UDC 535.2
DOI: 10.15587/1729-4061.2023.286464

DETERMINING THE EFFICIENCY OF INSTALLING FIXED SOLAR PHOTOVOLTAIC MODULES AND MODULES WITH DIFFERENT TRACKING OPTIONS

Gennadii Golub
Doctor of Technical Sciences, Professor*
Nataliya Tsyvenkova
Corresponding author
PhD, Associate Professor*
E-mail: nataliyatsyvenkova@gmail.com
Oksana Yaremenko**
Oleh Marus
PhD, Associate Professor
Department of Tractors, Cars and Bioenergy Resources*
Ivan Omarov
Postgraduate Student**
Anna Holubenko
Assistant
Department of Electrification, Automation of Production and Engineering Ecology
Polissia National University
Staryi blvd., 7, Zhytomyr, Ukraine, 10008
*Department of Tractors, Cars and Bioenergy Resources
National University of Life and Environmental Sciences of Ukraine
Heroyiv Oborony str., 15, Kyiv, Ukraine, 03041
**Department of Renewable Organic Energy Resources
Institute of Renewable Energy of the National Academy of Sciences of Ukraine
Hnata Khotkevycha str., 20-a, Kyiv, Ukraine, 02049

The object of this study is photovoltaic modules in various installation options. The physical model of the Earth's illumination by a parallel flow of solar rays has been refined. The dependence of the cosine of the angle of incidence of the Sun's rays on the angular length of the day, as well as the average annual efficiency of the installation of photovoltaic modules, both fixed and with various tracking options, was determined. Refinement of the physical model implies determining the angle of inclination as the angle between the inclined axis of the Earth and its projection on a vertical plane, perpendicular to the line connecting the centers of the Earth and the Sun. This line passes through the center of the Earth. The concept of the average annual efficiency of the installation of photovoltaic modules is introduced as the annual weighted average value of the cosine of the angle of incidence of solar rays on the plane of the photovoltaic module. Various options for installing photovoltaic modules were analyzed: fixed horizontal on the equator; stationary, installed at an angle to the horizon; one that performs tracking in horizontal (vertical) planes; with full tracking. The efficiency of installing a photovoltaic module at each latitude can be equal to the efficiency of installing this module at the equator, that is, 47.93 % when installing the module at an angle of inclination to the horizon equal to the latitude. Tracking in the vertical plane makes it possible to increase the efficiency of the photovoltaic module installation by up to 50 %. Compared to full tracking, tracking in the horizontal plane at an angle of latitude makes it possible to obtain the efficiency of the installation of the photovoltaic module at the level of 97.93 %.

The results could be used as a basis for evaluating the efficiency of the installation of photovoltaic modules at different latitudes with different techniques of their installation, as well as for the subsequent generation of electricity

Keywords: photovoltaic module, angle of incidence of solar rays, angle of inclination, angle of inclination to the horizon, efficiency of module installation

Received date 06.06.2023
Accepted date 17.08.2023
Published date 31.08.2023

***How to Cite:** Golub, G., Tsyvenkova, N., Yaremenko, O., Marus, O., Omarov, I., Holubenko, A. (2023). Determining the efficiency of installing fixed solar photovoltaic modules and modules with different tracking options. Eastern-European Journal of Enterprise Technologies, 4 (8 (124)), 15–25. doi: https://doi.org/10.15587/1729-4061.2023.286464*

1. Introduction

The production of solar energy is one of the urgent problems of modern civilization [1], when a significant part of energy needs is met by traditional types of fuel. As the reserves of fossil fuels, due to the high demand for energy, are rapidly decreasing, renewable energy sources, in particular, solar energy, are increasingly being considered as the energy of the future [2]. This substantiates the importance of scientific research in the area of increasing the efficiency of electricity production through the use of photovoltaic modules [3–5]. One of the ways of increasing the efficiency of photovoltaic modules, along with improving the efficiency of the photovoltaic modules themselves, is increasing the efficiency of their installation [6].

Scientists from different countries have carried out a number of studies on determining the efficiency of installing photovoltaic modules [6–8]. However, their results are based on experimental studies performed under different conditions at different latitudes, which makes their universal application impossible.

One expression from [9] is best suited for evaluating the efficiency of the installation of photovoltaic modules. Although it gives adequate calculation results in many applications, especially in a simplified form, it is, in fact, the result of mathematical transformations and does not reflect the physical essence of the phenomenon of the Earth being illuminated by the Sun.

Therefore, the task of determining the average annual efficiency of the installation of solar photovoltaic modules, both stationary and with various tracking options, remains relevant.

2. Literature review and problem statement

Scientists conducting research in the field of solar energy use a known expression from [10] to determine the efficiency of photovoltaic modules. According to this expression, the efficiency of the photovoltaic module depends directly on its power and inversely on the area, intensity of solar radiation, and the angle of its incidence on the solar panel. In this case, the angle of incidence of solar radiation on the panel is the angle between the direction of incidence of solar radiation and the perpendicular to the photovoltaic module and is determined in degrees.

The expression in [10] structurally defines the main directions of improvement of photovoltaic modules, namely: research into the nature of the intensity of solar radiation, improving the efficiency of the photovoltaic module itself, and increasing the efficiency of its installation.

The most accurate expression for determining the intensity of solar radiation is given in [11], where the specified parameter depends on the conditions of the passage of solar radiation through the Earth's atmosphere, temperature, and spectrum. Also, the authors of work [11] note that since solar radiation is a natural phenomenon, it is impossible to influence its value. Therefore, it is better to determine the efficiency of the photovoltaic module based on the type of semiconductors used and its design. However, issues related to the low efficiency of modern photovoltaic modules, which is only 18–22 %, still remain unresolved. Also, the authors of [11] note that modern technologies can ensure the growth of the indicated indicator, however, this requires significant expenditure of money and time.

In [12], it is noted that the angle of incidence of solar radiation on the panel of the module is a significant factor influencing the specific power of the photovoltaic module, except for the cases of using a photovoltaic module that follows the Sun. However, photovoltaic modules that follow the Sun are not often used due to their relatively high cost. Therefore, the task related to determining the efficiency of the photovoltaic module installation remains open.

One of the options for solving this problem is to establish the optimal range of the angle of incidence of solar radiation on the photovoltaic module, which was proposed by the authors of work [9]. In their work, the authors gave a mathematical expression for determining the specified angle and conducted a number of studies on the verification of this expression. The expression involves taking into account the following parameters: time angle (angular displacement of the Sun to the east/west from the local meridian); inclination angle; latitude of the installation point of the panel on the earth's surface; tilt angle of the photovoltaic module; azimuth of the photovoltaic module. The optimal ranges of values of the angle of incidence of solar radiation for various conditions and schemes of installation of solar panels of photovoltaic modules have been determined. However, the issue of determining the declination angle, that is, the angular position of the Sun at noon relative to the plane of the equator, remained incomplete. The mathematical expression used by the authors is quite cumbersome and inconvenient. The calculation error under some conditions could reach 3 %.

An option that makes it possible to avoid errors when determining the angle of inclination is the mathematical expression proposed in [13]. The expression has been successfully used to optimize methods for installing photovoltaic modules on irregularly shaped roofs by determining the time for which $\cos\theta_z$ is non-negative [14], as well as for the design and optimization of solar photovoltaic power plant structures [15] and for optimizing the tilt angles of photovoltaic modules depending on monthly periods [16].

In [17] it is emphasized that the efficiency of the photovoltaic module depends on the parameters of its installation. The installation of the module can be with the tracking of the Sun in the following planes: horizontal (from the South to the West); in the vertical (from South to North); both vertically and horizontally. Modules can also be installed using fixed panels installed at a certain angle to the horizontal plane. The authors of work [17] modeled different ways of installing photovoltaic panels and established that even when installing photovoltaic panel systems on tracks, it is possible to obtain 20–35 % more energy. However, the cited studies are of an exclusively theoretical nature. In practice, there are a number of factors that significantly distort analytical calculations and prevent achieving the desired result.

In order to avoid these distortions, considerable attention was paid in study [18] to determining the optimal value of the tilt angle and azimuth for the solar collector. Practical verification of calculations is presented on the example of six cities. For them, it was established that during the year, the angles of inclination of the modules were optimal in the range from 20° to 35°, and in the winter period – from 30° to 55°. For the winter period, the optimal azimuthal angles were from –5° to 10°, and during the year – from –10° to 10°. Current standards recommend that the azimuth angle should be in the range of –20° to +20°, and the tilt angle should be within ±25° of latitude. The maximum reduction in solar energy storage for tilt angles can reach 29.5 %, and for azimuth angles – 6.5 %. However, the research is incomplete because within the framework of the study, the authors did not investigate the issue of the production of electrical energy from the accumulated solar energy modules.

The study of the technical and economic potential of photovoltaic systems for electricity production is reported in [19]. The place of the study was the city of Guangzhou, located in the south of China at a latitude of 23°. According to [19], the optimal results of the operation of solar systems were observed when installing photovoltaic panels with an inclination angle of 20°. At the same time, the payback period of these systems was only 6 years, which is a good indicator for an alternative energy facility.

Work [20] also states that installing solar panels or collectors with optimal orientation and tilt angle intensifies energy production and is an important measure to increase the economic performance of solar systems. The authors of work [20] provided essential recommendations for the installation of solar systems aimed at the maximum annual production of solar energy in the intertropical region. The panels of these systems must be directed towards the equator at an angle of inclination equal to the local latitude.

It has been experimentally proven that in West and Central Africa, a moderate deviation (up to 20°) from the optimal orientation and inclination does not have a significant effect on the degree of solar radiation fall (decrease in the level of irradiation <5 %).

In addition to previous studies [18–20], the results of studying the performance of a system of four small photovoltaic modules installed with different tilt angles [21] are reported. Performance is estimated by analyzing actual measurement data in the city of Kitakyushu (latitude 33.89). The optimal angle of inclination for the installed capacity of the photovoltaic system during the year was about 32°. However, the practical results have a narrow application since, for example, the installation of this system at higher latitudes would most likely require an increase in the value of the optimal angle of inclination. However, this statement needs to be verified experimentally.

A significant practical contribution to establishing the dependence of the productivity of solar panels on the angle of their inclination during installation is highlighted in study [22]. Research was carried out in three cities of Albania: Tirana (latitude 41.3275°), Kuksi (latitude 42.0807°), and Vlore (latitude 40.4661°). The article is a successful example of the application of the Bernard-Menguy-Schwartz (BMS) model to determine the optimal value of the annual and seasonal tilt angles of solar panels. Determination of the optimal angle of inclination was carried out based on the fact that the solar energy reaching the panels has a maximum value at the optimal angle of inclination. Accordingly, these angles were: Tirana – 38°; Kuksi – 39°; Vlora – 37°. It should be noted that the annual optimal tilt angle of the solar panels based on the above mathematical model was almost equal to the latitude of the location. In addition, it is noted that the seasonal or adjusted tilt angles of solar panels, in order to increase their efficiency, should be compared with the annual optimal tilt angle. However, the procedure presented in [22] is not complete as it does not take into account the agreement of the tilt angle with the lunar period.

This omission was taken into account when performing the studies given in [16], where it is noted that the agreement of the tilt angle of the panels for the monthly period is better than for the annual period. A monthly-tuned panel angle installation increased the amount of energy stored by the panels by 12 kW/m^2 per year compared to an annual-tuned panel installation. This is quite a significant indicator.

The above-mentioned studies are supplemented by the results of experiments reported in [23, 24]. It was experimentally established that the use of a two-axis solar tracker increases the average annual production of electricity by 41 % [23]. In turn, the use of a uniaxial solar tracker increases the efficiency of the system by 57.4 % compared to a fixed solar panel installed at the optimal tilt angle [24].

Our review of literary sources [9–24] shows that their authors took the angular position of the Sun at noon relative to the plane of the Earth's equator as the angle of inclination of the Sun relative to the Earth, as recommended in [9]. It should be noted that this explanation of the angle of inclination suggests that the Sun changes its position by rotating around the Earth. This definition is valid only for the equator. It is more appropriate to determine the angle of inclination by the angular position of the Sun at noon relative to the plane of the Earth's latitude, where the photovoltaic panel is installed. In view of the above, the scientific task consists in determining the degree of increase in the efficiency of the installation of photovoltaic modules by refining the physical model of the Earth's illumination by the parallel flow of solar rays and determining the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

The results could make it possible to reduce the number and complexity of experimental studies, reduce time and investment in project work, and quantitatively evaluate the efficiency of installing solar photovoltaic modules at different latitudes with different istallation techniques.

3. The aim and objectives of the study

The purpose of our research is to determine options for tracking solar photovoltaic modules, which could ensure the maximum efficiency of their installation and make it possible to increase the average annual efficiency of electricity production by photovoltaic modules.

To achieve the goal, the following tasks were set:

– to determine the length of the daylight according to the refined physical model of the Earth's illumination by the parallel flow of the Sun's rays;

– to determine the dependence of the cosine of the angle of incidence of the Sun's rays on the angular length of the day;

– to determine the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options.

4. The study materials and methods

The efficiency of photovoltaic modules was calculated according to a known formula from [9], where the specific power of the module is:

$$P/S = I\cdot\eta\cdot\cos\theta_z, \tag{1}$$

where P is the power of the photovoltaic module, W; S – area of the photovoltaic module, m^2; I – intensity of solar radiation, W/m^2; η – the efficiency of the photovoltaic module, relative unit; θ_z – the angle of incidence of solar radiation (the angle between the direction of incidence of solar radiation and the perpendicular to the photovoltaic module), degree.

Schematic images of the ecliptic, that is, the visible movement of the Sun around the Earth, were used to calculate the inclination angle; Fig. 1.

The angle of incidence of sunlight (the angle between the direction of incidence of sunlight and the perpendicular to the photovoltaic module) is, according to [16], degrees:

$$\cos\theta_z = (\sin\varphi\cdot\cos\varphi - \cos\varphi\cdot\sin\beta\cdot\cos\gamma)\sin\delta + \\ + \begin{pmatrix} \sin\beta\cdot\sin\gamma\cdot\sin\omega + \cos\varphi\cdot\cos\beta\cdot\cos\omega + \\ + \sin\varphi\cdot\sin\beta\cdot\cos\gamma\cdot\cos\omega \end{pmatrix}\cdot\cos\delta, \tag{2}$$

where δ is the declination angle (angular position of the Sun at noon relative to the plane of the equator); ω – time angle (angular displacement of the sun to the east or west of the local meridian); φ – width of the installation point of the panel on the earth's surface; β – the angle of inclination of the photovoltaic module (the angle between the panel plane and the horizontal plane); γ – azimuth of the photovoltaic module (deviation from the meridian of the projection of the normal to the panel on the horizontal plane).

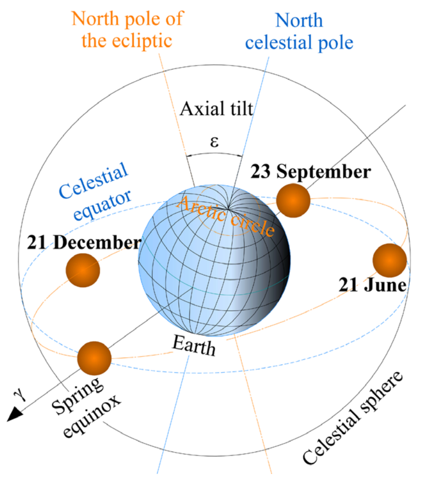

Fig. 1. Schematic representation of the ecliptic, that is, the visible movement of the Sun around the Earth [25]

To determine the angle of inclination, the dependence from[13] was used:

$$\delta = 23.45 \cdot \sin\left[\frac{360°}{365.25}(284.47 + n)\right], \quad (3)$$

where n is the day number starting from January 1.

The latitude and azimuth (position) of the Sun relative to the Earth were determined by mathematical expressions [25] obtained by mathematical transformations of the equations of the sphere model, in the center of which the Earth is located, and the Sun moves around the Earth on a sphere; Fig. 1. To refine the calculations, dependences from [26] were used, which include a model of the solar system where the Earth revolves around the Sun; Fig. 2.

For the conditions shown in Fig. 2, it is assumed that the constant angle between the perpendicular to the plane of Zela's orbit and the Earth's axis is 23.45°.

To link the physical parameters between the Sun and the Earth and perform various mathematical transformations, the scheme shown in Fig. 3 was used.

As a basis for research, a scheme is adopted, which assumes the rotation of the Earth around the Sun (Fig. 4). According to this scheme, the Earth rotates on its axis and moves in an almost circular orbit around the Sun and, as a result, is in a parallel stream of solar rays.

Connecting the centers of the Earth and the Sun (black line), passing through the center of the Earth a vertical plane (red line), perpendicular to the line connecting the centers of the Earth and the Sun, a cross section of the Earth is obtained; Fig. 4. This section divides the globe into two parts.

This plane separates the day side of the Earth from the side of the Sun, and the night side from the opposite side.

Using this scheme, a scientific definition of the angle of inclination is given according to expression (3), which in scientific literature is called the angle of inclination of the Earth, the angle of inclination of the Sun, or the angle of the position of the Sun at noon relative to the plane of the equator. In fact, the tilt angle is the angle between the inclined axis of the Earth and its projection on a vertical plane that is perpendicular to the line connecting the centers of the Earth and the Sun and passing through the center of the Earth. In summer, when the day is the longest (June 21), this angle is maximum and positive (the north pole is on the day side of the Earth), and in winter, when the day is the shortest (December 21), it is maximum and negative (the north pole is on the night side of the Earth). In spring and autumn, when the day is equal to the night (March 21 and September 21), the tilt angle is zero and this means that the Earth's axis is in a vertical plane that is perpendicular to the line connecting the centers of the Earth and the Sun. At the same time, the angle between the perpendicular to the plane of Zela's orbit and the Earth's axis remains constant and is 23.45°.

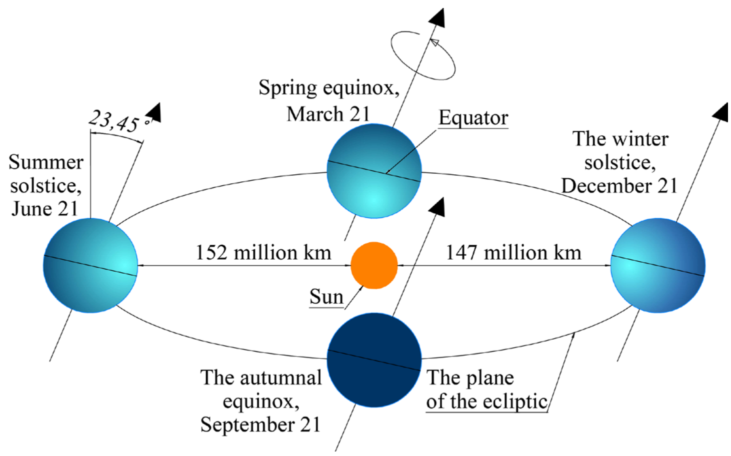

Fig. 2. Diagram of the image of the Earth revolving around the Sun [26]

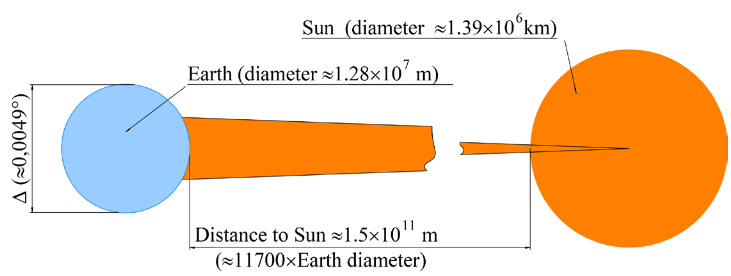

Fig. 3. Diagram of the relationship of physical parameters between the Sun and the Earth [26]

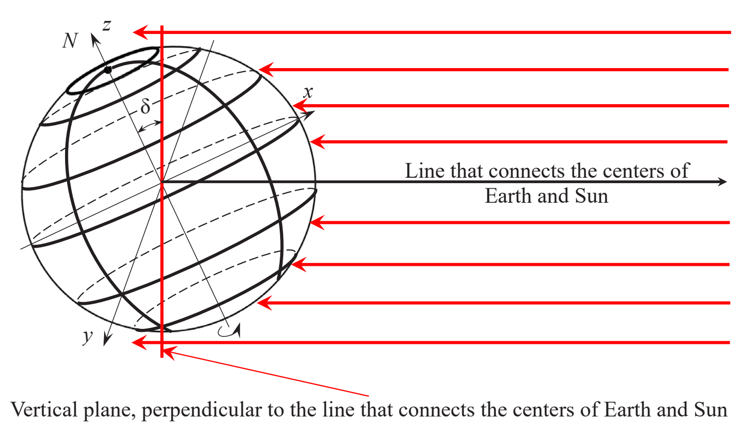

Fig. 4. Diagram of the Earth in a parallel stream of sunlight

It should be noted that similarly, the equatorial plane of the Earth also changes its position but not relative to the Sun, but relative to the parallel flow of the Sun's rays. In spring and autumn, when the day is equal to the night (March 21 and September 21), the equatorial plane of the Earth is placed in a parallel flow of the Sun's rays. At the same time, the tilt angle has a zero value, and this means that the Earth's axis is perpendicular to the line connecting the centers of the Earth and the Sun. However, this determination of the angle of inclination is not important for further research since in the equatorial plane day is equal to night.

Schemes 1–4 and equations (1) to (3) are used to refine the physical model of the Earth's illumination by the parallel flow of the Sun's rays and to determine the average annual efficiency of the installation of stationary and tracking modules with different options.

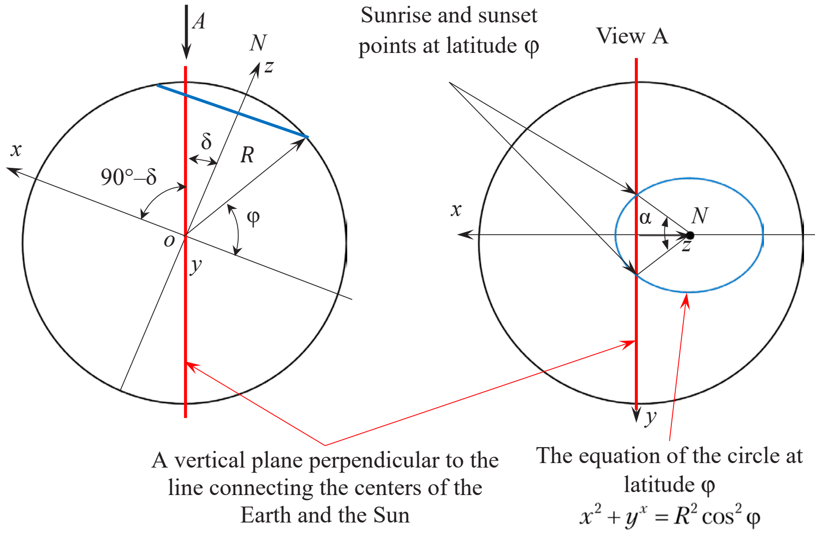

Fig. 5. Diagram of the Earth for determining the length of daylight hours

5. Results of investigating the ways to increase the efficiency of the installation of solar photovoltaic modules

5. 1. Determination of the length of daylight according to the refined physical model of the Earth's illumination by the parallel flow of the Sun's rays

Further detailing of the scheme of Fig. 4 is shown in Fig. 5. This scheme makes it possible to determine the length of the daylight in the form of the length of an arc at latitude φ, which is cut off from the meridian circle by a vertical plane that is perpendicular to the line connecting the centers of the Earth and the Sun.

It is obvious that the equation of the vertical line, which is the line of intersection of the vertical plane perpendicular to the line connecting the centers of the Earth and the Sun and the *xoz* plane (Fig. 5), is the following expression:

$$x = z \cdot \mathrm{tg}\delta = R \sin\varphi \cdot \mathrm{tg}\delta. \quad (4)$$

Substituting this expression into the equation of a circle at latitude φ, we get:

$$y^2 = R^2 \cdot \cos^2\varphi - x^2 = R^2 \cdot \cos^2\varphi - R^2 \cdot \sin^2\varphi \cdot \mathrm{tg}^2\delta, \quad (5)$$

$$y^2 = R^2 \cdot \left(\cos^2\varphi - \sin^2\varphi \cdot \mathrm{tg}^2\delta\right) = R^2 \cdot \left(1 - \mathrm{tg}^2\varphi \cdot \mathrm{tg}^2\delta\right). \quad (6)$$

Hence:

$$y = \pm R\sqrt{1 - \mathrm{tg}^2\varphi \cdot \mathrm{tg}^2\delta}. \quad (7)$$

Based on the coordinates of the place of sunrise and sunset at latitude φ, we get:

$$\mathrm{tg}\frac{a}{2} = \frac{y}{x} = \sqrt{\frac{\cos^2\varphi - \sin^2\varphi \cdot \mathrm{tg}^2\delta}{\sin^2\varphi \cdot \mathrm{tg}^2\delta}} = \sqrt{\frac{\mathrm{ctg}^2\varphi}{\mathrm{tg}^2\delta} - 1}. \quad (8)$$

From where the angular length of daylight *a* will be:

If,

$$\delta \prec 0 \rightarrow a = 2\mathrm{arctg}\sqrt{\frac{\mathrm{ctg}^2\varphi}{\mathrm{tg}^2\delta} - 1}, \quad (9)$$

if,

$$\delta \geq 0 \rightarrow a = 2\pi - 2\mathrm{arctg}\sqrt{\frac{\mathrm{ctg}^2\varphi}{\mathrm{tg}^2\delta} - 1}. \quad (10)$$

Dependences (9, 10) can function under the condition:

$$\frac{\mathrm{ctg}^2\varphi}{\mathrm{tg}^2\delta} - 1 \geq 0 \text{ or } \frac{\mathrm{ctg}^2\varphi}{\mathrm{tg}^2\delta} \geq 1. \quad (11)$$

The boundary condition in this case will be the following angle:

$$\mathrm{ctg}^2\varphi = \mathrm{tg}^2\delta \text{ або } \varphi = \mathrm{arctg}\frac{1}{\mathrm{tg}\delta} = 66.55°, \quad (12)$$

and this is the angle of inclination of the Earth's axis of rotation to the plane of the Earth's orbit around the Sun, due to which the seasons change.

The results of calculating the angular and hourly daylight length according to expressions (9), (10), for example, at latitude φ=50° (Kyiv, Ukraine), are given in Table 1.

Table 1

An example of calculating the angular and hourly length of daylight

Days of the year	Day number *n*	Angle of declination of the Earth δ, degree	Angular length of the day α, degree	Hourly length of the day *t*, hour		Deviation	
				Estimated	Actual	Δ, hour	Δ, %
21 March	80	–0.31	179.25	11.95	12.2	0.25	2.09
21 June	172	23.45	242.26	16.15	16.43	0.28	1.73
21 December	355	–23.45	117.74	7.85	7.98	0.13	1.66

The illumination scheme of a horizontal photovoltaic panel installed in the meridian plane at the equator and in the northern hemisphere is shown in Fig. 6.

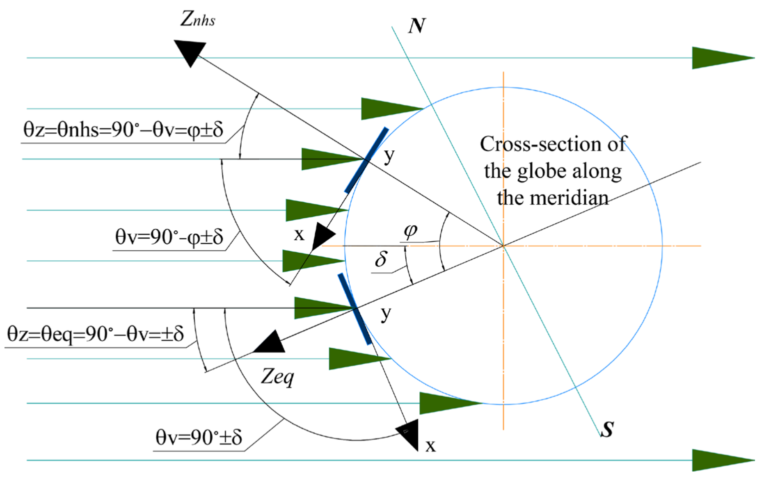

Fig. 6. Illumination scheme of a horizontal photovoltaic panel in the meridian plane, at the equator, and in the northern hemisphere

According to Table 1, the estimated angular length of a daylight day at the equator is 180°, and the hourly length is 12 hours. At the same time, in expressions (9), (10), the value of the latitude of the equator must be taken at a level minimally different from 0. A systematic error in the calculation of the angular length of daylight at the level of about 2 % is due to the inaccuracy of determining the angle of inclination according to expression (3). Also, each area has its own relief features that affect the actual length of the daylight hours. However, the calculations of the angular and hourly length of daylight according to expressions (9), (10) can be used to evaluate the efficiency of the installation of photovoltaic modules.

5. 2. Determination of the dependence of the cosine of the angle of incidence of the sun's rays on the angular length of the day

The following schemes have been developed for calculations:

– illuminance of a horizontal photovoltaic panel in the meridian plane, at the equator and in the northern hemisphere; Fig. 6;

– illuminance of a photovoltaic panel installed at an angle of latitude in the northern hemisphere; Fig. 7;

– illuminance of a photovoltaic panel installed in the equatorial or latitudinal plane; Fig. 8.

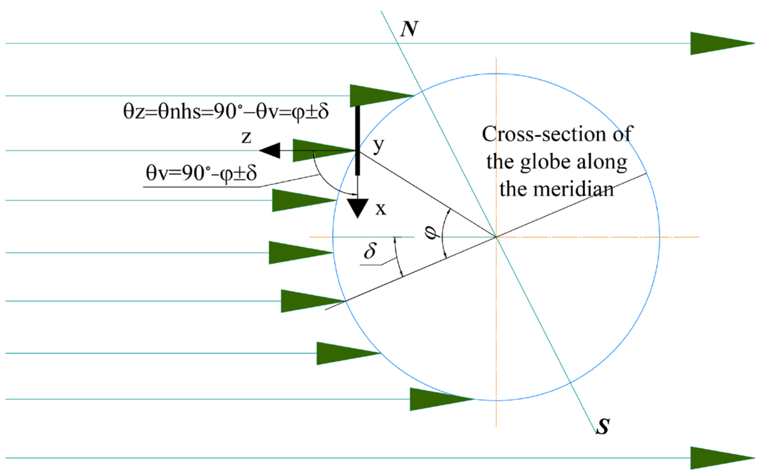

Fig. 7. Illumination diagram of a photovoltaic panel installed at an angle of latitude in the northern hemisphere

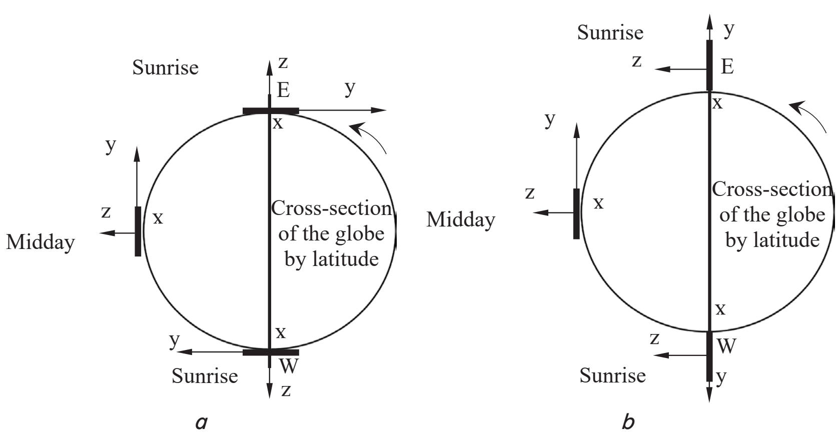

Fig. 8. Schemes of illumination of a photovoltaic panel in the equatorial or latitudinal plane:
a — horizontal, installed at an angle of latitude in the northern hemisphere and tracking the sun in a vertical plane;
b — tracking the Sun in the horizontal plane and full tracking of the Sun

The incidence of solar radiation on the surface of photovoltaic modules was determined by the position of the sun's rays in three-dimensional space (Fig. 6, 7) according to the formula of guiding cosines:

$$\cos^2\theta_H + \cos^2\theta_V + \cos^2\theta_Z = 1, \qquad (13)$$

where θ_H is the angle of incidence of the sun's rays relative to the y-axis, which is placed in the plane of the solar panel parallel to the equatorial plane; θ_V is the angle of incidence of the sun's rays relative to the x-axis, which is placed in the plane of the solar panel in the meridional plane; θ_Z is the angle of incidence of the sun's raysrelaive to the z-axis, which is perpendicular to the plane of the solar panel.

Whence the cosine of the angle of incidence of the sun's rays relative to the z-axis, which is perpendicular to the surface of the solar panel, will be:

$$\cos\theta_Z = \sqrt{1-\cos^2\theta_H - \cos^2\theta_V} = \sqrt{\sin^2\theta_H - \cos^2\theta_V}. \qquad (14)$$

If the sun's rays are parallel to the *xoz* plane (Fig. 5–8), it means that the sun's rays are parallel to the meridional plane and perpendicular to the *oy* axis, and therefore θ_H=90°, $\cos\theta_H$=0, $\sin\theta_H$=1. If the sun's rays are parallel to the *yoz* plane (Fig. 5–8), it means that the sun's rays are parallel to the equatorial plane and perpendicular to the *ox* axis, and therefore θ_V=90°, $\cos\theta_V$=0, $\sin\theta_V$=1. If both of these conditions are met, $\cos\theta_Z$=1, θ_Z=0, that is, the sun's rays will be perpendicular to the plane of the solar panel surface. The actual angle of incidence of the sun's rays on the plane of the solar panel was determined based on the values of the angles θ_H and θ_V during sunrise and sunset, as well as at noon. In other hours of daylight, these angles were determined on the basis of their uniform change from sunrise to noon and from noon to sunset since the angular speed of the Earth's rotation is constant.

The calculation of the cosine of the angle of incidence of the rays for various options for installing photovoltaic panels is given in Table 2.

The calculation of the cosine of the angle of incidence of the rays for a photovoltaic panel that tracks the Sun in a horizontal plane and complete tracking of the Sun are given in Table 3.

Based on the calculated data of the angular length of the day and the values of the cosine of the angle of incidence of the rays on the photovoltaic panel, the corresponding dependences shown in Fig. 9, 10 were constructed.

Table 2

Characteristic angles and calculation of the cosine of the angle of incidence of the beams for different installation options of PV panels

Time point		Sunrise	Noon	Sunset
Horizontal azimuth of the Sun (axis y), θ_H		0	90	180
Horizontal PV panel				
Vertical (x-axis) azimuth of the Sun at the equator φ=0°, θ_V	March 21	90	90	90
	June 21		113	
	December 21		67	
Cosine of the angle of incidence of rays at the equator φ=0°, $\cos\theta_Z$	March 21	0	1.000	0
	June 21		0.917	
	December 21		0.917	
Vertical (x-axis) azimuth of the Sun at latitude φ=50°, θ_V	March 21	90	40	90
	June 21		63	
	December 21		17	
Cosine of the angle of incidence of rays at latitude φ=50°, $\cos\theta_Z$	March 21	0	0.639	0
	June 21		0.895	
	December 21		0.285	
PV panel at an angle of latitude to the horizon				
Vertical (x-axis) azimuth of the Sun at latitude φ=50°, θ_V	March 21	90	90	90
	June 21		113	
	December 21		67	
Cosine of the angle of incidence of rays at latitude φ=50°, $\cos\theta_Z$	March 21	0	1.000	0
	June 21		0.917	
	December 21		0.917	
Tracking the Sun with a photovoltaic panel in a vertical plane				
Vertical (x-axis) azimuth of the Sun at the equator and latitude φ=50°, θ_V	Throughout the year	90	90	90
Cosine of the angle of incidence of rays at the equator and latitude φ=50°, $\cos\theta_Z$	Throughout the year	0	1.000	0

Table 3

Characteristic angles and calculation of the cosine of the angle of incidence of the rays for a photovoltaic panel that performs tracking of the Sun in the horizontal plane and full tracking of the Sun

Time point		Sunrise	Noon	Sunset
Horizontal azimuth of the Sun (axis y), θ_H		90	90	90
Tracking the Sun with a photovoltaic panel in a horizontal plane				
Vertical (x-axis) azimuth of the Sun at the equator and latitude φ=50°, θ_V	March 21	90	90	90
	June 21		113	
	December 21		67	
Cosine of the angle of incidence of rays at the equator and latitude φ=50°, $\cos\theta_Z$	March 21	1,000	1.000	1,000
	June 21		0.917	
	December 21		0.917	
Full Sun tracking by PV panel				
Vertical (x-axis) azimuth of the Sun at the equator and latitude φ=50°, θ_V	Throughout the year	90	90	90
Cosine of the angle of incidence of rays at the equator and latitude φ=50°, $\cos\theta_Z$	Throughout the year	1,000	1.000	1,000

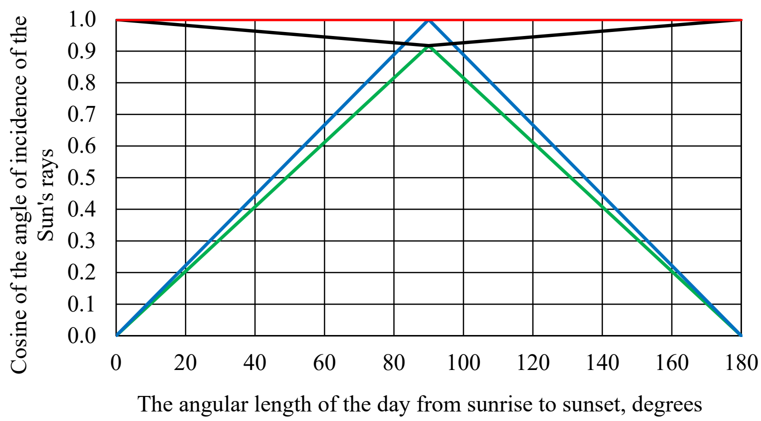

Fig. 9. Dependence of the cosine of the angle of incidence of the Sun's rays on the angular length of the day from sunrise to sunset at the equator for June 21

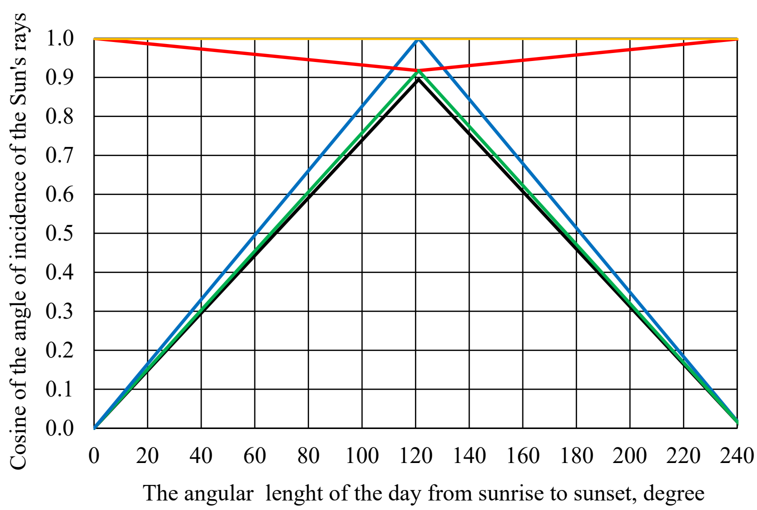

Fig. 10. Dependence of the cosine of the angle of incidence of the Sun's rays on the angular length of the day from sunrise to sunset at latitude φ=50° for June 21

The technique and angle of installation of photovoltaic modules has a significant impact on the production of electricity. The research results confirmed the insignificant efficiency (33.4 % at a latitude of φ=50°) of horizontal photovoltaic modules, which increases as it approaches the equator, where it reaches a value of 47.93 %. It has been proven that photovoltaic modules installed horizontally on the equator have the same installation efficiency as photovoltaic modules installed at a latitude of φ=50° at an angle of inclination to the horizon of 50°. This confirms the fact, proven by many experimental studies, that the angle of inclination of stationary photovoltaic modules to the horizon in the first approximation should correspond to the latitude of the area where they are installed.

5. 3. Determination of the average annual efficiency of the installation of stationary photovoltaic modules and modules with different tracking options

The annual efficiency of the installation of photovoltaic modules was determined as the annual weighted average value of the cosine of the angle of incidence of solar rays $\cos\theta_Z^{an}$ on the plane of the solar panel according to the following expression:

$$\cos\theta_Z^{an} = \frac{\sum_{i=1}^{365} a_i \cos\theta_{Zi}^d}{\sum_{i=1}^{365} a_i}, \quad (15)$$

where a_i is the angular length of the i-th day, degrees; $\cos\theta_{Zi}^d$ – daily efficiency of the installation of photovoltaic modules.

The daily efficiency of the installation of photovoltaic modules was determined as the weighted average daily value of the cosine of the angle of incidence of solar rays on the plane of the solar panel:

$$\cos\theta_{Zi}^d = \frac{\sum_{j=0}^{a_j} a_j \cos\theta_j}{\sum_{j=0}^{a_j} a_j}, \quad (16)$$

where a_j is the current value of the angular length of the j-th day from sunrise to sunset, degrees; $\cos\theta_j$ – corresponding to the angular length of the j-th day, the value of the cosine of the angle of incidence of the sun's rays.

The daily value of the efficiency of the installation of photovoltaic modules was determined according to expression (10) as the daily average weighted value of the cosine of the angle of incidence of solar rays on the plane of the solar panel. This is nothing but the ratio of the area under the lines of the cosine values of the angle of incidence of the sun's rays to the general plane of the graph.

The resulting values of the daily efficiency of installing photovoltaic modules on June 21 at the equator and at latitude φ=50° are given in Table 4.

Having determined the daily efficiency of the installation of photovoltaic modules for each day of the year and for each type of their installation, both at the latitude φ=50° and at the equator, the value of the annual efficiency of their installation was obtained. This value is the weighted average annual value of the cosine of the angle of incidence of the sun's rays $\cos\theta_Z^{an}$ on the plane of the solar panel. The results of these calculations are shown in Fig. 11, 12.

Table 4

Daily efficiency of installation of photovoltaic modules during June 21

The way to install the panel at latitude φ=50°				
Horizontal fixed	Fixed at an angle of latitude	Tracking in the vertical plane	Tracking in the horizontal plane at an angle of latitude	Full tracking
44.73	45.87	50	95.87	100
Method of installation of the panel at the equator				
Horizontal fixed		Tracking in the vertical plane	Tracking in the horizontal plane	Full tracking
45.87		50	95.87	100

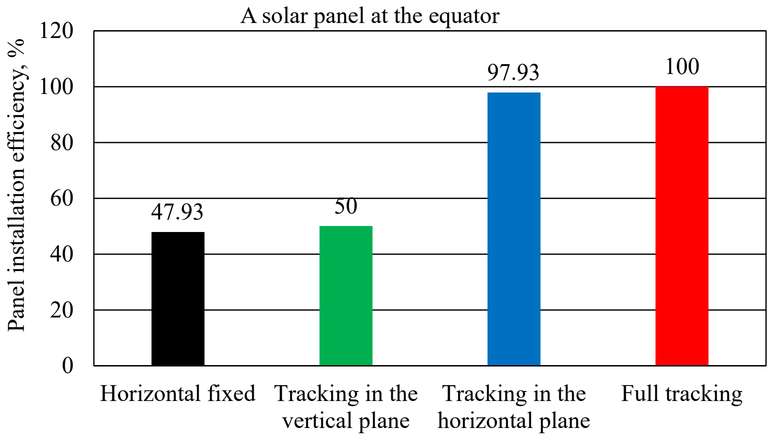

Fig. 11. Annual efficiency of installation of photovoltaic modules at the equator

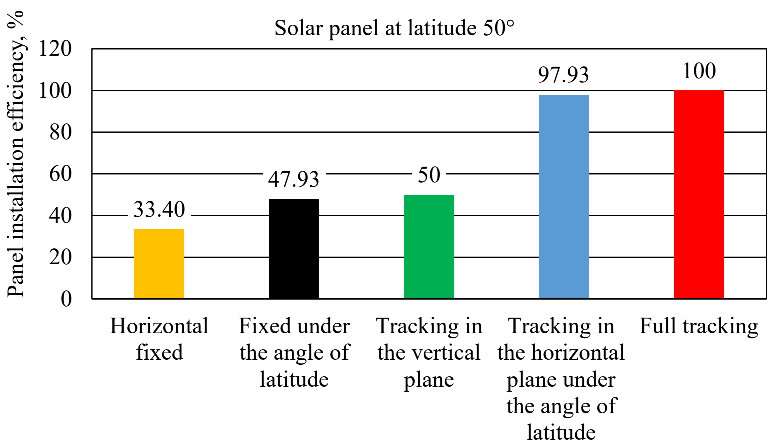

Fig. 12. Annual efficiency of installation of photovoltaic modules at latitude φ=50°

The results of studies on the calculation of the installation efficiency of photovoltaic modules showed that tracking in the vertical plane for photovoltaic modules installed at any latitude would have an installation efficiency of 50 %. This is only 2.07 % more than for stationary photovoltaic modules. This indicates the impracticality of using photovoltaic modules with tracking in the vertical plane.

At the same time, the analysis of the research results reveals that the installation efficiency of photovoltaic modules with tracking in the horizontal plane (rotation of the photovoltaic modules in the horizontal plane to track the Sun along the horizon) for the photovoltaic modules installed at an arbitrary latitude, will have an installation efficiency of 97.93 %. This is 2.07 % less than for photovoltaic modules installed with the possibility of full tracking (turning of photovoltaic modules in vertical and horizontal planes to track the Sun along the horizon and in height). This may indicate the impracticality of using photovoltaic modules with full tracking and its replacement by tracking in the horizontal plane, which simplifies the system of controlling the rotation of photovoltaic modules.

6. Discussion of results of the study of ways to improve the efficiency of the installation of solar photovoltaic modules

Our results regarding the efficiency of the installation of photovoltaic modules allow establishing a relationship between the annual efficiency of the installation of photovoltaic modules and the option of their installation. To evaluate the efficiency of the installation of photovoltaic modules, the value of the annual average weighted by the cosine of the angle of incidence of solar rays on the plane of the solar panel was used (15), (16). The value of the efficiency of the installation of photovoltaic modules with full tracking at the level of 100 % means that the cosine of the angle of incidence of solar rays on the plane of the solar panel is 1. The angle of incidence of the rays with respect to the x-axis, which is perpendicular to the plane of the photovoltaic module, will be zero, which means the perpendicularity of the solar rays and the plane of the photovoltaic module. Smaller values of the efficiency of the installation of photovoltaic modules for other installation options are explained by the shorter time when the sun's rays are perpendicular to the plane of the photovoltaic module or the complete absence of such a phenomenon. The results are similar to the results reported in works [18, 19], in which the authors confirm in practice the dependence of the efficiency of the installation of photovoltaic modules on the season, the angle of inclination of the panel, the angle of azimuth, etc.

The peculiarity of the proposed method is the determination of the weighted average annual value of the cosine of the angle of incidence of solar rays on the plane of the panel based on the values of the average weighted daily cosine of the angle of incidence of solar rays on the plane of the panel (14). The results could be used to calculate the efficiency of installing photovoltaic modules at different latitudes. It should be noted that these results have numerous experimental confirmations in the scientific literature [14–20].

Limitations of the study: the results on the efficiency of the installation of photovoltaic modules allow establishing a relationship between the annual efficiency of the installation of photovoltaic modules and the option of their installation on the earth's surface from the equator to the value of latitude 66.55°.

The main drawback of this study is that it does not explain the slight deviation of the optimal installation angles of photovoltaic modules, established experimentally, from the installation angles determined according to this procedure, and which have values corresponding to the values of the latitude of the installation site of photovoltaic modules.

The further development of this study should consist in determining the optimal values of the installation angles of photovoltaic modules, which would fully coincide with the experimental data. Also, the performed studies could be the basis for further technical and economic substantiation of techniques for installing photovoltaic modules at different latitudes.

7. Conclusions

1. The length of the daylight hours was determined according to the refined physical model of the Earth's illumination by the parallel flow of the Sun's rays. Refinement of the physical model was carried out by determining the tilt angle as the angle between the inclined axis of the Earth and

its projection on a vertical plane, which is perpendicular to the line connecting the centers of the Earth and the Sun and passing through the center of the Earth.

2. A mathematical expression is presented that describes the dependence of the cosine of the angle of incidence of the Sun's rays on the angular length of the day. The expression takes into account the angles of incidence of the sun's rays relative to the axes, which are placed in the plane of the solar panel parallel to the equatorial and meridional planes. This makes it possible to determine the annual weighted average value of the cosine of the angle of incidence of solar rays on the plane of the solar panel and to evaluate the efficiency of the photovoltaic module installation. According to the expression, it is established that the greatest efficiency of installing photovoltaic modules is achieved when the angle of inclination of stationary photovoltaic modules to the horizon in the first approximation corresponds to the latitude of the area where they are installed. The proposed methodology for evaluating the installation of photovoltaic modules can be extended to arbitrary latitudes and techniques of installation of photovoltaic modules, and our data could be used to perform an economic evaluation of the use of photovoltaic modules for electricity generation.

3. The efficiency of installing photovoltaic modules at each latitude can be equal to the efficiency of installing photovoltaic modules at the equator, namely 47.93 % in the case of installing photovoltaic modules at an angle of inclination to the horizon equal to the latitude. The tracking of photovoltaic modules in the vertical plane makes it possible to increase the efficiency of the installation of photovoltaic modules by up to 50 %. Compared to full tracking, tracking in the horizontal plane at an angle of latitude makes it possible to obtain the efficiency of installing photovoltaic modules at the level of 97.93 %.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

The data will be provided upon reasonable request.

References

1. Dib, M., Nejmi, A., Ramzi, M. (2020). New auxiliary services system in a transmission substation in the presence of a renewable energy source PV. Materials Today: Proceedings, 27, 3151–3156. doi: https://doi.org/10.1016/j.matpr.2020.03.820
2. Satpathy, R., Pamuru, V. (2020). Solar PV Power: Design, Manufacturing and Applications from Sand to Systems. Academic Press. doi: https://doi.org/10.1016/c2018-0-02530-x
3. Umar, S., Waqas, A., Tanveer, W., Shahzad, N., Janjua, A. K., Dehghan, M. et al. (2023). A building integrated solar PV surface-cleaning setup to optimize the electricity output of PV modules in a polluted atmosphere. Renewable Energy, 216, 119122. doi: https://doi.org/10.1016/j.renene.2023.119122
4. Lu, Y., Li, G. (2023). Potential application of electrical performance enhancement methods in PV/T module. Energy, 281, 128253. doi: https://doi.org/10.1016/j.energy.2023.128253
5. Jha, V. (2023). Generalized modelling of PV module and different PV array configurations under partial shading condition. Sustainable Energy Technologies and Assessments, 56, 103021. doi: https://doi.org/10.1016/j.seta.2023.103021
6. Barbosa de Melo, K., Kitayama da Silva, M., Lucas de Souza Silva, J., Costa, T. S., Villalva, M. G. (2022). Study of energy improvement with the insertion of bifacial modules and solar trackers in photovoltaic installations in Brazil. Renewable Energy Focus, 41, 179–187. doi: https://doi.org/10.1016/j.ref.2022.02.005
7. Dhimish, M., Ahmad, A., Tyrrell, A. M. (2022). Inequalities in photovoltaics modules reliability: From packaging to PV installation site. Renewable Energy, 192, 805–814. doi: https://doi.org/10.1016/j.renene.2022.04.156
8. Peters, I. M., Hauch, J. A., Brabec, C. J. (2022). The role of innovation for economy and sustainability of photovoltaic modules. IScience, 25 (10), 105208. doi: https://doi.org/10.1016/j.isci.2022.105208
9. Duffie, J. A., Beckman, W. A. (2013). Solar Engineering of Thermal Processes. John Wiley & Sons. doi: https://doi.org/10.1002/9781118671603
10. Wald, L. (2021). Fundamentals of Solar Radiation. CRC Press. doi: https://doi.org/10.1201/9781003155454
11. Grygiel, P., Tarłowski, J., Prześniak-Welenc, M., Łapiński, M., Łubiński, J., Mielewczyk-Gryń, A. et al. (2021). Prototype design and development of low-load-roof photovoltaic modules for applications in on-grid systems. Solar Energy Materials and Solar Cells, 233, 111384. doi: https://doi.org/10.1016/j.solmat.2021.111384
12. Winter, C.-J., Sizmann, R. L., Vant-Hull, L. L. (Eds.) (1991). Solar Power Plants. Springer Berlin, 425. doi: https://doi.org/10.1007/978-3-642-61245-9
13. Cooper, P. I. (1969). The absorption of radiation in solar stills. Solar Energy, 12 (3), 333–346. doi: https://doi.org/10.1016/0038-092x(69)90047-4
14. Barbón, A., Ghodbane, M., Bayón, L., Said, Z. (2022). A general algorithm for the optimization of photovoltaic modules layout on irregular rooftop shapes. Journal of Cleaner Production, 365, 132774. doi: https://doi.org/10.1016/j.jclepro.2022.132774

15. Konovalov, Y. V., Khaziev, A. N. (2022). Computer technology applications to calculate the insolation of photoelectric power plant. IOP Conference Series: Earth and Environmental Science, 990 (1), 012048. doi: https://doi.org/10.1088/1755-1315/990/1/012048
16. Yassir, A. (2019). Genetic Algorithm as a Solutions Optimization of Tilt Angles for Monthly Periods of Photovoltaic Installation. IOP Conference Series: Materials Science and Engineering, 536 (1), 012100. doi: https://doi.org/10.1088/1757-899x/536/1/012100
17. Khadidja, B., Dris, K., Boubeker, A., Noureddine, S. (2014). Optimisation of a Solar Tracker System for Photovoltaic Power Plants in Saharian Region, Example of Ouargla. Energy Procedia, 50, 610–618. doi: https://doi.org/10.1016/j.egypro.2014.06.075
18. Jin, Z., Xu, K., Zhang, Y., Xiao, X., Zhou, J., Long, E. (2017). Installation Optimization on the Tilt and Azimuth Angles of the Solar Heating Collectors for High Altitude Towns in Western Sichuan. Procedia Engineering, 205, 2995–3002. doi: https://doi.org/10.1016/j.proeng.2017.10.225
19. Pan, D., Bai, Y., Chang, M., Wang, X., Wang, W. (2022). The technical and economic potential of urban rooftop photovoltaic systems for power generation in Guangzhou, China. Energy and Buildings, 277, 112591. doi: https://doi.org/10.1016/j.enbuild.2022.112591
20. N'Tsoukpoe, K. E. (2022). Effect of orientation and tilt angles of solar collectors on their performance: Analysis of the relevance of general recommendations in the West and Central African context. Scientific African, 15, e01069. doi: https://doi.org/10.1016/j.sciaf.2021.e01069
21. Shu, N., Kameda, N., Kishida, Y., Sonoda, H. (2006). Experimental and Theoretical Study on the Optimal Tilt Angle of Photovoltaic Panels. Journal of Asian Architecture and Building Engineering, 5 (2), 399–405. doi: https://doi.org/10.3130/jaabe.5.399
22. Buzra, U., Mitrushi, D., Serdari, E., Halili, D., Muda, V. (2022). Fixed and adjusted optimal tilt angle of solar panels in three cities in Albania. Journal of Energy Systems, 6 (2), 153–164. doi: https://doi.org/10.30521/jes.952260
23. Nikitenko, G. V., Konoplev, E. V., Salpagarov, V. K., Danchenko, I. V., Masyutina, G. V. (2020). Improving the energy efficiency of using solar panels. IOP Conference Series: Earth and Environmental Science, 613 (1), 012092. doi: https://doi.org/10.1088/1755-1315/613/1/012092
24. Kuttybay, N., Saymbetov, A., Mekhilef, S., Nurgaliyev, M., Tukymbekov, D., Dosymbetova, G. et al. (2020). Optimized Single-Axis Schedule Solar Tracker in Different Weather Conditions. Energies, 13 (19), 5226. doi: https://doi.org/10.3390/en13195226
25. Jäger, K., Isabella, O., Smets, A. H. M., Swaaij, R. A., Zeman, M. (2014). Solar Energy, Fundamentals, Technology, and Systems. Delft: Delft University of Technology, 408. Available at: http://web.kpi.kharkov.ua/ief/wp-content/uploads/sites/39/2020/05/solar_energy_1.pdf
26. Renewable Energy. Solar Power (2009). Courseware Sample by the staff of Lab-Volt Ltd., 115.