The object of this study is photovoltaic modules in various installation options. The physical model of the Earth’s illumination by a parallel flow of solar rays has been refined. The dependence of the cosine of the angle of incidence of the Sun’s rays on the angular length of the day, as well as the average annual efficiency of the installation of photovoltaic modules, both fixed and with various tracking options, was determined. Refinement of the physical model implies determining the angle of inclination as the angle between the inclined axis of the Earth and its projection on a vertical plane, perpendicular to the line connecting the centers of the Earth and the Sun. This line passes through the center of the Earth. The concept of the average annual efficiency of the installation of photovoltaic modules is introduced as the annual weighted average value of the cosine of the angle of incidence of solar rays on the plane of the photovoltaic module. Various options for installing photovoltaic modules were analyzed: fixed horizontal on the equator; stationary, installed at an angle to the horizon; one that performs tracking in horizontal (vertical) planes; with full tracking. The efficiency of installing a photovoltaic module at each latitude can be equal to the efficiency of installing this module at the equator, that is, 47.93% when installing the module at an angle of inclination to the horizon equal to the latitude. Tracking in the vertical plane makes it possible to increase the efficiency of the photovoltaic module installation by up to 50%. Compared to full tracking, tracking in the horizontal plane at an angle of latitude makes it possible to obtain the efficiency of the installation of the photovoltaic module at the level of 97.93%.

The results could be used as a basis for evaluating the efficiency of the installation of photovoltaic modules at different latitudes with different techniques of their installation, as well as for the subsequent generation of electricity.

Keywords: photovoltaic module, angle of incidence of solar rays, angle of inclination, angle of inclination to the horizon, efficiency of module installation

1. Introduction

The production of solar energy is one of the urgent problems of modern civilization [1], when a significant part of energy needs is met by traditional types of fuel. As the reserves of fossil fuels, due to the high demand for energy, are rapidly decreasing, renewable energy sources, in particular, solar energy, are increasingly being considered as the energy of the future [2]. This substantiates the importance of scientific research in the area of increasing the efficiency of electricity production through the use of photovoltaic modules [3–5]. One of the ways of increasing the efficiency of photovoltaic modules, along with improving the efficiency of the photovoltaic modules themselves, is increasing the efficiency of their installation [6].

Scientists from different countries have carried out a number of studies on determining the efficiency of installing photovoltaic modules [6–8]. However, their results are based on experimental studies performed under different conditions at different latitudes, which makes their universal application impossible.
One expression from [9] is best suited for evaluating the efficiency of the installation of photovoltaic modules. Although it gives adequate calculation results in many applications, especially in a simplified form, it is, in fact, the result of mathematical transformations and does not reflect the physical essence of the phenomenon of the Earth being illuminated by the Sun. Therefore, the task of determining the average annual efficiency of the installation of solar photovoltaic modules, both stationary and with various tracking options, remains relevant.

2. Literature review and problem statement

Scientists conducting research in the field of solar energy use a known expression from [10] to determine the efficiency of photovoltaic modules. According to this expression, the efficiency of the photovoltaic module depends directly on its power and inversely on the area, intensity of solar radiation, and the angle of its incidence on the solar panel. In this case, the angle of incidence of solar radiation on the panel is the angle between the direction of incidence of solar radiation and the perpendicular to the photovoltaic module and is determined in degrees.

The expression in [10] structurally defines the main directions of improvement of photovoltaic modules, namely: research into the nature of the intensity of solar radiation, improving the efficiency of the photovoltaic module itself, and increasing the efficiency of its installation.

The most accurate expression for determining the intensity of solar radiation is given in [11], where the specified parameter depends on the conditions of the passage of solar radiation through the Earth’s atmosphere, temperature, and spectrum. Also, the authors of work [11] note that since solar radiation is a natural phenomenon, it is impossible to influence its value. Therefore, it is better to determine the efficiency of the photovoltaic module based on the type of semiconductors used and its design. However, issues related to the low efficiency of modern photovoltaic modules, which is only 18–22 %, still remain unresolved. Also, the authors of [11] note that modern technologies can ensure the growth of the indicated indicator, however, this requires significant expenditure of money and time.

In [12], it is noted that the angle of incidence of solar radiation on the panel of the module is a significant factor influencing the specific power of the photovoltaic module, except for the cases of using a photovoltaic module that follows the Sun. However, photovoltaic modules that follow the Sun are not often used due to their relatively high cost. Therefore, the task related to determining the efficiency of the photovoltaic module installation remains open.

One of the options for solving this problem is to establish the optimal range of the angle of incidence of solar radiation on the photovoltaic module, which was proposed by the authors of work [9]. In their work, the authors gave a mathematical expression for determining the specified angle and conducted a number of studies on the verification of this expression. The expression involves taking into account the following parameters: time angle (angular displacement of the Sun to the east/west from the local meridian); inclination angle; latitude of the installation point of the panel on the earth’s surface; tilt angle of the photovoltaic module; azimuth of the photovoltaic module. The optimal ranges of values of the angle of incidence of solar radiation for various conditions and schemes of installation of solar panels of photovoltaic modules have been determined. However, the issue of determining the declination angle, that is, the angular position of the Sun at noon relative to the plane of the equator, remained incomplete. The mathematical expression used by the authors is quite cumbersome and inconvenient. The calculation error under some conditions could reach 3 %.

An option that makes it possible to avoid errors when determining the angle of inclination is the mathematical expression proposed in [13]. The expression has been successfully used to optimize methods for installing photovoltaic modules on irregularly shaped roofs by determining the time for which cosθ is non-negative [14], as well as for the design and optimization of solar photovoltaic power plant structures [15] and for optimizing the tilt angles of photovoltaic modules depending on monthly periods [16].

In [17] it is emphasized that the efficiency of the photovoltaic module depends on the parameters of its installation. The installation of the module can be with the tracking of the Sun in the following planes: horizontal (from the South to the West); in the vertical (from South to North); both vertically and horizontally. Modules can also be installed using fixed panels installed at a certain angle to the horizontal plane. The authors of work [17] modeled different ways of installing photovoltaic panels and established that even when installing photovoltaic panel systems on tracks, it is possible to obtain 20–35 % more energy. However, the cited studies are of an exclusively theoretical nature. In practice, there are a number of factors that significantly distort analytical calculations and prevent achieving the desired result.

In order to avoid these distortions, considerable attention was paid in study [18] to determining the optimal value of the tilt angle and azimuth for the solar collector. Practical verification of calculations is presented on the example of six cities. For them, it was established that during the year, the angles of inclination of the modules were optimal in the range from 20° to 35°, and in the winter period – from 30° to 55°. For the winter period, the optimal azimuthal angles were from –5° to 10°, and during the year – from –10° to 10°. Current standards recommend that the azimuth angle should be in the range of –20° to +20°, and the tilt angle should be within ±25° of latitude. The maximum reduction in solar energy storage for tilt angles can reach 29.5 %, and for azimuth angles – 6.5 %. However, the research is incomplete because within the framework of the study, the authors did not investigate the issue of the production of electrical energy from the accumulated solar energy modules.

The study of the technical and economic potential of photovoltaic systems for electricity production is reported in [19]. The place of the study was the city of Guangzhou, located in the south of China at a latitude of 23°. According to [19], the optimal results of the operation of solar systems were observed when installing photovoltaic panels with an inclination angle of 20°. At the same time, the payback period of these systems was only 6 years, which is a good indicator for an alternative energy facility.

Work [20] also states that installing solar panels or collectors with optimal orientation and tilt angle intensifies energy production and is an important measure to increase the economic performance of solar systems. The authors of work [20] provided essential recommendations for the installation of solar systems aimed at the maximum annual production of solar energy in the intertropical region. The panels of these systems must be directed towards the equator at an angle of inclination equal to the local latitude.
It has been experimentally proven that in West and Central Africa, a moderate deviation (up to 20°) from the optimal orientation and inclination does not have a significant effect on the degree of solar radiation fall (decrease in the level of irradiation <5%).

In addition to previous studies [18–20], the results of studies on the performance of a system of four small photovoltaic modules installed with different tilt angles [21] are reported. Performance is estimated by analyzing actual measurement data in the city of Kitakyushu (latitude 33.89°). The optimal angle of inclination for the installed capacity of the photovoltaic system during the year was about 32°. However, the practical results have a narrow application since, for example, the installation of this system at higher latitudes would most likely require an increase in the value of the optimal angle of inclination. However, this statement needs to be verified experimentally.

A significant practical contribution to establishing the dependence of the productivity of solar panels on the angle of their inclination during installation is highlighted in study [22]. Research was carried out in three cities of Albania: Tirana (latitude 41.3275°), Kuksi (latitude 42.0807°), and Vlore (latitude 40.4661°). The article is a successful example of the application of the Bernard-Menguy-Schwartz (BMS) model to determine the optimal value of the annual and seasonal tilt angles of solar panels. Determination of the optimal angle of inclination was carried out based on the fact that the solar energy reaching the panels has a maximum value at the optimal angle of inclination. Accordingly, these angles were: Tirana – 38°; Kuksi – 39°; Vlora – 37°. It should be noted that the annual optimal tilt angle of the solar panels based on the above mathematical model was almost equal to the latitude of the location. In addition, it is noted that the seasonal or adjusted tilt angles of solar panels, in order to increase their efficiency, should be compared with the annual optimal tilt angle. However, the procedure presented in [22] is not complete as it does not take into account the agreement of the tilt angle with the lunar period.

This omission was taken into account when performing the studies given in [16], where it is noted that the agreement of the tilt angle of the panels for the monthly period is better than for the annual period. A monthly-tuned panel angle installation increased the amount of energy stored by the panels by 12 kWh/m² per year compared to an annual-tuned panel installation. This is quite a significant indicator.

The above-mentioned studies are supplemented by the results of experiments reported in [23, 24]. It was experimentally established that the use of a two-axis solar tracker increases the average annual production of electricity by 41% [23]. In turn, the use of a uniaxial solar tracker increases the average annual production of electricity by 12 kW/m². In [22] the refined physical model of the Earth’s illumination by the parallel flow of solar rays and determining the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

The results could make it possible to reduce the number and complexity of experimental studies, reduce time and investment in project work, and quantitatively evaluate the efficiency of installing solar photovoltaic modules at different latitudes with different installation techniques.

3. The aim and objectives of the study

The purpose of our research is to determine options for tracking solar photovoltaic modules, which could ensure the maximum efficiency of their installation and make it possible to increase the average annual efficiency of electricity production by photovoltaic modules.

To achieve the goal, the following tasks were set:

- to determine the length of the daylight according to the refined physical model of the Earth’s illumination by the parallel flow of the Sun’s rays;
- to determine the dependence of the cosine of the angle of incidence of the Sun’s rays on the angular length of the day;
- to determine the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

4. The study materials and methods

The efficiency of photovoltaic modules was calculated according to a known formula from [9], where the specific power of the module is:

\[ P/S = \eta \cos \theta, \]  

(1)

where \( P \) is the power of the photovoltaic module, \( W \); \( S \) – area of the photovoltaic module, \( m^2 \); \( I \) – intensity of solar radiation, \( W/m^2 \); \( \eta \) – the efficiency of the photovoltaic module, relative unit; \( \theta \) – the angle of incidence of solar radiation (the angle between the direction of incidence of solar radiation and the perpendicular to the photovoltaic module), degree.

Schematic images of the ecliptic, that is, the visible movement of the Sun around the Earth, were used to calculate the inclination angle; Fig. 1.

The angle of incidence of sunlight (the angle between the direction of incidence of sunlight and the perpendicular to the photovoltaic module) is, according to [16], degrees:

\[ \cos \theta_j = \left( \sin \phi \cos \delta - \cos \phi \sin \beta \cos \omega \right) \sin \alpha \cos \delta + \left( \sin \beta \sin \gamma \sin \omega - \sin \phi \cos \beta \cos \omega \right) \cos \delta, \]  

(2)

where \( \delta \) is the declination angle (angular position of the Sun at noon relative to the plane of the Earth’s equator as the angle of inclination of the Sun relative to the Earth, as recommended in [9]). It should be noted that this explanation of the angle of inclination suggests that the Sun changes its position by rotating around the Earth. This definition is valid only for the equator. It is more appropriate to determine the angle of inclination by the angular position of the Sun at noon relative to the plane of the Earth’s latitude, where the photovoltaic panel is installed. In view of the above, the scientific task consists in determining the degree of increase in the efficiency of the installation of photovoltaic modules by refining the physical model of the Earth’s illumination by the parallel flow of solar rays and determining the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

In addition to previous studies [18–20], the results of studies on the performance of a system of four small photovoltaic modules installed with different tilt angles [21] are reported. Performance is estimated by analyzing actual measurement data in the city of Kitakyushu (latitude 33.89°). The optimal angle of inclination for the installed capacity of the photovoltaic system during the year was about 32°. However, the practical results have a narrow application since, for example, the installation of this system at higher latitudes would most likely require an increase in the value of the optimal angle of inclination. However, this statement needs to be verified experimentally.

A significant practical contribution to establishing the dependence of the productivity of solar panels on the angle of their inclination during installation is highlighted in study [22]. Research was carried out in three cities of Albania: Tirana (latitude 41.3275°), Kuksi (latitude 42.0807°), and Vlore (latitude 40.4661°). The article is a successful example of the application of the Bernard-Menguy-Schwartz (BMS) model to determine the optimal value of the annual and seasonal tilt angles of solar panels. Determination of the optimal angle of inclination was carried out based on the fact that the solar energy reaching the panels has a maximum value at the optimal angle of inclination. Accordingly, these angles were: Tirana – 38°; Kuksi – 39°; Vlora – 37°. It should be noted that the annual optimal tilt angle of the solar panels based on the above mathematical model was almost equal to the latitude of the location. In addition, it is noted that the seasonal or adjusted tilt angles of solar panels, in order to increase their efficiency, should be compared with the annual optimal tilt angle. However, the procedure presented in [22] is not complete as it does not take into account the agreement of the tilt angle with the lunar period.

This omission was taken into account when performing the studies given in [16], where it is noted that the agreement of the tilt angle of the panels for the monthly period is better than for the annual period. A monthly-tuned panel angle installation increased the amount of energy stored by the panels by 12 kWh/m² per year compared to an annual-tuned panel installation. This is quite a significant indicator.

The above-mentioned studies are supplemented by the results of experiments reported in [23, 24]. It was experimentally established that the use of a two-axis solar tracker increases the average annual production of electricity by 41% [23]. In turn, the use of a uniaxial solar tracker increases the average annual production of electricity by 12 kW/m². In [22] the refined physical model of the Earth’s illumination by the parallel flow of solar rays and determining the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

The results could make it possible to reduce the number and complexity of experimental studies, reduce time and investment in project work, and quantitatively evaluate the efficiency of installing solar photovoltaic modules at different latitudes with different installation techniques.

3. The aim and objectives of the study

The purpose of our research is to determine options for tracking solar photovoltaic modules, which could ensure the maximum efficiency of their installation and make it possible to increase the average annual efficiency of electricity production by photovoltaic modules.

To achieve the goal, the following tasks were set:

- to determine the length of the daylight according to the refined physical model of the Earth’s illumination by the parallel flow of the Sun’s rays;
- to determine the dependence of the cosine of the angle of incidence of the Sun’s rays on the angular length of the day;
- to determine the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

4. The study materials and methods

The efficiency of photovoltaic modules was calculated according to a known formula from [9], where the specific power of the module is:

\[ P/S = \eta \cos \theta, \]  

(1)

where \( P \) is the power of the photovoltaic module, \( W \); \( S \) – area of the photovoltaic module, \( m^2 \); \( I \) – intensity of solar radiation, \( W/m^2 \); \( \eta \) – the efficiency of the photovoltaic module, relative unit; \( \theta \) – the angle of incidence of solar radiation (the angle between the direction of incidence of solar radiation and the perpendicular to the photovoltaic module), degree.

Schematic images of the ecliptic, that is, the visible movement of the Sun around the Earth, were used to calculate the inclination angle; Fig. 1.

The angle of incidence of sunlight (the angle between the direction of incidence of sunlight and the perpendicular to the photovoltaic module) is, according to [16], degrees:

\[ \cos \theta_j = \left( \sin \phi \cos \delta - \cos \phi \sin \beta \cos \omega \right) \sin \alpha \cos \delta + \left( \sin \beta \sin \gamma \sin \omega - \sin \phi \cos \beta \cos \omega \right) \cos \delta, \]  

(2)

where \( \delta \) is the declination angle (angular position of the Sun at noon relative to the plane of the Earth’s equator as the angle of inclination of the Sun relative to the Earth, as recommended in [9]). It should be noted that this explanation of the angle of inclination suggests that the Sun changes its position by rotating around the Earth. This definition is valid only for the equator. It is more appropriate to determine the angle of inclination by the angular position of the Sun at noon relative to the plane of the Earth’s latitude, where the photovoltaic panel is installed. In view of the above, the scientific task consists in determining the degree of increase in the efficiency of the installation of photovoltaic modules by refining the physical model of the Earth’s illumination by the parallel flow of solar rays and determining the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.
To determine the angle of inclination, the dependence from [13] was used:

$$\delta = 23.45 \sin \left[ \frac{360}{365.25} (284.47 + n) \right]. \tag{3}$$

where $n$ is the day number starting from January 1.

The latitude and azimuth (position) of the Sun relative to the Earth were determined by mathematical expressions [25] obtained by mathematical transformations of the equations of the sphere model, in the center of which the Earth is located, and the Sun moves around the Earth on a sphere; Fig. 1. To refine the calculations, dependences from [26] were used, which include a model of the solar system where the Earth revolves around the Sun; Fig. 2.

For the conditions shown in Fig. 2, it is assumed that the constant angle between the perpendicular to the plane of Zela’s orbit and the Earth’s axis is 23.45°.

To link the physical parameters between the Sun and the Earth and perform various mathematical transformations, the scheme shown in Fig. 3 was used.

As a basis for research, a scheme is adopted, which assumes the rotation of the Earth around the Sun (Fig. 4). According to this scheme, the Earth rotates on its axis and moves in an almost circular orbit around the Sun and, as a result, is in a parallel stream of solar rays.

Connecting the centers of the Earth and the Sun (black line), passing through the center of the Earth a vertical plane (red line), perpendicular to the line connecting the centers of the Earth and the Sun, a cross section of the Earth is obtained; Fig. 4. This section divides the globe into two parts.

This plane separates the day side of the Earth from the side of the Sun, and the night side from the opposite side.

Using this scheme, a scientific definition of the angle of inclination is given according to expression (3), which in scientific literature is called the angle of inclination of the Earth, the angle of inclination of the Sun at noon relative to the plane of the equator. In fact, the tilt angle is the angle between the inclined axis of the Earth and its projection on a vertical plane that is perpendicular to the line connecting the centers of the Earth and the Sun and passing through the center of the Earth. In summer, when the day is the longest (June 21), this angle is maximum and positive (the north pole is on the day side of the Earth), and in winter, when the day is the shortest (December 21), it is maximum and negative (the north pole is on the night side of the Earth). In spring and autumn, when the day is equal to the night (March 21 and September 21), the tilt angle is zero and this means that the Earth's axis is in a vertical plane that is perpendicular to the line connecting the centers of the Earth and the Sun. At the same time, the angle between the perpendicular to the plane of Zela's orbit and the Earth's axis remains constant and is 23.45°.
5. Results of investigating the ways to increase the efficiency of the installation of solar photovoltaic modules

5.1. Determination of the length of daylight according to the refined physical model of the Earth’s illumination by the parallel flow of the Sun’s rays

Further detailing of the scheme of Fig. 4 is shown in Fig. 5. This scheme makes it possible to determine the length of the daylight in the form of the length of an arc at latitude \( \varphi \), which is cut off from the meridian circle by a vertical plane that is perpendicular to the line connecting the centers of the Earth and the Sun.

It is obvious that the equation of the vertical line, which is the line of intersection of the vertical plane perpendicular to the line connecting the centers of the Earth and the Sun and the \( xoz \) plane (Fig. 5), is the following expression:

\[
x = z \cdot \tan \delta = R \sin \varphi \cdot \tan \delta.
\]  

(4)

Substituting this expression into the equation of a circle at latitude \( \varphi \), we get:

\[
y^2 = R^2 \cdot \cos^2 \varphi - x^2 = R^2 \cdot \cos^2 \varphi - R^2 \cdot \sin^2 \varphi \cdot \tan^2 \delta,
\]  

(5)

\[
y^2 = R^2 \cdot (\cos^2 \varphi - \sin^2 \varphi \cdot \tan^2 \delta) = R^2 \cdot (1 - \tan^2 \varphi \cdot \tan^2 \delta).
\]  

(6)

Hence:

\[
y = \pm R \sqrt{1 - \tan^2 \varphi \cdot \tan^2 \delta}.
\]  

(7)

Based on the coordinates of the place of sunrise and sunset at latitude \( \varphi \), we get:

\[
tg \frac{a}{2} = \frac{y}{x} = \frac{\cos^2 \varphi - \sin^2 \varphi \cdot \tan^2 \delta}{\sin^2 \varphi \cdot \tan^2 \delta} = \frac{\tan^2 \varphi}{\tan^2 \delta - 1}.
\]  

(8)

From where the angular length of daylight \( a \) will be:

\[
\delta < 0 \rightarrow a = 2 \arctg \frac{\tan^2 \varphi}{\tan^2 \delta - 1},
\]  

(9)

if,

\[
\delta \geq 0 \rightarrow a = 2 \pi - 2 \arctg \frac{\tan^2 \varphi}{\tan^2 \delta - 1}.
\]  

(10)

Dependences (9, 10) can function under the condition:

\[
\frac{\tan^2 \varphi}{\tan^2 \delta} \geq 1.
\]  

(11)

The boundary condition in this case will be the following angle:

\[
\arctg \frac{\tan^2 \varphi}{\tan^2 \delta} = \alpha = \arctg \frac{1}{\tan \delta} = 66.55^\circ,
\]  

(12)

and this is the angle of inclination of the Earth’s axis of rotation to the plane of the Earth’s orbit around the Sun, due to which the seasons change.

The results of calculating the angular and hourly daylight length according to expressions (9), (10), for example, at latitude \( \varphi = 50^\circ \) (Kyiv, Ukraine), are given in Table 1.

<table>
<thead>
<tr>
<th>Days of the year</th>
<th>Day number ( n )</th>
<th>Angle of declination of the Earth ( \delta ), degree</th>
<th>Angular length of the day ( a ), degree</th>
<th>Hourly length of the day ( t ), hour</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 March</td>
<td>80</td>
<td>-0.31</td>
<td>179.25</td>
<td>11.95</td>
<td>12.2</td>
</tr>
<tr>
<td>21 June</td>
<td>172</td>
<td>23.45</td>
<td>242.26</td>
<td>16.15</td>
<td>16.43</td>
</tr>
<tr>
<td>21 December</td>
<td>355</td>
<td>-23.45</td>
<td>117.74</td>
<td>7.85</td>
<td>7.98</td>
</tr>
</tbody>
</table>

Table 1: An example of calculating the angular and hourly length of daylight.
The illumination scheme of a horizontal photovoltaic panel installed in the meridian plane at the equator and in the northern hemisphere is shown in Fig. 6.

According to Table 1, the estimated angular length of a daylight day at the equator is 180°, and the hourly length is 12 hours. At the same time, in expressions (9), (10), the value of the latitude of the equator must be taken at a level minimally different from 0. A systematic error in the calculation of the angular length of daylight at the level of about 2% is due to the inaccuracy of determining the angle of inclination according to expression (3). Also, each area has its own relief features that affect the actual length of the daylight hours. However, the calculations of the angular and hourly length of daylight according to expressions (9), (10) can be used to evaluate the efficiency of the installation of photovoltaic modules.

5.2. Determination of the dependence of the cosine of the angle of incidence of the sun’s rays on the angular length of the day

The following schemes have been developed for calculations:
- illuminance of a horizontal photovoltaic panel in the meridian plane, at the equator and in the northern hemisphere; Fig. 6;
- illuminance of a photovoltaic panel installed at an angle of latitude in the northern hemisphere; Fig. 7;
- illuminance of a photovoltaic panel installed in the equatorial or latitudinal plane; Fig. 8.

Fig. 6. Illumination scheme of a horizontal photovoltaic panel in the meridian plane, at the equator, and in the northern hemisphere

Fig. 7. Illumination diagram of a photovoltaic panel installed at an angle of latitude in the northern hemisphere

Fig. 8. Schemes of illumination of a photovoltaic panel in the equatorial or latitudinal plane:
\( a \) — horizontal, installed at an angle of latitude in the northern hemisphere and tracking the sun in a vertical plane;
\( b \) — tracking the Sun in the horizontal plane and full tracking of the Sun
The incidence of solar radiation on the surface of photovoltaic modules was determined by the position of the sun’s rays in three-dimensional space (Fig. 6, 7) according to the formula of guiding cosines:

\[ \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1, \]  
(13)

where \( \theta_y \) is the angle of incidence of the sun’s rays relative to the \( y \)-axis, which is placed in the plane of the solar panel parallel to the equatorial plane; \( \theta_y \) is the angle of incidence of the sun’s rays relative to the \( x \)-axis, which is placed in the plane of the solar panel in the meridional plane; \( \theta_z \) is the angle of incidence of the sun’s rays relative to the \( z \)-axis, which is perpendicular to the plane of the solar panel.

Whence the cosine of the angle of incidence of the sun’s rays relative to the \( z \)-axis, which is perpendicular to the surface of the solar panel, will be:

\[ \cos \theta_z = \sqrt{1 - \cos^2 \theta_x - \cos^2 \theta_y} = \frac{1}{\sqrt{1 - \sin^2 \theta_x - \sin^2 \theta_y}}. \]  
(14)

If the sun’s rays are parallel to the \( xoz \) plane (Fig. 5–8), it means that the sun’s rays are parallel to the meridional plane and perpendicular to the \( oz \) axis, and therefore \( \theta_y = 90^\circ, \cos \theta_y = 0, \sin \theta_y = 1 \). If the sun’s rays are parallel to the \( yoz \) plane (Fig. 5–8), it means that the sun’s rays are parallel to the equatorial plane and perpendicular to the \( ox \) axis, and therefore \( \theta_x = 90^\circ, \cos \theta_x = 0, \sin \theta_x = 1 \). If both of these conditions are met, \( \cos \theta_x = 1, \cos \theta_y = 0, \sin \theta_y = 1 \), that is, the sun’s rays will be perpendicular to the plane of the solar panel surface. The actual angle of incidence of the sun’s rays on the plane of the solar panel was determined based on the values of the angles \( \theta_x \) and \( \theta_y \) during sunrise and sunset, as well as at noon. In other hours of daylight, these angles were determined on the basis of their uniform change from sunrise to noon and from noon to sunset since the angular speed of the Earth’s rotation is constant.

The calculation of the cosine of the angle of incidence of the rays for various options for installing photovoltaic panels is given in Table 2.

The calculation of the cosine of the angle of incidence of the rays for a photovoltaic panel that tracks the Sun in a horizontal plane and complete tracking of the Sun are given in Table 3.

Based on the calculated data of the angular length of the day and the values of the cosine of the angle of incidence of the rays on the photovoltaic panel, the corresponding dependences shown in Fig. 9, 10 were constructed.
the angle of incidence of solar rays to the general plane of the graph.

The resulting values of the daily efficiency of installing photovoltaic modules on June 21 at the equator and at latitude $\varphi=50^\circ$ are given in Table 4.

Having determined the daily efficiency of the installation of photovoltaic modules for each day of the year and for each type of their installation, both at the latitude $\varphi=50^\circ$ and at the equator, the value of the annual efficiency of their installation was obtained. This value is the weighted average annual value of the cosine of the angle of incidence of the sun’s rays to the general plane of the solar panel. The results of these calculations are shown in Fig. 11, 12.

### Table 4: Daily efficiency of installation of photovoltaic modules during June 21

<table>
<thead>
<tr>
<th>The way to install the panel at latitude $\varphi=50^\circ$</th>
<th>Horizontal fixed</th>
<th>Fixed at an angle of latitude</th>
<th>Tracking in the vertical plane</th>
<th>Tracking in the horizontal plane at an angle of latitude</th>
<th>Full tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.73</td>
<td>45.87</td>
<td>50</td>
<td>95.87</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

5.3. Determination of the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options

The annual efficiency of the installation of photovoltaic modules was determined as the annual weighted average value of the cosine of the angle of incidence of solar rays $\cos^\varphi \theta$ on the plane of the solar panel according to the following expression:

$$\cos^\varphi \theta = \frac{\sum_{j=1}^{365} a_j \cos \theta_j}{\sum_{j=1}^{365} a_j}$$

(15)

where $a_j$ is the angular length of the $j$-th day, degrees; $\cos \theta_j$ – corresponding to the angular length of the $j$-th day, the value of the cosine of the angle of incidence of the sun’s rays.

The daily efficiency of the installation of photovoltaic modules was determined as the weighted average daily value of the cosine of the angle of incidence of solar rays on the plane of the solar panel:

$$\cos \theta_j = \frac{\sum_{j=1}^{365} a_j \cos \theta_j}{\sum_{j=1}^{365} a_j}$$

(16)

where $a_j$ is the current value of the angular length of the $j$-th day from sunrise to sunset, degrees; $\cos \theta_j$ – corresponding to the angular length of the $j$-th day, the value of the cosine of the angle of incidence of the sun’s rays.

The technique and angle of installation of photovoltaic modules has a significant impact on the production of electricity. The research results confirmed the insignificant efficiency (33.4 % at a latitude of $\varphi=50^\circ$) of horizontal photovoltaic modules, which increases as it approaches the equator, where it reaches a value of 47.93 %. It has been proven that photovoltaic modules installed horizontally on the equator have the same installation efficiency as photovoltaic modules installed at a latitude of $\varphi=50^\circ$ at an angle of inclination to the horizon of 50°. This confirms the fact, proven by many experimental studies, that the angle of inclination of stationary photovoltaic modules to the horizon in the first approximation should correspond to the latitude of the area where they are installed.
The results of studies on the calculation of the installation efficiency of photovoltaic modules showed that tracking in the vertical plane for photovoltaic modules installed at any latitude would have an installation efficiency of 50%. This is only 2.07% more than for stationary photovoltaic modules. This indicates the impracticality of using photovoltaic modules with tracking in the vertical plane.

At the same time, the analysis of the research results reveals that the installation efficiency of photovoltaic modules with tracking in the horizontal plane (rotation of the photovoltaic modules in the horizontal plane to track the Sun along the horizon) for the photovoltaic modules installed at an arbitrary latitude, will have an installation efficiency of 97.93%. This is 2.07% less than for photovoltaic modules installed with the possibility of full tracking (turning of photovoltaic modules in vertical and horizontal planes to track the Sun along the horizon and in height). This may indicate the impracticality of using photovoltaic modules with full tracking and its replacement by tracking in the horizontal plane, which simplifies the system of controlling the rotation of photovoltaic modules.

6. Discussion of results of the study of ways to improve the efficiency of the installation of solar photovoltaic modules

Our results regarding the efficiency of the installation of photovoltaic modules allow establishing a relationship between the annual efficiency of the installation of photovoltaic mod-
its projection on a vertical plane, which is perpendicular to the line connecting the centers of the Earth and the Sun and passing through the center of the Earth.

2. A mathematical expression is presented that describes the dependence of the cosine of the angle of incidence of the Sun’s rays on the angular length of the day. The expression takes into account the angles of incidence of the sun’s rays relative to the axes, which are placed in the plane of the solar panel parallel to the equatorial and meridional planes. This makes it possible to determine the annual weighted average value of the cosine of the angle of incidence of solar rays on the plane of the solar panel and to evaluate the efficiency of the photovoltaic module installation. According to the expression, it is established that the greatest efficiency of installing photovoltaic modules is achieved when the angle of inclination of stationary photovoltaic modules to the horizon in the first approximation corresponds to the latitude of the area where they are installed. The proposed methodology for evaluating the installation of photovoltaic modules can be extended to arbitrary latitudes and techniques of installation of photovoltaic modules, and our data could be used to perform an economic evaluation of the use of photovoltaic modules for electricity generation.

3. The efficiency of installing photovoltaic modules at each latitude can be equal to the efficiency of installing photovoltaic modules at the equator, namely 47.93 % in the case of installing photovoltaic modules at an angle of inclination to the horizon equal to the latitude. The tracking of photovoltaic modules in the vertical plane makes it possible to increase the efficiency of the installation of photovoltaic modules by up to 50 %. Compared to full tracking, tracking in the horizontal plane at an angle of latitude makes it possible to obtain the efficiency of installing photovoltaic modules at the level of 97.93 %.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

The data will be provided upon reasonable request.

References