UDC 004.942:004.75
DOI: 10.15587/1729-4061.2023.286187

CONSTRUCTING THE MATHEMATICAL MODEL OF A RECOMMENDER SYSTEM FOR DECENTRALIZED PEER-TO-PEER COMPUTER NETWORKS

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Recommender systems make it easier to search with a large amount of content, supplementing or replacing the classic search output with recommendations. In P2P networks, their use can have additional benefits. Because of indexing and search problems, previously added files may not be available to P2P network users. If the user cannot find the file he is looking for, one can provide him with a list of recommendations based on his preferences and search query.

The object of research is the process of creating recommendations for users of decentralized P2P networks to facilitate data search.

The urgent task of increasing the accuracy of mathematical modeling of recommender systems by taking into account the requirements for reliability and data security during changes in the structure of a decentralized P2P network is solved.

An analytical model of the recommender system of a decentralized P2P network has been developed, the main feature of which is taking into account the requirements of reliability and security of recommendation messages. This was done by introducing the following indicators into the general model of the decentralized recommender system – the probability of reliable packet transmission and the probability of safe packet transmission. The developed analytical model makes it possible to conduct a comparative analysis of different methods of operation of recommender systems and to set acceptable parameters under which the degree of relevance does not fall below a certain threshold.

The developed mathematical model of the system based on the GERT scheme differs from the known ones by taking into account the reliability and security requirements during changes in the structure of the decentralized P2P network. This has made it possible to improve the accuracy of simulation results up to 5 %.

The proposed mathematical model could be used for prototyping recommender systems in various fields of activity

Keywords: recommender system, decentralized computer network, peer-to-peer network, GERT network, information security

Received date 29.05.2023
Accepted date 07.08.2023
Published date 30.08.2023

***How to Cite:** Mikhav, V., Semenov, S., Meleshko, Y., Yakymenko, M., Shulika, Y. (2023). Constructing the mathematical model of a recommender system for decentralized peer-to-peer computer networks. Eastern-European Journal of Enterprise Technologies, 4 (9 (124)), 24–35. doi: https://doi.org/10.15587/1729-4061.2023.286187*

1. Introduction

Recommender systems are a relevant and popular tool that can increase work efficiency and expand the possibilities of the information space for a significant part of users and organizations [1–3].

Different network models can be used to build information systems, which depend on the specifics of tasks and available resources. The studies [4, 5] showed that the most common models are centralized networks of the "client-server" type, centralized and decentralized peer-to-peer networks, and hybrid networks.

Currently, the advantages of decentralized systems (ease of implementation, high scalability, efficiency, etc.) increase the relevance and demand for their implementation in comparison with centralized and hybrid systems.

Building a recommender system for decentralized P2P networks, both unstructured and structured, is a challenging task. Since there are no servers in such systems, each individual computer will independently calculate recommendations based on the information available to it [4, 6, 7]. As a result, there may be problems with the reliability and data security of the information system of the network [5, 8], in particular, the recommender system. After all, new nodes connecting to the network can be controlled by attackers or be affected by malicious software. Such nodes can be programmed to distort or intercept recommender system data, and such information attacks are quite common [9–11].

Models of P2P networks can be divided into the following three types:

– centralized P2P networks, for example, BitTorrent [12];

– unstructured decentralized P2P networks most often use the "Flooding" search algorithm, a typical representative is the "Gnutella" network [13];

– structured decentralized P2P networks, most often built on the basis of distributed hash tables, for example, on the basis of the "Kademlia" [14] or "Chord" [15] algorithm.

The study of decentralized peer-to-peer networks showed that the issue of creating recommender systems for such an architecture is quite relevant but in practice it was hardly considered. Although in P2P networks, the issue of information search is very acute [16, 17]. Recommender systems with different data filtering algorithms could make it much easier for users to find useful content. They would also increase interest in decentralized peer-to-peer networks for distributed content storage. At the same time, in decentralized P2P networks, the classic methods of operation of recommender systems will not be able to work without some adaptation and changes due to the peculiarities of the architecture of these networks.

Thus, it can be emphasized that one of the main advantages of using decentralized peer-to-peer networks for recommender systems is their high scalability and fault tolerance, which in general has a positive effect on the accuracy of recommendations. However, in such networks it can be more difficult to control the authenticity of the data and its security. The risks of threats to information security and, as a result, low-quality services of recommender systems in them are quite high [9, 16].

One of the ways to eliminate these risks is the modeling of the recommender system, the assessment of probabilistic and technical characteristics, taking into account the requirements of reliability and information security, and further configuration and adjustment of the network.

From a practical perspective, this is the first step towards improving recommender systems for decentralized peer-to-peer computer networks. But without this step, it is impossible to substantiate new solutions for further improvement and development of appropriate hardware and software components of the recommender system. The evaluation of probability-time characteristics will make it possible to determine the influence of the main parameters of recommender systems on the duration of the processes of formation and change of recommendations. This, in turn, will make it possible to optimize these parameters taking into account the requirements for their reliability and security.

Thus, the issue of adequate assessment of probabilistic-temporal characteristics of the processes of forming and changing recommendations, with a rational choice of its main parameters, remains relevant at present. Solving this issue will increase the safety and reliability of recommendations.

2. Literature review and problem statement

For centralized peer-to-peer networks, it is possible to apply standard methods for building recommender systems since such networks have a server that stores links to files and statistics. Difficulties arise when trying to create recommender systems for decentralized peer-to-peer networks because it is necessary to take into account their architecture and adapt known methods and approaches to it. In particular, in decentralized networks, all information is distributed among all computers in the network. These are files, routing tables, statistics, etc. Therefore, there is a problem of collecting information for the recommender system and considering the possibility of collecting not all the information but the minimum necessary, so as not to survey all the computers in the network.

A review of current scientific research [18–27] was conducted regarding the possibility of developing recommender systems for peer-to-peer networks.

Article [18] provides an overview of current research in the field of recommender systems. The authors review the existing methods and models used to build various recommender systems. The cited article presents various methods of filtering data and generating recommendations used in recommender systems. However, the cited article is analytical in nature, and also does not consider the specifics of creating recommender systems for different computer network architectures. In the cited article, the issue of assessing the probabilistic and technical characteristics of data processing remained unresolved.

Paper [19] proposes a decentralized recommender system for a social network. The authors point out that social networks contain a lot of valuable information, but the growth of data is one of the biggest challenges, and this has led to the need for recommender systems. Collaborative filtering and a trust metric along with a user similarity metric are used to implement the recommender system. A decentralized approach is also proposed that calculates similarity and trust relationships between users in a distributed manner. It is noted that the decentralized approach contributes to better performance of the system in creating recommendations. The cited work shows the possibility of developing decentralized recommender systems, but the testing of the proposed model takes place in a centralized hierarchical computer network. Therefore, the proposed recommendation network model, although it contains decentralized computing, is not adapted to the architecture and features of decentralized peer-to-peer computer networks. Also, the issue of assessing the accuracy of the developed model and the reliability of the obtained results remained unresolved in the paper.

Paper [20] considers the problem of high computational complexity and the problems of representing the space of distributed data, which make collaborative data clustering, which, in particular, is used as an important component of recommender systems, in distributed peer-to-peer networks, impossible. The authors propose a new series of collaborative fuzzy clustering algorithms based on random features. In the most basic algorithm, each node in a distributed P2P network first maps its data into a low-dimensional space of random features with an approximation of a given kernel using the random Fourier feature mapping method. Each node then independently searches for clusters with its local data and collaborative knowledge from its neighboring nodes. And distributed clustering is performed among all network nodes until a global consensus result is reached, that is, all nodes have the same cluster centers. The following open questions remain: the choice of optimal kernels for clustering, dependence on random signs, and the need to achieve a global consensus. Also, the cited article does not provide a model of a recommender system for a decentralized network, although the proposed collaborative clustering algorithms will be useful in its development.

In [21], an algorithm for building a distributed recommender system based on collaborative filtering and distributed DHT hash tables is proposed, which allows finding the nearest "neighbors" for users for whom recommendations are calculated in a decentralized network. The algorithm searches for information about the user's "neighbors" according to the "fuzzy critical value" generated by the user's "extreme score". The work is focused on finding the most suitable "neighbors" for the user to qualitatively implement collaborative filtering in a decentralized network and does not consider other problems of developing distributed decentralized recommender systems, in particular, their reliability and information security.

In [22], a method of recommending a trusted node for inter-domain collaborative filtering based on SDN is proposed. Using the cross-domain collaborative filtering method, trusted network nodes (mobile devices) are searched for, which demonstrate trustworthiness based on their actions. Simulation experiments confirm that the method proposed in the cited paper has high recommendation accuracy and short running time when selecting a reliable node. The cited work can be useful in general for the development of decentralized recommender systems. But it is focused on a specific area of application of recommender systems and solves a narrow specific problem, therefore, it may not be suitable for solving other problems of using recommender systems. In addition, in the cited article, the issue of assessing the accuracy of the developed model, taking into account the requirements for reliability and security, remained unresolved.

Article [23] proposes an approach to the development of decentralized recommender systems based on the use of blockchain technology and smart contracts. The proposed system is built on the basis of Distributed Ledger technology, which works without any centralized authority and supports both decentralized ratings and ranking of various elements. Experimentation of the proposed system took place on the Ethereum platform and demonstrated the feasibility of the proposed approach in terms of performance and cost. However, as in [23], the authors do not consider some possible limitations of the application of blockchain technology in recommender systems, such as the high complexity of algorithms, long calculation time, and insufficient universality of the proposed approach.

In [24], a decentralized recommender system for a peer-to-peer network is proposed, based on a popular model of collaborative filtering and blockchain technologies. It is used to create a system of smart contracts for enterprises and increase the information security of their online activities. The authors propose a new algorithm that allows solving the problem of centralized data storage and processing in recommender systems. Only one specific method of building blockchain-based decentralized recommender systems is described. This reduces the practical value of the cited work for a number of specific recommender systems in which blockchain technology is not effective. Examples of such systems are those that require high data processing speed and quick decision-making, systems with a high degree of data confidentiality, etc.

The P2PCF recommender system is proposed in [25]. The authors use a collaborative filtering approach to recommend content in P2P social networks. According to the authors of the cited article, P2PCF ensures confidentiality. At the same time, the proposed approach assumes that the rating matrix is distributed among users in such a way that each user sees only the interactions made by his friends on his timeline. This significantly reduces the reliability of the input data and reduces the accuracy of the simulation results.

In [26], on the contrary, the authors pay more attention to the reliability of the obtained data, increasing the possible sources of information and considering the possibility of their combined use. But the issue of security is not considered in the proposed model. In addition, the technical and practical possibilities of using the entire range of proposed sources of recommendation data are not substantiated in the cited article. From this, it can be concluded about the need for filtering and generalization of input data, modeling and optimization according to the criterion of maximum accuracy, taking into account safety and reliability requirements.

In [27], a method for calculating trust in P2P based on the analysis of network behavior is proposed. The method relies on the basic elements of social psychology to establish intimate relationships between people, considers the influence of node behavior on trust, and uses aggregate trust, current trust, feedback trust, and aggregate abuse trust to calculate and update the trust value to improve dynamic adaptability and fault tolerance of peer-to-peer networks. In the future, the elements considered in the cited article can be used in the model of trust in recommendations. But the paper does not show how most of the elements can be combined in a single model of the recommender system. And therefore, these materials need to be refined in the area of adaptation of certain given elements of trust to the requirements of safety and reliability of recommendation data in the generalized model of the recommender system for decentralized peer-to-peer computer networks.

Perhaps the issues of the accuracy of the results of simulation of peer recommender systems remained unresolved because the used mathematical apparatus does not always provide an opportunity to find effective solutions. When solving the problems of increasing the accuracy of results, researchers are forced to deal with the problems of introducing additional input data and complicating models. In this case, the experience of using GERT network modeling technology can be useful. GERT uses a graphical notation that makes it possible to easily represent the sequence and dependences of events in a project. This facilitates the modeling and analysis of the project, in particular the determination of probabilistic and temporal characteristics. The GERT network technology allows the uncertainty of probabilistic variables, such as event execution times or delays, to be taken into account using random variables. This allows for more realistic modeling of different scenarios and taking into account possible risks and uncertainties in the results. In addition, GERT-network modeling allows consideration of simultaneous or parallel events that may occur at the same time. This is important to accurately determine the execution time of processes.

Also, for different architectures of computer networks, recommender systems must be developed differently, and existing recommender systems are focused on centralized hierarchical networks, and there are practically no mathematical models of their operation for decentralized peer-to-peer networks.

Thus, the scientific problem of increasing the accuracy of estimating the probabilistic-temporal characteristics of recommender systems in decentralized peer-to-peer computer networks remains unsolved.

3. The aim and objectives of the study

The purpose of this study is to construct a mathematical model of the recommender system to solve the scientific problem of increasing the accuracy of the assessment of probabilistic time characteristics of the processes of forming and changing recommendations.

Based on this, in order to achieve the set goal, the following tasks must be solved:

– to form the structure of the recommender system of a peer-to-peer decentralized network, taking into account the increased requirements for data reliability and security;

– to formalize and evaluate the probability-time characteristics of the process of forming and changing recommendations;

– to conduct comparative studies of the GERT-model of the recommender system for a decentralized peer-to-peer computer network and evaluate the accuracy of the simulation results.

4. The study materials and methods

The object of our research is the process of creating recommendations for users of decentralized P2P networks to facilitate data search.

The research was carried out using the computer algebra system from the MathCad class of automated design systems of the international software development company PTC. (Developer headquarters is in Boston, Massachusetts, USA).

To assess the accuracy of the developed model, approximate estimates based on data processing from the open Netflix Prize data set were used.

The accuracy of the simulation results is evaluated using the indicator of the confidence probability of falling into the "averaged" confidence interval.

Implementation of the GERT-network modeling technology and the issue of ensuring the accuracy of modeling results require a number of preliminary and immediate stages.

The first stage is the formation of a structure that meets the specified safety and reliability requirements and the definition of input data. At this stage, the main rules of network operation, its characteristics, such as time parameters, probability distributions, and other parameters affecting the system's operation, are defined. The second stage is the construction of the GERT model and the calculation of probability-time characteristics. The GERT model should include events and connections between them. At the same time, it is necessary to apply mathematical methods to calculate the characteristics of the network, such as average times for the formation of recommendations, transfer of a file for different "datasets" and so on.

Among the main assumptions taken into account in our work is that recommender systems have vulnerabilities that allow attackers to gain unauthorized access to data or influence recommendations. In addition, objective and subjective external factors are sources of uncertainty of input data and affect the reliability of the results of the formation of recommendations.

The main limitation under consideration is the P2P network format. This applies to such indicators as scalability, node autonomy, connectivity, reliability, and others.

Our article considers a numerical method for finding the continuous density of the probability distribution of the GERT-network transit time, provided that the set of distributions that can be characterized by individual arcs of the model includes an exponential distribution.

The proposed method is based on the transition from the equivalent W-function $W_E(s)$ of the GERT network to its characteristic function $X_E(\xi)$ and the use of the rotation formula [28].

The density of the probability distribution of the GERT network transit time is determined by the following expression:

$$\zeta(x)=\frac{1}{2\pi}\int_{-i\infty}^{i\infty} e^{-i\xi x} X_E(\xi)\,\mathrm{d}\xi.$$

The characteristic function $X_E(\xi)$ is based on Mason's topological equation [28] by replacing the variable s in the equivalent derivative function of moments $M_E(s)$ with $i\zeta$, where ζ is a real variable.

To ensure the conditions of integration, the factor exp$(-0.5\zeta^2)$ was introduced into the integrand expression [28]. This is equivalent to adding to the GERT network a sequential branch described by a normally distributed random variable ζ_2 with zero mathematical expectation and variance equal to unity. A dummy branch can be enabled immediately after the source s of the network. If the random variable ζ_1 is the transit time of the GERT network, then the density distribution $\tilde{\zeta}(x)$ of the sum $\zeta_1+\zeta_2$ is determined by the expression $\widetilde{X_E}(\xi)=X_E(\xi)\exp(-0.5\xi^2)$.

After finding the density $\tilde{\zeta}(x)$, the desired distribution density $\zeta(x)$ must be found. This is achieved using a numerical method of transformation of distribution laws based on the solution of a system of linear equations.

5. Constructing the mathematical model of a recommender system for decentralized peer-to-peer computer networks

5. 1. The structure of the recommender system of a peer-to-peer decentralized computer network taking into account the increased requirements for data reliability and security

When modeling a recommender system of a decentralized peer-to-peer computer network, a large number of tasks arise that can be solved using various GERT models. They can be used both independently of each other and in combined systems. Several new varieties of GERT models may be made available to the user. In particular, high-dimensional homogeneous networks, heterogeneous networks, networks with application aging, random GERT networks, etc.

In order to determine the input parameters of the GERT model and form the corresponding GERT network, taking into account the requirements of data reliability and security in recommender systems, we shall introduce several assumptions and suggestions.

Multi-node security monitoring of objects of the recommender system is a process whose purpose is to reliably determine the security state of the monitoring object Θ. Such monitoring occurs to detect changes within the existing position in the state field F of the monitoring nodes and aims to record state changes security p_Θ.

The change in the security status of the recommender system object p_Θ shall be understood as the ordered set of pairs:

$$p_\Theta=\{x_i,t_i\}:\forall i=1\ldots n,\ \ x_i\in F,$$

where x_i is an indicator that captures the object's security status; t – moment of observation of the monitoring object Θ; i is the observation number of the monitoring object Θ.

Let, for the purpose of ensuring the decentralized performance of the function of multi-node monitoring of objects of the recommender system, the internal subsystem of analytical monitoring of the safety of objects has the architecture of a peer-to-peer network of intelligent monitoring nodes that form the set R. For network addressing in such a subsystem, we can use the technology of classless IP addressing. Then the mapping $C^{(R\to R)}$ of the set R determines the structure of logical connections of the elements of this set. At the same time, the result of the procedure for assigning IP addresses from some set A to the elements of this set R can be described by the bijective mapping: $C^{(R\to A)}$. In addition, the surjective mapping $C^{(R\to\Psi)}$ in the model can formalize the connection of the elements of the set R with the elements of the set of their direct connection Ψ on the monitoring plane.

Then, in order to perform the function of multi-node security monitoring of objects of the recommender system, the elements of the set R must implement the mechanisms of access and distributed data storage, which represent the set $\aleph(\psi_r,\partial_r),\ \psi_r\in\Psi,\ \ \partial_r\in A,\ \ r\in R.$

Assume:

$$\Re:\{\aleph(\psi_r,\partial_r)\}\xrightarrow{ON}R. \tag{1}$$

Then $\forall r_i\in R,$ using the logical connections given by the mapping $C^{(R\to R)}$, it is possible to add $\{\aleph(\psi_r,\partial_r)\}$ to relation (1) and search:

$$\partial_{r_j}:(x,y)\in\psi_{r_j},\left(\left(\partial_{r_j},\psi_{r_j}\right),r_k\in\Re\right), \tag{2}$$

where (x, y) is the search query argument.

The procedure for performing the function of multi-node security monitoring of objects is determined by an algorithm formalized using the following function:

$$L^{(MMO)}=$$
$$=\left\{\bigcup_{i=1\ldots|R|}L_{r_i}^{(MMO)}:L_{r_i}^{(MMO)}=\ldots=L_{r_{|r|}}^{(MMO)}\right\}:\{(MOD,IDS,NET)\}\to TR, \tag{3}$$

where $L_{r_i}^{(MMO)}$ is the formalization of the algorithm for multi-node safety monitoring of objects of intelligent nodes r_i of the recommender system; MOD is a set of data flows from monitoring nodes from the set R; IDS – a set of security identification features of the monitored object; NET – a peer-to-peer network of intelligent monitoring nodes; TR is the set of all possible security states of the monitored object.

In order to increase the reliability and security of data in recommender systems, we shall establish the following order of distributed data storage in individual elements of the R network:

$$\bigcup_{i=1\ldots|R|}S_{r_i}\equiv S,$$

where $\bigcup_{i=1\ldots|R|}s_{r_i}$ is the set of regions of responsibility of the elements of R, given by the bijective mapping.

The rule of anchoring $\{\aleph(\psi_r,\partial_r)\}$ by the elements of the set S is as follows: if the arguments are $\forall\left(x_{\psi_{r_i}},y_{\psi_{r_i}}\right)\in s_{r_j},$ the mechanisms of access and distributed data storage $\{\aleph(\psi_{r_i},\partial_{r_i})\}\xrightarrow{ON}\{r_j\}.$

In order to increase the reliability of the recommendations, all elements of the R network should be able to carry out "informational interaction" among themselves. Also, in order to reduce the load on the communication channels that occurs during the operation of the recommender system and increase its security, the mapping $C^{(R\to R)}$ should be injective.

Our studies showed that this condition can be achieved transitively – with the help of the composition of the mappings of logical connections, that is, the condition "$\{r_i,r_j\}$ must be fulfilled:

$$\exists C^{(r_i\to r_j)}=\left(C^{(r_i\to R)}\circ\ldots\circ C^{(R\to r_j)}\right):\{r_i\}\to\{r_j\}, \tag{4}$$

where $\left(C^{(r_i\to R)}\ldots C^{(R\to r_j)}\right)\subset C^{(R\to R)}.$

Suppose that the $C^{(R\to R)}$ structure is determined by the presence of common bonds in the elements of S. The relationship between such nodes of the recommender system, which are considered "neighbors", can be described as follows:

$$C^{(R\to R)}=\bigcup_{i=1\ldots|R|}C^{(r_i\to R)}=\left\{(r_i,r_j):s_{r_i}\cap s_{r_j}\right\}:R\to R, \tag{5}$$

where $C^{(r_i\to R)}$ is part of the data about the closest connections of the recommender system. Recommender system nodes in this relationship know each other's IP addresses and areas of responsibility.

Then the relationship defining the connection between the "parent" nodes of the recommender system and the "child" nodes, information about the state of which is stored in the "parent" nodes, can be written in the following form:

$$C_{data}^{(R\to R)}=\bigcup_{i=1\ldots|R|}C_{data}^{(r_i\to R)}=\left\{(r_i,r_j):s_{r_i}\cap\psi_{r_j}\right\}:R\to R, \tag{6}$$

where $C_{data}^{(r_i\to R)}$ is the local information of the "parent" node r_i about its "child" nodes. Participants of such relationships have data on each other's address information.

Fig. 1 shows an example of the structure of a decentralized peer-to-peer network with a recommender system, formed taking into account the increased requirements for data reliability and security by linking the areas of responsibility of "parent" and "child" nodes.

It is proposed to use the concept of mobile agents for a schematic representation of the future prototype of the recommender system model of a decentralized peer-to-peer computer network.

A set of recommender system nodes is considered as a multi-agent system. In the system, each node is represented by an agent object. It has functions related to the achievement of a local goal – tracking changes in the states of objects from the available set of "child" objects within the limits of its responsibility.

In addition, it is responsible for transmitting data about the "child" node in the presence of such changes. In order to increase the reliability and security of data, as well as to decentralize the multi-agent system when performing the function of multi-node monitoring of objects, the network infrastructure of the recommender system will be represented in the form shown in Fig. 2.

Such a recommender system consists of parts of two types: "agent" – a software and hardware platform that implements the functions of exchanging identification data about the state of nodes of the recommender system, and a trigger – a platform that initiates the formation of subsets of "child" and "parent" nodes.

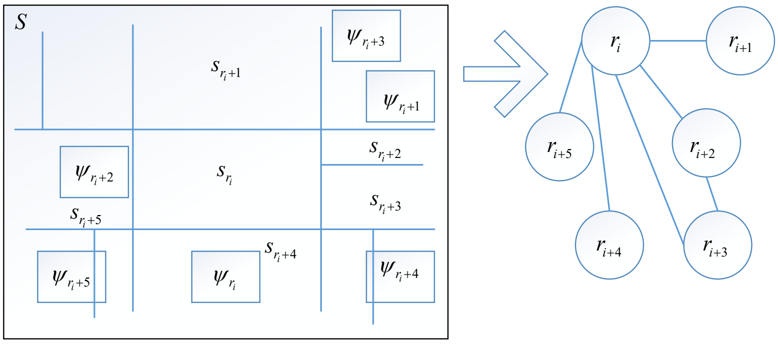

Fig. 1. An example of the structure of a peer-to-peer recommender system formed taking into account increased requirements for data reliability and security by linking the areas of responsibility of "parent" and "child" nodes

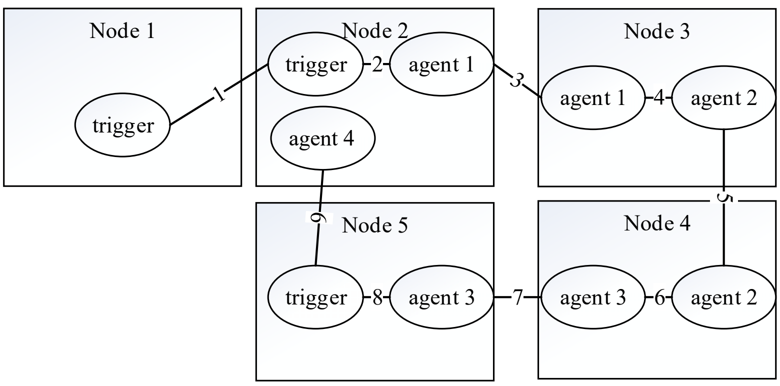

Fig. 2. An example of the process of interaction of nodes of a multi-agent recommender system

5. 2. Formalization and evaluation of probabilistic-temporal characteristics of the processes of formation and change of recommendations

The GERT scheme for identifying the state of nodes of the recommender system in the process of forming and changing recommendations is shown in Fig. 3.

In the diagram, state 1 corresponds to the "Initial" status, in which there are no changes. State 2 corresponds to the status "Changes occurred". In this state, the recommendation "portrait" may change in one of the nodes, that is, a new recommendation ideology may be formed. State 4 corresponds to the "Changes have been reported" status. In this state, the "father" node notices changes in the recommendation ideology (recommendation "portrait" of the "child" node). State 5 – status "Checked and recorded about state changes". In this state, the "portrait" of the "child" node in the "parent" node is fixed. State 3 corresponds to the status "Fixed in all nodes". In this state, the "portrait" of the "child" node is fixed in other nodes.

The features of this scheme are the consideration of states 4 and 5 in the developed model. This consideration will make it possible to, firstly, display the proposed structure of the peer-to-peer network recommender system (Fig. 1) in the GERT model. Secondly, it made it possible to mathematically formalize the communication processes between the "parent" and "child" nodes. Under conditions where the responsibility for the safety and reliability of input data can be placed on the "parent" node, such "deepening" of the GERT scheme for identifying the state of the nodes of the recommender system will improve the accuracy of the simulation results.

The characteristics of the branches of the GERT scheme are given in Table 1.

In Table 1, $q_1=1-p_1-p_2$; $q_2=1-p_3$; $q_3=1-p_4$; $q_4=1-p_5$.

The equivalent W-function [28, 29] of the time of identification of the state of the nodes of the recommender system in the process of forming and changing recommendations is equal to:

$$W_E(s)=\frac{W_{13}+W_{14}W_{43}+W_{15}W_{53}}{1-W_{15}W_{51}-W_{14}W_{41}-W_{12}W_{21}}=$$

$$=\left(\frac{\dfrac{p_3\lambda_3}{(\lambda_3-s)}+\left(\dfrac{p_2\lambda_2}{(\lambda_2-s)}\times\dfrac{p_4\lambda_5}{(\lambda_5-s)}\right)+\left(\dfrac{p_5\lambda_5}{(\lambda_5-s)}\times\dfrac{q_1\lambda_2}{(\lambda_2-s)}\right)}{1-\left(\dfrac{q_1\lambda_2}{(\lambda_2-s)}\times\dfrac{q_4\lambda_4}{(\lambda_4-s)}\right)-\left(\dfrac{p_2\lambda_2}{(\lambda_2-s)}\times\dfrac{q_3\lambda_4}{(\lambda_4-s)}\right)-\left(\dfrac{p_1\lambda_1}{(\lambda_1-s)}\times\dfrac{q_2\lambda_4}{(\lambda_4-s)}\right)}\right). \quad (7)$$

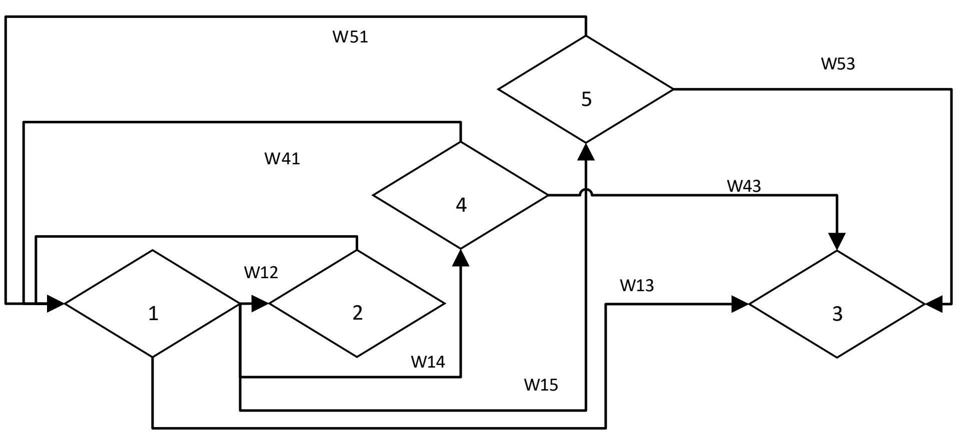

Fig. 3. GERT-scheme of identification of the state of the nodes of the recommender system in the process of forming and changing recommendations

Table 1

Characteristics of the branches of the GERT scheme for identifying the state of the nodes of the recommender system in the process of forming and changing recommendations

W-function	Probability	Producing function of moments
W_{12}	p_1	$\lambda_1/(\lambda_1-s)$
W_{14}	p_2	$\lambda_2/(\lambda_2-s)$
W_{15}	q_1	$\lambda_2/(\lambda_2-s)$
W_{13}	p_3	$\lambda_3/(\lambda_3-s)$
W_{21}	q_2	$\lambda_4/(\lambda_4-s)$
W_{43}	p_4	$\lambda_5/(\lambda_5-s)$
W_{41}	q_3	$\lambda_4/(\lambda_4-s)$
W_{53}	p_5	$\lambda_5/(\lambda_5-s)$
W_{51}	q_4	$\lambda_4/(\lambda_4-s)$

After analyzing expression (13), we obtain the following ratio:

$$W_E(s)=\frac{(\lambda_4-s)\times\left(\lambda_2\lambda_5 q_5(\lambda_3-s)+p_3\lambda_3(\lambda_2-s)(\lambda_5-s)\right)}{\left((\lambda_3-s)(\lambda_5-s)\times \times\left((\lambda_2-s)\left((\lambda_4-s)(\lambda_1-s)-p_1q_2\lambda_1\lambda_4\right)-\lambda_2\lambda_4 q_6(\lambda_1-s)\right)\right)}, \quad (8)$$

where $q_5=(p_2p_4+q_1p_5)$, $q_6=(q_1q_4-q_3p_2)$.

Recent studies have shown that there are no simple methods for finding singular points of a function $\Im_E(z)$ in complex GERT networks with possible loops. This is due to the fact that finding such points requires solving nonlinear equations, and the more complex the structure of the GERT network, the more difficult it is to solve the corresponding equation. Therefore, during modeling and performing complex transformations, you can replace real variables $(z=-i\varsigma)$, where ς is a real variable, and get the following:

$$\Im(z)=\frac{bz^3+az^2+uz+k}{(\lambda_3+z)(\lambda_5+z)\times(z^3+wz^2+dz+r)}, \quad (9)$$

where:

$$b=q_5\lambda_5\lambda_2+p_3\lambda_3,$$

$$a=q_5\lambda_5\lambda_2(\lambda_3+\lambda_4)+p_3\lambda_3(\lambda_4+\lambda_2+\lambda_5),$$

$$u=q_5\lambda_5\lambda_2(\lambda_3\lambda_4+\lambda_4+\lambda_3)+ +p_3\lambda_3(\lambda_2\lambda_4+\lambda_5\lambda_4+\lambda_2\lambda_5),$$

$$k=\lambda_2\lambda_3\lambda_4\lambda_5(q_5+p_3),$$

$$w=\lambda_1+\lambda_4+\lambda_2,$$

$$d=\lambda_2\left(\lambda_4+\lambda_1+\lambda_4(q_1q_4\lambda_1-q_2q_3)\right)+\lambda_1\lambda_4(1-p_1q_2),$$

$$r=\lambda_1\lambda_4\lambda_2(1-p_1q_2-q_1q_4+p_2q_3).$$

The following can be highlighted as a feature of the solution of this equation. If the function $\Im_E(z)$ within $Rez<0$ satisfies the requirements of Jordan's lemma, then the integral taken along the Bromwich contour is the sum of calculations of the function $\Im_E(z)$ with respect to all its features $\zeta(x)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}e^{zx}\Im_E(z)\mathrm{d}z=\sum_{k=1}^{n}Res\left[e^{zx}\Im_E(z)\right]$.

To fulfill the conditions of Jordan's lemma, it is necessary that the function $\Im_E(z)$ in the left half-plane be analytic with the exception of a finite number of poles and uniformly approach zero with respect to the argument z as $|z|\to\infty$.

The function $\Im_i(z)=\frac{\lambda}{\lambda+z}$, representing the exponential distribution uniformly tends to zero in the argument z as $|z|\to\infty$. In addition, the function has a simple pole at the point $z=-\lambda$.

As a feature of the model, one can also note the possibility of its adaptation to various distribution functions. For example, if the Erlang distribution is used as a basis, the function $\Im_i(z)=\frac{\lambda^\alpha}{(\lambda+z)^\alpha}$ uniformly converges to zero depending on the argument z at $|z|\to\infty$. In addition, there is a pole of multiplicity α at the point $z=-\lambda$.

Probability distribution density of the identification time of the nodes of the recommender system in the process of forming and changing recommendations:

$$\zeta(x)= =\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}e^{zx}\frac{bz^3+az^2+uz+k}{(\lambda_3+z)(\lambda_5+z)\times(z^3+wz^2+dz+r)}\mathrm{d}z, \quad (10)$$

where the integration is performed with the Bromwich contour.

According to the research results, it was found that the integration method depends on whether the function $\Im(z)$ has only simple poles or poles of a higher order. If the function $\Im(z)$ has only simple poles, then the expression $e^{zx}\Im(z)$ can be written in the form:

$$e^{zx}\Im(z)=\frac{e^{zx}(bz^3+az^2+uz+k)}{(z^5+g_4z^4+g_3z^3+g_2z^2+g_1z+g_0)}=\frac{\alpha(z)}{\beta(z)}, \quad (11)$$

where

$$g_4=\lambda_5+\lambda_3+w,$$

$$g_3=\lambda_5\lambda_3+w(\lambda_5+\lambda_3)+d,$$

$$g_2=\lambda_5\lambda_3 w+d(\lambda_5+\lambda_3)+r,$$

$$g_1=\lambda_5\lambda_3+r(\lambda_5+\lambda_3),$$

$$g_0=\lambda_5\lambda_3 r.$$

Then the probability distribution of the time for identifying the state of the nodes of the recommender system in the process of forming and changing recommendations:

$$\zeta(x)=\sum_{n=1}^{5}Res\left[e^{zx}\Im(z)\right]=\sum_{n=1}^{5}\frac{\alpha(z_n)}{\beta(z_n)}= =\sum_{n=1}^{5}\frac{e^{z_nx}(z_n^3b+z_n^2a+z_nu+k)}{(5z_n^4+4g_4z_n^3+3g_3z_n^2+2g_2z_n+g_1)}. \quad (12)$$

The function $\Im(z)$ can have poles of a higher order, in addition to simple poles, which are determined by the roots of the equation $z^3+wz^2+dz+r=0$. This is possible if the values of λ_3 and λ_5 coincide or are equal to the values of the roots z_3, z_4, z_5. In such cases, the density of the time distribution $\xi(x)$ of identifying the state of the recommender system nodes in the process of forming and changing recommendations can be found using the formula that uses z_n poles of order m:

$$\gamma_{-1}=\frac{1}{(m-1)!}\lim_{z\to z_n}\frac{d^{m-1}\left[\left(z-z_n\right)^m e^{zx}\Im(z)\right]}{dz^{m-1}}.$$

Expression (9) can be written as a function consisting of a numerator and a denominator with a higher power than the numerator. This allows us to use the conditions of Jordan's lemma for this function. The function $\Im(z)$ contains poles at the points $z_1=-\lambda_3$, $z_2=-\lambda_5$, as well as three more poles generated by the polynomial z^3+vz^2+rz+c. To find the solution of the equation, we used:

$$z^3+wz^2+dz+r=0, \tag{13}$$

and received three more special points z_3, z_4, z_5.

As can be seen, the coefficients of equation (13) are greater than zero. Therefore, if this equation has real roots, then all of them are negative. If equation (13) has negative real roots, then they must not be of the second or third multiplicity. If we assume that the polynomial z^3+wz^2+dz+r generates a pole of the third order at the point $z=-\iota$, then $(z+\iota)^3=z^3+3\iota z^2+3\iota^2 z+\iota^3$. Since $\iota<0$, then $i^3<0$, which contradicts the condition $\iota^3=c=\lambda_1\lambda_4\lambda_2\left(1-p_1q_2-q_1q_4+p_2q_3\right)>0$. Therefore, the polynomial z^3+wz^2+dz+r does not can generate poles of the third order.

It can be assumed that z^3+wz^2+dz+r generates a second-order pole at the point $z=-i_1$ and a first-order pole at the point $z=-i_2$, then the relation must hold:

$$\begin{aligned}&\left(z+\alpha_1\right)^2\left(z+\alpha_2\right)=\\&=z^3+\left(2\iota_1+\iota_2\right)z^2+\left(2\iota_1^2+2\iota_1\iota_2\right)z+\iota_1^2\iota_2.\end{aligned} \tag{14}$$

Given $i_2<0$ and $\iota_1^2>0$, then $\iota_1^2\iota_2<0$.

If we conduct an analysis, it becomes clear that there is a discrepancy since $\iota_1^2\iota_2=c=\lambda_1\lambda_4\lambda_2\left(1-p_1q_2-q_1q_4+p_2q_3\right)>0$. Therefore, expression (9) cannot generate poles of the second order.

According to the fundamental theorem of algebra, any entire rational function of the nth power has n zeros. Therefore, the polynomial z^3+wz^2+dz+r can have either three different negative real zeros, or one negative real zero and two complex conjugate zeros.

Using these postulates, one of the key characteristics of the quality of the recommender system can be found – the time to identify the state of the nodes of the recommender system in the process of forming and changing recommendations.

Thus, the GERT model of a recommender system for a decentralized peer-to-peer computer network has been developed. The mathematical model differs from the known ones by taking into account the requirements for the reliability and security of information of the recommender system during changes in the structure of the decentralized peer-to-peer network. The model can be used to study the processes of formation and change of recommendations to users of decentralized peer-to-peer computer networks. It can also be used in the development of new methods, algorithms, and ways for organizing recommender systems that use a decentralized peer-to-peer structure of a computer network.

5. 3. Comparative studies of the GERT-model of a recommender system for a decentralized peer-to-peer computer network

To find the probability density distribution of the time of identification of the state of the recommender system nodes in the process of forming and changing recommendations $\xi(x)$, the defining characteristics of the model are: λ_1=0,4, λ_2=0.2, λ_3=0.8, λ_4=0.3, λ_5=0.99, p_1=0.92, p_2=0.01, p_3=0.9, p_4=0.6, p_5=0.5.

With the help of expressions (11), (12), we found that w=0.9, d=0.247, r=0.022.

After solving the equation in the denominator of expression (10) with the Vieta-Cardano method, it was determined that the function $\Im(z)$ has simple poles: $z_1=-0.8$, $z_2=-0.9$, $z_3=-0.483$. In addition to real roots, there are two complex conjugated roots: $z_4=-0.208+i\times0.04$, and $z_5=-0.208-i\times0.04$.

According to relation (12), $\xi(x)$ looks as follows:

$$\begin{aligned}&\sum_{n=1}^{5}\mathrm{Res}\left[e^{zx}\Im(z)\right]=\\&=\frac{e^{(\beta+\delta i)_n x}\left((\beta+\delta i)^3 b+(\beta+\delta i)^2 a+(\beta+\delta i)\ u+k\right)}{\left(5(\beta+\delta i)^4+4g_4(\beta+\delta i)^3+3g_3(\beta+\delta i)^2+2g_2(\beta+\delta i)\ +g_1\right)}+\\&+\frac{e^{(\beta-\delta i)_n x}\left((\beta-\delta i)^3 b+(\beta-\delta i)^2 a+(\beta-\delta i)\ u+k\right)}{\left(5(\beta-\delta i)^4+4g_4(\beta-\delta i)^3+3g_3(\beta-\delta i)^2+2g_2(\beta-\delta i)\ +g_1\right)}.\end{aligned} \tag{15}$$

The sum of the values of any small-rational function:

$$f(z)=\frac{\varpi_m z^m+\varpi_{m-1}z^{m-1}+\ldots+\varpi_1 z+\varpi_0}{\ell_m z^m+\ell_{m-1}z^{m-1}+\ldots+\ell_1 z+\ell_0},\quad \varpi_m\neq0,\quad \ell_m\neq0,$$

for values of complex arguments can be represented in the form $\frac{\tau+i\partial}{\wp+i\hbar}+\frac{\tau-i\partial}{\wp-i\hbar}$, where τ, ∂, $\wp$, $\hbar$ are some coefficients.

This proposition can be proved by the method of mathematical induction. After that, using Euler's formulas, you can get:

$$\begin{aligned}&\sum_{n=1}^{5}\mathrm{Res}\left[e^{zx}\Im(z)\right]=e^{(\beta+\delta i)}\frac{\tau+i\partial}{\wp+i\hbar}+e^{(\beta-\delta i)}\frac{\tau-i\partial}{\wp-i\hbar}=\\&=\frac{2e^{\beta x}}{\wp^2+\hbar^2}\left[\left(\tau\wp+\hbar\right)\cos(\delta x)+\left(\tau\wp-\hbar\right)\sin(\delta x)\right].\end{aligned} \tag{16}$$

where

$$\tau=\beta b-3\beta\delta^2 b+\beta^2 a-\delta^2 a+\beta u+k,$$

$$\partial=3\beta^2\delta b-\delta^3 b+2\beta\delta a+\delta u,$$

$$\begin{aligned}&\wp=5\beta^4-30\beta^2\delta^2+5\delta^4+4\beta^3-\\&-12\beta\delta^2+3\beta^2 w-3\delta^2 w+2\beta d+r,\end{aligned}$$

$$\hbar=20\beta^3\delta-20\beta\delta^3+12\beta^2\delta-4\delta^3+6\beta\delta w+2\delta d.$$

Taking into account the above, the probability distribution function of the time of identification of the state of the nodes of the recommender system in the process of forming and changing recommendations is defined as:

$$\zeta(x)= \frac{2e^{-0.208x}\left[\frac{(9.224\cos(0.043x))}{10^4}+\frac{7.922\sin(0.043x)}{10^4}\right]}{1.321\times10^3}.$$

This expression is convenient for further analysis of the process of identification of the state of nodes of the recommender system during the formation and change of recommendations, taking into account the ratio of the precedence of operations.

Table 2 gives the values of the density of the probability distribution of the identification time of the nodes of the recommender system in the process of forming and changing recommendations.

We obtained the given values by calculating $\xi_b(x)$, using the Lagrange polynomial of the second power, which was constructed by interpolating the integrand function. The accuracy of calculating the values of $\xi_b(x)$ depends on the accuracy of determining the roots z_3, z_4, z_5, while the same density $\xi_u(x)$ was used.

The method of interpolation by the Lagrange polynomial of the second power has its natural error, which affects the accuracy of the calculation of the function $\xi_u(x)$. But the absolute error of calculating the density values does not exceed 0.002, which is sufficient for most practical applications related to the set goal of our work. However, for cases where greater accuracy is required, it is possible to increase the accuracy of calculations by switching to interpolating polynomials of higher powers.

The density of distribution of file transfer time is shown in Fig. 4.

It can be seen from the chart (Fig. 4) that the highest values of the density of the probability distribution of the identification time of the nodes of the recommender system in the process of forming and changing recommendations fall on the interval from 1 to 10 measurement units.

Theoretical studies and practical experience in the administration of recommender systems show that accurate quantitative assessment of the reliability and safety of recommender messages is impossible due to some uncertainty in the input data.

The accuracy of a mathematical model can be assessed by comparing its predictions with real data or experimental results. If the developed model predicts the results observed in practice properly, then it can be claimed that it is accurate. In addition, various statistical methods can be used to compare forecasting results with real data, such as correlation analysis, root mean square deviation, and others.

To assess the accuracy of the developed model, we used approximate estimates based on data processing from the Netflix Prize data dataset [30]. In the work, the well-known approach of mathematical statistics is used as a basis for reliability assessment, based on testing according to Pearson's χ^2 agreement criterion.

Table 2

The values of the probability distribution density of the time of identification of the state of the nodes of the recommender system in the process of formation and change of recommendations, obtained by calculations and interpolation

No.	$\xi_b(x)$	$\xi_u(x)$
1	0.281	0.294
2	0.21	0.223
3	0.194	0.207
4	0.178	0.191
5	0.16	0.173
6	0.143	0.156
7	0.126	0.139
8	0.109	0.122
9	0.093	0.106
10	0.078	0.091
11	0.064	0.077
12	0.05	0.063
13	0.038	0.051
14	0.036	0.049
15	0.035	0.048
16	0.033	0.046
17	0.028	0.041
18	0.027	0.04
19	0.026	0.039

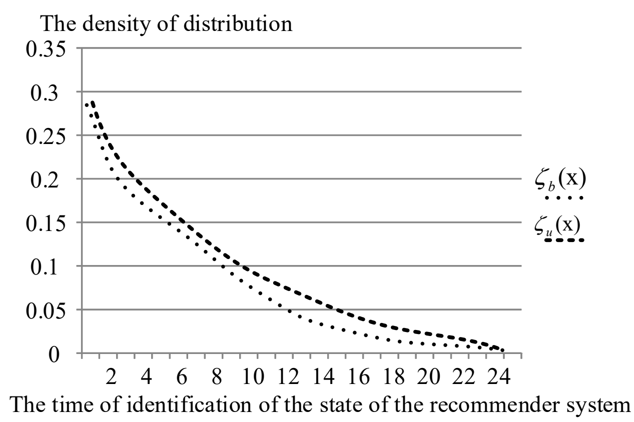

Fig. 4. The probability distribution density of the time of identification of the state of the nodes of the recommender system in the process of forming and changing recommendations

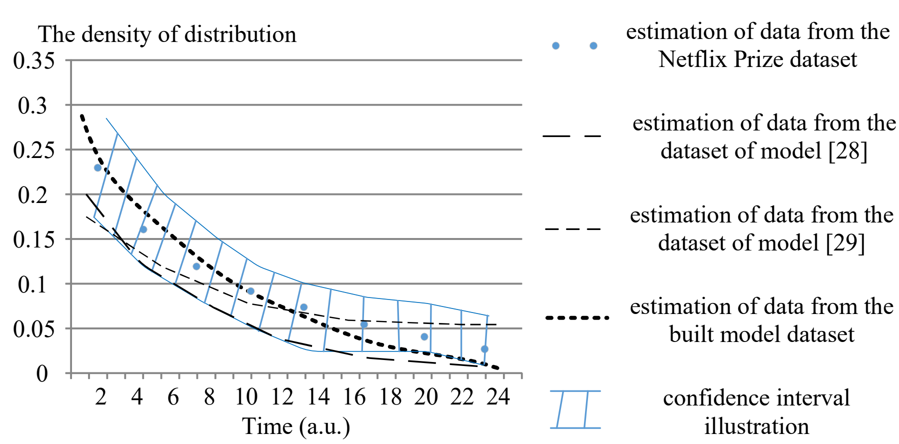

Fig. 5. Chart of distribution density dependence on file transfer time for different datasets

The results of the accuracy study indicate that the value of the identification time of the nodes of the recommender system in the process of forming and changing recommendations falls with a confidence probability of 0.94 into the "averaged" confidence interval (Fig. 5). In addition, the results of a comparative analysis of the developed model with known models [10, 29] show an improvement of this accuracy indicator to 5 %.

6. Discussion of results of the development of a mathematical model of the recommendation system for a decentralized peer-to-peer computer network

The developed model of the decentralized recommender system has made it possible to evaluate the probabilistic and technical characteristics of the data treatment process in the recommender system of the decentralized peer-to-peer computer network.

The framework of the developed recommender system was based on the multi-agent peer-to-peer CAN network paradigm. At the same time, the main innovative component of this advancement was the construction of a structure in which a set of monitoring nodes is considered as a multi-agent system. In this structure, each node is represented by an agent-object that has functions related to tracking and identifying changes in the state of monitoring objects within its field of view. In order to achieve the autonomy of agent objects and decentralization of the multi-agent system to perform the function of multi-node monitoring of objects, the structure of the peer-to-peer network recommender system was developed (Fig. 1).

One of the main results of the formalization of the processes of forming and changing recommendations is obtaining an analytical expression (12). With its help, it is possible to estimate the density of the probability distribution of the identification time of the nodes of the recommender system. In turn, the results of research and evaluation provide an opportunity to determine other characteristics. For example, the mathematical expectation of the identification time of the nodes of the recommender system or the time of file transfer for different "datasets". Equation (12) is solved using expressions (13), (14). A practical example of the application of the developed GERT model is given using equations (15), (16).

Also, one of the advantages of this analytical expression is the possibility of using it for different "datasets" of recommender systems. It is only necessary to empirically obtain the values of the main characteristics (λ and p) of the GERT network.

Taking into account the requirements for the reliability and safety of advisory messages in comparison with known models has made it possible to increase the accuracy of the simulation results. Evaluation and research results show an improvement in accuracy of up to 5 %. This was proven by means of a comparative analysis of the developed model with known models [10, 29].

This consideration was enabled by introducing the following indicators into the general model of the decentralized recommender system: the probability of reliable packet transmission and the probability of safe packet transmission. This makes it possible to analyze various methods of recommender systems from the point of view of reliability and information security and to set acceptable parameters, at which the extent of relevance does not fall below a certain threshold.

The results obtained in the comparative studies of the GERT model can be explained by the correct selection of indicators of the effectiveness of the recommender system, namely the probability of reliable transmission of the package and the probability of safe transmission of the package. In addition, the implementation of the proven method of modeling using GERT networks in the form of the proposed scheme in Fig. 3 also allowed us to improve the quality of the results.

As the research findings showed, despite the obvious advantages of the proposed model, there are some ways to improve it. This especially applies to the definition of the input characteristics of the model and the choice of the exponential distribution in the practical part of the research. The choice of other distribution laws when describing individual transitions from state to state in the future will provide an opportunity to improve the accuracy of the simulation results.

In part, these shortcomings have already been eliminated in the current work by using the GERT-network modeling approach.

The proposed GERT network model also has limitations. Of course, a slight increase in its elements-nodes will lead to a significant complication of the analytical expression (7). Therefore, it is necessary to use the technologies of rational representation of GERT schemes during modeling.

Further development of this issue is possible with the help of the following proposals:

1. Expanding functionality: it is possible to expand the functionality of the system by adding new parameters and algorithms. For example, one could investigate the effectiveness of using deep learning or neural networks to improve the accuracy of recommendations.
2. Evaluation of model stability: it is possible to investigate how resistant the model is to changes in the input data. For example, it is possible to analyze the influence of various external factors, in particular, cyber interference or failure of individual system components.
3. Conducting additional experiments: you can conduct experiments to test the accuracy of the model under different conditions. For example, we can conduct experiments on different types of input data using different datasets.

7. Conclusions

1. The structure of the recommender system for a peer-to-peer decentralized computer network has been formed. The basis of the proposed structure is a multi-agent system in which each node is represented by an agent object. The main purpose of such a structure is to monitor changes in the states of objects from the existing set of "child" objects within the limits of its responsibility, taking into account reliability and security indicators.

2. Probability-time characteristics of the process of formation and change of recommendations were formalized and evaluated. The mathematical formalization of the dynamics of the studied processes differs from the known ones by taking into account the requirements for the reliability and security of recommendation data during changes in the structure of the peer-to-peer network of the recommender system. This has made it possible to increase the accuracy of the assessment of probabilistic-time characteristics of the processes of formation and change of recommendations.

3. Comparative studies of the GERT model of the recommender system for a decentralized peer-to-peer computer network were conducted. The results of the accuracy study indicate that the value of the identification time of the nodes

of the recommender system in the process of forming and changing recommendations falls with a confidence probability of 0.94 in the "averaged" confidence interval. In addition, the results of a comparative analysis of the developed model with known models show an improvement of this accuracy indicator to 5 %.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available in the main text of the manuscript.

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