

*The object of this study is the process that forms fill materials from crushed rock under load for managing the condition of side rocks in a coal-bearing massif with preparatory workings. Deformation properties of crushed rock under laboratory conditions were evaluated on the basis of a study of compressive compression of the fill material. It was registered that there is a quadratic functional dependence between the change in the bulk density of crushed rock of different granulometric composition and the specific potential energy of deformation. It was experimentally established that the specific potential energy of deformation reaches the limit values at the maximum compression of crushed rock when the fill material consists of parts of different sizes.*

*For experimental samples with different thickness of the rock layer  $h_0$  (m), there was a linear relationship between their longitudinal deformation  $\Delta h$  (m) and the external load  $F$  (kN), which determined the behavior of the deforming body at critical levels. Under such conditions, with a relative change in the volume of the fill material  $\delta V=0.36$ , that is, with the same relative deformation at any values of the parameter  $h_0$  (m) and the compaction coefficient of the crushed rock  $k_{con}=1.57$ , the maximum stiffness of the rock supports was ensured.*

*With a limited amount of external static load on the experimental samples, in the process of their deformation when the parameter  $h_0$  was reduced by 2 times before their compression, the compaction coefficient of the crushed rock increased from  $k_{con}=1.33$  to  $k_{con}=1.57$ . At the same time, the specific potential energy of deformation increased by 40 %, which made it possible to ensure the maximum rigidity of the fill material at the minimum value of the longitudinal deformation  $\Delta h$  (m) of the experimental samples*

*Keywords: crushed rock, fill material, compressive compression, potential energy, bearing capacity*

# DETERMINING THE DEFORMATION PROPERTIES OF CRUSHED ROCK UNDER COMPRESSIVE COMPRESSION CONDITIONS

**Daria Chepiga**

*Corresponding author*

PhD, Associate Professor\*

E-mail: daria.chepiha@donntu.edu.ua

**Serhii Pakhomov**

Director

State Enterprise «Myrnogradvugilya»

Soborna, str., 1, Myrnograd, Ukraine, 85323

**Vitalii Hnatiuk**

Production Director

PJSC «Pokrovske Mine Management»

Shybankova sq., 1a, Pokrovsk, Donetsk region, Ukraine, 85300

**Maksym Hryhorets**

Postgraduate Student\*

**Yaroslav Liashok**

Doctor of Economic Sciences, Professor\*

**Serhii Podkopaiev**

Doctor of Technical Sciences, Professor\*

\*Department of Mining Management and Labour Protection

Donetsk National Technical University

Potebni str., 56, Luts'k, Ukraine, 43003

Received date 06.04.2023

Accepted date 21.06.2023

Published date 31.08.2023

**How to Cite:** Chepiga, D., Pakhomov, S., Hnatiuk, V., Hryhorets, M., Liashok, Y., Podkopaiev, S. (2023). Determining the deformation properties of crushed rock under compressive compression conditions. *Eastern-European Journal of Enterprise Technologies*, 4 (1 (124)), 85–95. doi: <https://doi.org/10.15587/1729-4061.2023.284386>

## 1. Introduction

During the development of coal deposits, especially in the Donbass, a landscape was formed, part of which are rock dumps. Mine tailings are the main source of environmental pollution in coal-mining regions. Environmental problems caused by the activities of coal enterprises are based on the technology of coal mining, related to the delivery and storage of rock on the surface and its composition. The rock remaining in the mine is necessary not only to solve environmental problems but also to increase the safety of mining operations.

When mining coal in underground conditions, it is important to take into account a number of factors that lead to

increased danger for people and production facilities. At the enterprises of the coal industry, rock collapses occur every year, which leads to injuries to miners and the collapse of mine workings. An analysis of injuries in the coal industry shows that a significant part of accidents (34 % of accidents) occurred in underground conditions and were associated with the collapse of rocks in mining areas.

The world experience of mine operation shows that filling the worked-out space with rock has a positive effect on the condition of side rocks and prevents their collapse in excavation areas. The foundation massifs perform the functions of load-bearing structures and ensure smooth deflection of the layered stratum of rocks in the produced space of the coal massif with preparatory works.

Obviously, the nature of the improvement of the condition of the side rocks when filling the created space is explained by the deformation properties of the crushed rock and the nature of the interaction of the roof and sole with the fill material. The base material, which is used for security structures in the form of supports, will limit the displacement of lateral rocks after compression. The original material must have certain characteristics, which will make it possible when it is used to ensure a smooth deflection of the roof in the created space of the excavation site. When substantiating the parameters of safety structures, taking into account the deformation properties of fill materials will make it possible to keep the workings in operational condition. This approach also makes it possible to increase the safety of mining operations at excavation sites by preventing the collapse of side rocks. Therefore, research aimed at studying the deformation properties of crushed rock is relevant.

---

## 2. Literature review and problem statement

---

In the practice of mining operations in the mines of the Central District of Donbas, preference is given to the use of traditional methods of managing mountain pressure – maintaining the roof on wooden chocks. In works [1, 2], it is suggested to use coal pillars, racks from wooden stands, or wooden chocks for the protection of preparatory workings. It is important to note that with this technique of managing rock pressure, the fastening used in the mining pit is not always able to resist the shift of the side rocks in the coal massif after their stratification. The techniques of protection of preparatory workings, which are currently used, do not provide reliable protection of rollback stretches from harmful manifestations of mountain pressure, including collapses of stratified layers. The suddenness of the collapse of stratified lateral rocks is predetermined not only by mining and geological factors but also by mining and technical factors. The latter include the inconsistency of the applied techniques of managing rock pressure and protection of workings to the specific mining and geological conditions of excavation sites.

It was established that the fill massif prevents the development of the movement of stratified lateral rocks in the coal array. It was shown [3] that the mechanism of interaction of the compacting crushed rock with the sedimentary rocks involves the closing of the cracks in the roof above the fill massif. But for a clearer representation of this process, it is necessary to establish the deformation properties of crushed rock. It should be noted that this happens because crushed rock after compression provides a smooth deflection of the roof and creates zones of rock stability in the produced space of the excavation site.

During the underground development of coal seams, the main factors that significantly worsen the ecological situation of coal-mining areas are the release of large volumes of rock from mines and the underworking of surface structures. About 40 % of surface and underground transport of a coal mine and more than 30 % of transport workers are used to deliver rock to the surface [4]. It is believed that with such indicators, the rock issue becomes a factor that has a significant impact not only on the development of the mining operation but also on the normal functioning of the coal enterprise.

In work [5], it was established that up to 4 billion tons of solid waste, which is not used as a rock resource, has accumulated in the dumps of coal mining enterprises in Donbas. It is

noted that the influence of mining production on the natural landscape of the Donetsk region is related to the disturbance of the soil cover, the withdrawal of part of the agricultural land from farms. The reason for this is that rock dumps are a potential resource for industrial production, in particular, the production of fill materials.

In work [6], it is proposed to use a hardening filler, the components of which can be materials from rock dumps, for the preservation of mining products in the excavation areas of coal mines. It is this approach that leads to a solution that will reduce the subsidence of the processed layer of rocks and limit its movement into the produced space of the coal massif. At the same time, the issue of preparation of multicomponent mixtures and regulation of the hardening process of the fill material remains unresolved.

It is believed [7] that leaving the rock in the mine would solve the complex issues of managing geomechanical processes in the coal massif with workings. It is noted that filling the worked-out space with crushed rock ensures the least shrinkage of the processed layer and limits the movement of side rocks in the worked-out space. It is important to note that when ascertaining such factors, there are no requirements for crushed rock, which should reflect the deformation characteristics of the fill materials.

It is believed [8] that one of the important aspects in the maintenance of rollbacks in the mining areas of a deep coal mine is the monitoring of the convergence of side rocks. Taking into account these requirements will make it possible to assess the stability of the roof and sole as the main load-bearing elements of the coal massif. At the same time, it is also necessary to take into account both the deformation characteristics of safety facilities and the relative stiffness of structures, taking into account their critical condition.

The concept of the critical state of deformable systems is widely used in science and technology. It is believed [9] that reaching a critical state means that the system parameters have reached their limit values, after exceeding which the system collapses or changes the patterns of its behavior. If we consider the fill massifs from crushed rock, then the transition through the critical levels for them means the compaction of the original material, which helps increase the bearing capacity of protective structures.

Possessing certain physical-mechanical properties, fill massifs from crushed rock perform the role of safety structures. Under certain conditions (for example, compressive compression), the use of full or partial filling of the worked-out space will limit the movement of side rocks in the coal massif. Filling the worked-out space has a positive effect on the geomechanical situation around the workings and will increase the safety of mining operations. All this gives reason to assert that it is expedient to conduct a study into the deformation properties of crushed rock under conditions of compressive compression. Such studies will make it possible to evaluate the deformation properties of fill materials under load, which, when managing the state of side rocks, excludes their collapse.

---

## 3. The aim and objectives of the study

---

The purpose of this study is to determine the deformation properties of the crushed rock under conditions of compressive compression during the formation of fill materials to limit the convergence of side rocks to prevent their collapse in the

excavation areas of coal mines. This will make it possible to keep the preparatory workings in operational condition and increase the safety of mining operations.

To achieve the goal, the following tasks were defined:

- to investigate the deformation characteristics of experimental samples from crushed rock of different granulometric composition and perform a comparative analysis to assess the bearing capacity of fill materials;
- to investigate the deformation characteristics of experimental samples from crushed rock of heterogeneous (in terms of particle size) granulometric composition under increasing external load;
- to study the deformation characteristics of experimental samples from crushed rock of heterogeneous (in terms of particle size) granulometric composition under conditions where the magnitude of the external load is limited.

#### 4. The study materials and methods

The object of our research is the processes of formation of protective structures from crushed rock for managing the condition of side rocks in a coal massif with preparatory workings. Deformation properties of crushed rock under laboratory conditions were evaluated on the basis of studying the compression of the fill material. To this end, crushed rock of different granulometric composition was used.

When selecting crushed rock for experimental samples, the following ratio [10] was used:

$$t_g \rho_m = t_g \rho_n, \tag{1}$$

where  $\rho_m = \rho_n$  is the angle of internal friction between the material of the model and nature, degrees.

To study the physical and mechanical properties of the fill material, an analysis of the granulometric composition was performed. Determination of the percentage composition of fractions in the initial material of crushed rock was carried out after sieving and weighing on technical scales with an accuracy of 0.01 g. Sieving of the original material was carried out through a set of standard sieves with openings of 5, 4, 3, 2, 1, and 0.1 mm. The sum of the masses of all fractions differed from the initial mass of the starting material by 1%, which meets the requirements set in [11]. Laboratory samples dried to a constant weight were used for testing.

To study the deformation characteristics of fill materials, crushed rock with the following size of the fractions was used:

- (4–5) mm, which occupied 100 % of the volume of the fill material;
- (0.1–1) mm, which occupied 100 % of the volume of the fill material;
- non-homogeneous (in terms of particle size) granulometric composition.

The data of sieve analysis of crushed rock of heterogeneous (by particle size) granulometric composition are given in Table 1.

The coefficient of heterogeneity of crushed rock was determined as in [11]. In these studies, the heterogeneity coefficient is equal to  $k_h = 4.8$ , which indicates the diversity of the parts of the crushed rock in terms of size in the total volume of the fill material.

The bulk density of the crushed rock (after sieving) and the cavity content before compressive compression were determined according to [11]. The results of laboratory studies

on the determination of bulk density  $\rho_{b.d.}$  ( $\text{kg}/\text{m}^3$ ) and hollowness  $M$  (%) of the fill material are given in Table 2.

Table 1

Sieve analysis data of crushed rock

Fraction size, mm	% in total volume
>5	4
4–5	16
3–4	19
2–3	24
1–2	18
0.1–1	14
<0.1	5

Table 2

Data from laboratory studies to determine the bulk density and hollowness of the fill material

Fraction size, mm	$\rho_{b.d.}$ , $\text{kg}/\text{m}^3$	$M$ , %
0.1–5	1790	15
4–5	1660	20
0.1–1	1910	10

A sample of crushed rock was placed in a steel cylinder with a height of  $h_c = 0.075$  m and a diameter of  $d_c = 0.075$  m and was loaded using a rigid steel punch. During the simulation, the initial thickness of the rock layer  $h_0$  (m) in the metal cylinder was changed. The dimensions of the experimental samples are given in Table 3.

Table 3

Dimensions of experimental samples for modeling

No. of entry	The thickness of the rock layer $h_0$ , m	Cross-sectional area $S$ , $\text{m}^2$	Volume of fill material $V$ , $\text{m}^3$
1	0.063	0.0044	0.00027
2	0.055		0.00024
3	0.046		0.000202
4	0.04		0.000173
5	0.0315		0.000138

Fig. 1 shows a P-50 hydraulic press for testing and a steel cylinder with crushed rock.



Fig. 1. Photographs of experimental equipment: a – hydraulic press P-50; b – steel cylinder and die for compressive compression of crushed rock

During the tests, a P-50 hydraulic press was used as experimental equipment, designed for loading experimental samples with a static load (Fig. 1, a). A steel cylinder filled with crushed rock with a determined thickness of the rock layer  $h_0$  (m) and a stamp (Fig. 1, b) was installed between the parallel plates of the press. After that, a vertical load  $F$  (kN) was applied. Under the action of an external static force  $F$  (kN), compressive loads occur in the sample, which causes compaction of the fill material and stamp sediment. When a static load is applied from the outside, the energy is completely spent on the deformation of the experimental samples.

Compressive compression was a special case of uniaxial compression with additional boundary conditions, which implies the impossibility of lateral expansion. In this case, the compaction coefficient of crushed rock  $k_{con}$  was calculated by the ratio of the volume occupied by the original material before compaction to the volume after compaction [12].

The relative deformation  $\lambda$  of a crushed rock sample under compression was determined by the expression:

$$\lambda = \frac{\Delta h}{h_0}, \tag{2}$$

where  $\Delta h$  is the longitudinal strain, i. e., the amount of compression of crushed rock in a metal cylinder, m.

Stiffness  $C$  (N/m) of the fill material was determined from Hooke's law according to the following expression from [12]:

$$C = \frac{F}{\Delta h}. \tag{3}$$

During compressive compression, a change in the volume  $\Delta V$  (m<sup>3</sup>) of the fill material is observed, and in this case the following ratio holds [12]:

$$\frac{\Delta V}{V} = \frac{\Delta h}{h_0}, \tag{4}$$

where  $V$  is the initial volume of crushed rock in the metal cylinder, m<sup>3</sup>.

The modulus of deformation  $E_g$ , (MN/m<sup>2</sup>) of protective structures was determined by the following expression from [12]:

$$E_g = \frac{F}{S} \cdot \frac{h_0}{\Delta h}, \tag{5}$$

where  $F/S$  is the mechanical stress, which under the conditions of compressive compression is considered pressure, N/m<sup>2</sup>.

The work of an external force, which is performed during the deformation of experimental samples, can be determined by the following expression from [7]:

$$A = \frac{E_g \cdot S}{h_0} \cdot \frac{\Delta h^2}{h_0} = \frac{E_g \cdot V}{2} \cdot \lambda^2. \tag{6}$$

It is believed [13, 14] that any deformable body with given geometric dimensions and material characteristics has

a set of critical levels of deformation potential energy. They corresponded to critical parameters; after increasing their values, the body changes its behavior.

The amount of potential energy  $U$  (J) stored in the experimental sample was determined by the following expression from [15]:

$$U = \frac{\sigma^2}{2E_g} \cdot V, \tag{7}$$

where  $\sigma^2/2E_g$  is the energy density or specific potential energy of deformation, J/m<sup>3</sup>.

In this study, the modulus of deformation  $E_g$  (N/m<sup>2</sup>) does not need to be identified with the modulus of elasticity  $E$  (N/m<sup>2</sup>) since it characterizes not only elastic but also residual deformations of the compressible (deforming) material.

The internal potential energy of objects in the mechanics of a solid deformed body, regardless of whether a static or dynamic problem is considered, has the property of periodicity. The property of periodicity is expressed in the fact that the energy has a number of critical values of the potential energy levels of deformation, which determine the stressed-deformed state of the body [13, 14, 16].

## 5. Results of studying the deformation properties of crushed rock under compressive compression conditions

### 5.1. Results of studying the deformation characteristics of experimental samples from crushed rock of different granulometric composition

Experimental samples from crushed rock of different granulometric composition were considered. For experiments, we used:

- sample No. 1, crushed rock consisted of fractions (0.1–5) mm in size;
- sample No. 2, crushed rock consisted of fractions with a size of (4–5) mm;
- sample No. 3, crushed rock consisted of fractions with a size of (0.1–1) mm.

Table 4 gives the results of research on compressive compression of experimental samples from crushed rock of heterogeneous (by particle size) granulometric composition.

In Table 4, the value of the external force  $F$  (kN) corresponded to the maximum longitudinal deformation  $\Delta h$  (m) of the experimental samples. After that, the relative deformation  $\lambda$ , stiffness  $C$  (N/m), and compaction coefficient of crushed rock  $k_{con}$  were determined.

Fig. 2 shows a plot of change in the specific potential energy of deformation  $\sigma^2/2E_g$  of the fill material depending on the crushed rock bulk density  $\rho_{b.d.}$ .

Table 4

Results of studies of compressive compression of experimental samples from crushed rock of heterogeneous (in size of parts) particle size distribution

No.	Fraction, mm	$F$ , kN	$\Delta h$ , m	$\lambda$	$\sigma$ , MPa	$E_g$ , MPa	$\sigma^2/2E_g$ , MJ/m <sup>3</sup>	$C \cdot 10^6$ , N/m	$k_{con}$
1	0.1–5	150	0.023	0.36	34.0	92.8	6.2	6.52	1.57
2	4–5	130	0.019	0.31	29.5	97.6	4.46	6.84	1.43
3	0.1–1	70	0.008	0.15	15.9	125.1	1.01	8.75	1.14

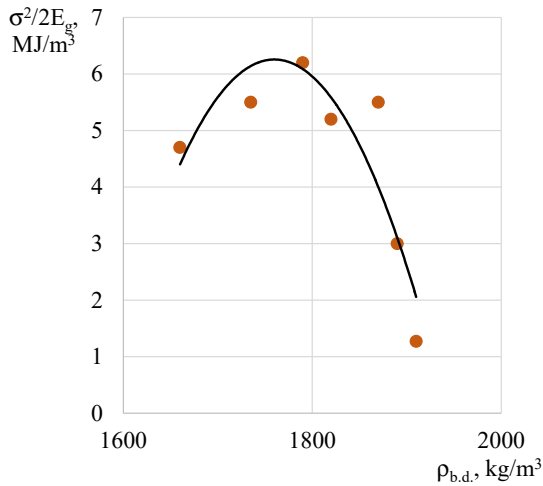


Fig. 2. The plot of change in the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material depending on the bulk density  $\rho_{b.d.}$  (kg/m<sup>3</sup>) of crushed rock

It can be seen from the plot (Fig. 2) that with an increase in the bulk density of crushed rock from  $\rho_{b.d.}=1660 \text{ kg/m}^3$  to  $\rho_{b.d.}=1790 \text{ kg/m}^3$ , the specific potential energy of deformation of the fill material increased from  $\sigma^2/2E_g=4.46 \text{ MJ/m}^3$  to  $\sigma^2/2E_g=6.22 \text{ MJ/m}^3$ . It should be noted that with a bulk density of  $\rho_{b.d.}=1660 \text{ kg/m}^3$ , the fill material consists of homogeneous (by the size of the parts) crushed rock of a coarse fraction (4–5) mm. At a bulk density of  $\rho_{b.d.}=1790 \text{ kg/m}^3$ , the fill material consists of heterogeneous (by the size of the parts) rock with fractions (0.1–5) mm in size. With an increase in the bulk density of the fill material to  $\rho_{b.d.}=1910 \text{ kg/m}^3$ , when the crushed rock is represented by a small fraction (0.1–1) mm, the specific potential deformation decreases to the value  $\sigma^2/2E_g=1.01 \text{ MJ/m}^3$  (Fig. 2).

After processing the experimental data, between the parameters  $\sigma^2/2E_g$  and  $\rho_{b.d.}$ , a quadratic dependence of the following form was established:

$$\sigma^2/2E_g = -0.0002\rho_{b.d.}^2 + 0.6551\rho_{b.d.} - 570.1695, \quad (8)$$

with a correlation coefficient of 0.89 [17].

After the maximum compression of the experimental samples, the external load increased. The data from experimental studies of compressive compression of experimental samples are given in Table 5.

It was registered that with an increase in the external force  $F$  (kN) over time, the longitudinal deformation  $\Delta h$  (m) of the experimental samples remained unchanged, and the stiffness of the fill material increased by 2–6 % (Table 5).

Table 5

The results of studies of compressive compression of experimental samples from crushed rock of heterogeneous (in terms of particle size) granulometric composition after maximum compression with increasing external load

No.	Fraction, mm	$F$ , kN	$\Delta h$ , m	$C \cdot 10^6$ , N/m
1	0.1–5	154	0.023	6.69
2	4–5	138	0.019	7.26
3	0.1–1	72	0.008	9.0

### 5. 2. Results of studying the deformation characteristics of experimental samples at the maximum stiffness of the fill material

Let us consider experimental samples from crushed rock with the size of fractions (0.1–5) mm, which had different initial thickness of the rock layer  $h_0$  (m) and were subjected to uniaxial compression in a steel cylinder. Table 6 gives the results of research on compressive compression of experimental samples. It is important to note that the value of the vertical load  $F$  (kN) corresponds to the maximum value of the external force under which, under the conditions of compression of crushed rock, the maximum rigidity of the fill material is ensured. During the experiments, the amount of compression of crushed rock  $\Delta h$  (m) was recorded at different values of the thickness of rock layer  $h_0$  (m) before deformation.

Fig. 3 shows the plot of change in the thickness of rock layer  $\Delta h$  (m) in the steel cylinder depending on the magnitude of the external force  $F$  (kN) under the conditions of compressive compression.

The change in the value of longitudinal deformation  $\Delta h$  (m) of experimental samples under the action of external load  $F$  (kN) was considered. The thicknesses of the rock layer in the experimental samples varied from  $h_0=0.0315 \text{ m}$  to  $h_0=0.063 \text{ m}$ . The registered value  $F$  (kN) provided the maximum stiffness of the fill material. It can be seen from the plot (Fig. 3) that under the action of an external force with a value of  $F=78 \text{ kN}$  for a sample with a parameter  $h_0=0.0315 \text{ m}$ , the longitudinal deformation was  $\Delta h=0.0115 \text{ m}$ . With an increase in the strength of the rock layer to  $h_0=0.063 \text{ m}$ , the static external force of  $F=150 \text{ kN}$  ensures the maximum rigidity of the fill material during the longitudinal deformation of the experimental sample  $\Delta h=0.023 \text{ m}$  (Fig. 3).

The relationship between the longitudinal deformation  $\Delta h$  (m) of the experimental samples and the external static load  $F$  (kN) is described by a pairwise linear dependence of the form:

$$\Delta h = -0.0005 + 0.00016F. \quad (9)$$

Table 6

Results of compressive compression studies of experimental samples from crushed rock

No.	$F$ , kN	$h_0$ , m	$\Delta h$ , m	$h$ , m	$\lambda$	$\sigma$ , MPa	$E_g$ , MN/m <sup>2</sup>	$\sigma^2/2E_g$ , MJ/m <sup>3</sup>	$U$ , J	$A$ , J	$C \cdot 10^6$ , N/m	$k_{con}$	$\Delta V$ , m <sup>3</sup>
1	150	0.063	0.023	0.04	0.36	34	92.8	6.22	1681.6	1616.1	6.52	1.57	$9.7 \cdot 10^{-5}$
2	131	0.055	0.02	0.035	0.36	29.7	81.6	5.4	1308	1273.6	6.55		$8.7 \cdot 10^{-5}$
3	114	0.046	0.017	0.029	0.36	25.9	69.9	4.79	962.2	910.7	6.7		$7.2 \cdot 10^{-5}$
4	96	0.04	0.015	0.025	0.36	21.8	58.1	4.08	705.8	683.4	6.4		$6.4 \cdot 10^{-5}$
5	78	0.0315	0.0115	0.02	0.36	17.7	48.4	3.29	446.6	429.3	6.78		$4.9 \cdot 10^{-5}$

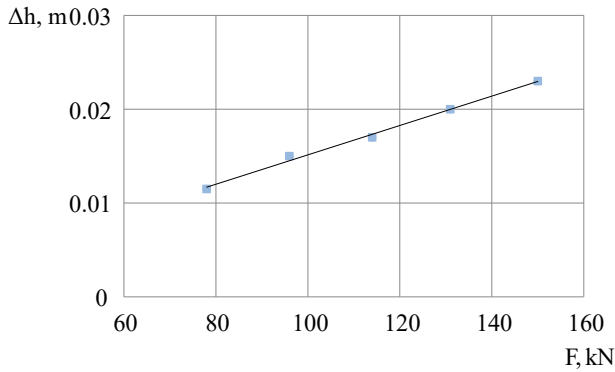


Fig. 3. The plot of changes in longitudinal deformation  $\Delta h$  (m) of rock layer in a steel cylinder depending on the magnitude of the external load  $F$  (kN) under compressive compression conditions

The correlation coefficient is 0.99, the relationship between the studied features is direct, and the closeness of the relationship according to the Chaddock scale is functional [17].

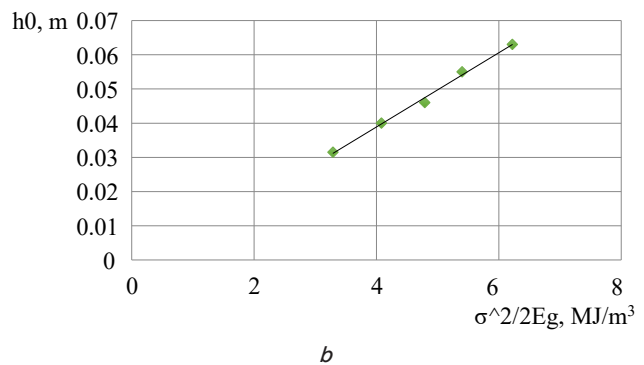
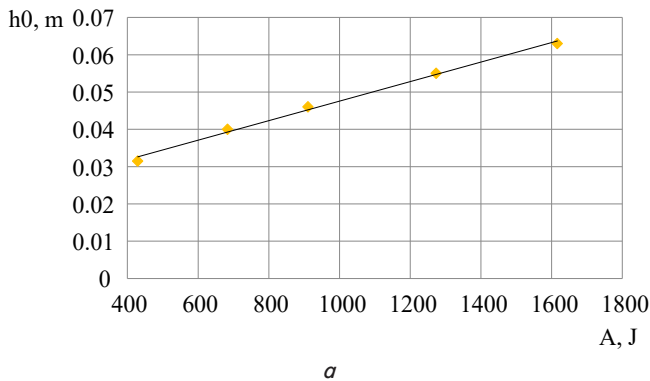


Fig. 4. Change plots: *a* – compression work  $A$  (J); *b* – specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material depending on the initial thickness  $h_0$  (m) of the rock layer

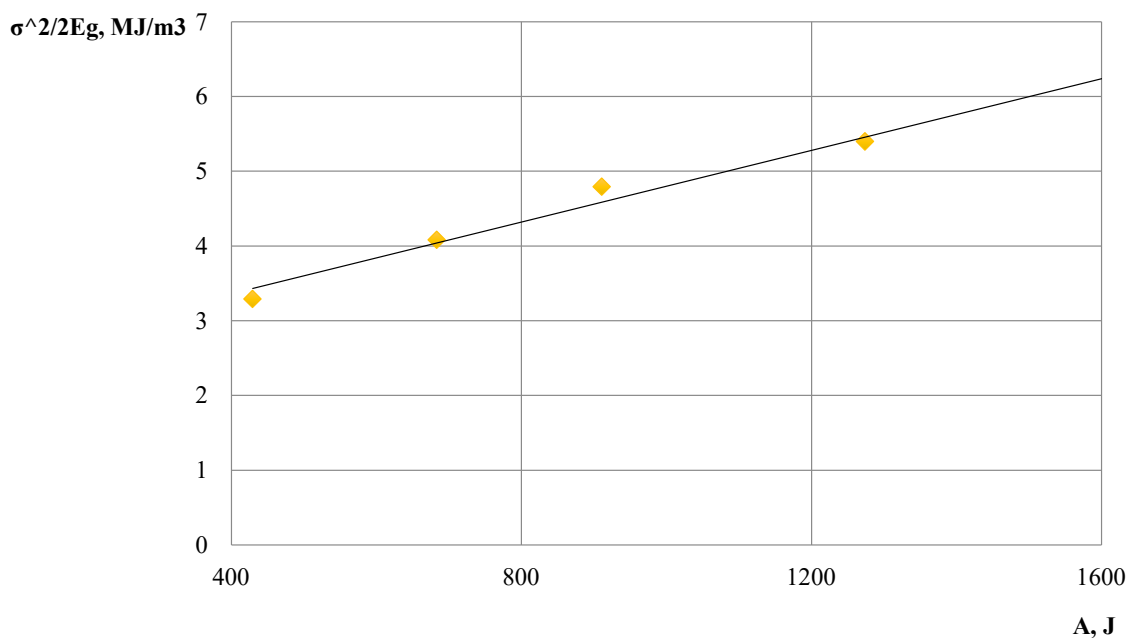


Fig. 5. The plot of change in the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material due to the work of compression  $A$  (J)

Fig. 4 shows the plots of changes in the work of compression  $A$  (J) and the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material depending on the initial thickness  $h_0$  (m) of the rock layer.

Fig. 4, *a* demonstrates that with an increase in the initial thickness of the rock layer from  $h_0=0.0315$  m to  $h_0=0.063$  m, the compression work of crushed rock increases from  $A=429.3$  J to  $A=1616.1$  J. With an increase in the parameter  $h_0$  (m), the specific potential energy and deformation of the fill material also increases. At  $h_0=0.0315$  m –  $\sigma^2/2E_g=3.23$  MJ/m<sup>3</sup>, and at  $h_0=0.063$  m –  $\sigma^2/2E_g=6.22$  MJ/m<sup>3</sup> (Fig. 4, *b*).

Fig. 5 shows the plot of change in the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material depending on the work of compression  $A$  (J).

Under the conditions of compressive compression of crushed rock with an increase in the specific potential energy of deformation from  $\sigma^2/2E_g=3.23$  MJ/m<sup>3</sup> to  $\sigma^2/2E_g=6.22$  MJ/m<sup>3</sup>, the work of compression increased from  $A=429.3$  J to  $A=1616.4$  J (Fig. 4). This regularity explains the compaction of the crushed rock and the increase in the rigidity of the fill material in relation to the change in the specific potential energy of the deformable body.

**5.3. Results of studying the deformation characteristics of experimental samples at a limited value of the external load**

Let's consider experimental samples from crushed rock with the size of fractions (0.1–5) mm. Such samples had different initial thickness of the rock layer  $h_0$  (m) and were subjected to uniaxial compression in a steel cylinder. An external load of  $F=78$  kN was applied to the samples. Table 7 gives the results of research on compressive compression of experimental samples.

Fig. 6 shows the plot of change in stiffness  $C$  (N/m) of the fill material depending on the value of the longitudinal deformation  $\Delta h$  (m) and the relative deformation  $\lambda$  of the experimental samples.

It can be seen from the plot (Fig. 6, a) that with a decrease in the value of the longitudinal deformation  $\Delta h$  (m) of experimental samples, their stiffness  $C$  increased. It was registered that at  $\Delta h=0.016$  m, the stiffness of the fill material was equal to  $C=4.87 \cdot 10^6$  N/m. When reducing the longitudinal deformation of the experimental sample to the value  $\Delta h=0.0115$  m, the stiffness of the fill material is equal

to  $C=6.78 \cdot 10^6$  N/m. This is explained by the effect of the limited external load and the change in the initial thickness of the rock layer.

It was registered that with an increase in the relative deformation from  $\lambda=0.25$  to  $\lambda=0.36$ , the stiffness of the fill material increases from  $C=4.87 \cdot 10^6$  N/m to  $C=6.78 \cdot 10^6$  N/m (Fig. 6, b).

Fig. 7 shows the plots of changes in the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material depending on the stiffness  $C$  (N/m) and the relative deformation  $\lambda$  of the experimental samples.

As the stiffness of the fill material increases from  $C=4.87 \cdot 10^6$  N/m to  $C=6.78 \cdot 10^6$  N/m, the specific potential energy increases from  $\sigma^2/2E_g=2.24$  MJ/m<sup>3</sup> to  $\sigma^2/2E_g=3.23$  MJ/m<sup>3</sup> (Fig. 7, a). This is due to the compaction of crushed rock and the repacking of parts of different sizes in the total volume of the fill material.

From the plot shown in Fig. 7, b, it can be seen that with an increase in the relative deformation of the experimental samples from  $\lambda=0.25$  to  $\lambda=0.36$ , the specific potential energy of deformation increases from  $\sigma^2/2E_g=2.24$  MJ/m<sup>3</sup> to  $\sigma^2/2E_g=3.23$  MJ/m<sup>3</sup>.

Table 7

Results of studies of compressive compression of experimental samples from crushed rock at a limited value of external load

No.	$F$ , kN	$h_0$ , m	$\Delta h$ , m	$h$ , m	$\lambda$	$\sigma^2/2E_g$ , MJ/m <sup>3</sup>	$U$ , J	$A$ , J	$C \cdot 10^6$ , N/m	$k_{con}$	$\Delta V$ , m <sup>3</sup>
1	78	0.063	0.016	0.047	0.25	2.24	606.8	564.5	4.87	1.33	$6.7 \cdot 10^{-5}$
2		0.055	0.015	0.04	0.27	2.41	579.2	545.1	5.2	1.36	$6.4 \cdot 10^{-5}$
3		0.046	0.014	0.032	0.3	2.69	544.6	544.6	5.57	1.42	$6 \cdot 10^{-5}$
4		0.04	0.013	0.028	0.32	2.75	459.3	459.3	6.5	1.46	$5.5 \cdot 10^{-5}$
5		0.0315	0.0115	0.02	0.36	3.23	446.6	446.6	6.78	1.57	$4.9 \cdot 10^{-5}$

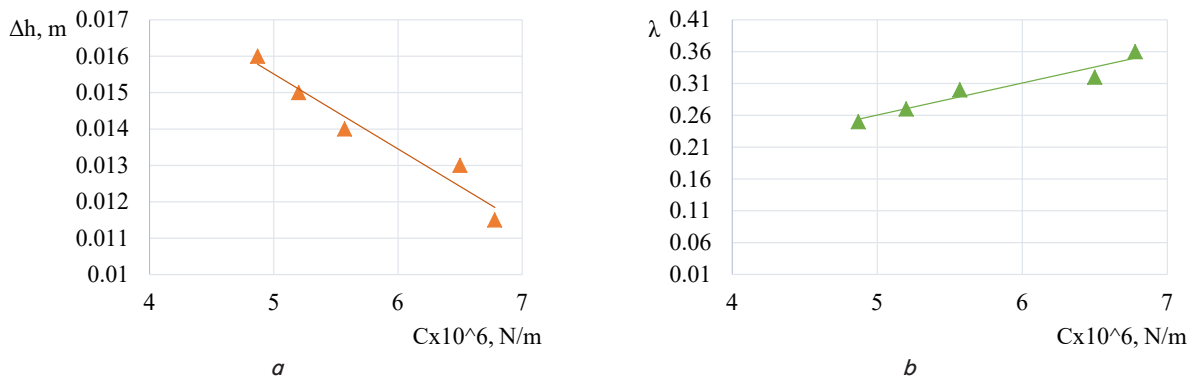


Fig. 6. The plot of change in stiffness  $C$  (N/m) of the fill material depending on: a – the amount of longitudinal deformation  $\Delta h$  (m); b – relative deformation  $\lambda$  of experimental samples

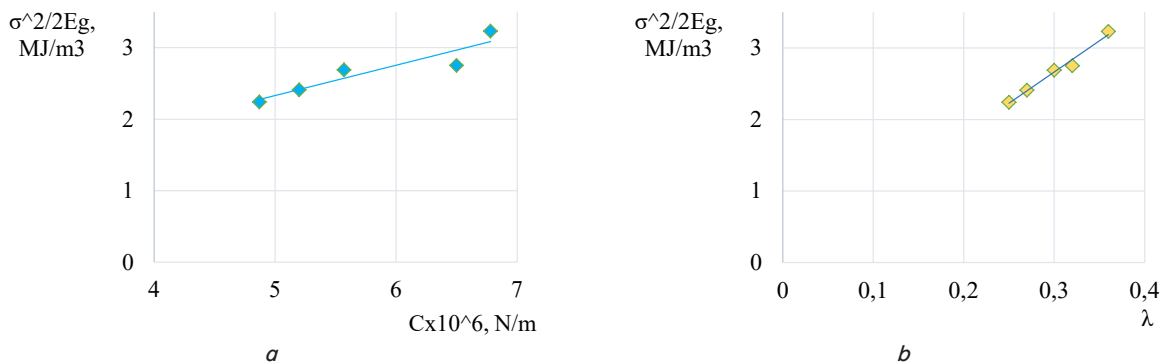


Fig. 7. Plots of change in the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material depending on: a – stiffness  $C$  (N/m); b – relative deformation  $\lambda$

Fig. 8 shows a plot of change in the compaction coefficient  $k_{con}$  of crushed rock depending on the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material under compressive compression conditions.

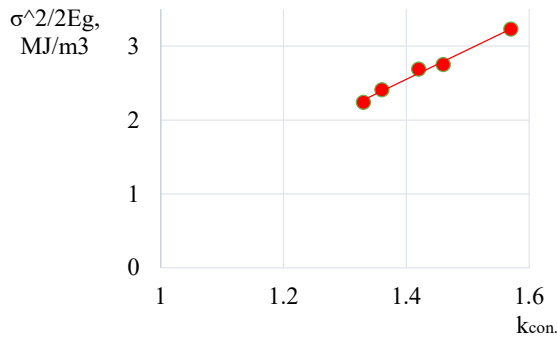


Fig. 8. The plot of change in the compaction coefficient  $k_{con}$  of crushed rock depending on the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material under conditions of compressive compression

It can be seen from the plot that with an increase in the specific potential energy of deformation from  $\sigma^2/2E_g=2.24$  MJ/m<sup>3</sup> to  $\sigma^2/2E_g=3.23$  MJ/m<sup>3</sup>, the compaction coefficient of crushed rock increases from  $k_{con}=1.33$  to  $k_{con}=1.57$  (Fig. 8).

The relationship between the compaction factor  $k_{con}$  of crushed rock and the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material is described by the dependence:

$$\frac{\sigma^2}{2E_g} = -3.501 + 4.33k_{con}. \tag{10}$$

The correlation coefficient is equal to 0.99, the relationship between the research features is direct, and the closeness of the relationship according to the Chaddock scale is functional [17].

Fig. 9 shows plots of changes in the compaction coefficient  $k_{con}$  of crushed rock depending on the relative vertical  $\Delta h/h_0$  and longitudinal  $\Delta h$  (m) deformation of the rock layer under compressive compression conditions.

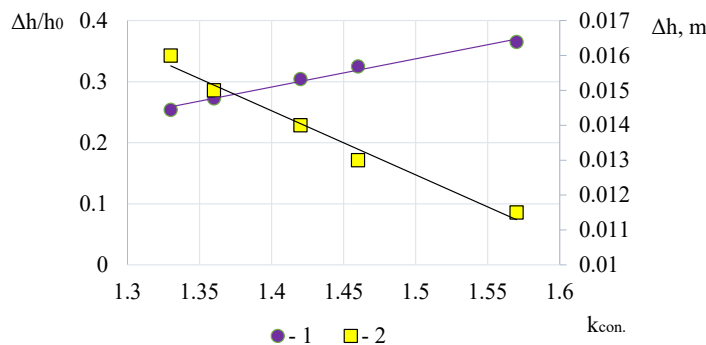


Fig. 9. Plots of change in the compaction coefficient  $k_{con}$  of crushed rock depending on the relative vertical  $\Delta h/h_0$  and longitudinal  $\Delta h$  (m) deformation of the rock layer under compressive compression conditions: 1 –  $\Delta h/h_0$ ; 2 –  $\Delta h$  (m)

It was registered that with an increase in the relative vertical deformation of the rock layer from  $\Delta h/h_0=0.25$  to  $\Delta h/h_0=0.36$ , the compaction coefficient  $k_{con}$  of crushed rock increases from  $k_{con}=1.33$  to  $k_{con}=1.57$ . At the same time, the

longitudinal deformation of the rock layer decreases from  $\Delta h=0.016$  m to  $\Delta h=0.0115$  m (Fig. 9). It is obvious that at the maximum values of the relative vertical deformation of the experimental samples, when their longitudinal deformation  $\Delta h$  (m) is minimal, the maximum stiffness of the fill material is ensured with the compaction coefficient of crushed rock  $k_{con}=1.57$ .

## 6. Discussion of results of studying the deformation properties of crushed rock under conditions of compressive compression

As a result of our studies into the deformation properties of crushed rock during compressive compression, the conditions under which the efficiency of its use is achieved were determined. Taking into account these conditions will limit the convergence of side rocks and prevent the collapse of the roof and sole in the created space of the coal massif with preparatory works. In these studies, the compression tests of the crushed rock were tests on the compression of a cylindrical specimen with a compressive load uniformly distributed on the end surface. Crushed rock was placed in a steel cylinder with a removable bottom and a rigid stamp.

The granulometric composition of the crushed rock determined the bulk density of the fill material (Table 2). During the experiments, it was established that under the conditions of compressive compression, there was a quadratic functional dependence between the bulk density of crushed rock and the specific potential energy of deformation of the fill material (Fig. 2, expression (8)).

The maximum value of the specific potential energy of the deformation of the fill material  $\sigma^2/2E_g=6.22$  MJ/m<sup>3</sup> was recorded during the compression of crushed rock with the size of fractions (0.1–5) mm, i. e., heterogeneous granulometric composition. In this case, the compaction coefficient of the initial material  $k_{con}=1.57$  (Fig. 2, Table 4).

For crushed rock of a shallow fraction (0.1–1) mm, the specific potential energy of deformation of the fill material reaches values of  $\sigma^2/2E_g=1.01$  MJ/m<sup>3</sup>, and the compaction coefficient of crushed rock  $k_{con}=1.14$ . In the presence of a large fraction (4–5) mm in size, which occupies 100 % of the volume of the fill material, the specific potential energy of deformation is equal to  $\sigma^2/2E_g=4.46$  MJ/m<sup>3</sup>, and the compaction coefficient  $k_{con}=1.43$  (Table 4).

When comparing the deformation characteristics, it was established that the greatest effect from the use of fill materials is achieved when their volume (composition) contains crushed rock of heterogeneous (in terms of fraction size) granulometric composition (Table 4, Fig. 2). In this case, maximum compression of the crushed rock is ensured with minimal stiffness of the fill material.

It is obvious that the critical level of the specific potential energy of the deformation of the fill materials is determined by the value of the limit values of the compaction coefficient of the crushed rock depending on the percentage content of fractions of different sizes in the total volume. The transition of the critical level means an increase in the bearing capacity of the fill material, which is associated with the compaction of crushed rock.

It is advisable to estimate the bearing capacity of the fill materials by the value of the compaction coefficient of the crushed rock. The stiffness of fill materials does not need to be considered as a constant physical characteristic as it is



a variable value that depends on the magnitude of the external load at maximum compression of the crushed rock. This is confirmed by the results of experiments in determining the deformation characteristics of fill materials with an increase (over a period of time) of force  $F$  (Table 5). It was registered that with an increase in external force, the stiffness of the fill material increased by 2–6 %.

Next, the crushed rock of different granulometric composition (heterogeneity coefficient  $k_h=4.8$ ) was used to study the deformation properties of fill materials. Such samples had various initial thicknesses of the rock layer  $h_0$  (m) and were subjected to uniaxial compression (Table 3). A separate case of uniaxial compression with an additional boundary condition, which implies the impossibility of lateral expansion of the rock, is compressive compression of the fill material (Tables 1, 2).

In order to evaluate the deformation characteristics of the experimental samples, we considered indicators reflecting the dependence of the longitudinal deformation  $\Delta h$  (m) on the increasing or limited external load  $F$  (kN). At the same time, the change in the compaction coefficient of crushed rock  $k_{con}$  was taken into account. In the process of compressive compression of the fill material, the amount of compression of the crushed rock  $\Delta h$  (m), i. e., the longitudinal deformation, reflecting the new state of equilibrium caused by the action of the applied load  $F$  (kN), was recorded. In the analysis of experimental data, the fact that each deformed body has its own structure of connections and corresponding field of deformations was taken into account. This field is determined by the dimensions of the body and the mechanical characteristics of the bonds, the level of the internal potential energy of the deformation.

When the crushed rock was compressed in the compression device, the diameter of the sample did not change. Therefore, the relative vertical deformation  $\Delta h/h_0$  of the fill material is equal to the relative change in volume  $\Delta V/V$  (expression (4), Tables 6, 7). In this case, compaction of crushed rock took place due to the reorientation of parts of different sizes while simultaneously reducing the hollowness of the fill material.

A body that deforms has a critical level of potential energy, after exceeding which the fill material changes its behavior under load. The transition through the critical level is associated with a change in the volume of the fill material, which means a change in hollowness.

For different initial thickness  $h_0$  (m) of the rock layer in the experimental samples, there is a linear relationship between the longitudinal deformation  $\Delta h$  (m) and the external load  $F$  (kN). Under such conditions, with the range of external force values of  $78 \text{ kN} \leq F \leq 150 \text{ kN}$ , maximum compression of the fill material is ensured. In this case, the relative change in volume for all samples subjected to compression corresponded to  $\Delta V=0.36$ , and with the same longitudinal deformation, the compaction coefficient of crushed rock is  $k_{con}=1.57$  (Table 6).

As the initial strength of the rock layer decreases from  $h_0=0.063 \text{ m}$  to  $h_0=0.0315 \text{ m}$ , the potential energy of deformation decreases from  $\sigma^2/2E_g=6.22 \text{ MJ/m}^3$  to  $\sigma^2/2E_g=3.23 \text{ MJ/m}^3$  and the compression work  $A=1616.1 \text{ J}$  to  $A=429.2 \text{ J}$ . Under such conditions, after the compaction of crushed rock in the total volume of the fill material, the bearing capacity of the support increases. It is obvious that the maximum effect from the use of crushed rock to limit the convergence of side rocks and prevent their collapse is ensured by reducing the initial height  $h_0$  (m) of the rock layer. Under real conditions – the thickness of the coal seam.

When limiting the external compressive load to  $F=78 \text{ kN}$ , there is a linear relationship between the compaction coefficient  $k_{con}$  of crushed rock and the value of the specific potential energy of deformation  $\sigma^2/2E_g$  ( $\text{MJ/m}^3$ ). Under such conditions, the relative change in the volume of the fill material was from  $\Delta V=0.25$  to  $\Delta V=0.36$ . When the volume of crushed rock in the compression process changed from  $\Delta V=0.000067 \text{ m}^3$  to  $\Delta V=0.000049 \text{ m}^3$ , the value of the compaction coefficient increased, from  $k_{con}=1.33$  to  $k_{con}=1.57$ . Moreover, the minimum value of relative deformation  $\lambda=0.25$  corresponds to the minimum value of  $k_{con}=1.33$  and vice versa, the maximum value of  $\lambda=0.36$  corresponds to the maximum value of  $k_{con}=1.57$ . Compaction is inevitably associated with the accumulation of potential energy in the fill material during compressive compression. The limit of energy accumulation in a deforming body is the achievement of a certain density of potential energy, which ensured maximum compression of the fill material.

One of the main requirements for fill arrays is their rigidity. With the longitudinal deformation of the rock layer  $\Delta h=0.016 \text{ m}$  and a constant load on the support, its stiffness is equal to  $C=4.87 \cdot 10^6 \text{ N/m}$ , and the compaction coefficient of the crushed rock is  $k_{con}=1.33$ . With a decrease in longitudinal deformation to the value  $\Delta h=0.011 \text{ m}$ , the stiffness of the fill material (support) reaches maximum values when  $C=6.78 \cdot 10^6 \text{ N/m}$ . At the same time, the compaction coefficient of crushed rock increased to  $k_{con}=1.57$ . The increase in the specific potential energy of deformation is 40 %, which can be explained by the increase in the stiffness of the fill material due to the compaction of the crushed rock after repacking its parts of different sizes (Fig. 7, 8).

Reaching a critical state means that the system parameters have reached their limit values. At the same time, the maximum compression of the fill material is ensured, and the deformed body has qualitatively changed the patterns of its behavior. It is obvious that the rigidity of protective structures under the action of external load depends on the magnitude of the longitudinal deformation of the rock layer.

In order to ensure the maximum rigidity of the fill mass, the specific potential energy of deformation of crushed rock must increase. This is due to the work of compressing crushed rock, changing its volume and repacking parts of different sizes in the total volume of the fill material. When the longitudinal deformation  $\Delta h$  of the experimental samples decreases, when their relative deformation reaches maximum values, and the external load is limited, the efficiency of this process increases (Fig. 9). This is determined by the height of the samples, as well as the expediency of filling the worked-out space when reducing the thickness of the coal seam.

Thus, having defined physical and mechanical characteristics, ground materials made of crushed rock can be used for flexible safety structures that perform the role of load-bearing structures. The internal potential energy of the deforming protective structures, when the crushed rock is compacted and the maximum compression is reached, reaches a critical level, the transition through which ensures their stable state. The maximum effect is achieved under conditions where, with minimum longitudinal deformation and maximum values of relative vertical deformation, the maximum stiffness of the bearing supports is ensured.

The results of research on the deformation properties of the fill material can be used to substantiate the parameters of flexible protective structures when supporting preparatory works in the excavation areas of coal mines. Such protective

structures, when they achieve maximum compression, can limit the convergence of side rocks in the worked-out space of the coal massif. In contrast to [18], the established result makes it possible to evaluate the deformation properties of crushed rock, as a system of fractions of fill materials, in the process of its compaction. In order to clarify the parameters of safety structures, it is advisable to carry out further research under natural conditions, which will make it possible to investigate the processes of formation of fill materials. At the same time, it will be possible to analyze the state of preparatory products in the coal-bearing massif.

The current study allows us to evaluate the deformation features of fill materials made of crushed rock under compressive compression. Such solutions are suitable for fill arrays when there is no possibility of lateral movement of the compressible source material.

---

## 7. Conclusions

---

1. It was experimentally established that under conditions of compression, between the bulk density  $\rho_{b.d.}$  of crushed rock and the specific potential energy of deformation  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) of the fill material, there is a quadratic functional dependence. The limiting value of the parameter  $\sigma^2/2E_g$  (MJ/m<sup>3</sup>) was recorded during the compression of crushed rock of heterogeneous (by particle size) granulometric composition, when the compaction coefficient of the initial material is  $k_{con}=1.57$ . The critical level of the specific potential energy of the deformation of the fill material is determined by the value of the limit values of the compaction coefficient of the crushed rock, depending on the percentage content of fractions of different sizes in the total volume of the fill material. The transition of the critical level determines the increase in the bearing capacity of the fill material, which is associated with the compaction of crushed rock.

2. It was experimentally established that during the compression of crushed rock, there is a linear relationship between the longitudinal deformation  $\Delta h$  (m) of protective structures and the external static load  $F$  (kN). Under such conditions, the convergence of the roof and the sole is limited in the worked-out space of the coal massif. For rock supports

with different initial thickness  $h_0$  (m) of the rock layer, with a decrease in the parameter  $h_0$  by a factor of 2 before their compression, the specific potential energy of deformation decreases simultaneously with the work of compression. With a relative change in the volume of the rock strips  $\Delta V=0.36$ , the compaction coefficient of the crushed rock reaches  $k_{con}=1.57$ . Due to this, with the same relative deformation of protective structures with different values of the parameter  $h_0$  (m), the maximum rigidity of the fill material is ensured.

3. It was experimentally established that under the conditions of compressive compression of crushed rock under a static load limited in magnitude, between the compaction coefficient  $k_{con}$  and specific potential energy of deformation, there is a linear functional dependence. Under such conditions, with a decrease in the initial thickness  $h_0$  (m) of the rock layer to compressive compression by a factor of 2, the compaction coefficient of crushed rock increases from  $k_{con}=1.33$  to  $k_{con}=1.57$  in the range of the relative change in the volume of rock supports  $0.25 \leq \Delta V \leq 0.36$ . At the same time, the specific potential energy of deformation increases by 40 %, due to which the maximum rigidity of the fill material is ensured with the minimum longitudinal deformation  $\Delta h$  (m) of the supports and makes it possible to evaluate their bearing capacity.

---

## Conflicts of interest

---

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

---

## Funding

---

The study was conducted without financial support.

---

## Data availability

---

All data are available in the main text of the manuscript.

---

## References

- Iordanov, I., Buleha, I., Bachurina, Y., Boichenko, H., Dovgal, V., Kayun, O. et al. (2021). Experimental research on the haulage drifts stability in steeply dipping seams. *Mining of Mineral Deposits*, 15 (4), 56–67. doi: <https://doi.org/10.33271/mining15.04.056>
- Galvin, J. M. (2016). *Ground Engineering – Principles and Practices for Underground Coal Mining*. Springer, 684. doi: <https://doi.org/10.1007/978-3-319-25005-2>
- Podkopaiev, S., Gogo, V., Yefremov, I., Kipko, O., Iordanov, I., Simonova, Y. (2019). Phenomena of stability of the coal seam roof with a yielding support. *Mining of Mineral Deposits*, 13 (4), 28–41. doi: <https://doi.org/10.33271/mining13.04.028>
- Petlovanyi, M., Malashkevych, D., Sai, K Zubko, S. (2020). Research into balance of rocks and underground cavities formation in the coal mine flowsheet when mining thin seams. *Mining of Mineral Deposits*, 14 (4), 66–81. doi: <https://doi.org/10.33271/mining14.04.066>
- Maydukov, G. L. (2007). Kompleksnoe ispol'zovanie ugol'nykh mestorozhdeniy Donbassa kak osnova ekologicheskoy bezopasnosti i energoberezhnyy v regione. *Ekonomichnyi visnyk Donbasu*, 4, 12–19.
- Krupnik, L. A., Shaposhnik, Yu. N., Shaposhnik, S. N., Tursunbaeva, A. K. (2013). Backfilling technology in Kazakhstan mines. *Journal of Mining Science*, 49 (1), 82–89. doi: <https://doi.org/10.1134/s1062739149010103> Bachurin, L. L., Iordanov, I. V., Simonova, Yu. I., Korol, A. V., Podkopaiev, Ye. S., Kaiun, O. P. (2020). Experimental studies of the deformation characteristics of filling massifs. *Technical Engineering*, 2 (86), 136–149. [https://doi.org/10.26642/ten-2020-2\(86\)-136-149](https://doi.org/10.26642/ten-2020-2(86)-136-149)

7. Bachurin, L., Iordanov, I., Kohtieva, O., Dovgal, V., Boichenko, H., Bachurina, Y. et al. (2021). Estimation of stability of roadways surrounding rocks in a coal-rock stratum considering a deformation characteristics of secondary support structures. *JOURNAL of Donetsk Mining Institute*, 1, 64–74. doi: <https://doi.org/10.31474/1999-981x-2021-1-64-74>
8. Czichos, H. (2013). *Physics of Failure. Handbook of Technical Diagnostics*, 23–40. doi: [https://doi.org/10.1007/978-3-642-25850-3\\_3](https://doi.org/10.1007/978-3-642-25850-3_3)
9. Nasonov, I. D. (1978). *Modelirovanie gornyx protsessov*. Moscow: Nedra, 256.
10. *Laboratorniy praktikum po kursu «Mekhanika gornyx porod»* (2012). Donetsk. Available at: <http://ea.donntu.edu.ua/bitstream/123456789/15314/1/Подкопаев%20С.В.%20Гавриш%20%20Н.Н.%20Деглин%20Б.М.%20Каме-нец%20В.И.%20Зинченко%20С.А.%20Лабораторный%20практикум%20по%20курсу%20%28Механика%20горных%20пород%29.pdf>
11. Iordanov, I., Novikova, Y., Simonova, Y., Yefremov, O., Podkopayev, Y., Korol, A. (2020). Experimental characteristics for deformation properties of backfill mass. *Mining of Mineral Deposits*, 14 (3), 119–127. doi: <https://doi.org/10.33271/mining14.03.119>
12. Stupishin, L. U. (2014). Variational Criteria for Critical Levels of Internal Energy of a Deformable Solids. *Applied Mechanics and Materials*, 578-579, 1584–1587. doi: <https://doi.org/10.4028/www.scientific.net/amm.578-579.1584>
13. Stupishin, L. Yu. (2011). Variatsionnyy kriteriy kriticheskikh urovney vnutrenney energii deformiruemogo tela. *Promyshlennoe i grazhdanskoe stroitel'stvo*, 8, 21–23.
14. Meshkov, Yu. Ya. (2001). The Concept of a Critical Density of Energy in Models of Fracture of Solids. *Uspehi Fiziki Metallov*, 2 (1), 7–50. doi: <https://doi.org/10.15407/ufm.02.01.007>
15. Tkachuk, O., Chepiga, D., Pakhomov, S., Volkov, S., Liashok, Y., Bachurina, Y. et al. (2023). Evaluation of the effectiveness of secondary support of haulage drifts based on a comparative analysis of the deformation characteristics of protective structures. *Eastern-European Journal of Enterprise Technologies*, 2 (1 (122)), 73–81. doi: <https://doi.org/10.15587/1729-4061.2023.272454>
16. Dekking, F. M., Kraaikamp, C., Lopushaä, H. P., Meester, L. E. (2005). *A Modern Introduction to Probability and Statistics*. Springer Texts in Statistics. doi: <https://doi.org/10.1007/1-84628-168-7>
17. Iordanov, I., Simonova, Y., Kayun, O., Podkopayev, Y., Polozhii, A., Boichenko, H. (2020). Substantiation of the stability of haulage drifts with protective structures of different rigidity. *Eastern-European Journal of Enterprise Technologies*, 3 (7 (105)), 87–96. doi: <https://doi.org/10.15587/1729-4061.2020.202483>