consignee).

For the transport and logistics scheme shown in Fig. 1, the delivery time of the cargo \( T_D \) is determined as follows:

\[
T_D = \sum_{i=1}^{1} t_0 + \sum_{i=1}^{m} t_i' + \sum_{i=1}^{1} t_{dept} + \sum_{i=1}^{1} t_{1435} + ... + \sum_{i=1}^{1} t_{trans} + \sum_{i=1}^{1} t_{avt} + \sum_{i=1}^{1} t_{dist} + \sum_{i=1}^{1} t_{P},
\]  

where \( \sum_{i=1}^{1} t_i' \), \( \sum_{i=1}^{1} t_p \), is the sum of production downtimes at the points of origin and finalizing of cargo flows; \( \sum_{i=1}^{1} t_{dept} \) – the total time of cargo delivery from the warehouse of the consignor to the station of departure and from the customs office of destination to the warehouse of the consignee; \( \sum_{i=1}^{1} t_{trans} \) – the total time of cargo from railroad wagons to cars; \( \sum_{i=1}^{1} t_{dist} \) – the total time of the cargo at the departure station, at the transshipment point, and at the customs office of destination; \( \sum_{i=1}^{1} t_{P} \) – the total time of cargo transportation in wagons of the European gauge and in cars, respectively.

---

Fig. 1. Scheme of transport and logistics chain of movement of international cargo flows alternately by rail and road transport
A diagram of time elements is proposed, which consists of the total time of cargo flows at the points of interaction of various types of transport during international multimodal transportation (Fig. 2).

In Fig. 2, the following designations of time elements are adopted, which make up the total idle time of cargo flows at points of interaction of various types of transport $T_{\text{inter}}$ during international multimodal transportation:

- $t_{\text{trans}}$ – the time of transshipment of goods with a direct option from a railroad car to a car;
- $t_{\text{carg}}$ – cargo reloading time in the case of using wagons as a «warehouse on wheels»;
- $t_{\text{unld}}$ – cargo unloading time from 1435 mm gauge wagons to the warehouse;
- $t_{\text{stor}}$ – storage time of goods in the warehouse;
- $t_{\text{load}}$ – the time of loading cargo into motor vehicles.

At interaction points, transshipment of goods from wagons to cars is mostly carried out with the help of loading and unloading mechanisms (HRM) on specially equipped transshipment fronts [12].

Sometimes the goods are not transshipped according to the direct option, from the wagon to the car, but are delayed for a certain time in the warehouse for various reasons. The main ones are the absence of cars for loading, commercial shortage, lack of documents or their incorrect registration, delay due to the results of customs operations, etc. All this has a negative impact on the logistics chain as a whole, in shortage, lack of documents or their incorrect registration, for a certain time in the warehouse for various reasons. The above indicators.

Then the Monte Carlo method can be used to determine the service time of applications is random and divided by law. The following was accepted: on the loading front of bulk cargoes, one crane is equipped with a special mechanical bucket for reloading bulk cargoes from wagons into cars. It is a single-channel mass service system with rejections that receives a Poisson flow of applications. In this case, the application means either a railroad car arriving at the same front for loading. The time between applications is random and distributed according to an exponential law. The following was accepted: on the loading front of bulk cargoes, one crane is equipped with a special mechanical bucket for reloading bulk cargoes from wagons into cars. It is a single-channel mass service system with rejections that receives a Poisson flow of applications.

The service time of applications is random and distributed according to an exponential law. The following was accepted: on the loading front of bulk cargoes, one crane is equipped with a special mechanical bucket for reloading bulk cargoes from wagons into cars. It is a single-channel mass service system with rejections that receives a Poisson flow of applications.

The technological process of bulk cargo transshipment from wagons to cars, according to the method based on mass service theory, was represented as a single-channel system with rejections with an incoming Poisson flow of requests. In (2), the indexes $a, b, d, f, q, r, e, u, y$ indicate the number of individual operations, which are used to calculate the time spent for the relevant elements of the cargo flow service technology at the point of interaction of different types of transport.

Each of the elements of the logistics chain may also have time costs for interoperational downtime, which result from the adopted technology of the point of interaction.

It can be seen from (2) that all the given parameters are multifactorial and have a vague formalization.

The total time of cargo flows at the points of interaction of different types of transport will consist of the following elements:

$$
\begin{align*}
\sum_{s} t_{\text{wold}} & = \sum_{s} t_{\beta} + \sum_{s} t_{\text{sort}} + \alpha \sum_{s} t_{\text{dep}} + \beta \sum_{s} t_{\text{carg}} + \\
& + \gamma \left( \sum_{s} t_{\text{wold}} + \sum_{s} t_{\text{carg}} + \sum_{s} t_{\text{load}} \right) + \sum_{s} t_{\text{dep}},
\end{align*}
$$

where $\sum_{s} t_{\beta}$ – time of stay of wagons with cargo in the acceptance fleet; $\sum_{s} t_{\text{sort}}$ – time of stay of wagons with cargo in the sorting fleet; $\sum_{s} t_{\text{dep}}$ – time of stay of wagons with cargo in the departure fleet; $\sum_{s} t_{\text{dep}}$ – time of location of cargo flows at the point of transshipment according to the direct variant; $\sum_{s} t_{\text{carg}}$ – the time of finding cargo flows on the reserve track of wagons used as «warehouses on wheels»; $\sum_{s} t_{\text{wold}}$ $\sum_{s} t_{\text{wold}}$ – time of unloading of goods from wagons of European track to the warehouse and time of loading them with goods from the warehouse in the opposite direction, respectively; $\sum_{s} t_{\text{carg}}$ – time of cargo in stock; $\sum_{s} t_{\text{load}}$ – time of loading cargo from warehouse to car; $\alpha$ – the share of cargo traffic that is reloaded by direct option; $\beta$ – the share of cargo traffic in «warehouses on wheels»; $\gamma$ – the share of cargo traffic that is reloaded through the warehouse.

In turn, narrow-gauge wagons arriving at transshipment fronts are divided into those that return empty and those that are used for return cargo flows. The shares of such cars will be $\mu$ and $\varphi$, respectively.

The following assumptions will be valid for car shares:

$$
\begin{align*}
0 & < \alpha < 1; 0 < \beta < 1; 0 < \gamma < 1; \\
\alpha + \beta + \gamma & = 1; 0 < \mu < 1; 0 < \varphi < 1; \mu + \varphi = 1.
\end{align*}
$$

The calculation results are given in Table 1 and Fig. 3–5 show the results of calculations of the above indicators.

![Fig. 2. Diagram of time elements that make up the total downtime of different modes of transport](image)

<table>
<thead>
<tr>
<th>Test number $i$</th>
<th>Received applications $N_{serv}$</th>
<th>Applications served $N_{serv}$</th>
<th>Duration of service $t_{serv}$</th>
<th>Average service time $t_{serv}$</th>
<th>Probability of service $P_{serv}$</th>
<th>Probability of rejection $P_{rejc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>3:43:37</td>
<td>0:44:43</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>5</td>
<td>4:21:45</td>
<td>0:52:21</td>
<td>0.71</td>
<td>0.29</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>13</td>
<td>12:19:17</td>
<td>0:56:52</td>
<td>0.72</td>
<td>0.28</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>9</td>
<td>8:53:40</td>
<td>0:59:18</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>9</td>
<td>4:11:26</td>
<td>0:27:56</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>6</td>
<td>5:36:06</td>
<td>0:56:61</td>
<td>0.85</td>
<td>0.15</td>
</tr>
<tr>
<td>Total</td>
<td>64</td>
<td>47</td>
<td>39:05:51</td>
<td>4:57:12</td>
<td>0.74</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Control processes

Average number of applications served

\[ \sum \] (8)

7.8

Average servicing time per application

\[ 0.738 \]

Average service probability

Average probability of rejection

\[ N \]

\[ 1 \text{— test number.} \]

Average servicing time per application

\[ t_{ser} = \frac{\sum t_{ser,avr}}{i} \] (6)

Average service probability

\[ P_{ser,avr} = \frac{\sum P_{ser}}{i} \] (7)

Table 2

<table>
<thead>
<tr>
<th>No. of entry</th>
<th>Indicator</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average number of applications served ( N_{ser} )</td>
<td>7.8</td>
</tr>
<tr>
<td>2</td>
<td>Average servicing time per application ( t_{ser} )</td>
<td>0.4932</td>
</tr>
<tr>
<td>3</td>
<td>Average service probability ( P_{ser,avr} )</td>
<td>0.738</td>
</tr>
<tr>
<td>4</td>
<td>Average probability of rejection ( P_{rejc,avr} )</td>
<td>0.262</td>
</tr>
</tbody>
</table>

Thus, the analysis of the results of six tests shows that on the cargo front of Terminal Karpaty LLC, one crane, equipped with a special mechanical bucket for overloading bulk cargo, will be able to serve 7.8 applications. The average service time for one request will be 49 minutes and 32 seconds. Approximately 73.8% of applications will be served and 26.2% of applications will be denied. Such a percentage of refusals may be related, for example, to inconsistency in customs clearance (inconsistency in customs regimes, weight of the cargo, vehicle, etc.). Thus, one service channel will be quite sufficient under the conditions of a stable import cargo flow of bulk cargo serviced on the territory of Terminal Karpaty LLC.

5.2. Obtaining the optimal value of overload time parameters based on fuzzy sets

A fuzzy model was built based on binary fuzzy relations \( A \) and \( B \). To construct the first relation, two basic sets \( X \) and \( Y \) were used. For the second, \( Y \) and \( Z \). It is assumed that the set \( X \) describes a finite set of parameter properties: \( X = \{x_1, x_2, x_3, x_4, x_5, x_6\} \). \( Y \) is a set of attributes of quality indicators of overload parameters (delivery speed, price, storage period, etc.). \( Y = \{y_1, y_2, y_3, y_4, y_5, y_6\} \). \( Z \) is a set of indicators, \( Z = \{z_1, z_2, z_3, z_4, z_5, z_6\} \).

In the context of the above problem, attitude \( A \) described the properties of the parameters, and attitude \( B \) described the qualitative indicators of overload parameters.

The degree of correspondence of the properties of the known parameters from the side of the quality indicators of the parameters will be significantly higher (1.0...0.9...0.8) compared to the variant of the parameter that is not used often (0.8...0.7). If there is a choice between the properties of the parameters, the same properties will more likely be attributed to the same parameter. A slightly lower degree of correspondence (0.6...0.4) will be to the parameter with which there is no direct correspondence of properties, but which has high qualitative indicators of the parameter. Accordingly, the lowest degree of correspondence (0.3...0) will be to the rarely used parameter. Therefore, instead of subjective indicators, other indicators were considered for compliance – qualitative indicators of overload parameters. It was assumed that the values of the membership functions were obtained by expert means and as a result of machine learning (Tables 3, 4).
Average probability of rejection
Average servicing time per application
Duration of service
Average number of applications received
Average service probability
Average number of applications served

Based on these tables, matrices of fuzzy relationships were compiled:

$$M_A = \begin{bmatrix}
0.9 & 0.9 & 0.8 & 0.5 & 0.4 & 0.4 \\
0.5 & 0.9 & 0.7 & 0.8 & 0.7 & 0.7 \\
0.9 & 0.7 & 0.9 & 0.8 & 0.6 & 0.6 \\
0.8 & 0.8 & 0.5 & 0.9 & 0.8 & 0.8 \\
0.7 & 0.8 & 0.9 & 0.6 & 0.9 & 0.9 \\
0.7 & 0.9 & 0.8 & 0.7 & 0.7 & 0.7 \\
\end{bmatrix} \quad (9)$$

$$M_B = \begin{bmatrix}
0.9 & 0.7 & 0.8 & 0.7 & 0.4 & 0.4 \\
0.8 & 0.8 & 0.6 & 0.8 & 0.7 & 0.7 \\
0.6 & 0.7 & 0.8 & 0.8 & 0.7 & 0.7 \\
0.4 & 0.8 & 0.5 & 0.9 & 0.8 & 0.8 \\
0.7 & 0.9 & 0.6 & 0.6 & 0.9 & 0.7 \\
0.7 & 0.8 & 0.9 & 0.6 & 0.8 & 0.8 \\
\end{bmatrix} \quad (10)$$

Then, the composition of two fuzzy binary relations is formed $A \otimes B$. Then the accessibility function will be written as follows:

$$\mu_{A \otimes B} \left( \{x_i, x_j\} \right) = \max_{x_i, x_j} \left( \min \left\{ \mu_A \left( \{x_i, x_j\} \right), \mu_B \left( \{x_i, x_j\} \right) \right\} \right). \quad (11)$$

According to this formula, the matrix with the result of fuzzy composition will be composed as follows: first, the minimum values of the membership function of all pairs of elements of the first row of the first matrix and the first column of the second matrix are searched. Then we get: $\min(0.9, 0.9) = 0.9$, $\min(0.9, 0.8) = 0.8$, $\min(0.8, 0.6) = 0.6$, $\min(0.5, 0.4) = 0.4$, $\min(0.4, 0.7) = 0.4$; then the maximum value from the obtained values is found:

$$\mu_{A \otimes B} \left( \{x_i, x_j\} \right) = \{0.9, 0.8, 0.6, 0.7, 0.4, 0.4\} = 0.9.$$ 

By analogy, other elements of the matrix are found. Then the matrix of the resulting fuzzy relation will look like this:

$$M_{A \otimes B} = \begin{bmatrix}
0.9 & 0.8 & 0.8 & 0.7 & 0.7 \\
0.8 & 0.8 & 0.7 & 0.8 & 0.8 \\
0.9 & 0.8 & 0.8 & 0.8 & 0.8 \\
0.8 & 0.9 & 0.8 & 0.9 & 0.8 \\
0.8 & 0.9 & 0.7 & 0.8 & 0.8 \\
\end{bmatrix}. \quad (12)$$

Using this algorithm, it is possible to compile tables of advantages of various objects and their attributes, on the basis of which decisions can be made regarding the set of indicators. In addition, this algorithm is relatively easy to program and can be implemented in expert decision support systems. Also, when making decisions, this simple algorithm will make it possible to introduce the properties of adaptability of logistics flows (primarily material).

If we convert matrix (12) into a tabular form, make a substitution and bring it to quantitative indicators of cargo overload, then round the result of calculations with an acceptable error of no more than three percent, then we will get the resulting table based on a fuzzy composition of relations.

### Table 5

<table>
<thead>
<tr>
<th>No. of entry</th>
<th>Indicator</th>
<th>Rounding result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average number of applications received $N_{recv}$</td>
<td>10.6</td>
</tr>
<tr>
<td>2</td>
<td>Average number of applications served $N_{serv}$</td>
<td>7.8</td>
</tr>
<tr>
<td>3</td>
<td>Average duration of service $t_{serv}$</td>
<td>6:30</td>
</tr>
<tr>
<td>4</td>
<td>Average servicing time per application $t_{serv}$</td>
<td>0.49</td>
</tr>
<tr>
<td>5</td>
<td>Average service probability $P_{serv.rej}$</td>
<td>0.74</td>
</tr>
<tr>
<td>6</td>
<td>Average probability of rejection $P_{recv.rej}$</td>
<td>0.26</td>
</tr>
</tbody>
</table>

This result completely coincides with the result obtained when using the model based on the mass service system.

In accordance with the chosen approach, the toolkit of the theory of fuzzy sets is the basis of the description of the parameters of the logistics model of the enterprise, as the main link of the system of information and analytical support of logistics management. The logistics model of an industrial enterprise allows you to reasonably choose a combination of key performance indicators of logistics management.

### 6. Discussion of the results of model development to optimize vehicle downtime

When the effectiveness of using the proposed approach was determined, the main idea was to first find the optimal values of the overload time parameters based on the method from mass service theory, and then to find the same optimal values based on the theory of fuzzy sets. This study is aimed
at the application of fuzzy logic methods in relation to multimodal transportation.

Thus, Fig. 1 shows a conventional diagram of the transport and logistics chain of international cargo flows alternately by rail and road transport. The term of cargo delivery \( T_d \) is determined by formula (1). The problem of synthesis by finding the optimal values of the time parameters of the overload with given real input data is solved. Determined: the average number of serviced applications; average service time of one application; probability of service; probability of rejection. Tables 1, 2 and Fig. 3–5 show the results of indicator calculations.

Since most of the factors belong to the subjective category, practical results can be processed both by tools based on the theory of mass service systems and simulation modeling methods, and by methods using the apparatus of fuzzy sets.

The procedure for building a fuzzy model based on binary fuzzy relations is as follows. A set of attitudes \( A \) is given that describes the properties of the parameters, and attitude \( B \) describes the qualitative parameters of the overload parameters. From the condition of obtaining membership functions on the basis of expert evaluations in combination with the results of machine learning (Tables 3, 4), matrices of fuzzy relations (9), (10) are formed. Next, the composition of two fuzzy binary relations \( AB \) is formed, and the accessory function (11) is determined. The resulting matrix (12) is converted into tabular form and reverse substitution is carried out to the quantitative indicators of overload. In this way, the result represented in Table 5, can be obtained, on the basis of a fuzzy composition of relations.

The analysis of the obtained results allows us to state that the optimal values of the overload time parameters found on the basis of the method based on the theory of mass service and on the basis of the theory of fuzzy sets practically do not differ from each other. The calculation results are rounded with an acceptable error of no more than three percent.

When applied in practice, there may be some limitations associated with the expert assessment of the degree of correspondence of the properties of the parameters from the qualitative indicators of the parameters for the formation of the membership function. This is due to the use of a ranking scale by experts. The admissibility of the monotonic transformation of the values of the characteristic measured in the rank scale means that these values can be changed arbitrarily, provided that the ordering of the objects or the equivalence classes of these objects is preserved. The second limitation is related to the assumption regarding the finding of expert evaluations based on average values according to Kolmogorov and/or Cauchy. We are talking about the application of averages in a specific scale with obtaining sustainable results. In further engineering studies, it is planned to resolve these limitations and provide methodological recommendations for use in a specific reference to the field of multimodal freight transportation.

As prospects for the development of this research, the proposed approach to modeling can be extended to more generalized multimodal networks with a large number of docking stops and overloading fronts.

It should be noted that when using the formal apparatus of fuzzy algebra and fuzzy logic for the formalization of the obtained values, without destroying the blurring properties, some limitations and rounding of the calculation results may occur. To obtain qualitative values of numerical parameters, it is enough to round the result within three to five percent of the error. When taking into account different opinions during the experiments, a coefficient of the quality of the assessment is introduced, which corresponds to the subjective degree of confidence of the expert.

7. Conclusions

1. As a result, based on the real data of the «Terminal Karpaty» enterprise, the optimization practical task of reducing the idle time of vehicles based on the mass service theory method was solved. It is assumed that the congestion system is a single-channel mass service system with rejections, which receives a Poisson flow of requests. The time between the arrival of two consecutive applications is distributed according to the Poisson law, the service time of applications is random and distributed according to the exponential law. Using the Monte Carlo method, operational indicators were optimized (average number of serviced applications \( N_{serv}=7.8 \), average service time of one application \( t_{serv}=0.49:32 \), average service probability \( P_{serv,avr}=0.738 \), average rejection probability \( P_{rej,avr}=0.262 \), on the basis of which the economic indicators of the enterprise are formed.

2. Based on the company’s data, a fuzzy model based on binary fuzzy relations is built. It was assumed that the values of the membership functions were obtained by expert means and as a result of machine learning. On the basis of a fuzzy composition of relations, operational indicators were optimized (average number of received applications \( N_{recv}=10:6 \), average number of serviced applications \( N_{serv}=7.8 \); average service duration \( t_{serv}=6:30 \); average service time of one application \( t_{serv}=0.49 \); average service probability \( P_{serv,avr}=0.74 \); average rejection probability \( P_{rej,avr}=0.26 \), on the basis of which the economic indicators of the enterprise are formed. This result completely coincides with the result obtained when using the model based on the mass service system.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

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