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ENSURING THE STABILITY OF MACHINING WHEN USING END MILLS

Yuri Petrakov
Corresponding author
Doctor of Technical Sciences, Professor*
E-mail: ypetrakov.86@gmail.com
Olexander Ohrimenko
Doctor of Technical Sciences, Professor*
Maxim Sikailo
Postgraduate Student*
*Department of Mechanical Engineering Technology
National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»
Beresteiskyi (Peremohy) ave., 37, Kyiv, Ukraine, 03056

This paper reports a new approach to ensuring the stability of the end milling process, due to vibration-free cutting modes, which are determined from the stability lobes diagram of the dynamic machining system. An application program for automatic calculation of the stability lobes diagram in the coordinates «mill spindle speed – feed» has been developed, which is a tool for the technologist-programmer when designing control program for numerically controlled machines. The mathematical model underlying the application program represents a dynamic machining system as a single-mass system with two degrees of freedom, covered by negative feedback in the direction of two coordinates. The trailing machining is represented as positive feedback loops with a delay function in each. The mathematical model is given in the form of state variables, which allows applying numerical modeling methods to determine both transient and frequency responses. The software developed includes a separate module for automatic design of the stability lobes diagram whose algorithm uses the features of the location of the Nyquist diagram on the complex plane. Since the functioning of the developed program requires a priori information about the dynamic parameters of the machining system, a procedure for their experimental identification is presented. The stiffness of the machining system in two coordinates was determined with the help of a dynamometer, and the frequency responses were determined by the impulse response function, which was obtained with an impact hammer. The research results were confirmed experimentally both by computer simulation and milling on a machine tool and could be recommended for determining the cutting mode at end milling

Keywords: end milling, stability of the cutting process, identification of dynamic parameters of the machining system, stability lobes diagram

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1. Introduction

The end milling process is a common manufacturing operation for various parts, particularly in the aviation industry.

It takes place in a flexible technological machining system (TMS) and is always accompanied by vibrations. Such vibrations, depending on the amplitude, always have negative consequences in terms of ensuring the necessary roughness of the machined surface, stability of the tool, and even stability of the entire cutting process.

To eliminate vibrations or reduce their amplitude, methods of passive control (Passive Chatter Control – PCC) or active control (Active Chatter Control – ACC) are used [1]. Passive methods are based mainly on the application of dynamic vibration compensators or on the use of special control programs based on general methods of controlling the spindle speed variation (SSV). Active methods are aimed at introducing into the TMS artificial harmonic signals that have the opposite phase compared to the exciting oscillations. In this way, oscillations in the cutting zone are damped. It is clear that such methods are based on the construction of automatic control systems with appropriate sensors, exciting devices, usually based on piezo elements, and spectrum analyzers with electronic control units and autonomous power subsystems [2]. It is obvious that such systems require significant modernization of existing equipment and significant financial costs.

At the same time, the reserves based on the definition of the so-called «vibration-free» cutting modes have not yet been exhausted. The widespread use of CNC machines and new tool materials makes it possible to hope for significant efficiency from the implementation of this approach for most cutting processes. To determine such modes, stability lobes diagrams (SLD) are used, which are projected in the coordinates «spindle speed – cutting depth». However, the construction of such diagrams causes certain difficulties of both theoretical and practical nature.

First, for their design, it is recommended to use complex algorithms based on algebraic stability criteria, which are found when solving the characteristic equations of a dynamic system of the second order with a single degree of freedom (SDOF). The second obstacle of a practical nature is the need to use the actual dynamic parameters of TMS and the dependence of the cutting force on the mode for SLD design. Therefore, resolving such a complex scientific and technical problem for end milling operations is an urgent task of cutting processing on CNC machines.

2. Literature review and problem statement

To devise a control strategy for any process, it is necessary to imagine its model, which can adequately represent the causes of problems that must be eliminated as a result of control formation. The most widely used is the method of control based on a priori information, which is based on the determination of control during process modeling using objective data on dynamic and power parameters of TMS.

Currently, two categories of oscillations during cutting are distinguished: primary and secondary [1]. The primary vibration is caused by the cutting process itself, nonlinear friction in the cutting zone, or the cross-effect of elastic shifts of TMS along the coordinate axes. Secondary vibration is caused by the regenerative effect of the waviness of the surface of the workpiece, which was formed in the previous pass. This regenerative effect is considered to be the most important reason for the self-excitation of TMS.

To represent the model of this process, the wavy shape of the chip thickness is considered, which is determined by the passage of the previous tooth of the cutter and the trajectory of the cutting edge of the tool during oscillations on the current passage. It is determined that the chip thickness, and therefore the cutting force, changes depending on the phase difference between these waves. Such a term as «dynamic chip thickness» is introduced, which is adopted in the model for determining the constancy of the cutting process [1]. At the same time, the adequacy of the representation is lost due to the lack of a general model of the cutting process taking into account the feedback loop in the elastic TMS.

An extended approach to the geometry of the chip thickness with reference to the closedness of the processes in TMS during cutting is reported in [3]. In addition, when determining stability, processing «behind the trail» is taken into account, which is represented by the delay function in the positive feedback of the closed TMS. This approach leads to the construction of a model that is used in determining the stability lobes diagram (SLD) in the «spindle speed – depth of cut» coordinates. However, the design of such a diagram is based on algebraic stability criteria, which are determined from the characteristic equation, and involves the use of a multi-step algorithm, which prevents its application in practice.

To design SLD, even when representing a dynamic model in the form of a one-mass system with one degree of freedom, objective data on the power characteristics of the cutting process and dynamic parameters of the system are required. Such parameters can be obtained experimentally using the methods reported in paper [4].

In the cited work, a method for identifying end milling parameters using linear predictive coding (LPC) and extended Kalman filtering was devised. The milling simulation model is validated by comparing simulated and experimental forces for a variety of end milling cuts. Modeling regenerative cutting forces, tool/workpiece vibration, and surface waviness requires identification of tool-workpiece system parameters, i.e., damping factor, natural frequency, and stiffness. To obtain these system parameters, cutting force measurements are used using a dynamometer (Kistler), which is installed on the bed under the workpiece.

However, the simulation is again based on regenerative chip thickness, phase change, and its effect on system dynamics. The authors proposed a linear model of milling forces taking into account the geometry of the tool. In addition, the methodology for calculating the cutting force and deflections of the tool/workpiece is explained.

To improve the adequacy of the model, one-mass dynamic systems are considered, but with two degrees of freedom [5]. It is proposed to take into account the dynamic stiffness as a function of the frequency response for each subsystem along the coordinate axes. It is proposed to form matrices of TMS transfer functions both directly along the coordinate axes and cross transfer functions, which enhances the adequacy of the model. However, the construction of stability lobes diagrams is also based on the concept of «dynamic chip thickness» and involves a complex algorithm containing several successive calculation steps. At the last step, it is suggested to repeat the analysis of oscillation frequencies on the entire range of studies. It is claimed that such an algorithm, despite the longer calculation time, would ensure the elimination of the problem of estimating very similar, but different modes of behavior of a dynamic system.

It should be noted that when applying the concept of «dynamic chip thickness», the representations of the previous elastic deviation and the current depth of cut are often inadequate because the amplitude of the elastic deviation is assumed to be almost equal to the depth of cut. Such a representation indicates either a completely insufficient rigidity of TMS, which rarely corresponds to real milling processes, or a deliberate hyperbolization of the process.

It is advisable to study dynamic processes during end milling by means of simulation in the time and frequency domains at the same time [6]. The model takes into account the closedness of the cutting process in the elastic machining system due to feedback in the form of elastic shifts along the coordinate axes. The dynamics of the system were represented by a one-mass model with two degrees of freedom. A structural diagram of the milling process using transfer functions, which reflects the cross-connections of a real machining system, has been compiled. It is shown that the main reason for the occurrence of regenerative oscillations is machining behind the track. The possibility of constructing the process SLD by comparing frequency responses in the form of Nyquist diagrams and time responses is shown; however, a generalized design algorithm and a reliable software tool are not available.

When identifying a dynamic system, the representation of force characteristics that connect the cutting mode and the components of the cutting force along the coordinate axes plays an important role. Such studies differ in the approach used: from the application of the finite element method [7] to the desire to preserve the physics of the cutting process during milling. In [8], a model was developed that takes into account the features of the end milling process with a spiral arrangement of the cutting edge along the height of the mill. In the model, each tooth of the helical end mill is discretized into several sections along the axis of the cutter to take into account the influence of the inclination angle on the cutting force. However, all analytical models require the use of empirical coefficients obtained by procedures for statistical treatment of experimental data [9].

Feeling the need to increase the adequacy of SLD [10], an experimental method for its determination during milling was devised. The methodology is based on empirical research, when during milling, the depth of cut is gradually increased in the feed direction, and the spindle speed remains constant on the pass and varies from pass to pass. The cutting process is stopped as soon as vibration is detected and thus the boundary between stable and unstable cutting is identified, which is fixed in the diagram. The procedure is designed to identify such a boundary using a microphone and special software.

Currently, a new SLD design method for turning is proposed, based on a new criterion of stability of closed TMSs with a delay function in positive feedback [11]. This method is embodied in a software tool that makes it possible to design

SLD and determine the vibration-free cutting mode quickly and efficiently, even on existing equipment, when performing a specific operation.

Thus, it can be stated that the results of the above numerous studies are currently little used in the programming of processing with end mills on CNC machines, despite the possibility of choosing a cutting mode in a wide range. Therefore, research aimed at building simple means of determining vibration-free cutting modes during end milling is appropriate.

3. The aim and objectives of the study

The purpose of this study is to devise a methodology and software tools for determining the stability lobes diagram when machining with end mills, which could ensure the possibility of operative assignment of a chatter-free cutting mode.

To achieve the goal, the following tasks were set:

– to build a mathematical model of the milling process with end mills, which reproduces the real processes of the elastic system taking into account the closedness of TMS, cross-connections along two coordinates, and machining along the trace;

– to devise a procedure of identification of dynamic parameters of TMS and force characteristics during end milling;

– to construct software tools for SLD design in the coordinates of the components of the cutting mode;

– to experimentally verify the results to be obtained.

4. The study materials and methods

The object of our study is the milling process with end mills, which takes place in an elastic closed TMS and takes into account machining after the trace. When analyzing the stability of the process, a new criterion was adopted for closed systems with a delay function in additional positive feedback, which is based on the features of the location of the Nyquist diagram on the complex plane.

The assessment methodology is based on a mathematical model, which, based on a system approach, represents TMS in the form of a connection of separate blocks. This makes it possible to obtain a mathematical model in the form of differential equations in variable states, maximally adapted to modeling by numerical methods. The new stability criterion has made it possible to build an algorithm and a corresponding software tool for the automatic construction of SLD of the end milling process.

For the practical application of the developed SLD design program, a procedure for identifying the dynamic and power parameters of TMS and the cutting process was devised. The methodology presents experimental studies on the determination of stiffness, frequency characteristics, and the dependence of the cutting force on mode. During the research, a three-component dynamometer (MCS 10-005-3C made by HBM Germany) was used, which was connected to ClipX BM40 amplifiers also made by HBM (Germany). An impact hammer (Impact Hammer Model 086C03 by PCB Piezotronics Inc. USA) and a recording oscilloscope (SDS 1022DL) were used for dynamic research. The adequacy of the proposed solutions and the convergence of the developed algorithm are confirmed by software as a result of modeling both the transient process and frequency responses, as well as the results of field tests.

5. Results of the study of the milling process for stability

5. 1. Mathematical model

The end milling process takes place in a closed elastic TMS, which is affected by the main perturbation in the form of a cutting force. Since elastic movements under the action of the cutting force occur mainly along two coordinates, for the analysis of dynamic processes it is quite sufficient to represent TMS as a one-mass one with two degrees of freedom (Fig. 1).

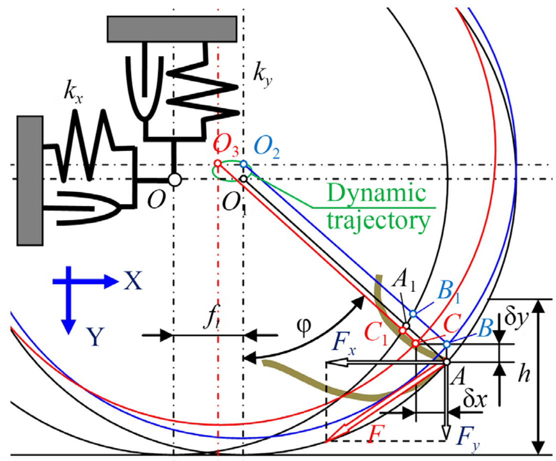


Fig. 1. Diagram of an elastic technological machining system

In turn, the cutting force F depends on many factors, among which the cutting mode can be distinguished, which determines the main geometric parameters of the allowance layer cut by the cutter tooth. Therefore, the main perturbation influence is the cutting force, which can be determined empirically when milling with end mills using the following dependence [6]:

$$F = C_p a^k b, \quad (1)$$

where a is the thickness, b is the cutting width; C_p, k are the empirical coefficient and exponent. For the conditions of further research, C_p=1150 N/mm, k=0.85.

Such a dependence is nonlinear and can be linearized with sufficient accuracy as a function of the components of the cutting mode:

$$F = k_f f_t + k_H h, \quad (2)$$

where f_t is the feed per tooth of the milling cutter, h is the cutting depth.

The linearization coefficients are defined as the values of the partial derivatives of the cutting force at the values of the variables at the linearization point [6]. The component cutting forces cause elastic movements of TMS along the corresponding coordinate axes (Fig. 1). These component forces acting on the cutter tooth can be determined according to the relationships reported in [12]. Since the tangential component of the cutting force acts normal to the front surface of the cutting wedge of the cutter tooth, the components along the coordinate axes can be defined as:

$$F_x = F\,\mathrm{Cos}(\varphi-\gamma-\beta),\; F_y = F\,\mathrm{Sin}(\varphi-\gamma-\beta), \quad (3)$$

where γ – rake angle (10 degress), and angle $\beta=\arctan(F_n/F_t)$= =arctan(0.5) (26.5 degrees).

In an absolutely rigid system, the cutting thickness during cylindrical milling depends on the cutting angle φ and feed per tooth f_t:

$$AA_1 = f_t \sin\varphi. \quad (3)$$

Here it is assumed that the cutter tooth moves along the arc of a circle. Taking into account the elastic movements in TMS under the action of the cutting force, it is also possible to determine the cutting depth from the geometric ratios of the scheme in Fig. 1. The elastic movement δ_y along the Y coordinate causes the center of the milling cutter to move from point O_1 to point O_2, and the cutting depth changes:

$$BB_1 = AA_1 - \delta y \cos\varphi. \quad (4)$$

Similarly, the elastic displacement on the X-coordinate is projected to the cutting depth as follows:

$$CC_1 = BB_1 - \delta x \sin\varphi. \quad (5)$$

To define a mathematical model that reflects dynamic processes that affect the stability of the entire TMS, it is enough to consider the structural diagram that was obtained in work [6] for one input. Therefore, at f_t=0, the structural diagram takes the form shown in Fig. 2.

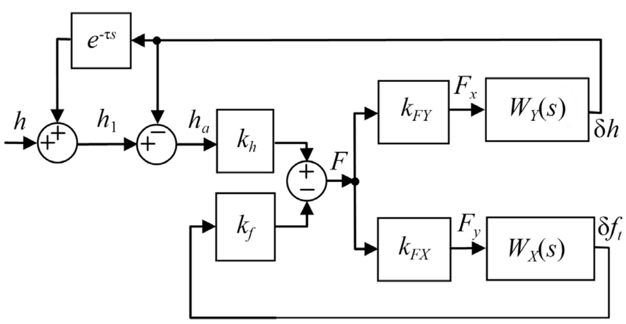


Fig. 2. Block diagram of the technological machining system by input *h*

The post-trace machining is represented by the delay function $e^{-\tau s}$ in positive feedback. The delay time τ is defined as the time between the passes of two adjacent cutter teeth:

$$\tau = 60 / z_m n_m, \quad (6)$$

where z_m is the number of cutter teeth, n_m is the spindle speed of the cutter (rpm).

In addition, cross-connections by coordinates are preserved in the structure, and the dynamic model is represented by two oscillating links:

– by the X coordinate:

$$W_X(s) = \frac{1/k_x}{T_x^2 s^2 + 2\xi_x T_x s + 1}, \quad (7)$$

– by the Y coordinate:

$$W_Y(s) = \frac{1/k_y}{T_y^2 s^2 + 2\xi_y T_y s + 1}, \quad (8)$$

where k_x, k_y are the stiffness along the X and Y axes, respectively, T_x, T_y, ξ_x, ξ_y are the natural oscillation periods and damping coefficients of the system oscillations along the X and Y axes, respectively.

Similarly, a mathematical model of the system can be built based on the feed input to the tooth f_t at h=0.

The components F_x and F_y of the cutting forces, in accordance with (3), can be determined by the coefficients:

$$k_{Fx} = \cos(\varphi_m - \gamma - \beta),\ k_{Fy} = \sin(\varphi_m - \gamma - \beta), \quad (9)$$

where φ_m is the average cutting angle.

To determine the change of elastic deformations in TMS along two coordinates in time, taking into account cross-connections, a mathematical model of the eighth order, represented in state variables, is used, the integration of which can be performed by standard numerical procedures. The implementation of the delay function for two inputs is performed at each step of integration according to recurrent dependences [6]:

$$h_j = h_0 + (\delta h)_{j-1},\ (f_t)_j = (f_t)_0 + (\delta f_t)_{j-1}, \quad (10)$$

where $(\delta h)_{j-1}$, $(\delta f_t)_{j-1}$ is the elastic movement of the system by depth and feed per tooth on the passage of the previous tooth of the cutter, j is the passage number.

Therefore, in the constructed mathematical model, the main factor of regenerative oscillations in the dynamic machining system is preserved. It consists in the presence of a delay function in positive feedback on two coordinates. Thus, during machining, the system will be affected by changes in the cutting force with a frequency equal to the frequency of its own oscillations and, of course, this «rocks» the dynamic system.

5. 2. Identification of dynamic parameters and force characteristics

As can be seen from the mathematical model, its adequacy depends entirely on the dynamic parameters of TMS and the force characteristics of the cutting process during milling. Such parameters include natural oscillation frequencies, oscillation damping coefficients, and stiffness of the system along two coordinate axes.

Determination of stiffness was carried out experimentally according to the measurement scheme shown in Fig. 3. A dynamometer is installed on the table of the milling machine, on which the workpiece is fixed. The purpose of the research is to determine the stiffness of TMS in the machining zone in the direction of the two coordinate axes. For measurements, an indicator is used, which measures the elastic shear on the cutter. The TMS loading is performed by manually moving the machine tables through the encoder when the cutter is in contact with the workpiece in transverse (X-axis) and longitudinal (Y-axis) feed. The magnitude of the force is indexed on the computer screen to which the outputs of the dynamometer amplifiers are connected.

To determine the frequency of natural oscillations of the system in its representation as a single mass with two degrees of freedom, experimental studies were performed using an impact hammer (Impact Hammer Model 086C03 from PCB Piezotronics Inc. USA), which was connected to a storage oscilloscope SDS 1022DL (Fig. 4).

The hammer signal is displayed on the oscilloscope screen and can be saved as a digital file as the impulse response of the dynamic machining system. According to this response, the amplitude-frequency response of TMS was calculated using a special program employing Fast Fourier Transform (FFT) (Fig. 5).

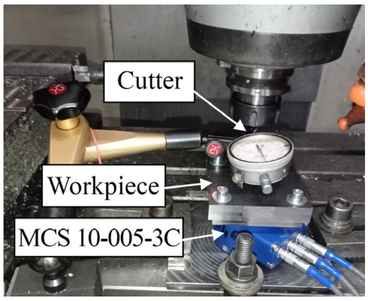


Fig. 3. Experimental determination of the rigidity of the technological machining system

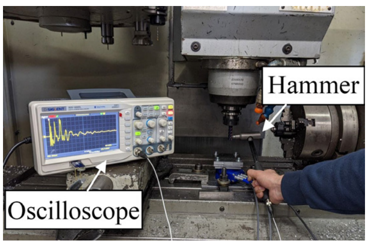


Fig. 4. Experimental determination of frequency responses of a technological machining system

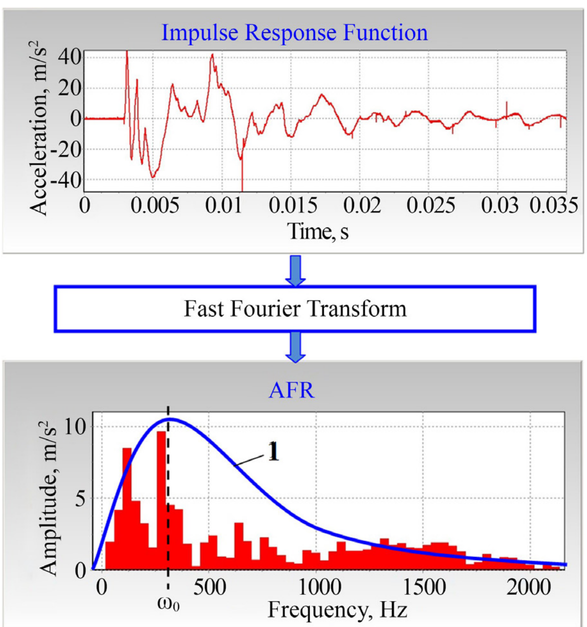


Fig. 5. Pulse response to frequency response conversion scheme

According to this response, when the dynamic system is initially represented as a one-mass one, the experimentally obtained characteristic of the spectrum can be approximated by the classical characteristic of the oscillating link with the frequency ω_0 of its natural oscillations (line 1 in Fig. 5). Such experiments were performed for each direction of the coordinate axes.

As a result of our experimental studies, a dynamic machining system was identified. The following parameters were obtained: stiffness k_x=2250 N/mm along the X axis; natural oscillation frequency $(\omega_0)_x$=260 Hz; stiffness k_y=2320 N/mm along the Y axis; natural oscillation frequency $(\omega_0)_y$=280 Hz; damping coefficient oscillations was taken as $\xi_x=\xi_y=0.05$ along both coordinate axes.

5. 3. End milling process stability lobes diagram

The constructed mathematical model represents a dynamic machining system at end milling as a one-mass system with two degrees of freedom. A certain adequacy of the results is achieved when using the parameters of the system as the initial data of the model, which were obtained experimentally according to the procedure described in the previous chapter.

An application program was built that allows modeling the response of the system in time due to the numerical integration of the differential equations of motion, represented in the form of state variables. The integration is performed by the fourth-order Runge-Kutta procedure. In addition, using special numerical procedures in the program, it is possible to determine frequency responses in the form of a Nyquist diagram. This provides an opportunity to evaluate the stability of the system according to a new stability criterion for closed systems with additional positive feedback through the delay function, the validity of which has been confirmed for turning [11].

The algorithm for finding the stability limit for each milling spindle speed is based on the automatic determination of the maximum amplitude $A_{max}(\omega)$ of the Nyquist diagram in the phase range $\varphi=0+360°i$, where i varies from 0 to n. If $A_{max}(\omega)<1$, the feed increases with a certain step; if $A_{max}(\omega)>1$, the feed decreases. The search is repeated by a new calculation of the Nyquist model and diagram using a numerical procedure, until $1-\delta<A_{max}(\omega)<1+\delta$. The value δ determines both the accuracy of the calculation and the time of the operation on the computer. As a result of the operation of the algorithm, a digital array of data is formed in the program, which forms a diagram of the stability of the system in the coordinates «mill spindle speed – feed» (Fig. 6).

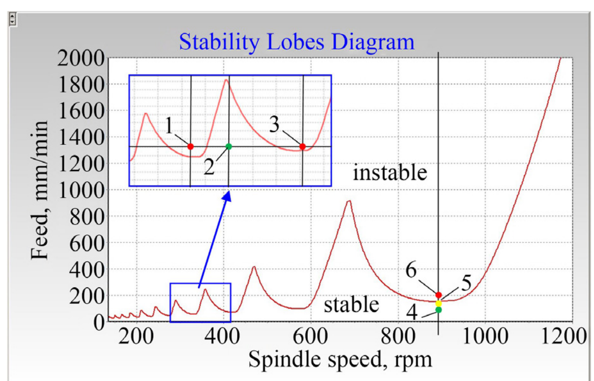


Fig. 6. Technological machining system stability lobes diagram

The stability lobes diagram divides the entire space of possible values of the cutting mode into two areas – a stable and unstable process (Fig. 6). The importance of our results for determining the cutting mode is obvious. So, the cutting mode corresponding to the combination of data in point 2 (feed 75 mm/min; spindle speed 365 rpm) guarantees a stable cutting process. Increasing the spindle speed to 420 rpm (point 1) or decreasing it to 335 rpm (point 3) can cause an unstable process. Similar results can be expected by changing the feed at the same spindle speed. Namely, 900 rpm: point 4 (feed 100 mm/min) is a stable process, point 5 (feed 150 mm/min) is the limit of stability, and point 6 (feed 200 mm/min) is a possible loss of stability.

It should be noted that the diagram forms a boundary, which is determined from the conditions of representation of the dynamic model as a one-mass one with two degrees of freedom, which is a certain approximation of the real process. Therefore, the adequacy of our results must be confirmed experimentally.

5. 4. Experimental verification of research results

To confirm the results, an approach using both simulation involving the developed program and a full-scale experiment of end milling on the machine is applied.

When simulating end milling in the devised program, one can observe the evolution of the process over time and compare the results with the location of the Nyquist diagram on the complex plane. Moreover, the results of the process evolution over time can be observed with the help of a virtual oscilloscope embedded into the program.

Fig. 7 shows the simulation results for cutting modes that correspond to points 1, 2, and 3 on the stability lobes diagram (Fig. 6). Simulating makes it possible to obtain both transient and frequency responses of TMS when applying the initial data obtained as a result of experimental studies. Our results convincingly prove the effectiveness of the new stability criterion for end milling operations: the process is stable when its Nyquist diagram does not cover the point with coordinates [+1,0] on the complex plane. On the screen of the virtual oscilloscope, line 1 represents the transient characteristic of TMS along the Y coordinate, which corresponds to the evolution in time of the elastic displacement δh, and line 2 represents the elastic displacement of TMS along the X coordinate – δf_t.

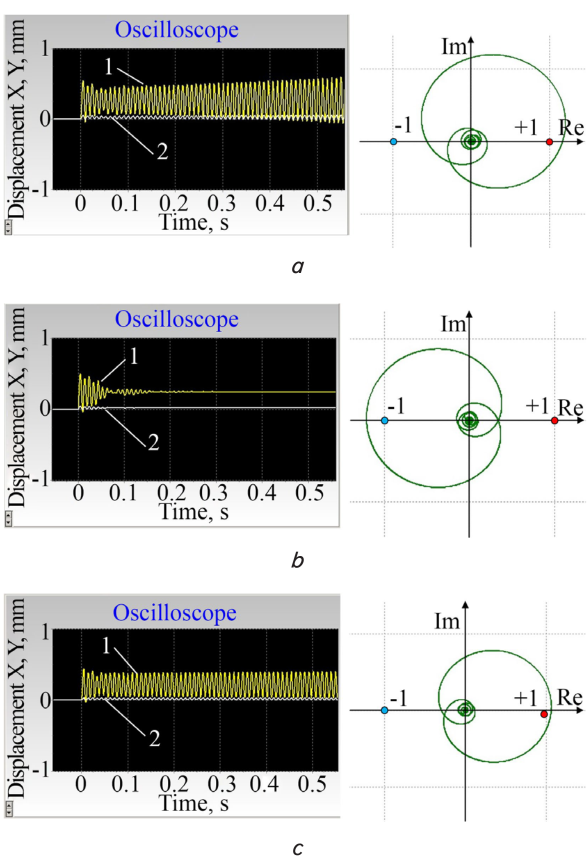


Fig. 7. Simulation results at a feed rate of 75 mm/min: *a* – milling spindle speed– 335 rpm; *b* – milling spindle speed – 365 rpm; *c* – milling spindle speed – 420 rpm

Simulation proves the adequacy of the proposed paradigm for the emergence of regenerative vibrations in TMS during cutting. The main reason is the multi-pass cutting process during end milling, where each subsequent tooth cuts an allowance distorted by system fluctuations during the previous pass. Such an effect is implemented in the simulating program due to recurrent dependences in the mathematical model represented by state variables for numerical integration.

The difference between the behavior of the system, which is closed due to the delay function in the positive feedback, which determines the model of post-trace machining, and the stability criteria of the system, which has a negative feedback, has been confirmed.

The ultimate confirmation of our results was obtained during a full-scale experiment. The corresponding experiments were performed when milling the workpiece on the XYZ VMC 1010 machine. The workpiece (Steel 45) was fixed according to the scheme that fully corresponds to the experimental studies for determining the dynamic responses of TMS (Fig. 4). Milling was performed with a ∅12 mm mill, four teeth, tooth spiral inclination 45°, according to the control program, on each side of the workpiece (Fig. 8).

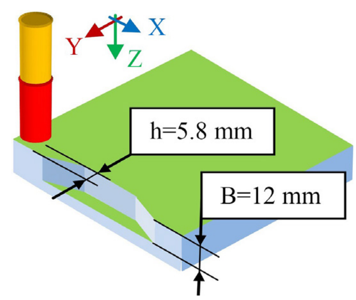
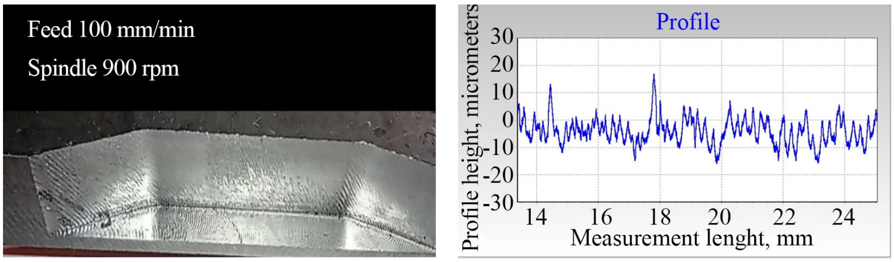


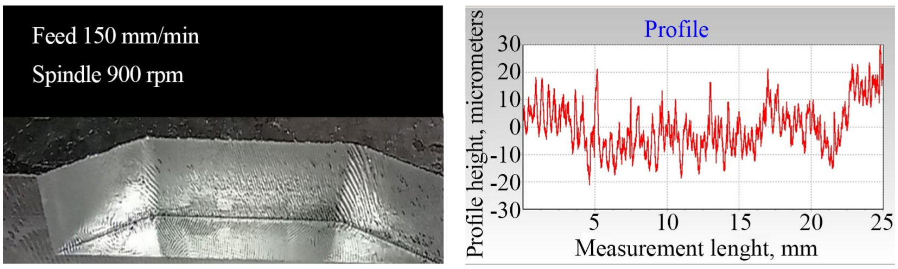
Fig. 8. Design diagram of the control machining program

After milling the blanks with cutting modes corresponding to points 1, 2, 3, 4, 5, and 6 on the stability lobes diagram (Fig. 6), the machined surfaces were scanned on a digital profilograph-profilometer. A comparison of the results proved that the roughness of the surfaces machined under the modes that correspond to points 1, 2, and 3 on the stability lobes diagram does not provide an opportunity to draw unequivocal conclusions about the stability of the process. However, modeling of such regimes shows significant differences: elastic displacements along the coordinate axes testify to the constancy of the process with regime point 2, regimes at points 1 and 3 lead to a gradual increase in the amplitude of oscillations. The study of the topography of the surfaces machined under the regimes corresponding to points 4, 5, and 6 on the stability lobes diagram proved their significant difference (Fig. 9).

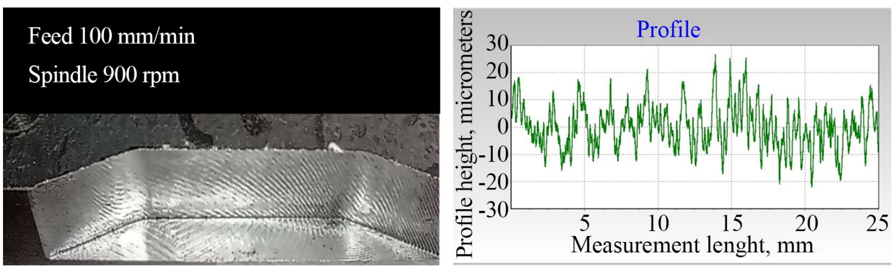
The roughness of the treated surfaces differs according to the variable Ra parameter: Ra 2.75 μm for mode 4; Ra 3.38 μm for mode 5, and Ra 5.43 μm for mode 6. In addition, the shape of their relief indicates the occurrence of significant fluctuations in TMS during the transition to the unstable cutting mode in accordance with the projected diagram (Fig. 9, *c*). Analysis of the surfaces machined at lower cutting speeds did not reveal significant changes in the roughness and character of the relief. Such results indicate an increase in the impact on the stability of the process from the designation of the cutting mode according to the diagram when moving to the region of higher cutting speeds.



a



b



c

Fig. 9. Machined surfaces and their profiles: *a* — feed — 100 mm/min, speed —900 rpm; *b* — feed — 150 mm/min, speed — 900 rpm; *c* — feed — 200 mm/min, speed — 900 rpm

The study reported here is aimed at determining a new method for ensuring the stability of the end milling process. The end-milling process is considered as occurring in a closed elastic dynamic system that is subject to positive feedback through the delay function. The delay time is equal to the time between the passage of the cutting zone of two adjacent cutter teeth. The dynamic system is represented as a single-mass system with two degrees of freedom, taking into account elastic displacement along two coordinate axes (Fig. 1).

The mathematical model is built from the state variables through the structural diagram of the dynamic system (Fig. 2), and the function of the delay argument is implemented in the form of a recurrent relation (3). This approach allows applying numerical modeling methods to obtain the response of the system in time and frequency response in the form of a Nyquist diagram. Therefore, the proposed model automatically takes into account the change in the thickness of the chips cut by each tooth and does not require separate geometric calculations to represent the response of the elastic system. It is on the basis of such calculations that certain stability conditions are determined in many studies [1–3].

Based on a new stability criterion for dynamic machining systems [11], the proposed algorithm for determining the stability lobes diagram was adapted to the end milling process. The developed software automatically projects a stability lobes diagram in the form of digital arrays in the coordinates «mill spindle speed – feed» (Fig. 6). Since the determination of the stability lobes diagram requires a priori information about the cutting process and the dynamic machining system, a procedure was devised, and experimental studies were carried out to establish such parameters.

The testing of the proposed solutions was carried out both by computer simulation and by field experiment. The simulation results (Fig. 7) fully confirmed the validity of the new stability criterion and the reliability of the functioning of the developed algorithm for the automatic design of the stability lobes diagram. Milling of workpieces under cutting modes determined according to the projected stability lobes diagram and measurements of the roughness of the treated surfaces also confirmed the effectiveness of the proposed solutions (Fig. 9).

It should be noted that the developed algorithm for automatic design of the stability lobes diagram uses frequency criteria, unlike the existing ones [3], which are based on algebraic criteria. Therefore, it can be used in production under the condition of operational determination of dynamic parameters of the machining system based on the vibration signal obtained directly during milling. Thus, our advancements could form the basis of a future integral system of

It is obvious that the concepts of «stability» and «instability» differ from those accepted in the classical theory since the mathematical model of the process differs in its structure and accepted assumptions from the real process. In classical terms, an unstable process leads to a complete loss of the ability to function as intended by the system. However, despite the accepted approximation of the experimental frequency response, linearization of the coefficients, and other assumptions, the applied model still generally corresponds to the actual process. The results of experimental studies confirm the theoretically obtained conclusions.

6. Discussion of results of investigating the stability of the end milling process

To eliminate the negative consequences of vibrations during cutting, various methods are used [2], among which the simplest one is the assignment of chatter-free cutting modes. In production, such a mode is most often found experimentally, by trial and error. It is possible to determine the chatter-free cutting mode in advance, at the stage of preparing the control program, according to the stability lobes diagram. However, simple effective methods and computer tools that can be used directly in production have not yet been devised to solve such a problem. The results of our research can form the basis for a practical solution to the specified problem as a whole.

designing control programs with the purpose of chatter-free cutting modes, or a system of automatic correction of the mode recorded in the control program during processing.

During the experimental studies, some features of the theoretically obtained results were revealed, namely in the area of low spindle speeds. In the indicated range (up to 400 rpm), it was not possible to confirm the adequacy of our technological solutions. This imposes certain limitations on the practical application of the developed software tools for setting the chatter-free cutting mode during end milling in practice.

7. Conclusions

1. A mathematical model of the end milling process was built, which takes into account the closedness of an elastic dynamic system in the form of a single-mass system with two degrees of freedom and an additionally closed one due to positive feedback along two coordinates through the delay function. The mathematical model was implemented in an application program that allows modeling processes in both time and frequency space.

2. A procedure of experimental determination of dynamic parameters of a dynamic machining system has been devised. We have proposed stiffness measurement schemes using a three-component dynamometer. Determination of the natural frequency of oscillations of a dynamic system in the representation as a single mass can be performed by its impulse response obtained from an impact hammer connected to an oscilloscope through fast Fourier transforms.

3. We have proposed an algorithm for automatic construction of the stability lobes diagram (SLD) in the «spindle speed – feed» coordinates according to the stability criterion, which uses the frequency responses of the dynamic system in the form of a Nyquist diagram. The algorithm is embedded into a computer program that automatically projects a given stability diagram, which predetermines its practical usefulness.

4. The adequacy of our results is confirmed both by computer simulation and a full-scale milling experiment at cutting modes that fall into the region of stability and instability on the SLD plot. The assessment of the level of vibrations in the system was carried out using roughness profilographs of surfaces treated under different modes. Thus, the machining experiment with a chatter-free cutting mode, determined by the stability lobes diagram, proved a decrease in the roughness of the machined surface from *Ra* of 5.45 μm to *Ra* of 2.75 μm.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

References

1. Quintana, G., Ciurana, J. (2011). Chatter in machining processes: A review. International Journal of Machine Tools and Manufacture, 51 (5), 363–376. doi: https://doi.org/10.1016/j.ijmachtools.2011.01.001
2. Yue, C., Gao, H., Liu, X., Liang, S. Y., Wang, L. (2019). A review of chatter vibration research in milling. Chinese Journal of Aeronautics, 32 (2), 215–242. doi: https://doi.org/10.1016/j.cja.2018.11.007
3. Altintas, Y. (2012). Manufacturing Automation. Cambridge: Cambridge University Press. doi: https://doi.org/10.1017/cbo9780511843723
4. Koh, M. H. (2012). Identification of system parameters for end milling force simulation with tool and workpiece compliance. University of New Hampshire, 761. Available at: https://scholars.unh.edu/thesis/761/
5. Xia, Y., Wan, Y., Luo, X., Liu, Z., Song, Q. (2021). An improved numerical integration method to predict the milling stability based on the Lagrange interpolation scheme. The International Journal of Advanced Manufacturing Technology, 116 (7-8), 2111–2123. doi: https://doi.org/10.1007/s00170-021-07311-z
6. Petrakov, Y., Sikailo, M. (2022). Simulation modeling of 2.5D milling dynamics by end mills. Bulletin of the National Technical University «KhPI» Series: New Solutions in Modern Technologies, 2 (12), 17–24. doi: https://doi.org/10.20998/2413-4295.2022.02.03
7. Sreenivasa Rao, A., Venkata Rao, K. (2017). A Study on Machining Characteristics in Milling of Ti-6Al-4V using Experimental and Finite Element Analysis. International Journal of Civil Engineering and Technology, 8 (7), 457–469. Available at: http://iaeme.com/Home/issue/IJCIET?Volume=8&Issue=7
8. Li, H. Z., Zhang, W. B., Li, X. P. (2001). Modelling of cutting forces in helical end milling using a predictive machining theory. International Journal of Mechanical Sciences, 43 (8), 1711–1730. doi: https://doi.org/10.1016/s0020-7403(01)00020-0
9. Stephenson, D. A., Agapiou, J. S. (2018). Metal Cutting Theory and Practice. CRC Press. doi: https://doi.org/10.1201/9781315373119
10. Quintana, G., Ciurana, J., Teixidor, D. (2008). A new experimental methodology for identification of stability lobes diagram in milling operations. International Journal of Machine Tools and Manufacture, 48 (15), 1637–1645. doi: https://doi.org/10.1016/j.ijmachtools.2008.07.006
11. Petrakov, Y., Danylchenko, M. (2022). A time-frequency approach to ensuring stability of machining by turning. Eastern-European Journal of Enterprise Technologies, 6 (2 (120)), 85–92. doi: https://doi.org/10.15587/1729-4061.2022.268637
12. Liang, S. Y. (2000). Manufacturing processes and equipment. Machining Science and Technology, 4 (2), 317–318. doi: https://doi.org/10.1080/10940340008945713