UDC 536.12;614.84;664
DOI: 10.15587/1729-4061.2023.287391

CONSTRUCTING A PHYSICAL-MATHEMATICAL MODEL OF GRAIN MASS SELF-HEATING BY A ROD SITE OF RECTANGULAR CROSS-SECTION

Maksym Slipchenko
PhD, Associate Professor
Department of Reliability and Durability of Machines and Structures named after V. Ya. Anilovich*

Vadym Bredykhin
PhD, Associate Professor
Department of Reliability and Durability of Machines and Structures named after V. Ya. Anilovich*

Andrey Pak
Corresponding author
Doctor of Technical Sciences, Associate Professor
Department of Physics and Mathematics*
E-mail: pak.andr1980@btu.kharkov.ua

Petro Gurskyi
PhD, Associate Professor
Department of Equipment and Engineering of Processing and Food Industries*

Oleksiy Alfyorov
Doctor of Technical Sciences, Associate Professor
Department of Engineering Systems Design
Sumy National Agrarian University
Herasyma Kondratieva str., 160, Sumy, Ukraine, 40000

Alina Pak
PhD, Associate Professor
Department of Marketing and Trade Entrepreneurship
Ukrainian Engineering Pedagogics Academy
Universitetskaya str., 16, Kharkiv, Ukraine, 610035

*State Biotechnological University
Alchevskih str., 44, Kharkiv, Ukraine, 61002

This paper considers the issue related to improving the energy and resource efficiency of the process of storing raw materials of plant origin, namely, preventing self-heating of grain masses in elevator silos. It was noted that the effectiveness of the analysis of self-heating of grain mass increases with the use of mathematical models of temperature fields of grain mass during storage together with data obtained experimentally. A physical-mathematical model has been built that describes a two-dimensional localized non-stationary temperature field of seed material generated by a homogeneous rod cell with a rectangular cross-section. A technique for accelerating the convergence of the series is proposed for the constructed analytical solution, which is based on the selection and analytical calculation of the sum of the component of slow convergence. The adequacy of the physical-mathematical model has been proven by calculations and by comparing the temperature of the self-heating site, obtained theoretically, and the temperature obtained under industrial setting. The established temperature kinetics of grain mass volumes during storage, obtained experimentally and theoretically, correlate with each other in the duration range from 0 to 30 days with a correlation coefficient of at least 0.98. This proves the possibility of applying forecasts of the temperature of self-heating sites in the volume of grain mass, obtained by using the physical-mathematical model built, under industrial setting. A limitation of the study is that the model is not universal. It is another stage on the way to a universal model. A limitation of the study is that for storage periods of more than 30 days, a new excess temperature forecast must be made

Keywords: self-heating of grain mass, temperature kinetics, model of a rod site of rectangular section

Received date 31.05.2023
Accepted date 08.09.2023
Published date 30.10.2023

How to Cite: *Slipchenko, M., Bredykhin, V., Pak, A., Gurskyi, P., Alfyorov, O., Pak, A. (2023). Constructing a physical-mathematical model of grain mass self-heating by a rod site of rectangular cross-section. Eastern-European Journal of Enterprise Technologies, 5 (5 (125)), 24–30. doi: https://doi.org/10.15587/1729-4061.2023.287391*

1. Introduction

The intensive development of agro-industrial production requires the search for solutions to the problems of efficient use of energy and material resources, as well as their scientific justification. At the same time, it is obviously relevant to comply with the requirements for the environmental friendliness of production [1].

One of the leading sectors of agro-industrial production is the production of grain material. One of the key final stages of post-harvest processing of grain material should be highlighted – its storage [2].

The arrangement of the material during storage depends on the varietal and sowing qualities [3]. Thus, seed material is stored in bags placed on wooden pallets. The grain mass, which has a lower class, is loaded into silos of elevators for long-term storage.

At the stage of grain mass storage, natural processes occur in it, which can have a harmful effect on its preservation. Self-heating of grain, the harmful effects of molds, mites, insects, and rodents on it reduce the quality of products.

Self-heating of grain material in elevator silos attracts special attention of researchers [4] from the point of view of energy and resource efficiency of the storage process. It leads to an increase in the temperature of the material, as a result of which the storage conditions deteriorate, and in some cases fires occur. The evolution of the self-heating process leads to significant changes in the quality and properties of the grain material, due to biochemical transformations, which often leads to its complete deterioration and loss [5].

The grain material that has just entered the elevator often has increased moisture, due to which it «breathes» intensively, which contributes to its self-heating. This process begins 1–2 hours after the seeds are unloaded from the vehicle. However, even a small degree of self-heating negatively affects its storability.

The effectiveness of solving this problem is determined by observing the appropriate humidity and temperature of the grain mass and the environment in which it is located, as well as by timely response to an increase in the temperature of the grain material due to self-heating. Therefore, studies that consider the development of procedures for studying the kinetics of moisture and temperature of plant raw materials during their storage are relevant.

2. Literature review and problem statement

An obvious increase relative to the norm of grain mass temperature due to self-heating during storage can be recorded using measuring equipment [6]. Temperature sensors are placed in the layer of raw materials and thus register the temperature of the grain mass. In agro-industrial production, silos designed to store raw materials weighing from tens to tens of thousands of tons are used. Analysis of the temperature of the corresponding volumes of grain masses, where the initial data is a point temperature measurement, is not effective. The effectiveness of the analysis improves if mathematical models of the temperature fields of the grain mass are used during storage, together with data obtained experimentally [7].

During the mathematical modeling of the temperature fields of the grain mass, models are used, which are divided into layer, nest, and rod models. The differences between these models are, first of all, in the shape of the self-heating site. Other differences are the number of spatial dimensions (one-dimensional, two-dimensional, or three-dimensional model) in which the simulation is carried out, and the methods used for the simulation.

Thus, in works [8, 9], a layer mathematical model is considered. In this case, based on the reported results, it is possible to calculate the temperature of the layer self-heating of the raw material, taking into account the heat dissipation to the walls of the silo, as well as to identify the parameters of the site. However, in the mode builtl, the heat transfer is assumed to be ideal, which is a certain drawback when applying this model under industrial setting.

In works [10, 11], a nested mathematical model of the temperature field in the volume of the grain mass was developed. A solution was obtained for the nest site depending on the distribution of heat sources in it. However, centrally symmetric self-heating of only the spherical region selected around the center of the site is considered. In [12], a model of a nest site of arbitrary shape was considered, but only two dimensions were taken into account.

Rod mathematical models of the temperature field in the volume of plant raw materials are considered in works [13, 14]. The calculation of the temperature increase in the rod site of the circular cross-section for an unlimited mass was obtained. The use of this model makes it possible to use it to estimate the time of reaching the fire-hazardous temperature in different types of plant material. However, closed solutions for a rectangular section were not obtained in [14], so the advantages of analytical representations over numerical solutions remained unrealized.

At the same time, the considered works use the differential equation of thermal conductivity with constant coefficients and solve it by both numerical [15] and analytical [16] methods.

It should be noted that there is no ideal mathematical model for describing the temperature fields in the volume of the grain mass during its storage. Obviously, for any mathematical model, there are limitations regarding its application under industrial setting due to different initial conditions (seeding, moisture, and initial temperature of raw materials, enthalpy, relative and absolute humidity of the surrounding gas medium, etc.) [17]. Based on this, it is necessary to construct a universal mathematical model of temperature fields in the volume of grain mass during storage. The way to solve this problem is to build mathematical models for different site shapes using different methods to find solutions.

3. The aim and objectives of the study

The purpose of this study is to build a physical-mathematical model of self-heating of the grain mass generated by a rod site of rectangular cross-section with a uniform distribution of heat sources. This will provide an opportunity to control and regulate grain mass storage regimes using the results of calculation and analysis of the kinetics of the excess temperature increase of its self-heating.

To achieve the goal, the following tasks were set:

– to obtain an analytical expression for calculating the increase in the excess temperature of the self-heating of the grain mass, generated by a rod site of a rectangular cross-section with a uniform distribution of heat sources;

– to check the adequacy of the constructed physical and mathematical model of self-heating of the grain mass, generated by a rod site with a rectangular cross-section with a uniform distribution of heat sources.

4. The study materials and methods

The object of our research is the increase in the excess temperature of the self-heating of the grain mass. The research assumes that the self-heating site has a rectangular section. That is, a rectangular cross-section of the silo with length l_1 and width l_2 is considered, where a self-heating rod site appears of the same cross-section as shown in Fig. 1.

The cross-sectional dimensions of the site are characterized by a length of $2u$ and a width of $2v$. The center of the site has coordinates x_0 and y_0 in the global system of rectangular coordinates xOy. The density of heat sources at the site is considered constant and equal to q_0.

An analytical method was used to build a physical-mathematical model of self-heating of the grain mass generated by a rod site with a rectangular cross-section with a uniform distribution of heat sources.

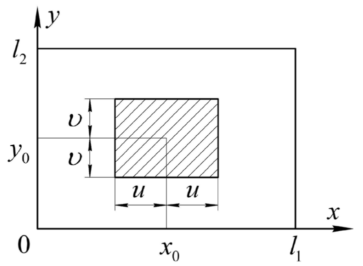

Fig. 1. Rod site of self-heating of grain mass of rectangular cross-section

To prove the adequacy of the model, experimental data were obtained in a silo filled with grain mass (wheat grain). The temperature in the heating site was measured using thermocouples. Registration of signals from thermocouples was carried out using analog-to-digital and digital-to-analog converters of the company DCON Utility (manufactured in the USA). The obtained experimental data were approximated and compared with the results of temperature increase calculations obtained using the physical-mathematical model built with the same initial parameters.

5. Results of simulation of the non-stationary temperature field generated by a rod site of rectangular cross-section

5. 1. Derivation of the expression for calculating the increase in the excess temperature of self-heating of the grain mass from the site of rectangular section

The non-stationary excess temperature field $T(x, y, t)$ is described by the differential equation:

$$\frac{\partial T}{\partial t}-\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{q(x,y)}{\rho c}H(t), \quad (1)$$

where $\alpha=\lambda/(\rho c)$ is the thermal conductivity coefficient of raw materials; λ – its thermal conductivity coefficient; c – specific weight of raw materials; ρ – specific heat capacity of raw materials; $H(t)$ – Heaviside unit function; t – time.

In the right-hand part of equation (1):

$$q(x,y)=\begin{cases} q_0 & \text{at} \quad x,y\in D; \\ 0 & \quad\quad x,y\notin D. \end{cases} \quad (2)$$

Here, D is the cross-sectional area of the site, where $x_0-u\le x\le x_0-u$, $y_0-v\le x\le y_0-v$.

Equation (1) is solved under the following boundary conditions:

$$T(0,y,t)=T(l_1,y,t)=T(x,0,t)=T(x,l_2,t)=0. \quad (3)$$

Considering:

$$T(x, y, 0)=0. \quad (4)$$

The solution is taken in the form of a double series:

$$T(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}(t)\sin(\alpha_m x)\sin(\beta_n y), \quad (5)$$

where $f_{mn}(0)=0$; $\alpha_m=\pi m/l_1$; $\beta_n=\pi n/l_2$.

It satisfies the given boundary conditions (3) and the initial condition (4).

Substitution (5) in (1), taking into account (2), leads to differential equations:

$$\frac{df_{mn}}{dt}+\alpha\gamma_{mn}^2 f_{mn}=\frac{b_{mn}}{\rho c}\cdot H(t). \quad (6)$$

Here:

$$\gamma_{mn}^2=\alpha_m^2+\beta_n^2;$$

$$b_{mn}=\frac{16q_0}{l_1 l_2\alpha_m\beta_n}\times$$
$$\times\sin(\alpha_m u)\sin(\beta_n \upsilon)\sin(\alpha_m x_0)\sin(\beta_n y_0).$$

Equations (6) have solutions:

$$f_{mn}(t)=\frac{b_{mn}}{\lambda\gamma_{mn}^2}\left[1-\exp\left(\alpha\gamma_{mn}^2 t\right)\right].$$

By substituting them in (5), the formula for the excess temperature of the raw material is obtained:

$$T(x,y,t)=\frac{16q_0}{\lambda l_1 l_2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\alpha_m\beta_n\gamma_{mn}^2}\times$$
$$\times\left[1-\exp\left(-\alpha\gamma_{mn}^2 t\right)\right]\times\sin(\alpha_m u)\sin(\beta_n\upsilon)\sin(\alpha_m x_0)\times$$
$$\times\sin(\beta_n y_0)\sin(\alpha_m x)\sin(\beta_n y). \quad (7)$$

In order to speed up convergence, expression (7) is split into two terms:

$$T(x,y,t)=T_A(x,y)-\frac{16q_0}{\lambda l_1 l_2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\exp\left(-\alpha\gamma_{mn}^2 t\right)}{\alpha_m\beta_n\gamma_{mn}^2}\times$$
$$\times\sin(\alpha_m u)\sin(\beta_n\upsilon)\sin(\alpha_m x_0)\times$$
$$\times\sin(\beta_n y_0)\sin(\alpha_m x)\sin(\beta_n y), \quad (8)$$

where

$$T_A(x,y)=\lim_{t\to\infty}T(x,y,t)=$$
$$=\frac{16q_0}{\lambda l_1 l_2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(\alpha_m u)\sin(\beta_n\upsilon)}{\alpha_m\beta_n\gamma_{mn}^2}\times$$
$$\times\sin(\alpha_m x_0)\sin(\beta_n y_0)\sin(\alpha_m x)\sin(\beta_n y). \quad (9)$$

The term $T_c(x, y)$ is the limiting value of the self-heating temperature.

Next, we limited ourselves to the case when $y=y_0=0.5l_2$. For it, $T_c(x, 0.5l_2)$ can be approximately represented in a closed form. Thus, using the results of work [11], it is possible to write:

$$T_A(x,\,0.5l_2)\approx\frac{q_0 u\upsilon}{\pi\lambda}\left[2\ln\left(\frac{2l_1}{\pi}\sin\frac{\pi x_0}{l_1}\right)-\ln\left(u^2+\upsilon^2\right)+\right.$$
$$\left.+3-\frac{u}{\upsilon}\mathrm{arctg}\frac{\upsilon}{u}-\frac{\upsilon}{u}\mathrm{arctg}\frac{u}{\upsilon}-\mathrm{K}\left(\frac{x_0}{l_1},\frac{l_2}{l_1}\right)\right]. \quad (10)$$

Here:

$$\mathrm{K}\left(\frac{x_0}{l_1},\frac{l_2}{l_1}\right)=4\sum_{m=1}^{\infty}\frac{\sin(\alpha_m x_0)\exp(-\alpha_m l_2/2)}{m\cdot\mathrm{ch}(\alpha_m l_2/2)}$$

is a series of rapid convergence. Its sum for various x_0/l_1 and l_2/l_1 ratios is reported in [11].

If $l_2/l_1 \geq 1$, then $K(x_0/l_1, 1)$ can also be calculated with an error of less than 1 % using the formula:

$$\mathrm{K}\left(\frac{x_0}{l_1},1\right)\approx$$
$$\approx 2\ln\frac{2\left[\mathrm{ch}(\pi l_2/l_1)+1\right]\left[\mathrm{ch}(\pi l_2/l_1)-\cos(2\pi x_0/l_1)\right]}{\mathrm{ch}(2\pi l_2/l_1)-\cos(2\pi x_0/l_1)}.$$

If the centers of the square cross-sections of the silo and site coincide ($l_1=l_2$, $u=v$, $x_0=y_0=0.5l_1$), formula (10) takes a more compact form:

$$T_c(0.5l_1, 0.5l_2)=\frac{q_0u^2}{\pi\lambda}\left(2\ln\frac{\sqrt{2}l_1}{\pi}+2.6684-\frac{\pi}{2}\right). \qquad (11)$$

To find out the actual errors of formula (10), a numerical example is considered. For calculations, the following are accepted: $l_1=l_2=10$ m, $u=2v=1$ m, and the ratio $x_0/l_1=0.2$; 0.3; 0.4; 0.5. The required $K(x_0/l_1, 1)$ values are recorded in Table 1.

Table 1

Values of $K(x_0/l_1, 1)$

x_0/l_1	0.2	0.3	0.4	0.5
K	0.1214	0.2237	0.3024	0.3316

They are borrowed from work [10].

The calculated dimensionless values $T_c^*=10\lambda q_0^{-1}T_c\left(x_0l_1^{-1};1\right)$ are recorded in Table 2.

For comparison, Table 3 gives T_c^*, calculated from formula (9), where N terms were kept in each of the series.

Table 2

Values of T_c^*, calculated from formula (10)

x_0/l_1	0.2	0.3	0.4	0.5
T_c^*	6.070	6.924	7.314	7.427

Table 3

Values of T_c^*, calculated from formula (9)

x_0/l_1	$N=5$	$N=10$	$N=30$	$N=50$	$N=200$
	Value of T_c^*				
0.2	5.202	5.989	6.061	6.048	6.047
0.3	6.178	6.855	6.930	6.918	6.917
0.4	6.425	7.248	7.325	7.313	7.312
0.5	6.701	7.363	7.440	7.427	7.427

Differences of the corresponding results in Tables 2, 3 are less than 0.5 %.

The described technique of accelerated convergence of dual series was earlier proposed in [16].

To calculate the increase in the temperature of the raw material in the center of the rectangular cross-section site located in the center of the silo, the following is additionally specified: $q_0=16$ W/m^3; $\lambda=0.15$ W/(m·K); $\alpha=1.8\cdot10^{-7}$ m^2/s. The obtained values of the increase at different points in time are listed in Table 4. In each of the calculation series, there are N terms. The results of calculations according to formula (10) are written in the numerators, and according to formulas (8), (9) in the denominators. The convergence of the solution series given by the numerators is significantly faster than in the solutions obtained in the denominators. This is especially true for large values of t when the self-heating process is developed. Then no more than five terms can be calculated in the series of accelerated convergence.

Table 4

Values of $T(0.5l_1; 0.5l_2; t)$

t, days	$N=3$	$N=5$	$N=10$	$N=50$	$N=100$
	Temperature value in the center of the site				
10	21.773 / 4.457	15.831 / 8.084	13.248 / 12.661	13.222 / 13.225	13.222 / 13.221
30	29.032 / 11.715	26.670 / 18.922	26.290 / 25.603	26.288 / 26.292	26.288 / 26.288
50	34.667 / 17.351	33.699 / 25.952	33.632 / 32.945	33.632 / 33.636	33.632 / 33.631
100	44.459 / 27.142	44.346 / 36.599	44.345 / 43.657	44.345 / 44.348	44.345 / 44.344
200	55.563 / 38.247	55.561 / 47.814	55.561 / 54.874	55.561 / 55.565	55.561 / 55.561

In order to calculate the increase in the excess temperature of self-heating, in addition to the thermophysical characteristics of the raw material and the dimensions of the silo, it is necessary to know the characteristics of the site (parameters q_0, u, v). For their definition, $u=v$, $l_1=l_2$, $x_0=y_0=0.5l_1$ are accepted. It is assumed that in the center of the site ($x=x_0$, $y=y_0$) the value of excess temperature is set at two moments of time at the beginning of the self-heating process. Let the value of T_1 be obtained at $t=t_1$, and the value of T_2 at $t=t_2$. The symbol z denotes their ratio ($z=T_2/T_1$). Then, according to (7), the following equation holds:

$$\Phi(\xi)-\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{\exp\left[-\alpha\pi^2l_1^{-2}\left(m^2+n^2\right)t_2\right]}{mn\left(m^2+n^2\right)}\sin(m\pi\xi)\sin(n\pi\xi)=$$
$$=z\left[\begin{array}{l}\Phi(\xi)-\\ -\displaystyle\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{\exp\left[-\alpha\pi^2l_1^{-2}\left(m^2+n^2\right)t_1\right]}{mn\left(m^2+n^2\right)}\sin(m\pi\xi)\sin(n\pi\xi)\end{array}\right], \qquad (12)$$

with unknown $\xi=ul_1^{-1}$.

The function $\Phi(\xi)$ is given by the following expression:

$$\Phi(\xi)=\frac{\pi^3\xi^3}{8}\left(\ln\frac{\sqrt{2}}{\pi\xi}+0.5488\right),$$

which approximates the sum of the double series:

$$S(\xi)-\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{\sin(m\pi\xi)\sin(n\pi\xi)}{mn\left(m^2+n^2\right)},$$

and follows from (11).

For comparison, Table 5 gives the $\Phi(\xi)$ and $S(\xi)$ values.

Table 5

Values of $\Phi(\xi)$ and $S(\xi)$

ξ	0.05	0.1	0.2	0.3	0.4
$\Phi(\xi)$	0.0266	0.0796	0.2109	0.3330	0.4136
$S(\xi)$	0.0266	0.0796	0.2109	0.3334	0.4185

From the data in Table 5 it follows that when $\xi < 0.3$ it is possible to accept $S(x) \approx \Phi(x)$.

Equation (12) was solved by numerical methods. But for an approximate definition of ξ, one can use the plots shown in Fig. 2.

Plot 1 in Fig. 2 was built at $t_2 = 3t_1 = 15$ days, and plot 2 – at $t_2 = 4t_1 = 16$ days. Upon determining ξ, we used the following formula:

$$q_0 = T_2 \left\{ \frac{16 l_1^2}{\lambda \pi^4} \left[\begin{array}{l} \Phi(\xi) - \\ - \sum\limits_{m=1,3,\ldots}^{\infty} \sum\limits_{n=1,3,\ldots}^{\infty} \dfrac{\exp\left(-\alpha \pi^2 l_1^{-2} \left(m^2 + n^2\right) t_2\right)}{mn\left(m^2 + n^2\right)} \sin(m\pi\xi) \sin(n\pi\xi) \end{array} \right] \right\}^{-1} \quad (13)$$

to calculate q_0.

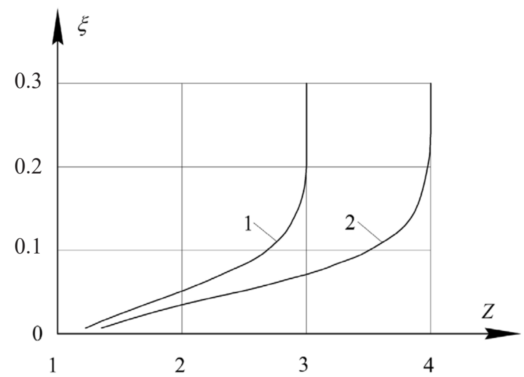

Fig. 2. Dependences of ξ on z

5. 2. Verification of the adequacy of the physical-mathematical model of self-heating of grain mass generated by a rod site of rectangular cross-section

Proving the adequacy of the physical-mathematical model of self-heating of grain mass generated by a rod site of rectangular cross-section was carried out in two stages.

At the first stage, the identification of the parameters of the site of square section in the silo, where $l_1 = l_2 = 10$ m, was considered.

Site parameters were calculated under the condition that at $t_1 = 5$ days, $T_1 = 4.8$ °C; and at $t_2 = 15$ days, $T_2 = 9.6$ °C. On plot 1 (Fig. 2), for the ratio $z = 2$, we find that $\xi = 0.05$, i.e., $u = v = 0.5$ m. Then, for the thermophysical characteristics of the raw material indicated above, $q_0 = 11$ W/m^3 was obtained according to formula (13). To check, by substituting the identified q_0 and u into formulas (8), (10), the following results were obtained: $T_1 \approx 4.84$ °C, $T_2 \approx 9.54$ °C, which differs from the temperature values used in the identification within the margin of error (<1 %).

Site parameters were also calculated when $t_1 = 4$ days, $T_1 = 8$ °C, and at $t_2 = 16$ days, $T_2 = 24$ °C. On plot 2 in Fig. 2 for the ratio $z = 3$, $\xi \approx 0.07$ was obtained, i.e., $u = v = 0.7$ m. According to formula (13), $q_0 \approx 19.9$ W/m^3 corresponds to these cross-sectional dimensions of the site. Calculation of the temperature increase according to formulas (8), (10) for the identified values of q_0 and u gives: $T_1 \approx 8.06$ °C, $T_2 \approx 24.05$ °C, which is also close to the values used in the identification.

At the second stage of proving the adequacy of the developed physical-mathematical model, the kinetics of the temperature of the self-heating site in the volume of the grain mass, obtained theoretically, and the kinetics obtained experimentally were compared. Experimental temperature kinetics were obtained for a self-heating site that was formed during storage in a silo filled with wheat grain. The increase in temperature of a certain volume of grain mass was recorded on the 5th day from the moment of filling the silo with wheat grain with an average moisture content of 19 %. Fig. 3 shows the temperature kinetics of the self-heating site in the volume of the grain mass obtained experimentally (blue line) and theoretically (red points).

The temperature kinetics represent a change in the temperature of the self-heating site ($T_{g.m.}$) of the grain mass over time (t) during storage.

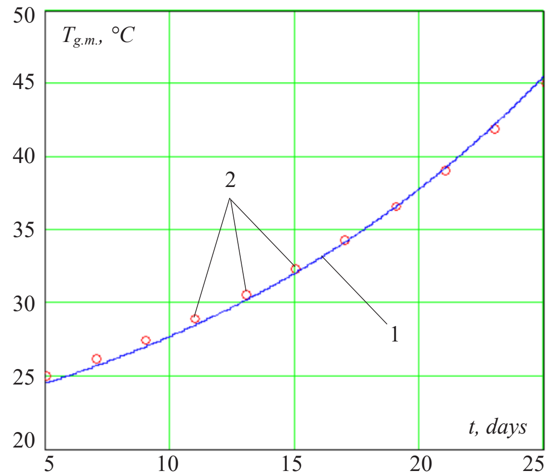

Fig. 3. Temperature kinetics of the self-heating site in the volume of the grain mass, obtained: 1 – by measuring the temperature under industrial setting; 2 – by calculation using the physical-mathematical model built

The experimentally obtained kinetics of the temperature of the self-heating site mean the kinetics obtained under industrial setting, the theoretically obtained kinetics are the kinetics calculated using the developed physical-mathematical model under the same starting conditions as the experimental ones.

Experimental temperature kinetics is an approximation function of data obtained under industrial setting. During storage (for Fig. 3 – 25 days), the temperature of the self-heating site was recorded discretely every 30 minutes. Next, the obtained experimental points were approximated by a polynomial function for clarity. This mathematical function was considered the experimental temperature kinetics.

6. Discussion of results of simulating the self-heating of grain mass generated by a rod site of rectangular cross-section

Using the analytical method, a physical-mathematical model of self-heating of grain mass generated by a rod site of rectangular cross-section with a uniform distribution of heat sources was built.

Our model makes it possible to determine the increase in the excess temperature of self-heating of the grain mass. The cross-section of the rod site (Fig. 1) has a rectangular shape with length l_1 and width l_2. At the same time, it is possible to change the characteristic dimensions of the self-heating site based on the size of the silo in which the grain mass is located, in contrast to the results reported in works [10, 11].

The ultimate result of the constructed physical-mathematical model is an expression for calculating the increase in the excess temperature of self-heating of the grain mass from the site of rectangular cross-section (7). At the same time, the expression assumes that heat sources are uniformly distributed in the volume of the self-heating site. This is an assumption closer to real cases than the assumption in which the heat exchange is considered ideal, as in works [8, 9].

The adequacy of the physical-mathematical model was proved by calculations and by comparing the temperature of the self-heating site obtained theoretically using the model and its temperature obtained under industrial setting.

According to the first stage of proving the adequacy of the model, the excess temperature of the site was calculated at different control points of the duration of storage of the grain mass in the silo with the site size $l_1=l_2=10$ m. At the same time, different values of the excess temperature of the self-heating site at the initial control points were taken into account. The calculation proved the possibility of an approximate determination of the parameters of the self-heating site using plots of the dependence of ξ on z (Fig. 2). The temperature increase determined from formulas (8), (10) for the identified values of q_0 and u gives values close to the values involved in the identification. At the same time, the difference between these values does not exceed 1 %, which is within the margin of error. The calculations prove the possibility of using the model to forecast the rise in temperature of self-heating sites of the grain mass during storage.

At the second stage of proving the adequacy of the developed physical-mathematical model, the temperature kinetics of the grain mass obtained under production conditions and the kinetics obtained using the model were compared. During the experimental studies, a set of thermocouples was placed over the volume of the silo filled with grain mass. The temperature kinetics obtained in this case are the registration of the signal from the thermocouple, the junction of which was located directly in the self-heating site. The theoretically obtained temperature kinetics were an array of points obtained by calculation using the expression for calculating the increase in the excess temperature of self-heating of the grain mass from the site of rectangular cross-section. Each value of the excess temperature of the self-heating site corresponds to the value of the duration of the storage process. At the same time, during simulation, the time t_1 changed, starting from 5 days, discretely with a step of 2 days with a corresponding change in t_2. In order to obtain exactly the temperature kinetics of the self-heating site, the base temperature obtained during experimental studies under industrial setting was added to the excess temperature values.

The given kinetics of the temperature of the site of self-heating in the volume of grain mass during storage, obtained experimentally and theoretically, correlate with each other with a correlation coefficient of at least 0.98 in the duration range from 0 to 30 days. This proves the possibility of applying forecasts of the temperature of self-heating sites in the volume of grain mass, obtained using the developed physical-mathematical model, under industrial setting.

The resulting physical-mathematical model could be used in agro-industrial production, namely, to improve the typical technological process of storing raw materials of plant origin. Using the proposed model would reduce the loss of agricultural products during storage.

The disadvantage of the study is that the model is not universal. That is, in any case, when it is used for different raw materials (different vegetable raw materials: corn grain, sunflower seeds, soy, etc.), the model requires a certain approbation. It represents another stage on the way to a universal model.

The limitation of the study is that the adequacy of the physical-mathematical model has been proven for forecasting periods in multiples of 30 days. For storage periods of more than 30 days, a new excess temperature forecast must be made. The increase in the range of prediction is a prospect for further research.

7. Conclusions

1. The localized temperature field is described by trigonometric series of accelerated convergence. A physical-mathematical model of self-heating of the grain mass generated by a rod site of rectangular cross-section with a uniform distribution of heat sources has been built, which makes it possible to identify the parameters of the internal heat source based on the data of raw material temperature measurements at certain moments of time. After identification, the formulae become suitable for forecasting the increase in self-heating temperature of raw materials over time.

2. The adequacy of the physical-mathematical model was proven by calculations and by comparing the temperature of the self-heating site obtained theoretically using the model and its temperature obtained under industrial setting. It was established that the temperature kinetics of the self-heating site in the volume of the grain mass during storage, obtained experimentally and theoretically, correlate with each other with a correlation coefficient of at least 0.98 in the duration range from 0 to 30 days. This proves the possibility of applying forecasts of the temperature of self-heating sites in the volume of grain mass, obtained using the developed physical-mathematical model, under industrial setting.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was carried out within the framework of the research topic, which is carried out under the state order «Increasing food security with the development of competitive technologies for obtaining high-quality seeds with improved biopotential» (0122U000810).

Data availability

All data are available in the main text of the manuscript.

References

1. Domaracka, L., Matuskova, S., Tausova, M., Senova, A., Kowal, B. (2022). Efficient Use of Critical Raw Materials for Optimal Resource Management in EU Countries. Sustainability, 14 (11), 6554. doi: https://doi.org/10.3390/su14116554
2. Chen, G., Hou, J., Liu, C. (2022). A Scientometric Review of Grain Storage Technology in the Past 15 Years (2007–2022) Based on Knowledge Graph and Visualization. Foods, 11 (23), 3836. doi: https://doi.org/10.3390/foods11233836
3. Olorunfemi, B. J., Kayode, S. E. (2021). Post-Harvest Loss and Grain Storage Technology- A Review. Turkish Journal of Agriculture – Food Science and Technology, 9 (1), 75–83. doi: https://doi.org/10.24925/turjaf.v9i1.75-83.3714
4. Cui, H., Zhang, Q., Wu, W., Zhang, H., Ji, J., Ma, H. (2022). Modeling and Application of Temporal Correlation of Grain Temperature during Grain Storage. Agriculture, 12 (11), 1883. doi: https://doi.org/10.3390/agriculture12111883
5. Wang, X., Wu, W., Yin, J., Zhang, Z., Wu, Z., Zhang, H. (2018). Analysis of wheat bulk mould and temperature-humidity coupling based on temperature and humidity field cloud map. Nongye Gongcheng Xuebao/Transactions of the Chinese Society of Agricultural Engineering, 34 (10), 260–266. doi: https://doi.org/10.11975/j.issn.1002-6819.2018.10.033
6. Yan, H., Chen, G., Zhou, Y., Liu, L. (2012). Primary study of temperature distribution measurement in stored grain based on acoustic tomography. Experimental Thermal and Fluid Science, 42, 55–63. doi: https://doi.org/10.1016/j.expthermflusci.2012.04.010
7. Ge, L Chen, E. (2021). Research on grain storage temperature prediction model based on improved long short-term memory. Journal of Computational Methods in Sciences and Engineering, 21 (5), 1145–1154. doi: https://doi.org/10.3233/jcm-204751
8. Quemada-Villagómez, L. I., Molina-Herrera, F. I., Carrera-Rodríguez, M., Calderón-Ramírez, M., Martínez-González, G. M., Navarrete-Bolaños, J. L., Jiménez-Islas, H. (2020). Numerical Study to Predict Temperature and Moisture Profiles in Unventilated Grain Silos at Prolonged Time Periods. International Journal of Thermophysics, 41 (5). doi: https://doi.org/10.1007/s10765-020-02636-5
9. Ol'shanskii, V. P. (2001). Temperature field of bedded self-heating of a bank in a silo. Combustion, Explosion and Shock Waves, 37, 53–56. doi: https://doi.org/10.1023/A:1002816725317
10. Ol'shanskii, V. P. (2004). Identification of the Parameters of a Nested Cylindrical Heat Source under Stationary Self-Heating of a Raw Material Mass of the Same Form. Journal of engineering physics and thermophysics, 77 (1), 242–246. doi: https://doi.org/10.1023/B:JOEP.0000020747.49072.8b
11. Jayanti, S., Valette, M. (2005). Calculation of dry out and post-dry out heat transfer in rod bundles using a three field model. International Journal of Heat and Mass Transfer, 48 (9), 1825–1839. doi: https://doi.org/10.1016/j.ijheatmasstransfer.2004.11.005
12. Ol'shanskii, V. P. (2002). Steady-state temperature field of a cylindrical mass of raw material when it is self-heated by an ellipsoidal source. Journal of Engineering Physics and Thermophysics, 75 (4), 954–956. doi: https://doi.org/10.1023/A:1020331622455
13. Olshanskiy, V. P., Slipchenko, M. V., Olshanskiy, O. V. (2021). To calculation and forecast of the temperature of formation self-heating of plant raw materials. Engineering of nature management, 3 (21), 66–72. doi: https://doi.org/10.5281/zenodo.7316973
14. Biliaiev, M. M., Berlov, O. V., Biliaieva, V. V., Kozachyna, V. A. (2022). Numerical simulation of the process of self-heating of plant raw materials for the purpose of determining the time of the fire initiation. Ukrainian Journal of Civil Engineering and Architecture, 6, 7–13. doi: https://doi.org/10.30838/j.bpsacea.2312.281221.7.809
15. Subrot Panigrahi, S., Singh, C. B., Fielke, J., Zare, D. (2019). Modeling of heat and mass transfer within the grain storage ecosystem using numerical methods: A review. Drying Technology, 38 (13), 1677–1697. doi: https://doi.org/10.1080/07373937.2019.1656643
16. Olshanskyi, B., Kharchenko, C., Slipchenko, M., Kovalyshyn, C., Mazurak, M. (2021). On calculation of the temperature of raw material self-heating in cylindrical tanks. Bulletin of Lviv National Environmental University. Agroengineering Research, 25, 21–27. doi: https://doi.org/10.31734/agroengineering2021.25.021
17. Askarov, A., Tlevlessova, D., Ostrikov, A., Shambulov, Y., Kairbayeva, A. (2022). Developing a statistical model for the active ventilation of a grain layer with high moisture content. Eastern-European Journal of Enterprise Technologies, 1 (11 (115)), 6–14. doi: https://doi.org/10.15587/1729-4061.2022.253038