

A method of correlation-interferometric direction finding has been improved, which effectively solves the problem of radio direction finding of radio emission sources under conditions of exposure to one or two masking interferences. The problem was solved using the selection of an unmasked fragment of the spatial spectrum of the signal and the reconstruction of the missing samples of its signal group. As a result of the synthesis of the proposed method, estimates of signal samples were obtained as exact solutions to the proposed energy balance equations. The resulting solutions provide a significant increase in the signal-to-interference ratio and, accordingly, direction-finding accuracy without increasing the number of reception channels of the antenna array. As a result of the simulation, the dependences of the standard deviation of the bearing estimate on the signal-to-noise ratio in the presence of interference were built. Under the influence of one or two masking interferences and a signal-to-interference ratio of 0 dB, the use of the known direction-finding method without interference selection produces an anomalously large direction-finding error of more than 0.42 degrees, which is practically independent of the signal-to-noise ratio. The direction-finding method with selection of spectral signal samples masked by interference reduces the direction-finding error to 0.22 degrees when exposed to one interference and to 0.3 degrees when exposed to two interferences. This is due to the presence of power losses of the usable signal during the selection of its samples masked by interference. The proposed method of direction finding with reconstruction of signal samples provides a significant gain in accuracy by 3–30 times compared to the method of selection of masked samples in the range of changes in the signal-to-noise ratio (–20.5) dB. The direction-finding error of the proposed method decreases with increasing signal/noise according to a hyperbolic dependence. It is advisable to use the proposed direction-finding method when masking no more than two samples of the signal group

Keywords: error variance, direction finding accuracy, signal spectrum reconstruction, analytical signal reconstruction

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IMPROVING THE ACCURACY OF A DIGITAL SPECTRAL CORRELATION-INTERFEROMETRIC METHOD OF DIRECTION FINDING WITH ANALYTICAL SIGNAL RECONSTRUCTION FOR PROCESSING AN INCOMPLETE SPECTRUM OF THE SIGNAL

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1. Introduction

Radio monitoring is an effective means of controlling the use of radio frequency resources. The number of radio-electronic devices is constantly increasing, especially mobile ones, with rapidly changing spatial and time-frequency parameters. Therefore, the most informative component of radio monitoring today is radio direction finding and deter-

tronic devices is constantly increasing, especially mobile ones, with rapidly changing spatial and time-frequency parameters. Therefore, the most informative component of radio monitoring today is radio direction finding and deter-

mination of the location of radio emission sources (RES), which provide spatial localization of radio-electronic equipment [1].

A promising direction in the implementation of radio direction finding is the use of passive broadband digital correlation radio direction finders, combining high speed and accuracy of direction finding in a complex electromagnetic environment (EME).

Simultaneous broadband reception and direction finding of several emissions may result in mutual masking interference. To increase noise immunity and direction-finding accuracy, it is advisable to use interference selection, for example, frequency, time, spatial. Increased noise immunity of radio direction finding is ensured using antenna arrays (AA) with a multi-lobe radiation pattern. Radio direction finders with AA provide high accuracy in determining the direction to RES while completely masking and blocking the signal with interference in frequency and time. When spatial partial or complete overlap of radio emissions of signal and interference occurs, the accuracy of radio direction finding and monitoring significantly deteriorates.

Therefore, an urgent task of improving radio monitoring tools is to increase the accuracy of direction finding in complex EME and the effect of masking interference in time, frequency, and direction of arrival.

2. Literature review and problem statement

Works [2, 3] report studies on the accuracy of estimating the direction to RES and method optimization. It is shown that, in contrast to classical direction-finding methods [1], the applied parallel spatial digital spectral analysis of radio signals and single-iteration delay estimation provide an accurate bearing estimation with minimal computational and time costs. The accuracy of direction finding is significantly affected by the type of spectral analysis window, signal-to-noise ratio, and signal-to-interference ratio at the inputs of radio channels. However, these studies do not consider cases of receiving a mixture of signals and interference. For monitoring practice, it is necessary to develop a direction-finding method using selection and compensation of masking interference.

In [4], studies were carried out on the accuracy of the digital method of correlation-interferometric direction finding with two-dimensional correlation processing of the spatial signal. A formula for the variance of bearing estimation is derived. It is shown that the main parameters influencing the accuracy of the estimate are the time of radiation analysis, the magnitude of the spatial shift for correlation processing, and the type of window weighting function for digital synthesis of the AA radiation pattern (RP). However, the simulation established dependences of the standard deviation (RMS) of the bearing estimate only on the signal-to-noise ratio. The influence of masking interference and possible modification of the method have not been studied.

In [5], a new configuration of the AA matrix was proposed to improve the accuracy of interferometric direction finding in the range of both low and high frequencies. Simulations have shown the effectiveness of the proposed antenna element array configuration. In [6], a technique was studied to compensate for a non-ideal modulating pulse for direction finding using a time modulation matrix. A reconstruction of the required envelope shape for the modulation pulse was

applied, and its formula was derived. However, the effectiveness of the proposed solutions is shown without taking into account the influence of masking interference in time, frequency, and direction of arrival.

In [7], a data model for a single-channel direction finding system with switching between antennas was studied. The work examined compensation for the mutual influence of antenna elements and the imperfection of the switch. However, the developed comprehensive model only includes noise modeling to improve the estimation accuracy. In [8], a new direction-finding method with spatial spectrum estimation is proposed. A linear array and one radio frequency reception channel are used. The autocorrelation matrix for the digital spectrum of the total array signal is estimated. An algorithm is proposed for assessing the directions of signals, including for determining the direction of coherent sources. This method has shown effectiveness for frequency-coherent interference. Also, in [9], a similar algorithm for estimating the direction of the signal to RES using the rotational invariance method (ESPRIT) is proposed. This work considers the presence of inhomogeneous noise and allows the estimation of up to $M-1$ uncorrelated signals. In [10], using subspace methods, a joint range estimation method is applied based on the unitary advanced multiple signal classification (MUSIC) algorithm, also known as the root-MUSIC algorithm. And in [11] they struggle with random deformations of signal arrays. The ESPRIT and root-MUSIC methods are generally used in the presence of a priori information about the number of signals in the mixture and lose stability at low signal-to-noise ratios. They also have high computational complexity, just like signal processing in [12].

Work [13] proposed a direction-finding method using a two-element AA. The time-modulated matrix does not synchronize samples and uses asymmetric time sequences, which simplifies the system. A radar with multiple inputs and outputs was also studied in [14]. An extended virtual AA with interference mutual communication is used. Algorithms are proposed for estimating the bearing under conditions of using a receiving matrix with homogeneous linear arrays. Interference is not compensated, but the simulation showed a low shift in bearing estimates for conditions of perturbation of the sensor position and gain inhomogeneity in the AA receiving channels.

In [15], masking interference is combated using an active emitting radar, which has both direction and range selection. In [16], a direction-finding method with protection against multipath propagation of radio waves is proposed. Study [17] improves the accuracy of parameter estimation for radars using orthogonal frequency division multiplexing (OFDM) technology, which also has low computational complexity due to the use of discrete Fourier transform. In [18], a new signal processing algorithm using a priori range information solves interference problems for an airborne active radar with a conformal AA. In [19], a new target detection algorithm is proposed for low signal-to-noise ratio conditions using a neural network. In general, the problems of active emitting radars are considered, which do not operate secretly and cannot always be used in radio monitoring and radio reconnaissance.

In [20], the problem is considered related to passive radar detection of ships for operation under real coastal conditions in the presence of interference from the sea and wind power plants. Adaptive approaches to pattern generation improve signal-to-noise ratios. Paper [21] proposes an efficient algo-

rithm for passive estimation of the largest angle of arrival for a hybrid mm-wave communication system. The computational complexity of processing algorithms is reduced. In [22], three efficient detectors are developed to detect distributed targets under unknown interference conditions. It improves passive detection performance. These methods enhance the performance of passive radar for certain conditions and type of system.

Thus, an unresolved part of the broad problem of increasing the efficiency of radio monitoring systems is improving the noise immunity and accuracy of passive ground-based direction finding under conditions of a priori uncertainty of the parameters of the received radio emission and the presence of masking interference.

3. The aim and objectives of the study

The goal of this work is to increase the accuracy of direction finding in complex EME under the influence of masking interference. This will improve the noise immunity and accuracy of radio monitoring.

To achieve the goal, the following tasks were set:

- to improve the digital spectral method of correlation-interferometric direction finding with reconstruction of the spatial analytical signal for conditions of processing an incomplete spatial spectrum of the signal;
- to perform an analytical assessment of the noise immunity of the proposed direction-finding method;
- to perform modeling of direction-finding algorithms, obtain the dependence of the standard deviation of the bearing estimate on the signal/interference and signal/noise ratios through modeling.

4. The study materials and methods

The object of our study is an improved digital spectral correlation-interferometric direction finder with reconstruction of a spatial analytical signal with the ability to process an incomplete spatial spectrum of the signal.

The adopted simplifications and assumptions are as follows. Radio emissions are generated by point RES and are received by a linear AA in the horizontal plane. Direction-finding channels of the linear AA have statistically independent additive stationary normal noise of their own with zero mathematical expectation and the same spectral power density within their bandwidth. The intrinsic noise of the radio channels of AA does not have inter-channel correlation and correlation with the received radio emissions. We also set that the direction-finding RES are located in the far zone, and there are no phase fluctuations along the path of propagation of radio emissions.

The research hypothesis is as follows. The temporal frequency spectra of radio emissions overlap, and the spatial spectra of radio emissions partially overlap (radio emissions come from close directions on the verge of spatial resolution).

Analytical studies were carried out to substantiate the proposed method and its noise immunity. The simulation simulates the reception of radio emissions by identical AA radio channels separated in space $Z=64$ with additive noise using a developed software model of a direction finder in the PTC MathCad environment. Hardware is a laptop for simulation. The application of the proposed method is appro-

priate for the conditions of direction finding of ground-based stationary point RES.

When solving the first problem, the theory of correlation analysis, the Hilbert transform, digital spectral analysis of received radio emissions, digital synthesis of a multi-beam RP of a multi-element linear AA were used, an algorithm was proposed for reconstructing lost spectral elements from a fragment of the signal spectrum undistorted by interference, and analytical methods of comparative analysis.

To solve the second problem, digital spectral analysis and analytical methods of comparative analysis were used.

To solve the third problem, the method of statistical radio engineering was used, a correlation method for estimating the bearing, a software modeling method, and a statistical method for estimating the standard deviation were proposed. A comparative analysis of the obtained dependences of the standard deviation of the bearing estimate was carried out for the proposed method of spectrum reconstruction and direction finding, as well as for known methods of amplitude interference selection.

5. Results of investigating the spectral method of correlation-interferometric direction finding for an incomplete signal spectrum

5.1. Development of a spectral method of correlation-interferometric direction finding for an incomplete signal spectrum

To ensure the possibility of passive direction finding of short-term broadband radio emissions in real time for conditions of complex EME, a search-free digital method of correlation-interferometric direction finding with restoration of spatial spectral signal groups distorted by interference was developed. Complex EME occurs when the temporal frequency spectra of radio emissions overlap, and the spatial spectra of radio emissions partially overlap (radio emissions come from close directions on the verge of spatial resolution).

Let a linear AA from Z identical direction-finding channels receive an additive mixture $S_z(t) = \sum_{l=1}^L S_l(t)$ of L random Gaussian quasi-continuous stationary real radio emissions $S_l(t)$ with a uniform energy spectrum $S_l^2(\omega) = \text{const}$. It is specified that radio emissions are generated by point RES and are received by a linear AA in the horizontal plane. Direction finding channels of a linear AA have statistically independent natural additive stationary normal noise $n_z(t)$ with zero mathematical expectation and the same power spectral density N within their band $\{\omega_L; \omega_H\}$ of transmission.

The intrinsic noise of the radio channels of AA does not have inter-channel correlation and correlation with the received $S_l(t)$ radio emissions. It is also specified that the direction-finding RES are located in the far zone, and there are no phase fluctuations along the path of propagation of radio emissions $S_l(t)$.

Thus, the initial research conditions can be represented as follows:

$$U_z(t) = \sum_{l=1}^L S_{z,l}(t - \tau_z) + n_z(t), \quad (1)$$

where $U_z(t)$ is a mixture of signals and self-noise received by the z -th direction finding channel;

$S_{z,l}(t-\tau_z)$ – l -th signal, which is received by the z -th direction finding channel of AA;

τ_z – signal delay in the z -th direction finding channel relative to the first reference channel.

Let estimates θ_l of directions to direction-finding RES be determined using non-search correlation processing, that is, by calculating only one value of the argument τ_l of the correlation function for each l -th radio emission $S_l(t)$.

It is specified that the probability density distribution $p_l(\omega)$ of radio emissions $S_l(t)$ over frequency ω within the band $\{\omega_L; \omega_H\}$ of transmission of direction-finding channels is uniform, that is $p_l(\omega)=1/(\omega_H/\omega_L)$. Also uniform is the distribution of the probability density $p_l(\theta)$ of radio emissions $S_l(t)$ in the direction θ in the horizontal plane. It is assumed that the bands of frequencies $\{\omega_{L,l}; \omega_{H,l}\}$ occupied by the Fourier spectra $S_l(j\omega)$ of radio emissions are known a priori.

It is assumed that the number L of received radio emissions $S_l(t)$ does not exceed the number Z of direction-finding channels of the linear AA. In this case, all L radio emissions received within the band $\{\omega_L; \omega_H\}$ of transmission of direction-finding channels belong to different resolution elements $\Delta_l(\omega_k; \theta_p)$ in time k -th frequencies ω_k and p -th directions θ_p :

$$\begin{aligned} S_l \quad t \in \Delta_l \quad \omega_k; \theta_p ; \\ S_{l+i} \quad t \in \Delta_{l+i} \quad \omega_k; \theta_p ; \\ \Delta_l \cap \Delta_{l+i} = 0 \Big|_{i=1, \dots, L-1}. \end{aligned} \quad (2)$$

For given initial conditions, it is advisable to use preliminary non-search reception of radio emissions at frequency ω_k and spatial direction θ_p , which is performed alternately. It is also advisable to search-free correlation assessment of the directions θ_l on RES using the reconstruction of the spatial analytical signal [3]. It is advisable to carry out search-free reception at frequency ω_k using parallel digital spectral analysis based on the fast Fourier transform (FFT) algorithm, implemented in each z -th direction finding channel of AA. It is advisable to implement non-search reception in the direction θ_p using the synthesis of a multi-lobe RP, which is also effectively carried out based on the FFT algorithm [3, 23].

It is advisable to reconstruct radio emissions $S_l(t)$ received by a linear AA by means of the Hilbert transform of samples of the corresponding signal groups $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$, formed at the output of AA with a multi-lobe RP [3].

In this case, the k -th signal group is a selected array $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ of complex responses of AA with a multi-lobe RP to the action of the k -th time spectral component $U_{z,l}(j\omega_k)$ with frequency ω_k of radio emission of the l -th RES received by the main lobes of overlapping adjacent partial RPs $U_{k,l}(j\Omega_p)$, where $p_{L,l}, p_{H,l}$ are the numbers of the lower and upper frequencies of the selected signal group, respectively.

When direction finding the l -th point RES by processing a mixture of radio emissions $S_l(t)$, other $(L-1)$ radio emissions will be station interference, which can significantly distort its signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$.

A search-free correlation estimate of the θ_l direction to the l -th RES is obtained, subject to the presence of masking spatial interference that distorts the readings of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ of direction-finding radio emission $S_l(t)$. For this purpose, an analysis of the features of the formation of signal groups and the corresponding masking interference at the output of AA with a multi-lobe RP was

performed. Digital synthesis of a multi-lobe RP is carried out using the FFT algorithm as follows [3, 24]:

$$U_{k,l} \quad j\Omega_p = \sum_{z=0}^{Z-1} U_{z,l} \quad j\omega_k \cdot \exp -j\Omega_p \cdot z \cdot W \quad z, \quad (3)$$

where $U_{k,l}(j\Omega_p)$ is the complex spatial spectrum for the k -th component of the time spectrum $U_{z,l}(j\omega_k)$ of the received mixture $U_z(t)$ of L radio emissions;

$\Omega_p = 2\pi \cdot p/d \cdot Z$ – the value of the spatial frequency that determines the direction of the p -th lobe of a multi-lobe RP, $p=0, 1, \dots, Z-1$;

d – distance between AA elements;

$W(z)$ is the weighting function of spatial spectral analysis, which determines the shape of the pattern lobe.

Analysis of equation (3) shows that the synthesis of a multi-lobe RP is equivalent to the action of a parallel set of spatially matched filters for harmonic spatial signals with the ability to select interference.

Adjacent p partial diagrams of a multi-lobe RP are significantly overlapped by the main lobes, which causes, under the action of the l -th quasi-harmonic radiation $U_{z,l}(j\omega_k)$, the formation at the output of the multi-lobe RP of a corresponding array of $m_l = p_{H,l} - p_{L,l}$ samples of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$.

The interference groups $\{U_{k,l-1}(j\Omega_p)\}_{p \in [p_{L,l-1}; p_{H,l-1}]}$ and $\{U_{k,l+1}(j\Omega_p)\}_{p \in [p_{L,l+1}; p_{H,l+1}]}$ are formed in a similar way at the output of the multi-lobe RP in response to the action at its input of the corresponding interference $S_{l-1}(t)$ и $S_{l+1}(t)$.

Diagrams explaining the formation of signal $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ and two masking interference groups $\{U_{k,l-1}(j\Omega_p)\}_{p \in [p_{L,l-1}; p_{H,l-1}]}$ and $\{U_{k,l+1}(j\Omega_p)\}_{p \in [p_{L,l+1}; p_{H,l+1}]}$, are shown in Fig. 1.

The interference $S_{l-1}(t)$ и $S_{l+1}(t)$ are called masking relative to the signal $S_l(t)$ if their interference groups $\{U_{k,l-1}(j\Omega_p)\}_{p \in [p_{L,l-1}; p_{H,l-1}]}$ and $\{U_{k,l+1}(j\Omega_p)\}_{p \in [p_{L,l+1}; p_{H,l+1}]}$ overlap in frequency Ω_p with the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$, that is:

$$\begin{aligned} p_{L,l} \leq p_{H,l-1}; \\ p_{H,l} \geq p_{L,l+1}. \end{aligned} \quad (4)$$

To ensure the noise immunity of the direction finder under the influence of masking interference, it is advisable for further analysis to use only a fragment of the signal group that does not contain elements distorted by interference that need to be rejected. This is ensured by using the resulting spatial selection of a fragment of the signal group that does not contain elements distorted by interference, as follows:

$$U_{kl,F}(j\Omega_p) = U_{k,l}(j\Omega_p) \cdot K_F(j\Omega_p), \quad (5)$$

where $K_F(j\Omega_p)$ is the complex transfer characteristic of the

spatial filter, and $K_F(j\Omega_p) = \begin{cases} 1 & |_{p_{H,l-1} < p_l < p_{L,l+1}}, \\ 0 & |_{p_{L,l+1} < p_l < p_{H,l-1}}; \end{cases}$

$U_{kl,F}(j\Omega_p)$ – a selected fragment of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$, which does not contain elements distorted by interference.

The boundary frequencies of a spatial filter with the characteristic $K_F(j\Omega_p)$ can be estimated from the ratio of the frequencies $\Omega_{\max,l}$ and $\Omega_{\max,l-1}, \Omega_{\max,l+1}$, corresponding to the local extrema of the l -th signal and $(l-1)$ th, $(l+1)$ th interference groups, respectively.

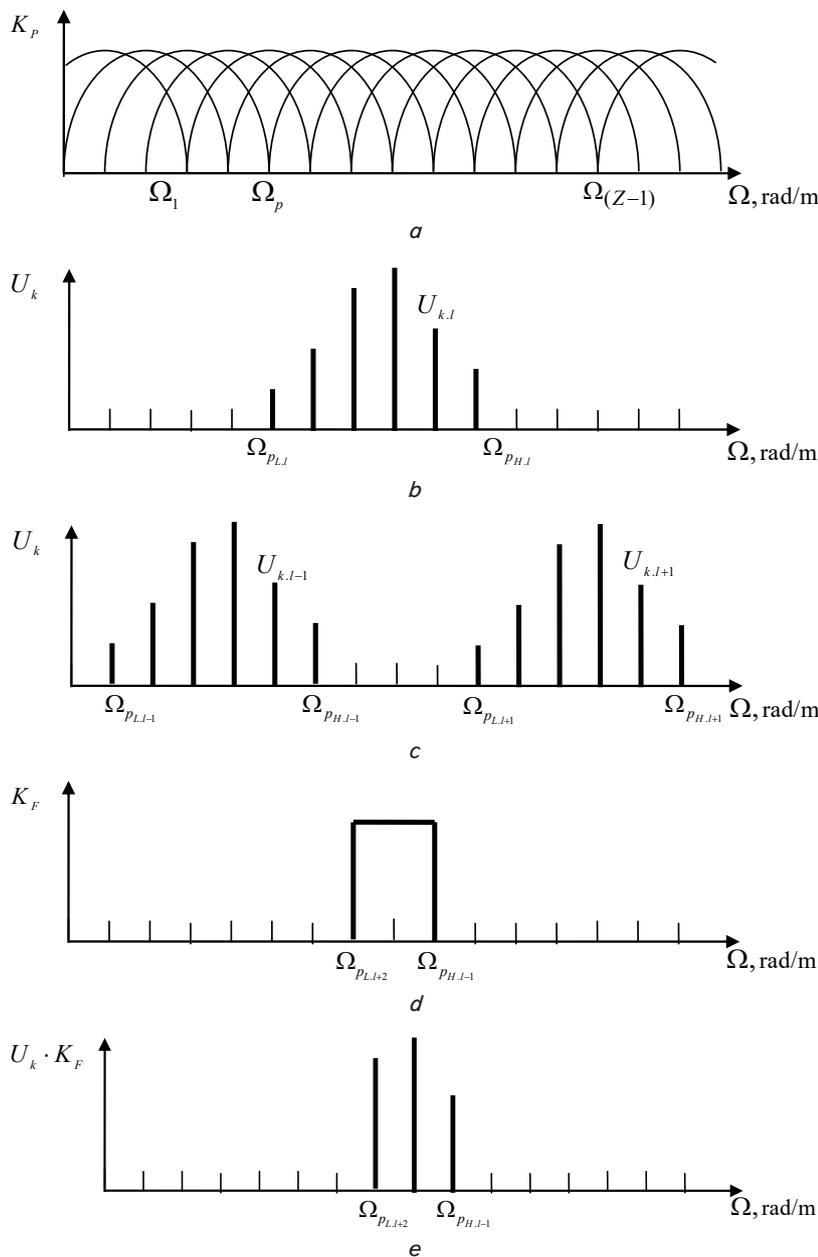


Fig. 1. Diagrams of the formation and selection of signal and interference groups: *a* – radiation pattern of a linear antenna array; *b* – spatial spectrum for the *k*-th component of the time spectrum of the received *l*-th radio emission; *c* – spatial spectrum for the *k*-th component of the time spectrum of the received (*l*–1)-th and (*l*+1)-th radio emissions; *d* – complex transfer characteristic of the spatial filter; *e* – selected fragment of the signal group, which does not contain elements distorted by interference

Spatial selection can significantly improve the signal-to-interference ratio provided that the power of the masking interference exceeds the power of the usable signal, that is, the signal-to-interference ratio at the input of the AA receiving channels is less than one. As a result, abnormally large direction-finding errors are eliminated. However, the disadvantage of using such direct selection is the significant loss of usable signal power and, accordingly, a significant increase in direction finding error.

Let's solve the problem of determining (reconstructing) the values of elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [pL,l;pH,l]}$

masked by noise using undistorted samples of the selected fragment $U_{k,l,F}(j\Omega_p)$. This, in turn, will allow for high-quality reconstruction of the spatial analytical signal $U_{A,kl}(j\Omega_p)$ and high-precision search-free correlation assessment of the θ_l direction to the *l*-th RES [3].

For this purpose, all possible options for generating masking interference are divided into two groups. The first group corresponds to two cases when one element of the *l*-th signal group with the lower $\Omega_{pL,l}$ or upper $\Omega_{pH,l}$ frequency is masked due to the action of one masking interference:

$$\begin{cases} \Omega_{pL,l} = \Omega_{pH,l+1}; \\ \Omega_{pH,l} = \Omega_{pL,l+1}. \end{cases} \quad (6)$$

The second group corresponds to four cases when two elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [pL,l;pH,l]}$ are masked due to the action of two $\{U_{k,l-1}(j\Omega_p)\}_{p \in [pL,l-1;pH,l-1]}$ and $\{U_{k,l+1}(j\Omega_p)\}_{p \in [pL,l+1;pH,l+1]}$ masking interference, that is:

$$\begin{cases} \Omega_{pL,l} = \Omega_{pH,l+1}; \\ \Omega_{pH,l} = \Omega_{pL,l+1}. \end{cases} \quad (7)$$

Equation (7) shows that the (*l*–1)th noise masks one sample of the signal group with the lower frequency $\Omega_{pL,l}$, and the (*l*+1)th noise masks 1 sample of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [pL,l;pH,l]}$ with the upper frequency $\Omega_{pH,l}$.

Thus, in the general case, to reconstruct a masked signal group, it is necessary to solve a system of two equations.

The computational costs of reconstructing a masked signal group have been minimized, for which equation (7) is represented using two unknown variables X_1 and X_2 .

To do this, an analysis of the features of the reconstruction of the spatial analytical signal $U_{A,kl}(j\Omega_p)$ was performed based on the corresponding signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [pL,l;pH,l]}$, determined based on the FFT algorithm:

$$\begin{aligned} \{U_{k,l}(j\Omega_p)\}_{p \in [pL,l;pH,l]} &= \\ &= \sum_{z=0}^{Z-1} U_z(j\omega_k) \cdot \exp(-j\Omega_p \cdot z) \cdot W_0(z) \Big|_{p \in [pL,l;pH,l]}, \end{aligned} \quad (8)$$

where $U_{k,l}(j\Omega_p)$ is the complex spatial (obtained as a result of processing the spatial implementation) spectrum for the *k*-th component of the array of time spectra $U_{z,l}(j\omega_k)$ of the accepted implementations $U_z(t)$ of the *z*-channel AA;

$W_0(z)$ – weighting function of spatial digital diagram formation (function of the “window” of spatial spectral analysis).

In turn, the spatial analytical signal $U_{A,kl}(j\Omega_p)$ is reconstructed within the aperture of the linear AA based on the discrete Hilbert transform and, taking into account (8), can be represented by the following model:

$$\begin{aligned} U_{Ak,l}(jz) &= \sum_{p=p_{L,l}}^{p_{H,l}} U_{k,l}(j\Omega_p) \cdot \exp(j\Omega_p \cdot z) = \\ &= \sum_{p=p_{L,l}}^{p_{H,l}} U_{k,l}(\Omega_p) \cdot \exp(j\Omega_p \cdot z + \Psi_{k,l} \Omega_p) = \\ &= U_{Ak,l}(z) \cdot W_\theta(z) \cdot \exp(j(\Omega_{A,kl} \cdot z + \Psi_{A,kl})), \end{aligned} \quad (9)$$

where $\Omega_{A,kl} = \omega_k \cdot \cos^2(\theta/c)$ – spatial frequency of the k -th spectral component of the l -th signal received by AA from the direction θ ;

$U_{Ak,l}(z)$, $\Psi_{A,kl}$ – amplitude and initial phase of the spatial analytical (complex, reconstructed from spectral components of only positive frequencies) signal $U_{A,kl}(j\Omega_p)$;

$U_{k,l}(\Omega_p)$, $\Psi_{k,l}(j\Omega_p)$ – module and argument of the p -th sample of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$.

Analysis of equation (9) shows that the distribution of spatial complex samples $U_{A,kl}(j\Omega_p)$ of the spatial analytical signal $U_{Ak,l}(jz)$ within the AA aperture corresponds to a harmonic spatial process with an unknown constant frequency $\Omega_{A,kl}$ and argument $\Psi_{A,kl}$. The distribution has amplitude modulation $U_{Ak,l}(z) \cdot W_\theta(z)$. This modulation is determined by the digital beamforming weighting function $W_\theta(z)$. Also, an essential feature of the samples of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ is the coherence of the arguments of its samples, that is,

$$\begin{aligned} \Psi_{k,l}(\Omega_{p_{L,l}}) &= \Psi_{k,l}(\Omega_{p_{L,l}+2n}); \\ \Psi_{k,l}(\Omega_{p_{H,l}}) &= \Psi_{k,l}(\Omega_{p_{H,l}-2n}); \\ \Psi_{k,l}(\Omega_{p_{L,l}+2n}) &= \Psi_{k,l}(\Omega_{p_{L,l}+2n+1}) \pm \pi, \end{aligned} \quad (10)$$

where $n=0, 2, 4$ are integers for a signal group of six samples.

Analysis of equation (10) shows that the initial phases $\Psi_{k,l}(\Omega_p)$ of all harmonic components of the analytical signal $U_{Ak,l}(jz)$ are significantly correlated and coherent within groups with even $p=2n$ and odd $p=(2n+1)$ frequency numbers.

Taking into account the indicated properties, to solve the problem, it is advisable to reconstruct the samples of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ from the selected samples of the undistorted fragment $U_{kl,F}(j\Omega_p)$ with minimal computational costs as follows. It is advisable to determine the reconstruction equation only for three special points of the AA aperture, for which the degree of coherence of the harmonic components $U_{k,l}(j\Omega_p) \cdot \exp(j\Omega_p \cdot z)$ is maximum: at $z=0$; $z=Z/2$; $z=Z/4$:

$$\begin{aligned} \left[\Psi_{k,l}(\Omega_{p_{L,l}+2n}) = \Psi_{k,l}(\Omega_{p_{L,l}+2n+1}) \pm \pi \right]_{z=0}; \\ \left[\Psi_{k,l}(\Omega_{p_{L,l}+2n}) = \Psi_{k,l}(\Omega_{p_{L,l}+2n+1}) \pm \pi \right]_{z=Z/2}; \\ \left[\Psi_{k,l}(\Omega_{p_{L,l}+2n}) = \Psi_{k,l}(\Omega_{p_{L,l}+2n+1}) \pm \pi/2 \right]_{z=Z/4, 3Z/4}. \end{aligned} \quad (11)$$

Taking into account certain features of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ and the procedure for reconstructing the corresponding spatial analytical signal $U_{Ak,l}(jz)$, it is advisable to determine the following system of equations. The solution to this system will ensure the reconstruction of the distorted (incomplete) signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ from its fragment $U_{kl,F}(j\Omega_p)$:

$$\begin{cases} \left| \sum_{p=p_{L,l}}^{p_{H,l}} U_{k,l}(j\Omega_p) \right| = W_\theta(0); \\ \left[\sum_{p=p_{L,l}}^{p_{H,l}} U_{k,l}(j\Omega_p) \cdot \exp(j\Omega_p \cdot Z/2) \right] \cdot W_\theta(Z/4) = \\ \left| \sum_{p=p_{L,l}}^{p_{H,l}} U_{k,l}(j\Omega_p) \cdot \exp(j\Omega_p \cdot Z/4) \right|. \end{cases} \quad (12)$$

When masking only one element of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ we have:

$$X_1 = U_{k,l}(j\Omega_{p_{L,l}}) \text{ или } X_1 = U_{k,l}(j\Omega_{p_{H,l}}). \quad (13)$$

When masking two elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$, the unknown variables X_1 and X_2 are determined according to equation (7) as follows:

$$\begin{cases} X_1 = U_{k,l}(j\Omega_{p_{L,l}}); \\ X_2 = U_{k,l}(j\Omega_{p_{L,l}}). \end{cases} \quad (14)$$

The solution of equations (13) determines the value of the modules of the reconstructed elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ as follows:

$$\begin{aligned} U_{k,l}(j\Omega_{p_{L,l}}) &= W_\theta(0) - \sum_{p=p_{L,l}+1}^{p_{H,l}} U_{k,l}(j\Omega_p) \cdot (-1)^p; \\ U_{k,l}(j\Omega_{p_{H,l}}) &= W_\theta(0) - \sum_{p=p_{L,l}}^{p_{H,l}-1} U_{k,l}(j\Omega_p) \cdot (-1)^p. \end{aligned} \quad (15)$$

The solution to the system of equations (14) is as follows:

$$\begin{aligned} X_1 &= \frac{-b - \sqrt{b^2 - 4ac}}{2a}; \\ X_2 &= U_{k,l}(j\Omega_{p_{L,l}+2}) - U_{k,l}(j\Omega_{p_{H,l}-2}) + X_1, \end{aligned} \quad (16)$$

where:

$$\begin{aligned} b &= 4U_{k,l}(j\Omega_{p_{H,l}-2}) - 8W_\theta^2(Z/4) \cdot U_{k,l}(j\Omega_{p_{L,l}+2}); \\ a &= 2 - 4W_\theta^2(Z/4); \\ c &= \left(2U_{k,l}(j\Omega_{p_{H,l}-2}) - U_{k,l}(j\Omega_{p_{L,l}+2}) \right)^2 - \\ &\quad - \left(U_{k,l}(j\Omega_{p_{L,l}+2}) \right)^2 \cdot (1 - 4W_\theta^2(Z/4)). \end{aligned}$$

The arguments of the reconstructed elements of equations (16) can be determined taking into account the coherence of the components of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ at the point $Z=Z/4$ of AA as follows:

$$\begin{aligned} \Psi_{X1} &= \Psi_{k,l}(\Omega_{p_{L,l}+2}); \\ \Psi_{X2} &= \Psi_{k,l}(\Omega_{p_{H,l}-2}). \end{aligned} \quad (17)$$

The values of the reconstructed elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ are determined similarly for other variants of condition (14) for masking it with noise.

As a result, the posed problem of reconstructing the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ from its undistorted fragment $U_{kl,F}(j\Omega_p)$ has been solved.

Taking into account the obtained values of the elements of the reconstructed signal group $\{\check{U}_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ according to the algorithm [3], an estimate of the spatial frequency Ω_l and directions θ_l to RES for the condition $\Delta z = Z/2$ is determined:

$$\hat{\Omega}_l = \frac{1}{Z/2} \times \text{arctg} \left[\frac{\sum_{k=k_L}^{k_H} S_{12A,k}(\Omega_p, Z/4, 3Z/4) \cdot \sin(\Delta\hat{\Psi}_{A,k}(\Omega_p, \Delta z) \cdot K_r(\omega_{S,k}))}{\sum_{k=k_L}^{k_H} S_{12A,k}(\Omega_p, Z/4, 3Z/4) \cdot \cos(\Delta\hat{\Psi}_{A,k}(\Omega_p, \Delta z) \cdot K_r(\omega_{S,k}))} \right];$$

$$\Delta\hat{\Psi}_{A,k}(\Omega_p, \Delta z) = \hat{\Psi}_{A,k}(\Omega_p, 3Z/4) - \hat{\Psi}_{A,k}(\Omega_p, Z/4); \quad (18)$$

$$z_1 = Z/4; \quad z_2 = 3Z/4; \quad \Delta z = Z/2;$$

$$\hat{\theta}_l = \arccos[\hat{\Omega}_l \cdot c / \omega_{S,l}].$$

Taking into account our results, the developed direction-finding method is implemented in the following sequence of actions. The additive mixture of radio emissions is received simultaneously by Z direction-finding channels of AA with a multi-lobe directional pattern (3). For each received radio emission, frequency and spatial selection of an array of fragments of signal groups is performed sequentially with rejection of elements masked by interference (5). Next, the complete signal groups are reconstructed taking into account the elements of their selected fragments, formulas (16) and (17). Next, taking into account the reconstructed signal groups according to (18), the final assessment of the direction to RES is determined in one cycle of correlation analysis. Thus, the task of developing a digital spectral method for correlation-interferometric direction finding with reconstruction of a spatial analytical signal when processing an incomplete spectrum of the signal has been solved.

5. 2. Analytical assessment of direction-finding method interference immunity

The noise immunity of the developed method was assessed in comparison with the well-known method that does not use the reconstruction of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ for conditions where the signal of the l -th point RES is masked by two noises. As an assessment of noise immunity, it is advisable to use the signal/interference ratio $\mu_{r,l}$, provided that the interference power P_r is much higher than the $P_r \gg P_n$ power P_n of the self-noise of direction-finding radio channels:

$$\mu_{r,l} = P_{S,l} / P_r, \quad (19)$$

where $P_{S,l}$ is the power of the l -th direction-finding RES; P_r is the total power of masking interference for the l -th signal.

To estimate the signal/interference ratio $\mu_{r,l}$, we take into account that masking interference with powers $P_{r,l-1}$ and $P_{r,l+1}$ is located in different sectors with respect to the partial RP $K_p(j\Omega_p)$, the output signals of which form the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$. For partial RP that form a

fragment $U_{kl,F}(j\Omega_p)$ of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ undistorted by interference, all interference is in the sector of their side lobes with a relative level $K_{SL}(\Omega_p)$. For other partial RPs $K_p(j\Omega_p)$, forming side elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ masked by interference, adjacent interference with powers $P_{r,l-1}$ and $P_{r,l+1}$ is in the sector of the main lobes with the relative level $K_{ML}(\Omega_p)$.

For the developed method of correlation-interferometric direction-finding using signal group reconstruction, the signal/interference ratio $\mu_{r,l}$ will be determined only by the relative level $K_{SL}(\Omega_p)$ of the side lobes of partial RPs. This was obtained by discarding the outer elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ masked by interference and replacing them with reconstructed elements from the fragment $U_{kl,F}(j\Omega_p)$ undistorted by interference. In the presence of two adjacent interference with powers $P_{r,l-1}$ and $P_{r,l+1}$, the ratio $\mu_{r,l}$ will be equal to:

$$\mu_{r,l} = \frac{P_{S,l}}{(P_{r,l-1} + P_{r,l+1}) \cdot K_{SL}^2 \cdot m_S}, \quad (20)$$

where $m_S = (p_{H,l} - p_{L,l} + 1)$ is the number of elements in the l -th signal group;

$P_{S,l} = \sum_{p=p_{L,l}}^{p_{H,l}} P_l \cdot K_{ML}^2(\Omega_p)$ – power of the l -th signal, consisting of the powers of the signal group samples.

For the well-known non-search digital method of correlation-interferometric direction finding with reconstruction of a spatial analytical signal [3], the signal/noise ratio $\mu_{r,2l}$ under equal research conditions will be as follows. It will be determined simultaneously by the relative levels of the side $K_{SL}(\Omega_p)$ and main $K_{ML}(\Omega_p)$ lobes of the partial radiation patterns of a multi-lobe RP:

$$\mu_{r,2l} = \frac{P_{S,l}}{(P_{r,l-1} + P_{r,l+1}) \cdot K_{SL}^2 \cdot m_S + \sum_{p=p_{L,l}}^{p_{L,l+1}} P_{r,l-1} \cdot K_{ML}^2(\Omega_p) + \sum_{p=p_{H,l}}^{p_{H,l+1}} P_{r,l+1} \cdot K_{ML}^2(\Omega_p)}. \quad (21)$$

The gain K_μ in terms of noise immunity of the proposed method is estimated in comparison with the well-known method that does not use the reconstruction of the signal group $\{\check{U}_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$ from the fragment $U_{kl,F}(j\Omega_p)$ undistorted by interference:

$$K_\mu = \frac{\mu_{r,l}}{\mu_{r,2l}} = \frac{(P_{r,l-1} + P_{r,l+1}) \cdot K_{SL}^2 \cdot m_S + \sum_{p=p_{L,l}}^{p_{L,l+1}} P_{r,l-1} \cdot K_{ML}^2(\Omega_p) + \sum_{p=p_{H,l}}^{p_{H,l+1}} P_{r,l+1} \cdot K_{ML}^2(\Omega_p)}{(P_{r,l-1} + P_{r,l+1}) \cdot K_{SL}^2 \cdot m_S}. \quad (22)$$

For example, let's estimate the gain K_μ in terms of noise immunity for conditions $L=3$ and the worst case of masking a signal group with two adjacent noises of equal power $P_{r,l-1} = P_{r,l+1}$, masking two extreme elements of the signal group $\{U_{k,l}(j\Omega_p)\}_{p \in [p_{L,l}; p_{H,l}]}$. The weighting function $W_\theta(z)$ of the 3-link Blackman beamforming function was used with the following parameters: side lobe level $K_{SL} = 58 \text{ dB} \approx 0.001$; the level of the main lobes $K_{ML}(\Omega_{pL}) = K_{ML}(\Omega_{pH}) = 40 \text{ dB} = 0.01$ at the lower Ω_{pL} and upper Ω_{pH} frequencies of the signal group is the same. Also, the level of

the main lobes $K_{ML}(\Omega_{pL+1})=K_{ML}(\Omega_{pH-1})=22\text{ dB}=0.1$ at the penultimate frequencies Ω_{pL+1} and Ω_{pH-1} of the signal group is also the same; number of elements in the signal group $m_S=6$. Then according to (22):

$$\begin{aligned} K_\mu &= \frac{2P_{r,l-1} \cdot K_{SL}^2 \cdot m_S + 2 \cdot \sum_{p=p_{l-1}}^{p_{l+1}} P_{r,l-1} \cdot K_{ML}^2(\Omega_p)}{2P_{r,l-1} \cdot K_{SL}^2 \cdot m_S} = \\ &= \frac{P_{r,l-1} \cdot K_{SL}^2 \cdot m_S + P_{r,l-1} \cdot (K_{ML}^2(\Omega_{p_l}) + K_{ML}^2(\Omega_{p_{l+1}}))}{P_{r,l-1} \cdot K_{SL}^2 \cdot m_S} = \\ &= \frac{K_{SL}^2 \cdot m_S + K_{ML}^2(\Omega_{p_l}) + K_{ML}^2(\Omega_{p_{l+1}})}{K_{SL}^2 \cdot m_S} = \\ &= \frac{(0.001)^2 \cdot 6 + (0.01)^2 + (0.1)^2}{(0.001)^2 \cdot 6} = 1,683 \approx 32.3\text{ dB}. \end{aligned} \quad (23)$$

Analysis of our results of estimating the gain K_μ in relative noise immunity shows its significant increase, all other conditions being equal. The efficiency of direction finding of RES also increases accordingly in the presence of powerful masking interference. Thus, the proposed method provides a significant increase in the signal-to-noise ratio and noise immunity of direction finding in general with a corresponding decrease in the variance of direction-finding error.

5. 3. Results of modeling the operation of a spectral correlation-interferometric direction finder

The direction-finding accuracy of the proposed method was studied according to formula (18) by simulation. The reception of radio emissions by identical AA radio channels separated in Z space with additive noise is simulated by using a developed software model in the PTC MathCad environment, USA. The initial modeling conditions and AA parameters are as follows:

- type of signal – with linear frequency modulation;
- operating frequency value $f_0=2$ GHz;
- signal spectrum width value $\Delta f_S=0.4$ MHz;
- antenna array pitch $d=0.05$ m;
- number of antenna elements $Z=64$;
- radio emission analysis time $T_a=0.1$ ms;
- type of internal noise – Gaussian with normal probability distribution;
- number of samples of the signal group of the harmonic spatial analytical signal $m_S=4$.

The dependence of the standard deviation of the bearing estimate σ_θ on the signal-to-noise ratio (SNR) in the presence of interference has been studied. The simulation will be performed for a pair of angles $[60, 52]^\circ$ of signal arrival and interference with a signal-to-interference ratio of 0 dB. With this ratio of the signal arrival and interference, their spatial spectra (signal groups) are overlapped by one extreme reading. Families of dependences on the signal-to-noise ratio are obtained.

Fig. 2 shows the dependence of the standard deviation of the bearing estimate on the signal-to-noise ratio (SNR, dB) for the case of one interference. The figure indicates: “P1grad” is the standard deviation

of the bearing estimate for algorithm (18) without selection of the masked spectral sample, and “P2grad” is the standard deviation of the bearing estimate when using selection (dropping) (5) of the spectral sample, which is the overlap of the signal and interference.

The standard deviation of the bearing estimate σ_θ was studied for the case of a signal arriving from the direction $[60]^\circ$ and two interference from the directions $[68, 52]^\circ$ with a signal-to-noise ratio of 0 dB. With this ratio, the catch of the signal arrival and two interferences, its spatial spectrum overlaps with the interference by two extreme readings - lower and upper. Fig. 3 shows the dependence of the standard deviation of the bearing estimate on the signal-to-noise ratio (SNR, dB) for the case of two interferences.

In Fig. 3, the following is indicated: “P3grad” is the standard deviation of the bearing estimate for the case without selection of two masked samples, and “P4grad” is the standard deviation of the bearing estimate for the case of selection (discarding) of two spectral samples, which are the overlap of the signal and two interference.

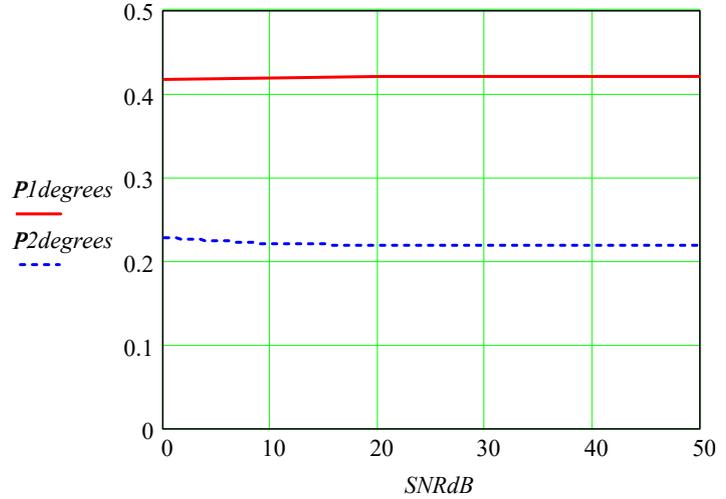


Fig. 2. Dependence of the standard deviation of the bearing estimate on the signal-to-noise ratio for one interference

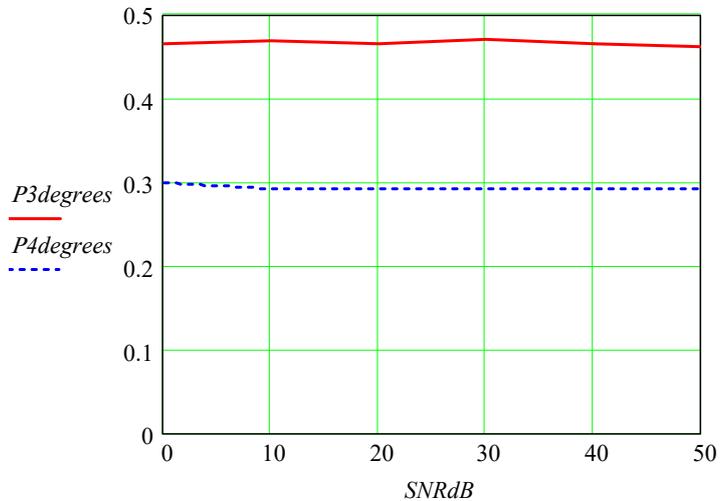


Fig. 3. Dependence of the standard deviation of the bearing estimate on the signal-to-noise ratio for two interference

The error in estimating the bearing for a pair of angles $[60, 52]^\circ$ of signal arrival and interference was studied when the signal-to-interference ratio changed. Let us set the noise level to be much lower than the interference level and can be neglected. Fig. 4 shows the dependence of the bearing estimation error on the signal-to-interference ratio (SIR, dB) for the case of one interference.

In Fig. 4, the following is indicated: “P5grad” is the error in estimating the bearing for the case of selection (5) (discarding) one spectral sample, which is the overlap of the signal and interference, and “P6grad” is the error in estimating the bearing for the case of restoring (15) one spectral sample masked by the interference.

The error in bearing estimation was studied for the case of a signal arriving from the direction $[60]^\circ$ and two interference from the directions $[68, 52]^\circ$ with a change in the signal-to-noise ratio. Let us set the noise level to be much lower than the interference level and can be neglected.

Fig. 5 shows the dependence of the bearing estimation error on the signal-to-interference ratio (SIR, dB) for the case of two interferences. In Fig. 5, the following is indicated: “P7grad” is the error in bearing estimation for the case of selection (discarding) of two spectral samples, upper and lower, which are the overlap of signal and interference, and “P8grad” is the error in bearing estimation for the case of restoration (16) of two spectral components masked by interference.

The standard deviation of the bearing estimate was studied for the case of a signal arriving from a direction of 60° and interference from a direction of 52° with a change in the signal-to-noise ratio. Let’s set the signal-to-noise ratio to 0 dB. Fig. 6 shows the dependence of the standard deviation of the bearing estimate on the signal-to-noise ratio (SNR, dB) in the presence of one interference.

In Fig. 6, the following is indicated: “P9grad” is the standard deviation of the bearing estimate for processing with selection (discarding) of one masked extreme upper spectral sample, and “P10grad” is the standard deviation of the bearing estimate for the proposed direction-finding method with restoration (15) of the spectral sample of the signal masked by interference.

As a result, from the obtained plots it was estimated that the direction-finding accuracy does not depend on the signal-to-noise ratio if the main error arises as a result of the overlap of the spatial spectra of the signal and interference, which occurs even with a difference of 8° in the directional angles of arrival. In the case where the signal and interference are located at a distance of more than 8° in direction, noise increases the influence on the standard deviation of the bearing estimate σ_θ . According to Fig. 2, 3, it is estimated that when selecting interference samples, the standard deviation of the bearing estimate decreases by 1.5–2 times. According to Fig. 4–6, it is clear that when reconstructing the discarded spectral components of the signal, the direction-finding error decreases by approximately an order of magnitude.

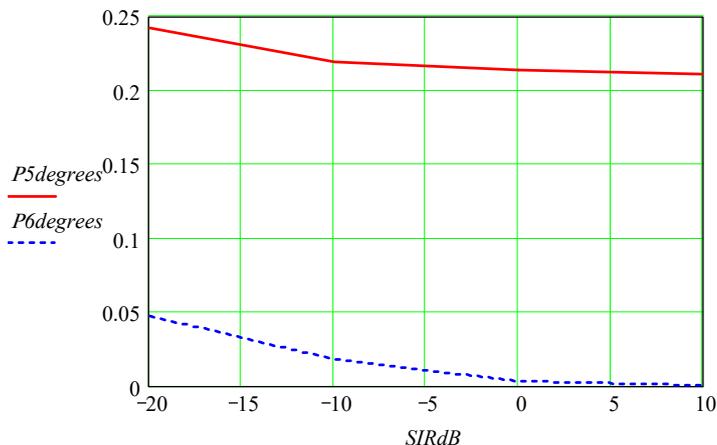


Fig. 4. Dependence of bearing estimation error on signal-to-noise ratio for one interference

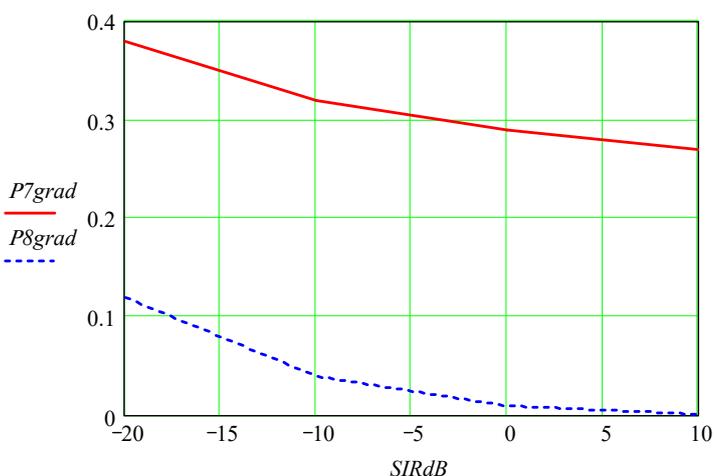


Fig. 5. Dependence of bearing estimation error on signal-to-interference ratio for two interferences

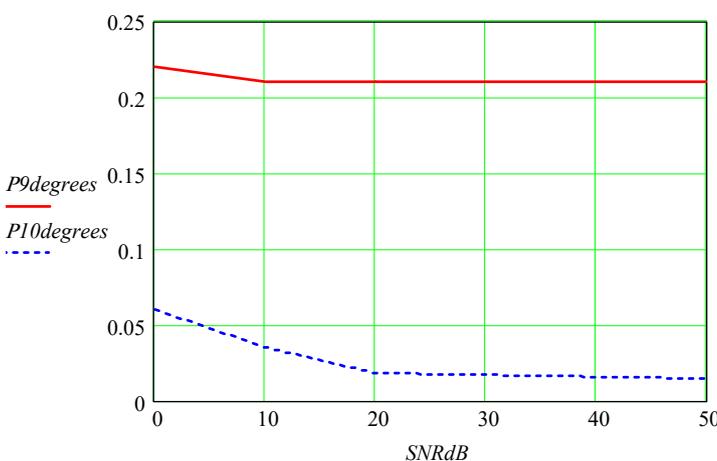


Fig. 6. Dependence of the standard deviation of the bearing estimate on the signal-to-noise ratio

6. Discussion of results of investigating the spectral method of correlation-interferometric direction finding for an incomplete signal spectrum

The constant increase in the number of radio-electronic equipment, their spatial distribution density, and mobility

requires an increase in the speed and accuracy of direction finding of the corresponding radio monitoring equipment. This problem is effectively solved by the use of digital broadband correlation radio direction finders with AA and the use of parallel spectral and spatial analysis. The high performance of such radio direction finders is ensured by methods of parallel selective reception in frequency and space and single-iteration estimation of the direction to RES with reconstruction of radiation in the antenna aperture. However, such known direction-finding methods provide the required combination of speed and accuracy in the absence of masking interference.

The proposed spectral method of correlation-interferometric direction finding for an incomplete signal spectrum effectively solves the problem of radio direction finding of RES under conditions of exposure to one or two masking interferences. This is achieved by using the reconstruction of signal group samples masked by noise. The proposed method is analytically synthesized using well-tested theory of statistical radio engineering and spatial parallel selective reception. As a result, the feasibility of reconstructing spatial signal samples masked by interference with subsequent one-iteration estimation of the direction to RES is substantiated. As a result of the synthesis of the proposed method, estimates of the reconstructed signal samples were obtained as exact solutions to the proposed energy balance equations. The resulting solutions provide a significant increase in the signal-to-noise ratio and direction-finding accuracy without increasing the number of AA reception channels. The analytical analysis of the noise immunity of the proposed method shows that its use significantly (more than 30 dB) increases the permissible signal-to-interference ratio SIR compared to known methods. The high performance of the proposed direction-finding method is ensured by the use of a new algorithm for single-iteration estimation of the values of masked samples of usable signal and the direction to its RES as a whole.

Simulation modeling of a radio direction finder was carried out using the proposed method for the conditions of the action of one or two masking interference and the presence of the antenna's natural noise. The simulation results are in good agreement with the results of the analytical analysis and confirm the significant gain of the proposed method at equal hardware costs compared to the well-known method without interference selection, using masked signal samples, as well as the method with selection of signal samples masked by interference.

Comparative analysis of dependences in Fig. 2–6 direction finding errors from the signal-to-noise ratio SNR of the proposed method and methods without selection and with selection of masked signal samples showed the following.

Under the influence of one or two masking interferences and the signal-to-interference ratio $SIR=0$ dB, the use of the known direction-finding method without interference selection generates an anomalously large direction-finding error (more than 0.42 degrees), Fig. 2, 3, which is practically independent of the signal-to-noise ratio SNR. This significantly reduces the effectiveness of this method in complex EME. The direction-finding method with selection (discarding) of signal samples masked by interference eliminates the anomalous value of the direction-finding error. However, the error of this method is quite significant (0.22 degrees), Fig. 4, and increases (up to 0.3 degrees), Fig. 5, under the influence of two interferences. This is due to the presence of power

losses of the usable signal during the selection of its samples masked by interference.

The proposed method of direction finding with reconstruction of signal samples provides a significant gain in accuracy by a factor of 3–30, Fig. 4, 5, in comparison with the method of selection (discarding) of masked samples in the range of SIR variations (–20;5) dB. The direction-finding error of the proposed method depends on the signal/interference value SIR and decreases with its increase according to a hyperbolic dependence. This is completely consistent with theoretical estimates.

The simulation results confirmed the possibility of reconstructing signal samples masked by interference and significantly increasing the accuracy of direction finding under the influence of powerful masking interference (SIR less than 0 dB), Fig. 5.

Thus, our results of analytical and experimental studies show that the proposed high-speed correlation method effectively solves the problem of direction finding in complex EME.

This study has the following limitations. The proposed method of correlation direction finding effectively solves the problem of radio direction finding of RES under conditions of exposure to one or two masking interference, provided that no more than two samples of the signal group are masked.

As a drawback of this study, it can be noted that the simulation was carried out under conditions of complete identity of the direction-finding channels, that is, assuming that the non-identity was previously eliminated.

It should be noted that when forming a multi-lobe RP AA of a direction finder, different types of weighting function $W(z)$ can be used. Therefore, in the future it is advisable to conduct studies of the influence on the direction-finding accuracy of the type of function $W(z)$ and the parameters of partial RPs for various conditions of the signal/noise, signal/interference, signal/(interference+noise) ratios and their parametric optimization.

It is also advisable to conduct research on correlation direction finding methods for different numbers of signal elements masked by interference when using different types of weighting function $W(z)$ and the action of powerful masking interference.

For conditions of very powerful (signal/interference less than –10 dB) interference, it is also advisable to conduct research and parametric optimization of methods for preliminary spatial resolution of received radiation at the output of AA and assess the degree of their mutual masking.

7. Conclusions

1. A high-speed digital spectral method of correlation-interferometric direction finding with reconstruction of a spatial analytical signal has been developed for conditions of processing an incomplete spatial spectrum of a signal under the influence of one or two masking noises. This is achieved by using the reconstruction of signal samples masked by noise. Analytical relations for estimating reconstructed signal samples at the output of the array were obtained. This ensures effective direction finding of RES using various types of weighting function $W(z)$ for synthesizing a multi-lobe radiation pattern, taking into account the power of masked interference. A combination of reconstruction of signal samples with subsequent single-iteration estimation

of the direction to RES is proposed, which ensures high accuracy and speed of the developed direction-finding method at a signal-to-interference ratio (-20 dB) without increasing the size of AA and the number of its receiving channels.

2. An analytical assessment of the noise immunity of the proposed method was carried out under conditions of exposure to powerful masking interference. Relationships for absolute and relative estimates of noise immunity were obtained. It is shown that the gain in noise immunity of the method with reconstruction compared to the known method without sample selection is more than 32 dB. The research results showed the effectiveness of the proposed method in complex EME.

3. Modeling of direction-finding algorithms was carried out using methods with reconstruction and selection of samples masked by interference. The dependence of the direction-finding error on the signal-to-noise ratio was studied at a signal-to-noise ratio of 0 dB. The use of methods with and without selection of samples produces an anomalously large direction-finding error of 0.42 degrees and 0.23 degrees, respectively. The method with reconstruction of readings provides an error of (0.02–0.06) degrees. The dependence of the direction finding of the proposed method on the signal-to-noise ratio was also studied. It is shown that the method with reconstruction of samples provides a significant gain in accuracy by 4–8 times compared to the selection method in the signal/interference range (-20 ; 10) dB. The simulation results confirmed the effectiveness of the method for re-

constructing signal samples masked by interference and are consistent with the results of analytical studies.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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