This paper considers the influence of higher harmonics in dynamic action systems due to their complex movement in the process of interaction with the technological load. The object of research is the process of propagation of oscillations in complex dynamic systems. One of the problems in the application of oscillatory processes is the consideration of higher harmonics in the overall movement of systems. To solve the problem, the idea of using a hybrid model that takes into account both discrete and distributed parameters was proposed. The resulting mathematical discrete model in the analytical equations of motion of the dynamic system preserves continuous properties in the form of wave coefficients. These coefficients in their analytical form take into account the contribution of higher harmonics of both the reactive (elastic-inertial) and active (dissipative) components of the resistance force. The studies were carried out on a model of a plant with a multimode spectrum of oscillations and a nonlinear dynamic system, which is a system with piecewise linear characteristics.A series of experimental studies with a wide variation of the change in the frequency of oscillations was carried out on the installation with a multimode spectrum of oscillations. Zones of manifestation of higher harmonics along the vertical axis of force action were revealed. The given spectrum at the exciter frequency of 35 Hz showed the manifestation of the spectrum component (around 70 Hz) along the X axis, which is an important result for practical application. For a system with piecewise linear characteristics, the manifestation of multimode, which manifests itself in the form of subharmonic and superharmonic oscillations, was determined. The contribution of each harmonic is determined by applying the obtained dependences. The results were used in the development of algorithms and calculation methods of a new class of dynamic action systems taking into account the contribution of higher harmonics

Keywords: dynamic system, technological load, continuous model, spectral characteristic, oscillation frequency

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DETERMINING THE INFLUENCE OF HIGHER HARMONICS OF NONLINEAR TECHNOLOGICAL LOAD IN DYNAMIC ACTION SYSTEMS

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1. Introduction

Systems of dynamic action occupy a prominent place in various industries, both to intensify the flow of one or another technological process, and to reduce the impact on the structures of various devices and structures. The effectiveness of their application is determined by the difference in modes and parameters, which is determined by a specific technology. Worth noting are significant achievements in modeling, theoretical and experimental research of dynamic action systems, which require wide application of software methods for calculating parameters of the work process. At the same time, in systems of complex structure, which generate energy for the flow of the technological process, there remain unsolved problems. One of these is the interaction of processing materials with working bodies, which can be of the same order of weight. In addition to the load, they change their physical and mechanical properties during the work process, which also affects the maintenance of the mode and parameters.Effective energy-saving modes are used extremely rarely, with the exception of modern small robots and manipulators. In the real practice of the processes, significant energy consumption occurs for the flow of work processes. So, for example, for resonant vibration machines of the construction industry, energy consumption is 5–7 times higher than resonant ones. The use of resonant energy-saving modes is restrained by the lack of generally accepted calculation models. Empirical or discrete models are mostly used in the calculations, which are adequate only within the limits of the conducted and obtained results. One of the ways to solve the problem is the idea of using a hybrid model that takes into account both discrete and distributed parameters. Mathematical solutions of similar problems are overcome by reducing discrete-continuous systems to discrete ones. Such a discrete model in the analytical equations of motion of the dynamic action system preserves the continuous properties of the processing media in the form of wave coefficients. These coefficients in their analytical form take into account both reactive and active resistance forces that take place in the real process of oscillations of any dynamic system. With this approach, it is possible to take into account the presence of harmonics higher than the fundamental one.

The search for solutions to reduce energy costs for the technological process is an actual area of research. The implementation of such studies will make it possible to clarify the calculation model, draw up a calculation algorithm on this basis and, as a result, reduce energy for the technological process.

2. Literature review and problem statement

Dynamic systems are used both to intensify the flow of one or another technological process, and to reduce the dynamic impact on the structures of various devices and structures. The basis of both the first and second directions of research are the classical theory of mechanical vibrations and the theory of solid media. For example, paper [1] reports the results of the study of the soil model with cubic nonlinearity, which can be used to describe seismic effects taking into account the heterogeneity of the soil massif. The authors of the work note the relevance and importance of taking into account layers with different dynamic characteristics, the possibility of resonance phenomena, as well as the possibility of strengthening and weakening dynamic action. This behavior of the dynamic system is due to the presence of wave phenomena and can be estimated on the basis of the energy analysis of the oscillation spectrum. Obviously, in the presence of such a spectrum, there is a need to take into account the influence of harmonics in the general behavior of the dynamic system. In [2], the problem of interaction between a moving train and a railroad track is considered. The main frequencies of oscillations of the railroad bed at different load speeds are determined and the parameters on which they depend are defined. In work [3], the spread of dynamic action from a railroad track in a soil massif was studied. The results of these studies indicate the presence of multi-frequency oscillations, the quantitative analysis of which from the point of view of energy indicators, the authors do not present in this study, but emphasize the importance of such an analysis. In [4], research is given on a complex dynamic system "source of vibrations – building structure" in which an additional element is used – a "damper", the purpose of which is damping of vibration action. It is quite obvious that the general system involves determining the main frequencies of oscillations and determining the response of such a system to external vibration.The solution to such a problem can be the assessment of the spectral characteristics of the external influence and the consideration of the general system as a frequency filter that extinguishes the negative effect. The second direction of research concerns the study of dynamic action systems with the search for parameters that can enhance the effect of oscillations due to energy accumulation, the use of complex forms of oscillations, etc. The research results relate to the main frequencies of oscillations, although the given multi-mass calculation scheme also has other harmonics of oscillations that need to be taken into account in the overall motion system. In [5], the search for such systems and the application of reliable models of real processes, which are based on the detection of patterns of change in the technological process of different physical properties of the working bodies of vibrating machines and processing environments, are reported. The cited paper proposes the idea of using a hybrid model that takes into account both discrete and distributed parameters.The authors of the work overcome the mathematical difficulties of solving such problems by reducing discrete-continuous systems to discrete ones. Such a discrete model in the analytical equations of motion of the "vibrating machine-processing environment" system preserves continuous properties in the form of wave coefficients. However, the given results do not take into account the higher harmonics of oscillations that are present in complex dynamic systems. The authors indicated the further development of research, in particular, consideration of the spectral analysis of dynamic action systems. Work [6] describes an increase in the intensity of ultrasonic impact on biological objects due to the concentration of ultrasonic energy. However, there is no analysis of the possible appearance of frequencies different from the main one. Work [7] considers the selection of forms of oscillations of a set of vibration modes on the example of beams and panels. The research results are obtained on the basis of numerical modeling, taking into account nonlinear effects, which are the source of the appearance of forms of oscillations different from the main one. The effectiveness of the application of such an approach requires an assessment of the energy component on each form of oscillation. The authors of works [8, 9] propose a scheme for the implementation of an oscillating system, which makes it possible to obtain an effect on the medium of oscillations with several different frequencies, which makes it possible to obtain the efficiency of compaction. At the same time, these works do not address the issue of the impact of higher vibration frequencies on the environment and technological equipment. The study of nonlinear characteristics and the application of the effects of combination modes are described in [10], where the problems of oscillatory action on nonlinear dynamic systems arising in various fields of science are considered. The occurrence of the phenomena of internal resonances of such systems can have significant applied and theoretical significance, in particular, due to the fact that the general properties of the systems can significantly affect the oscillatory actions.This approach requires clarification regarding the transition from the initial equations of motion to equations that describe the real motion of the system taking into account the spectrum of oscillations. The construction of a dynamic system with self-adaptive adjustment to external oscillations with different frequencies is considered in [11]. The authors demonstrate a system capable of converting mechanical energy into electrical energy based on the application of the internal properties of the system and the detection of internal resonances. At the same time, the authors determine the need for a more precise setting of the geometric parameters of the system to obtain a higher result. Experimental studies in the above areas generally confirm the need to study the phenomena of generation, transmission, and use of energy flows using the spectrum of oscillations. Experimental studies of measurements of accelerations in a dynamic model are reported in [12]. On the basis of the constructed vibration spectrum, natural vibration frequencies are determined. The described technique can be used in the study of more complex dynamic systems. The authors note the potential of parametric resonance at higher vibration frequencies, which is a promising area of research. Studies of the influence of man-made vibration within the city on the basis of experimental measurement of vibration accelerations are given in [13]. The authors proposed a method of finding and analyzing important points based on the construction of the spectrum of oscillogram records. This approach gives an idea of what happens during the technological process of tunnel drilling. The authors recommend monitoring the process based on spectral analysis and, if necessary, limiting the performance of work depending on the type of soil to be developed. This decision is determined by the different amount of energy transfer, which depends on the strength of the soil massifs and the distance to the monitoring object. Depending on such conditions, oscillations occur at different frequencies. As a result, resonant modes of the general dynamic model take place. Measurement of dynamic characteristics of systems for the purpose of diagnosis and monitoring of existing defects in metal structures is presented in work [14]. The method of application of experimental studies of vibration and their processing is described. It is proposed to improve the calculation model based on the obtained dynamic characteristics. The application of the method to nonlinear active vibration control systems is reported in [15]. According to the authors, this method makes it possible to obtain a result without significant knowledge about the dissipative properties of the system, which, of course, gives advantages and reduces the time for research. As for the means of measurement, various types of sensors can be used. Thus, in work [10], acceleration measurement sensors are used. Thus, the search for technical solutions for the implementation of energy-efficient modes of operation for the implementation of technological processes is an actual direction of research, as evidenced by the above review. It is obvious that in real dynamic systems there are complex processes due to nonlinear phenomena. Such phenomena cause oscillations with frequencies different from the fundamental frequency. As a rule, the consideration of such fluctuations in the general motion of a dynamic system is not considered at the level of mathematical models. Experimental models, on which such regimes are found, need to be refined taking into account such phenomena. Therefore, taking into account higher harmonics in the general process of oscillations will make it possible not only to assess their energy impact but also serve as a prerequisite for the construction of new systems of dynamic action.

3. The aim and objectives of the study

The purpose of this work is to determine the influence of higher harmonics that arise in the technologically loaded system of dynamic action, which will ensure a reduction in energy consumption and increase the efficiency of the technological process.

To achieve the goal, the following tasks were defined:

– to develop a calculation scheme of the dynamic action system taking into account the influence of non-linear technological load and justify the research methodology;

– to obtain and analyze the spectra of vibration frequencies as a response of the structure to external sources of vibration;

– to determine modes and rational parameters taking into account higher harmonics to reduce energy consumption in dynamic action systems.

4. The study materials and methods

The object of our research is the process of propagation of oscillations in complex dynamic systems.

The theoretical studies, conclusions, and proposals reported in this work are based on the fundamental laws of physics, the impact theory, generally accepted provisions of the classical theory of oscillations, discrete vibration systems, and continuous media. Physical and mathematical models are built on the basis of ideas about the change of elastic, inertial, and dissipative properties of the "machine - environment" system. The working hypothesis of the construction of energy-saving systems is based on the purposeful harmonization of the movement of the "machine - environment" system with the rational use of their internal properties.

The mathematical model of the structure of the vibrating installation is built according to the following assumptions. The metal structures of the working body are modeled with discrete parameters and perceive only elastic deformations. The technological load is modeled by a system with distributed parameters.

Experimental studies were carried out using a station for collecting information from measuring sensors (developed by us) and a three-axis station ZET 048C.

Processing of the results was carried out using the methods of mathematical statistics, Fourier analysis.

5. Results of investigating the influence of higher harmonics arising in the technological load on the movement of a dynamic action system

5. 1. Development of a calculation scheme of a dynamic system and a procedurefor researching its movement

An installation with a multi-mode spectrum of oscillations was adopted as the calculation scheme of the force action system (Fig. 1, *a*). To determine the resistance forces of the technological load taking into account the higher harmonics, a highlighted element of the technological load along the *Z* axis is given (Fig. 1, *b*).

The mathematical model of the motion of the technological load is adopted as a continuum system, described by a wave equation in the following form:

$$\frac{\partial \mathrm{I}\, u(z,t)}{\partial z \mathrm{I}} = \frac{\rho^*(z,t)}{E^*(z,t)} \cdot \frac{\partial \mathrm{I}\, u(z,t)}{\partial t \mathrm{I}}, \qquad (1)$$

where ρ is the density of the medium; E^* – complex modulus of elasticity; *u* – displacement of the technological load layer along the *Z* coordinate. If we accept the law of force change:

$$F(t)=\sum_{n=-\infty}^{+\infty} F_n e^{in\omega t}, \tag{2}$$

where $\omega=2\pi/T,\ n=\pm1,$

$$F_n=F_n^*=\int_{-\tau/2}^{\tau/2} F(\tau)e^{-in\omega t}\mathrm{d}\tau,$$

then the solution to the original equation, according to the Fourier method, can be represented by a complex wave function:

$$u(z,t)=\sum_{n=-\infty}^{+\infty}\left(U_n e^{k_n z}+U_n e^{-k_n z}\right)e^{in\omega t}. \tag{3}$$

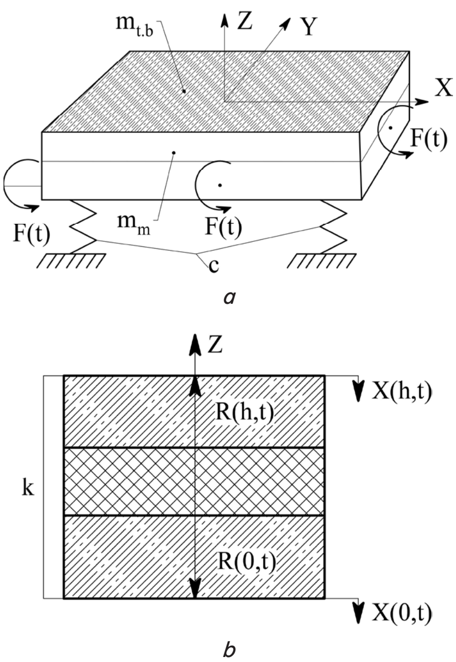

Fig. 1. System of dynamic action: *a* — calculation scheme; *b* — selected element of the technological load along the *Z* axis

The displacement U is determined by the product of two functions, one of which depends on the argument $z(z)=U_1 e^{nk_n z}-U_2 e^{-nk_n z}$, and the other – only on the argument $T_n(t)=e^{in\omega t}$.

In solution (3), U_{1n} and U_{2n} are constants determined from boundary conditions; n – harmonic number; k_n – complex wave number; $k_n=(\alpha_n+i\beta_n)$, where α_n, β_n are the coefficients obtained by substituting solution (3) into the wave equation (1):

$$\alpha_n=\frac{\omega_n}{c_v}\sqrt{\frac{\sqrt{1+\gamma^2-1}}{2\left(1+\gamma^2\right)}};$$

$$\beta_n=\frac{\omega_n}{c_v}\sqrt{\frac{\sqrt{1+\gamma^2}+1}{2\left(1+\gamma^2\right)}}. \tag{4}$$

When performing transformations (4), it is taken into account that the complex modulus of elasticity is expressed by the dependence:

$$E^*=E(1+i\gamma). \tag{5}$$

When finding dependence (6), the following equality was used:

$$(\alpha_n+i\beta_n)^2=-n^2\omega^2/\left[c_v^2(1+i\gamma)\right], \tag{6}$$

where c_v, as in dependence (4), determines the speed of propagation of waves in the technological load.

The reactive and active components of the resistance of the technological load are determined separately for a clearer representation of the idea and essence of the method for estimating the contribution of higher harmonics under the following condition:

$$x^*(t)=\begin{cases}-x_1\sin n\left(\dfrac{\pi}{\tau_1}t\right), 0\le t\le\tau_1\\[2ex] \hline x_2\sin n\left(\dfrac{\pi}{\tau_2}t\right), \tau_1\le t\le\tau_2\end{cases}. \tag{7}$$

Here, x_1 and x_1 are the amplitude of oscillations at the corresponding times of the system movement: T, τ_1, τ_2, n – the harmonic number.

According to Newton's law, the reactive component of the reaction of the medium is represented in the following form:

$$R_{mn}=-m_{mn}\ddot{x}, \tag{8}$$

where $\ddot{x}$ is the acceleration of the contact zone;

m_{mn} is part of the mass of the technological environment, which identifies the reactive component resistance.

On the other hand, the technological load resistance force:

$$R_{mn}=-ES\frac{\partial U}{\partial z}\Big|_{Z=0}, \tag{9}$$

where $\frac{\partial U}{\partial z}\Big|_{Z=0}$ – contact layer distortion (Fig. 1, *a*).

To determine the acceleration $\ddot{x}$, the original wave equation (1), written relative to the acceleration, was used:

$$\ddot{\chi}=\frac{\partial^2 u}{\partial t^2}\Big|_{Z=0}=c^2(1+i\gamma)\frac{\partial U}{\partial z}\Big|_{Z=0}. \tag{10}$$

Comparing (9) and (10), as well as taking into account (8), we obtained:

$$m_{mn}=\frac{-ES\dfrac{\partial U}{\partial z}\Big|_{Z=0}}{c^2(1+i\gamma)\dfrac{\partial^2 u}{\partial z^2}\Big|_{Z=0}}. \tag{11}$$

Thus, the task of finding the reactive resistance, or rather the coefficient m_c', which has the dimensionality of mass and determines the reactive resistance, is reduced to finding the deformation $\frac{\partial u}{\partial z}$ and its derivative $\frac{\partial^2 u}{\partial z^2}$ at the boundary $z=0$ (Fig. 1, *a*). From (10), we can determine the strain:

$$\frac{\partial u}{\partial z}\Big|_{Z=0}=(\alpha_1+i\beta_1)[U_{1n}-U_{2n}],$$

$$\frac{\partial u}{\partial z}\Big|_{Z=h}=0;\ U_{1n}e^{h(\alpha_1+i\beta_1)n}-U_{2n}e^{-h(\alpha_1+i\beta_1)n}=0; \tag{12}$$

hence:

$U_{1n} / U_{2n} e^{-2h(\alpha_1+i\beta_1)_n}$.

Then the derivative of deformation (12):

$$\frac{\partial^2 u}{\partial z^2}\Big|_{Z=0} = (\alpha_1 + i\beta_1)(U_{1n} + U_{2n}). \quad (13)$$

By substituting (12), (13) into (11), subject to (10), the formula for calculating the coefficient m'_c is obtained

$$m'_c = -\frac{ES}{(\alpha_1 + i\beta_1)} \frac{\sum_{n=-\infty}^{+\infty} nU_{2n}\left(\frac{U_{1n}}{U_{2n}} - 1\right)e^{in\omega t}}{\sum_{n=-\infty}^{+\infty} n^2 U_{2n}\left(\frac{U_{1n}}{U_{2n}} + 1\right)e^{in\omega t}} =$$

$$= \frac{ES}{(\alpha_1 + i\beta_1)} \frac{\sum_{n=-\infty}^{+\infty} nU_{2n}\left[1 - e^{2h(\alpha_1+i\beta_1)n}\right]e^{in\omega t}}{\sum_{n=-\infty}^{+\infty} n^2 U_{2n}\left[1 - e^{2h(\alpha_1+i\beta_1)n}\right]e^{in\omega t}}. \quad (14)$$

Taking the boundary conditions:

$$\frac{\partial U}{\partial Z}\Big|_{Z=h} = 0 \text{ and } U\big|_{Z=0} = x(t),$$

as well as the expansion of function (10), we obtain:

$$\sum_{n=-\infty}^{+\infty} U_{2n}\left[1 + e^{2h(\alpha_1+\beta_1)n}\right]e^{in\omega t} = \sum_{n=-\infty}^{+\infty} x_n e^{in\omega t}.$$

Then:

$$U_{2n} = \frac{x}{1 + e^{-2h(\alpha_1+\beta_1)n}}. \quad (15)$$

Taking into account (10), the formula for determination of m'_c takes the form:

$$m'_c = \frac{\frac{ESk_{1n}}{\omega^2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}(-n^2\omega^2)x_{1n}e^{i(n\omega t+\phi_n)} th\frac{k_{1n}h}{n}}{\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}(-n^2\omega^2)x_{1n}e^{i(n\omega t+\phi_n)}}. \quad (16)$$

It follows from (16) that it depends on time t, the shape of the pulse (x_{1n},φ_n), the height of the technological load in the direction of force action h, and the frequency ω.

The real part of dependence expression (11):

$$m'_c = \frac{\frac{ESk_{1n}}{\omega^2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}(-n^2\omega^2)x_{11}\cos(n\omega t+\phi_n)\frac{1}{n}\mathrm{Re}\left[k_{11}th(k_{1n}h)\right]}{\sum_{n=1}^{\infty}(-n^2\omega^2)x_{1n}\cos(n\omega t+\phi_n)} =$$

$$= \frac{\frac{ESk_{1n}}{\omega^2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}(-n^2\omega^2)\sqrt{\begin{matrix}\mu_n^2+\\+\nu_n^2\end{matrix}}\cos\left[n\omega t + \mathrm{arctg}\left(-\frac{dn}{\alpha n}\right)\right]\frac{1}{n}\mathrm{Re}\left[k_{11}th(k_{1n}h)\right]}{\sum_{n=1}^{\infty}(-n^2\omega^2)\sqrt{\mu^2+\nu^2}\cos\left[n\omega t + \mathrm{arctg}\left(-\frac{dn}{\alpha n}\right)\right]}.$$

After transforming this dependence, we can finally write the expression for equivalent reactance:

$$R_p = -m'_c\ddot{x},$$

where

$$m'_c = \frac{\frac{ES}{\omega^2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}(-n^2\omega^2)\sqrt{\mu_n^2+\nu_n^2}\cos\left[n\omega t + \mathrm{arctg}\left(-\frac{dn}{\alpha n}\right)\right]\frac{N_{11}}{n}}{\sum_{n=1}^{\infty}(-n^2\omega^2)\sqrt{\mu^2+\nu^2}\cos\left[n\omega t + \mathrm{arctg}\left(-\frac{dn}{\alpha n}\right)\right]}. \quad (17)$$

Here:

$$N_{11} = \frac{\alpha_{11}sh(2\alpha_{1n}h) - \beta_{11}\sin(2\beta_{1n}h)}{ch(2\alpha_{1n}h) + \cos(2\beta_{1n}h)};$$

$$\alpha_{1n} = \frac{n\omega}{c_v\sqrt[4]{1+\gamma^2}}\cos\left[\frac{1}{2}\mathrm{arctg}(-\gamma)\right];$$

$$\beta_{1n} = \frac{n\omega}{c_v\sqrt[4]{1+\gamma^2}}\sin\left[\frac{1}{2}\mathrm{arctg}(-\gamma)\right].$$

Now it is possible to determine the active resistance by taking the second component of equation (10):

$$R_a = iES\gamma\frac{\partial U}{\partial z}\Big|_{Z=0}. \quad (18)$$

Equating this force to the equivalent force of viscous resistance $F_c = b\dot{x}$, we obtained:

$$R_a = i\gamma ES\sum_{n=1}^{\infty}k_{1n}th(k_{1n}h)x_{1n}e^{in\omega t} =$$

$$= \frac{ES\gamma}{\omega}\sum_{n=1}^{\infty}ink_{11}\omega th(k_{1n}h)x_{1n}e^{in\omega t}. \quad (19)$$

By analogy with the procedure for finding the reactance, (10) can be represented as:

$$\frac{ES\gamma}{\omega}\sum_{n=1}^{\infty}ink_{11}\omega th(k_{1n}h)x_{1n}e^{in\omega t} = b_{ek}\sum_{n=1}^{\infty}i`n\omega x_{1n}e^{in\omega t},$$

where the equivalent resistance coefficient is:

$$b_{ek} = \frac{\frac{ES\gamma}{\omega}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}nk_{11}th(k_{1n}h)x_{1n}e^{in\omega t}i\omega}{\sum_{n=1}^{\infty}in\omega x_{1n}e^{in\omega t}} =$$

$$= \frac{\frac{ES\gamma}{\omega}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}nk_{11}th(k_{1n}h)x_{1n}e^{in\omega t}}{\sum_{n=1}^{\infty}nx_{1n}e^{in\omega t}}. \quad (20)$$

Having carried out the procedure of separation (20) into real and imaginary parts of this expression, the resistance coefficient of the medium is:

$$b_{ek.} = \frac{\frac{ES\gamma}{\omega}\sum_{n=1}^{\infty}n\sqrt{\mu_n^2+\nu_n^2}\cos\left[n\omega t + \mathrm{arctg}\left(-\frac{dn}{\alpha n}\right)\right]N_{11}}{\sum_{n=1}^{\infty}n^2\sqrt{\mu_n^2+\nu_n^2}\cos\left[n\omega t + \mathrm{arctg}\left(-\frac{dn}{\alpha n}\right)\right]}. \quad (21)$$

The resulting dependences (16) and (21) determine the reactive and active resistance of the technological load to oscillations, taking into account higher harmonics for systems of dynamic action with harmonic excitation. These coefficients take a discrete form, which greatly simplifies their representation in the joint equation of motion of the dynamic action system. It becomes possible to use the coefficients m_c' and b_{ek} in the equations of motion of hybrid dynamic systems, which include concentrated parameters of the power system (mass m_m, resistance coefficient b) and distributed parameters of the technological load:

$$\left(m_m + m_c'\right)\ddot{x} + \left(b + b_{ek}\right)\dot{x} + cx = F_0 e^{i\omega t}. \qquad (22)$$

For nonlinear dynamic systems, which are systems with piecewise linear characteristics, not only one, but also several stable periodic modes may occur. Such regimes are considered multi-mode. In nonlinear systems, multimodality manifests itself in the form of subharmonic and superharmonic oscillations, which are formed on the basis of free oscillations of the system, which are supported by an external forced force. The conditions for the implementation of these modes are as follows. If n periods of the forced force having a period T_ω approximately coincide with m periods of free oscillations:

$$nT_\omega = mT. \qquad (23)$$

Under the condition m=1 – subharmonic oscillations will be realized, and when n=1 and $m \geq 2$ – superharmonic oscillations.

Fulfillment of condition (23) at any values is a resonance condition (for example, with $m=n$ – the principal, main resonance). In the case of subharmonic resonance, large quasi-linear oscillations occur in the system, which are supported by external forced forces, the frequency of which is an integer times the frequency of free oscillations. In the case of superharmonic resonance, large quasi-linear oscillations are also manifested, which are supported by external forced forces, the frequency of which is an integer times less than the frequency of free oscillations. Although with superharmonic oscillations the oscillation period coincides with the forced force period, the contribution of higher harmonics to the solution is very significant. By choosing the main harmonic near $2\pi/\tau_0$, we can significantly increase the amplitude of the 3/2 harmonic, 5/2 harmonic, etc.; τ_0 is the impact time. To enhance the contribution of superharmonics, it is worth choosing frequencies near $2\pi/(\tau_0+\Delta)$. Such a real situation depends on the compelling force, elastic characteristics of oscillations, and inertial properties of masses.

The contribution of each harmonic is determined by the function:

$$\frac{\pi\left|x\right|}{2\left(x_1 \cdot x_2\right)} = \frac{\left|\sin\dfrac{n\omega\tau_0}{2}\right|}{n}. \qquad (24)$$

In order for the first n harmonics to be different from zero, it is necessary to satisfy the condition $N=1/(1+n)$, where $N=\tau_0/\mathrm{T}$ is the ratio of the impact time to the period. The rational number of harmonics can be determined from the followingcondition:

$$\frac{\left|\sin\left(\dfrac{n\omega_0 T_0}{2}\right)\right|}{n\left|\sin\left(\dfrac{\omega\tau_0}{2}\right)\right|} \leq N. \qquad (25)$$

The practical use of dependences (20) and (25) requires knowledge of the acoustic properties of the technological load. It is also necessary to know the shape of oscillations in the form of movement or acceleration of the contact zone. After all, (20) and (21) include the coefficients of the Fourier series. Therefore, in the case of a complex (non-harmonic) law of change of motion of the contact zone, it is worth approximating its oscillations by some function and finding the amplitude x_n at the frequency ω_n. The further calculation procedure is carried out using the same method, with the only difference that the search for wave coefficients is evaluated at each frequency ω.

5. 2. Analysis of the obtained oscillation frequency spectra as a structure response to external sources of vibration

The experimental model of the dynamic action system is represented in the form of an installation (Fig. 2, *a*), which is equipped with two, asymmetrically installed vibration exciters of oscillations (Fig. 2, *b*), equipped with sensors for the unbalanced mass position.

The oscillation frequency of the experimental setup could vary from 4 to 55 Hz. That range made it possible to obtain a set of experimental data, on the basis of which verification of theoretical studies was carried out.

A series of experiments was conducted, the recorded oscillograms of oscillations were analyzed in detail, the numerical values were entered in a table, followed by spectral analysis by the discrete Fourier transform method.

The obtained spectra were analyzed in order to determine the numerical values of the oscillation frequencies, which correspond to the main peaks on the spectrograms and are a consequence of the structure's response to external sources of vibration.

Spectral density was adopted as an evaluation criterion.

Fig. 3 shows vibrograms in the direction of the *X, Y*, and *Z* axes.

The resulting spectra (Fig. 4) indicate the predominant influence of oscillations and the transmission of maximum power precisely at the frequency of 23.4 Hz; the influence of superharmonics is practically not manifested in any of the directions.

Fig. 5 shows the vibrograms of the movement of the installation at the frequency of the forcing force of 35 Hz.

The vibrogram (Fig. 5, *a*) indicates a mode close to resonance, and the scaled vibrogram (Fig. 5, *b*) shows the effect of the high-frequency component.

a *b*

Fig. 2. Experimental setup: *a* – general view; *b* – vibration exciter with an unbalanced mass position sensor

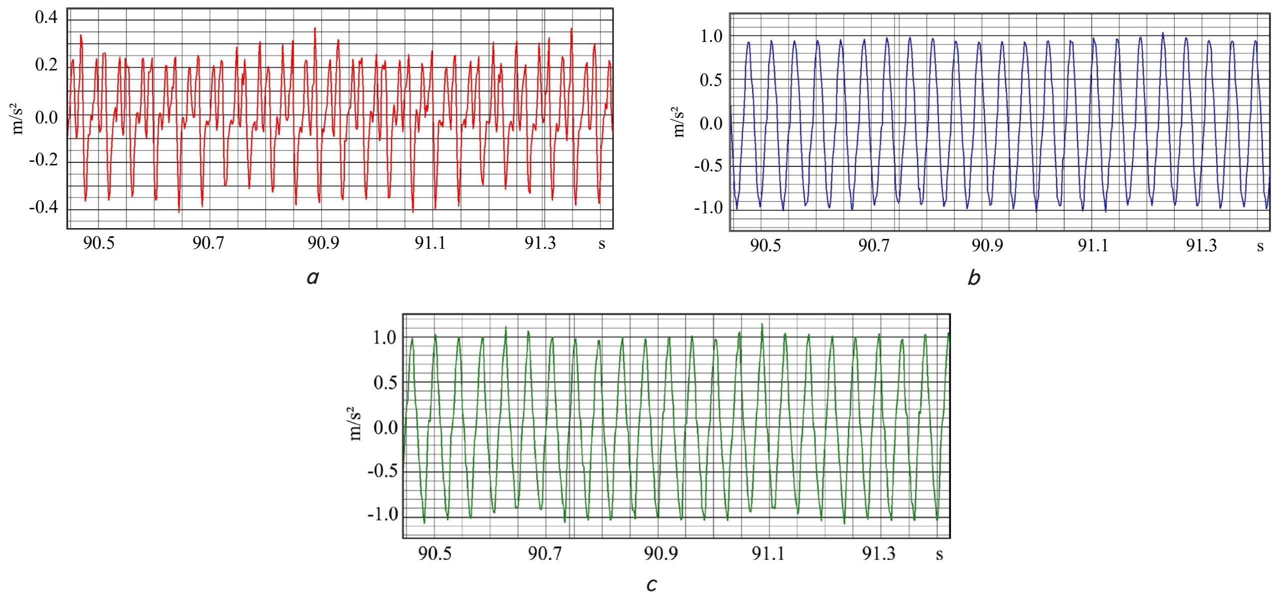

Fig. 3. Vibrogram of vibrations of the installation (23.4 Hz): *a* – *X* axis; *b* – *Y* axis; *c* – *Z* axis

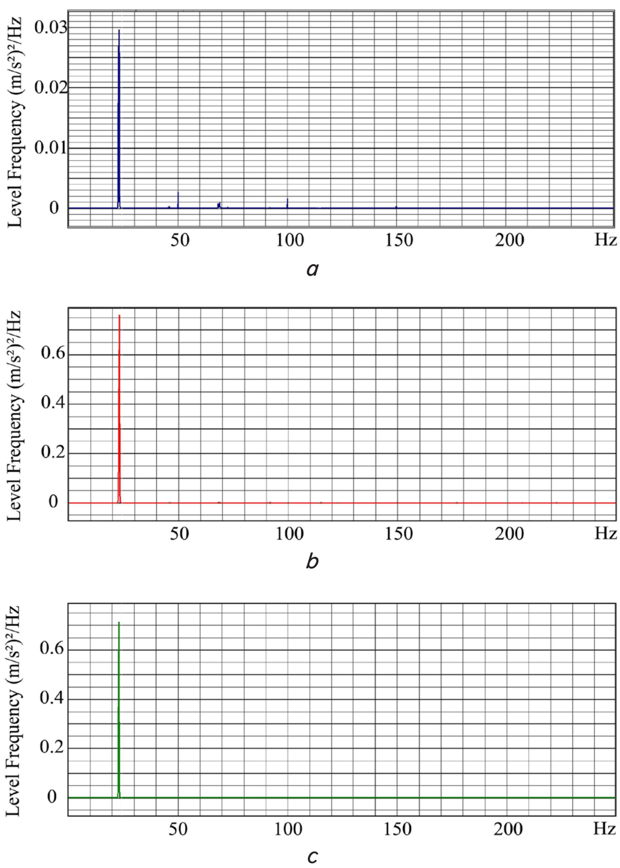

Fig. 4. Spectrum of vibrations of the installation (23.4 Hz): *a* – *X* axis; *b* – *Y* axis; *c* – *Z* axis

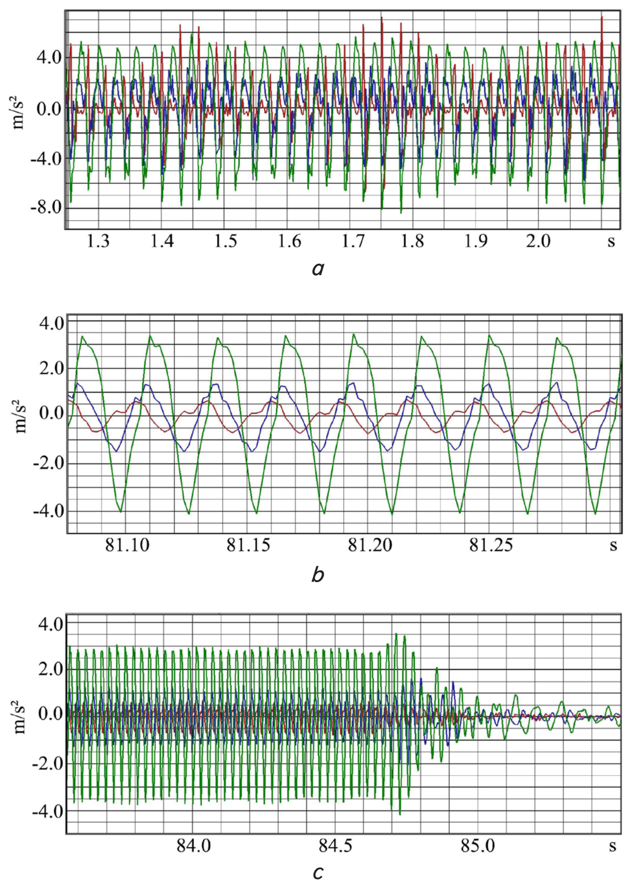

Fig. 5. Vibrogram of vibrations of the installation (35 Hz): *a* – general vibrogram of movement; *b* – scaled vibrogram; *c* – installation stop implementation

The given spectrum (Fig. 6) at the excitation frequency of 35 Hz shows the manifestation of the component of the spectrum (near 70 Hz) along the *X* axis, which is an important result of this experiment.

Vibrograms of installation's oscillations from stable mode to stop are shown in Fig. 7.

A wide spectrum of oscillations is noted in all three directions of oscillations of the installation (Fig. 7), which is confirmed by the spectrograms in Fig. 8.

Our studies confirm the possibility of effective use of spectral analysis for the assessment of dynamic processes in complex systems. Such evaluation criteria as spectral power and spectral density can be used for adequate analysis, identification of the main frequencies of oscillations, their harmonics, and allow taking into account the influence of higher harmonics on the process under study.

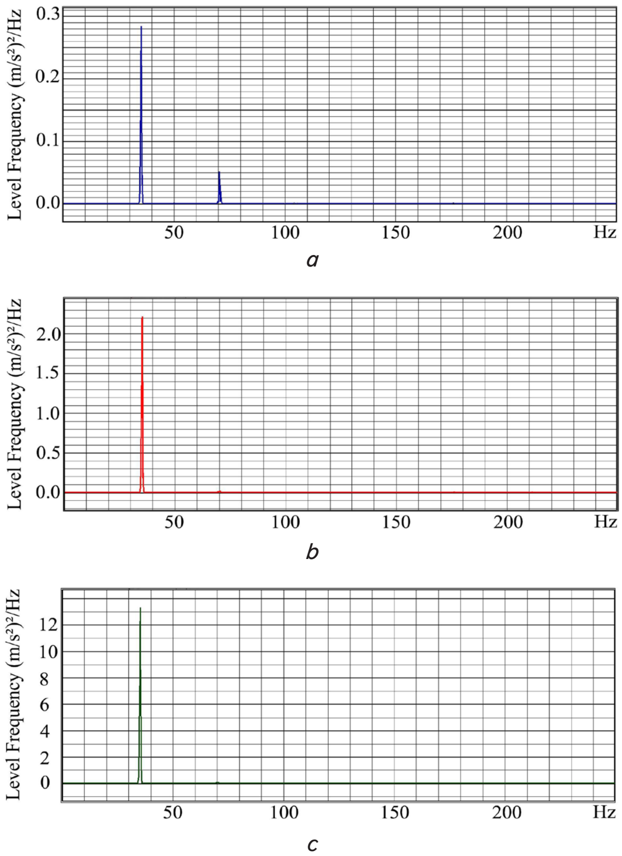

Fig. 6. Spectrum of vibrations of the installation (35 Hz): *a* – *X* axis; *b* – *Y* axis; *c* – *Z* axis

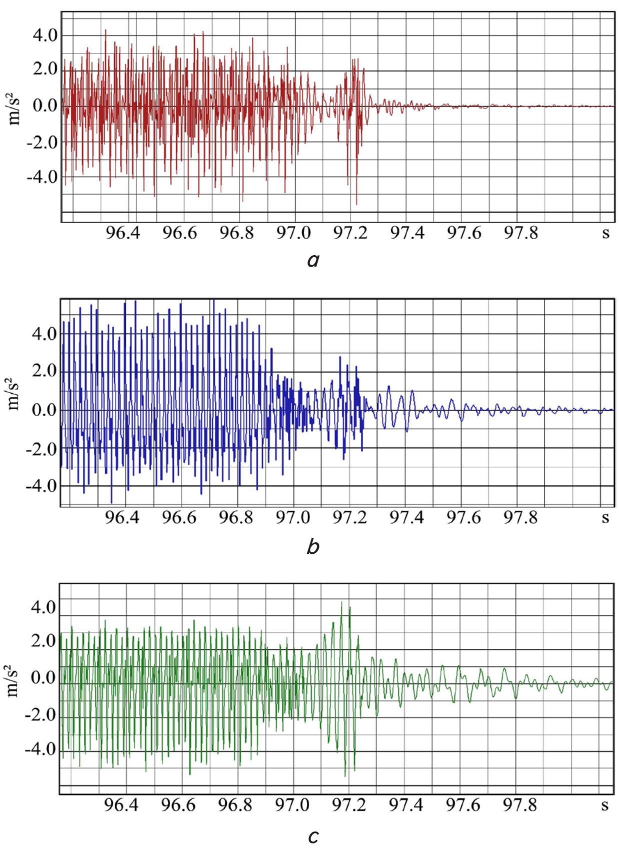

Fig. 7. Vibrogram of oscillations of the installation from stable mode to stop (49–0 Hz): *a* – *X* axis; *b* – axis *Y*; *c* – *Z* axis

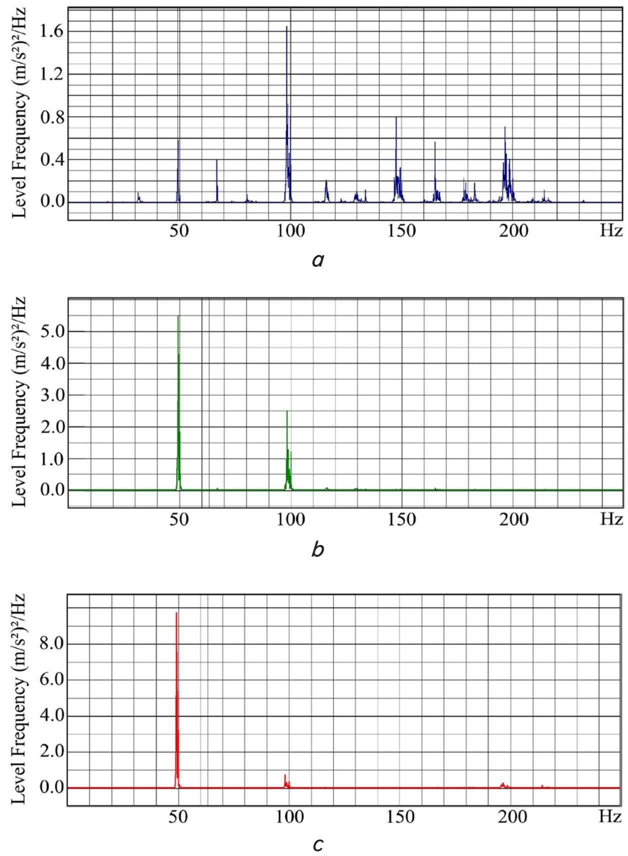

Fig. 8. Oscillation spectrum of the installation (frequency 49 Hz): *a* – *X* axis; *b* – axis *Y*; *c* – *Z* axis

5. 3. Determination of modes and rational parameters taking into account higher harmonics

Various spectra of detected frequencies (Fig. 4, 6, 8) different from the main, external frequency of oscillations were obtained by experimental studies on the installation with a multimode spectrum of oscillations (Fig. 2). At the same time, higher harmonics were not detected at some frequencies (23.4 Hz, Fig. 4, b, c). Thus, the obtained modes of operation can be marked as modes without significant influence of higher harmonics. The implementation of operating modes at higher frequencies made it possible to obtain the result of the influence of multi-mode oscillations. Thus, at the excitation frequency (35 Hz, Fig. 6), there is a peak on the Z axis in the spectrogram, which is a confirmation of the influence of oscillations with a frequency of 70 Hz. This result proves the existence of complex forms of oscillations with the contribution of higher harmonics.

Thus, in the calculations of vibrating machines, it is necessary to take into account not only the initial numerical values of the amplitude-frequency mode of the oscillator but also the form of oscillations, which is realized at the same time.

This fundamentally new result is evidence that significant energy savings can be obtained under this mode since it is significantly dependent on frequency. It is worth noting the significant manifestation of oscillations different from the main one on the frequency spectrum of the installation (Fig. 8) at a frequency of 49 Hz along all axes: *Z*, *X*, and *Y*. These modes are fundamentally new in the calculations and development of a similar class of dynamic action systems.

6. Discussion of research results and determination of modes and rational parameters, taking into account higher harmonics

As a result of theoretical studies, new dependences of wave coefficients (16) and (21) were obtained, which take into account the contribution of higher harmonics in the oscillation process. The application of such dependences opens up the possibility of representing their simple notation in the oscillation equation (22). Actually, these coefficients, having a discrete form, figuratively speaking, have a "memory" of their nature of preservation and consideration of wave phenomena in calculations. For nonlinear oscillations, on the example of vibrosystems with piecewise linear characteristics, dependenceswere obtained for determining the contribution of each harmonic (24) and their rational number (25). The resulting analytical dependences allow taking into account higher harmonics of oscillations in the general spectrum of oscillations. Experimentally, it is possible to evaluate such action [2, 3] and determine the main frequencies of oscillations. The result of our work makes it possible to estimate the impact of many frequency fluctuations based on an analytical solution that saves time. The construction of complex vibration systems requires calculation with given parameters, and this research allows us to solve such a problem. This approach is provided by promising research [5], the results obtained in this work partially reflect this within the scope of the implementation of the compaction process. The presence of higher harmonics in the studied system is due to the nonlinear effects of both the design of the installation and the technological load. In contrast to [7], our result was achieved by experimental studies taking into account physical nonlinearity. Thus, it becomes possible to take into account the features of dynamic systems and use such properties to implement more effective modes of oscillation. A similar result was obtained during the implementation of parametric resonance [10]. In contrast to that result, the proposed approach allows taking into account a larger number of frequencies, and therefore a larger number of such resonances. Also, the obtained dependences make it possible to take into account nonlinear phenomena that occur in complex dynamic systems.

Our research has limitations in terms of experimental determination of the contribution of higher harmonics for vibration systems with piecewise linear characteristics. The limited number of models in the part of experimental research can be attributed to the shortcomings of this study. After all, each dynamic system is special, so the obtained results can be applied only to systems similar to the one under study. Difficulties may arise in this aspect, which are associated with the need for additional research on other models of complex systems. Such studies are planned as a continuation of the considered topic based on the idea of comparing completed and planned studies. The proposed method of determining the contribution of higher harmonics can be successfully applied to such energy-intensive processes as grinding and sorting of materials. Such processes take place in mobile crushing and sorting plants, which are widely used. This is especially important for the construction of energy-saving equipment and for Ukraine under the conditions of restoration of destroyed objects and processing of materials.

7. Conclusions

1. We have devised the calculation scheme of the installation with a multimode spectrum of oscillations and substantiated a procedure for theoretical determination of the contribution of higher harmonics to dynamic action systems.

2. We have established and analyzed vibration spectra of the design of the experimental setup in the range from 4 to 55 Hz. The operating modes of the installation with oscillation frequencies of 23.4 Hz, 35.0 Hz, and 49.0 Hz, which correspond to close to resonant ones. Theresult confirms the hypothesis of using the internal properties of the dynamic system to implement energy-efficient modes of operation, taking into account higher harmonics.

3. Experimental studies at the installation with a multimode spectrum of oscillations confirmed the energy-saving mode at the excitation frequency of 35 Hz with the manifestation of higher harmonics of 70 Hz. Theresult is explained by the peculiarity of the studied system, in which manifestations of higher harmonics of oscillations are possible.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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