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CONSTRUCTION OF MATHEMATICAL MODEL OF PARTICLE MOVEMENT BY AN INCLINED SCREW ROTATING IN A FIXED CASING

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Screw conveyors are used to move bulk materials vertically upwards, horizontally, and at an angle to the horizon. The processes that take place when particles are moved by a screw conveyor in vertical and horizontal directions have been studied. There is a significant difference between them: for transportation in the vertical direction, the necessary conditions must be ensured (sufficient angular speed of rotation of the screw), and for horizontal transportation, the movement of the particle occurs at any angular speed of rotation of the screw. Therefore, when changing the inclination of the axis of the screw, there comes a moment when transportation becomes possible, while it was impossible in the vertical direction.

This paper considers the movement of a particle under the condition that it simultaneously contacts two surfaces: the moving surface of the screw and the stationary surface of the cylindrical casing in which the screw rotates. Their common line along which the particle slides is a helical line – the periphery of the screw. The particle slides along the helical line of the rotating screw, i.e., it is in relative motion with respect to it. At the same time, it slides along the surface of the casing, relative to which it is in absolute motion. The trajectory of the particle's absolute motion is its sliding track on the casing surface.

When constructing differential equations of the relative motion of particles, the forces applied to the particle were taken into account. The initial position was taken to be the vertical direction of the screw to transport the particle upwards. If an auger in a cylindrical casing is tilted from the vertical direction to a certain angle, then all applied forces (except the force of weight) will also tilt to this angle. On the basis of this, generalized differential equations of the relative motion of a particle during its transportation by an inclined screw were built. They made it possible to derive a generalized mathematical model of the movement of a particle by an inclined screw that rotates inside a fixed casing

Keywords: force applied to a particle, differential equations of motion, cylindrical casing, angle of inclination of the screw

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1. Introduction

The screw is a helical linear surface, which is formed by the helical movement of a rectilinear generatrix. During movement, the generatrix uniformly rotates around the axis and at the same time uniformly moves along it. The type of surface, which is also called a helicoid, depends on the position of this generatrix in relation to the axis. It can cross or not cross the axis, be perpendicular to it, or not. The most common in technology is a helicoid, in which the rectilinear generatrix intersects the axis at a right angle, that is, a screw, which is known in the scientific literature as a helical conoid.

The working surface of the screw is its compartment between two coaxial cylinders, and the inner cylinder can play the role of a shaft. The edges of the screw are helical lines of the same pitch, but with different angles of elevation. Of interest is the angle of elevation of the outer edge of the screw, that is, the helical line, which can be considered the line of intersection of the screw with the cylindrical casing. The axis of the casing screw can be vertical, inclined, or horizontal. The direction of material transportation depends on its position. The nature of the movement of an individual particle for different positions of the axis is different. This determines the relevance of the mathematical description of the generalized model of particle movement depending on the position of the axis.

2. Literature review and problem statement

The issue of increasing the wear resistance and durability of parts is being investigated by many scientists. Two parameters have a significant influence on these indicators: the quality of the coating of the working surfaces of machine parts, as well as the geometry of the contact surfaces. In addition, these indicators are closely interrelated. The quality of the surface treatment of the working surfaces of the parts affects their microgeometry [1].

In scientific studies, the existing and latest methods of surface treatment of parts are sufficiently and comprehensively covered, with a description of the quality characteristics of the coatings obtained with their help [2]. This approach to solving the issue of increasing wear resistance and durability of parts has a technological and structural nature. However, this issue can be solved with the help of geometrical methods. The basis of such methods is the construction of mathematical models that can be applied to simulations of various processes [3]. Thus, in [4], a refined mathematical model of a screw conveyor is presented. The basis of such a model is the coordination of the process of feeding the material to the conveyor and the geometric coefficient of filling its working space. The authors envisage the optimization of the parameters and operating modes of the conveyor, provided that it is installed at an angle to the horizon without taking into account other possible positions.

Article [5] examines the horizontal and inclined position of the screw, namely its angular parameter. This makes it possible to choose the speed of rotation of the casing in accordance with the conditions of transportation and the characteristics of the bulk cargo, while simultaneously increasing the efficiency of transportation. In addition, the cited work establishes an inversely proportional relationship between the angular parameter of the conveyor and the efficiency of the transportation process.

The results of research into the design parameters of screw conveyors for fluff cleaning are given in [6]. The purpose of the study was to ensure sufficient material cleaning efficiency with simultaneous high performance of the conveyor. However, its position was limited to horizontal. Similar studies of a horizontal screw conveyor for transporting and cleaning cotton are reported in [7]. The authors developed the design of the conveyor and substantiated its parameters.

The technical and economic rationale for increasing the productivity of inclined screw conveyors with rotary housings is presented in [8]. The authors propose rational parameters of the screw conveyor, which will ensure an increase in the overall economic indicator by more than 10 %. In addition, the optimal rotation frequency of the casing is set in the range of 460...620 rpm. However, the above studies are limited to the inclined position of the auger.

Since the transportation of loose materials using screw conveyors is widely used in various devices and mechanisms, there are different approaches in the mathematical models for describing the movement of material. One such approach is to move a single particle without taking into account its interaction with other particles. At the same time, the Fr net trihedron [9] or the Darboux trihedron [10] can be used to study the patterns of movement of particles on surfaces. With the help of Fr net trihedron, the complex motion of a point on a plane with a specified displacement of the latter was studied in [9]. Article [10] describes a procedure for determining the regularities of the formation of curves and surfaces using the Darboux trihedron.

Papers [11, 12] report the results of studies of particle transport by a vertical screw taking into account this limitation. Work [13] considered the transportation of grain products by a screw conveyor with a large inclination of its axis. The movement of material with a horizontal arrangement of the auger is considered in [14]. However, issues related to the transitional position of the screw axis: from horizontal to vertical remained unresolved. This allows us to state that it is appropriate to study the process of transporting a particle by an inclined screw rotating in a stationary casing when the angle of inclination of its axis is changed.

3. The aim and objectives of the study

The purpose of this work is to construct a generalized mathematical model of the movement of a particle by an inclined screw rotating in a stationary casing. This will make it possible to use it for any position of the screw: horizontal, vertical, or inclined.

To achieve the goal, the following tasks were set:

– to build a system of differential equations of particle motion for the general case of an inclined screw;

– to identify patterns of particle movement along the vertical axis of the screw;

– to investigate the regularities of particle movement along the horizontal axis of the screw;

– to find patterns of movement of the particle when the position of the screw axis changes from vertical to horizontal.

4. Materials and methods for studying the process of moving a particle by an inclined screw

The object of our research is the process of transporting loose material using an inclined screw that rotates inside a stationary cylindrical casing. At the same time, when constructing a mathematical model of the movement of loose material, a simplification was adopted, according to which the research was conducted for a single particle without taking into account its interaction with other particles. Such a model does not accurately reflect the real process of transportation; however, it allows us to identify certain regularities that can be transferred to the actual process. In addition, this practice of research is quite common, which indicates the legitimacy of this approach and its usefulness for practical application.

For research, the apparatus of differential geometry of surfaces, vector algebra, means of theoretical mechanics, and computer visualization of the results were used. Numerical integration of differential equations was carried out using the "Simulink" dynamic modeling system of the "MATLAB" environment, and mathematical transformations and simplifications were carried out using the "Mathematica" symbolic mathematics package. Verification of the adequacy of the model and the obtained results was carried out on known partial cases when the axis of the screw is located vertically or horizontally.

5. Results of investigating the process of particle movement by an inclined screw

5. 1. Construction of differential equations of particle motion for the general case of an inclined screw

The equation of motion of the particle was built according to the vector notation in the form $m\overline{a}=\overline{F}$, where m is the mass of the particle, $\overline{a}$ is the vector of its absolute acceleration, $\overline{F}$ is the resulting vector of forces applied to the particle. First, the forces applied to a particle located on the surface of a vertical screw were considered (Fig. 1, *a*). It simultaneously contacts the surface of the screw and the cylindrical casing. When the screw rotates, it is forced to slide along a helical line with an angle of elevation β – the periphery of the screw. Under the influence of gravity, it presses on the surface of the screw, and under the influence of centrifugal force – on the cylindrical casing. Accordingly, there are two reaction forces – N and N_c. The first is directed along the normal to the surface of the screw, the second – to the surface of the casing.

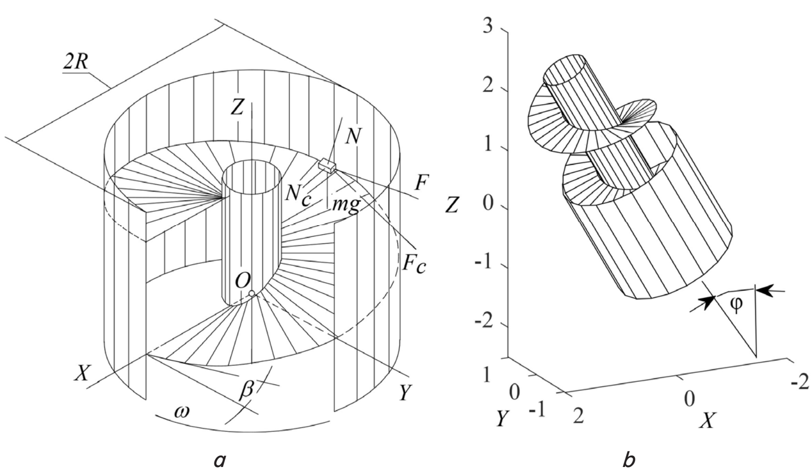


Fig. 1. An auger in a cylindrical casing: *a* — with a vertical axis and a diagram of the forces applied to the particle; *b* — with a rotation by an angle φ around the *OY* axis, the count of which starts from the *OZ* axis

As a result of the action of these forces, frictional forces arise when particles slide on surfaces. Each of them is the product of the surface reaction force and the corresponding coefficient of friction. For the screw $F=N{\cdot}f$, for the casing – $F_c=N_c{\cdot}f_c$, where f and f_c are the coefficients of particle friction on the surface of the screw and casing, respectively. These forces are tangential to the trajectories of particle sliding in the opposite direction from the direction of speed: force F – to the trajectory of sliding along the helical line of the screw, i.e., to the trajectory of relative motion, force F_c – to the trajectory of sliding along the surface of the cylindrical casing, i.e., to the trajectory of absolute motion. Both of these forces are located in the plane tangential to the cylindrical casing (Fig. 1, *a*). The weight of the particle is $m{\cdot}g$, where m is the mass of the particle, g=9.81 m/s^2 is the acceleration of free fall.

The process of lifting a particle occurs as follows. The screw rotates around its axis with a constant angular velocity ω (Fig. 1, *a*). If the particle was on the surface of the screw at the beginning of rotation, then under the action of centrifugal force it moves to its periphery and subsequently contacts both surfaces: the screw and the cylindrical casing. As a result of this interaction, it slides along both of them and rises up (or falls down if the angular velocity of rotation of the screw is insufficient). If the screw together with the casing is turned to an angle φ, then all the listed forces, except for the force of weight, must also be turned to this angle. This is explained by the fact that they are tied to the geometric parameters of surfaces and trajectories in addition to the downward force of gravity. With this in mind, the direction of action of the forces for the vertical screw can be determined, after which they, together with the screw and casing, must be turned by an angle φ.

The parametric equations of the helical line – the edges of the vertical screw take the following form:

$$x = R\cos\alpha;$$

$$y = R\sin\alpha;$$

$$z = R\alpha\,\mathrm{tg}\beta, \qquad (1)$$

where R is the radius of the cylindrical casing – a constant value;

β – the angle of rise of the helical line – a constant value;

α – an independent variable – the angle of rotation of the point of the helical line around its axis.

Through the helical line (1), the screw's rectilinear generatrixes pass in the direction of the axis perpendicular to it. Based on this formation of the screw, its parametric equations:

$$X = (R-u)\cos\alpha;$$

$$Y = (R-u)\sin\alpha;$$

$$Z = R\alpha\,\mathrm{tg}\beta, \qquad (2)$$

where u is the second independent variable of the surface – the length of the rectilinear generatrix of the screw. Its countdown starts from the spiral line (1). When $u=R-r$, where r is the radius of the screw shaft, we get the parametric equations of the helical line on the shaft, and when u=0 – the parametric equations of the helical line (1). To distinguish the equations of the surface from the equations of the line, it is customary to denote them with uppercase letters for the surface and lowercase letters for the line.

The particle is forced to slide on the two surfaces, remaining on the helical line (1), along which it also slides, since it belongs to the screw. Its position on this line depends on time t. Thus, $\alpha=\alpha(t)$, and under this condition, equations (1) become dependent on a new variable – time t, that is, they are equations of relative motion. The direction of rotation of the screw is chosen in such a way that the particle, sliding along the helical line, rises up in absolute motion. According to Fig. 1, a, the rotation is clockwise, that is, the rotation angle around the OZ axis is negative. Taking this into account, the rotation of the helical line (1) during time t by the angle $(-\omega \cdot t)$ according to the known rotation formulas is written:

$$x_a = R\cos\alpha\cos(-\omega t) - R\sin\alpha\sin(-\omega t) = = R\cos(\omega t-\alpha);$$

$$y_a = R\cos\alpha\sin(-\omega t) + R\sin\alpha\cos(-\omega t) = = -R\sin(\omega t-\alpha);$$

$$z_a = R\alpha\,\mathrm{tg}\beta. \tag{3}$$

Parametric equations (3) take into account the rotation of the helical line (1) around the OZ axis by the angle $(-\omega \cdot t)$ and by the angle $\alpha=\alpha(t)$ when the particle slides along the helical line in relative motion, i.e., they are the equations of the trajectory of the absolute motion of the particle. The dependence $\alpha=\alpha(t)$ is unknown, which must ultimately be found as a result of building the differential equations of motion. Equations (3) were differentiated with respect to time t, as a result of which absolute velocity projections were obtained:

$$x'_a = -R(\omega-\alpha')\sin(\omega t-\alpha);$$

$$y'_a = -R(\omega-\alpha')\cos(\omega t-\alpha);$$

$$z'_a = R\alpha'\,\mathrm{tg}\beta. \tag{4}$$

The projections of the absolute acceleration were obtained by differentiating the absolute velocity (4):

$$x''_a = R\alpha''\sin(\omega t-\alpha) - R(\omega-\alpha')^2\cos(\omega t-\alpha);$$

$$y''_a = R\alpha''\cos(\omega t-\alpha) + R(\omega-\alpha')^2\sin(\omega t-\alpha);$$

$$z''_a = R\alpha''\,\mathrm{tg}\beta. \tag{5}$$

The absolute trajectory (3) must be rotated about the OY axis by an angle of φ:

$$x_{a\phi} = z_a\sin\phi + x_a\cos\phi = = R\alpha\,\mathrm{tg}\beta\sin\phi + R\cos(\omega t-\alpha)\cos\phi;$$

$$y_{a\phi} = -R\sin(\omega t-\alpha);$$

$$z_{a\phi} = z_a\cos\phi - x_a\sin\phi = = R\alpha\,\mathrm{tg}\beta\cos\phi - R\cos(\omega t-\alpha)\sin\phi. \tag{6}$$

Similarly, it is necessary to turn the absolute acceleration vector at angle φ:

$$x''_{a\phi} = R\alpha''\left[\mathrm{tg}\beta\sin\phi + \sin(\omega t-\alpha)\cos\phi\right] - -R(\omega-\alpha')^2\cos(\omega t-\alpha)\cos\phi;$$

$$y''_{a\phi} = R\alpha''\cos(\omega t-\alpha) + R(\omega-\alpha')^2\sin(\omega t-\alpha);$$

$$z''_{a\phi} = R\alpha''\left[\mathrm{tg}\beta\cos\phi - \sin(\omega t-\alpha)\sin\phi\right] + +R(\omega-\alpha')^2\cos(\omega t-\alpha)\sin\phi. \tag{7}$$

Equations (7) are the projection on the coordinate axis of the vector $\bar{a}$ – the left-hand part of the vector equation $m\bar{a}=\bar{F}$. After that, we shall find the projections of the vectors of the applied forces, rotated by the angle φ (except for the weight force), which are included in the right-hand part of the vector equation. All these vectors for a vertical screw are given in work [1]. After turning by an angle φ, they take the following form:

– projections of the unit normal vector N to the surface of the screw:

$$N_x = \cos\beta\sin\phi - \sin\beta\cos\phi\sin(\omega t-\alpha);$$

$$N_y = -\sin\beta\cos(\omega t-\alpha);$$

$$N_z = \cos\beta\cos\phi + \sin\beta\sin\phi\sin(\omega t-\alpha); \tag{8}$$

– projections of the unit vector of the normal N_c to the casing surface:

$$N_{cx} = -\cos\phi\cos(\omega t-\alpha);$$

$$N_{cy} = \sin(\omega t-\alpha);$$

$$N_{cz} = \sin\phi\cos(\omega t-\alpha); \tag{9}$$

– projections of the unit vector V_r of the relative velocity of the particle:

$$V_{rx} = \sin\beta\sin\phi + \cos\beta\cos\phi\sin(\omega t-\alpha);$$

$$V_{ry} = \cos\beta\cos(\omega t-\alpha);$$

$$V_{rz} = \sin\beta\cos\phi - \cos\beta\sin\phi\sin(\omega t-\alpha); \tag{10}$$

– projections of the unit vector V_a of the particle's absolute velocity:

$$V_{ax} = \frac{\alpha'\sin\beta\sin\phi - (\omega-\alpha')\cos\beta\cos\phi\sin(\omega t-\alpha)}{\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}};$$

$$V_{ay} = -\frac{(\omega-\alpha')\cos\beta\cos(\omega t-\alpha)}{\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}};$$

$$V_{az} = \frac{\alpha'\sin\beta\cos\phi + (\omega-\alpha')\cos\beta\sin\phi\sin(\omega t-\alpha)}{\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}}; \tag{11}$$

– projections of the unit vector of the weight of the particle, the direction of which does not depend on the angle φ:

$$\{0;\ 0;\ -1\}. \tag{12}$$

The vector equation $m\bar{a}=\bar{F}$ must be written in projections on the axis of the $OXYZ$ coordinate system, taking into account that the friction force F is directed in the opposite direction of the vector V_r (10) of the relative velocity of the particle, and the friction force F_c is directed in the opposite direction of the vector V_a (11) of the absolute velocity of the particle:

$$mx''_{a\phi} = NN_x + N_cN_{cx} - fNV_{rx} - f_cN_cV_{ax};$$

$$my''_{a\phi} = NN_y + N_cN_{cy} - fNV_{ry} - f_cN_cV_{ay};$$

$$mz''_{a\phi} = NN_z + N_cN_{cz} - fNV_{rz} - f_cN_cV_{az} - mg. \qquad (13)$$

In equations (13), N and N_c are scalar quantities (magnitudes of reaction forces of the surface of the screw and casing, respectively), and the same symbols with additional axis indices are projections of unit vectors along which these forces are directed. If we substitute all the above expressions in (13), the result will be a nonlinear system of three differential equations with three unknown functions: $\alpha=\alpha(t)$, $N=N(t)$ i $N_c=N_c(t)$. Bringing this system to a form convenient for numerical integration gives the following result:

$$\alpha'' = \frac{g\cos\beta}{R}\begin{bmatrix}(\cos\beta - f\sin\beta)\sin\phi\sin(\omega t-\alpha)- \\ -(f\cos\beta+\sin\beta)\cos\phi\end{bmatrix}+$$
$$+\frac{f_c\cos\beta(\omega-2\alpha'+\omega\cos2\beta - f\omega\sin2\beta)}{2R\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}}\times$$
$$\times\left[R(\omega-\alpha')^2+g\sin\phi\cos(\omega t-\alpha)\right], \qquad (14)$$

$$N = mg\begin{bmatrix}\cos\beta\cos\phi+ \\ +\sin\beta\sin\phi\sin(\omega t-\alpha)\end{bmatrix}+$$
$$+\frac{mf_c\omega\sin\beta\cos\beta}{R\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}}\times$$
$$\times\left[R(\omega-\alpha')^2+g\sin\phi\cos(\omega t-\alpha)\right], \qquad (15)$$

$$N_c = mR(\omega-\alpha')^2 + mg\sin\phi\cos(\omega t-\alpha). \qquad (16)$$

The differential equation (14) can be solved independently using numerical methods. After finding the solution in the form of $\alpha=\alpha(t)$, it is possible to determine the reaction of the screw (15) and the cylindrical casing (16).

5. 2. Regularities of particle displacement at the vertical axis of the screw

At $\varphi=0$, the resulting equations (14) to (16) will describe the process of particle movement through a vertical screw. Equations (15) and (16) will not be taken into account since the differential equation (14) fully describes the motion of the particle. At $\varphi=0$, it takes a simplified form:

$$\alpha'' = f_R(\omega-\alpha')^2\times$$
$$\times\cos\beta\frac{\omega\cos\beta(\cos\beta - f\sin\beta)-\alpha'}{\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}}-$$
$$-\frac{g\cos\beta}{R}(f\cos\beta+\sin\beta), \qquad (17)$$

$$\alpha''=\alpha'=0.$$

It is obvious that the angular acceleration and speed of particle sliding α'', α' along the edge of the screw are influenced by constant values, including the value of the angular velocity ω of the screw rotation. If the angular velocity ω is insufficient, the particle will not be able to rise up and, on the contrary, will slide down. Therefore, there is a limiting value of the angular velocity ω between the rise and fall of the particle. It can be found theoretically. At the limit value of ω, the particle "sticks" on the helical line – the edge of the screw and rotates along with it. This means that there is no sliding along the helical line but there is sliding along the cylindrical casing, that is, there is no relative trajectory, and the absolute one is a circle on the cylinder. This happens under the condition that substituting these values into equation (17) and solving them with respect to ω leads to the result:

$$\omega_o = \sqrt{\frac{g}{f_cR}\cdot\frac{f\cos\beta+\sin\beta}{\cos\beta - f\sin\beta}}. \qquad (18)$$

For example, when $f=f_c=0.3$, $R=0.1$ m, $\beta=20°$, according to (18), we get: $\omega_o=15.6$ s^{-1}. As a result of numerical integration in Fig. 2, the relative (thick line) and absolute (thin line) trajectories of particle motion are plotted for $\omega<\omega_o$ (Fig. 2, *a*) and $\omega>\omega_o$ (Fig. 2, *b*).

After the transition period, the angular speed of rotation of the particle when it slides along the helical line becomes constant $(\alpha'=\text{const})$, which is shown in Fig. 3 for both cases (Fig. 2).

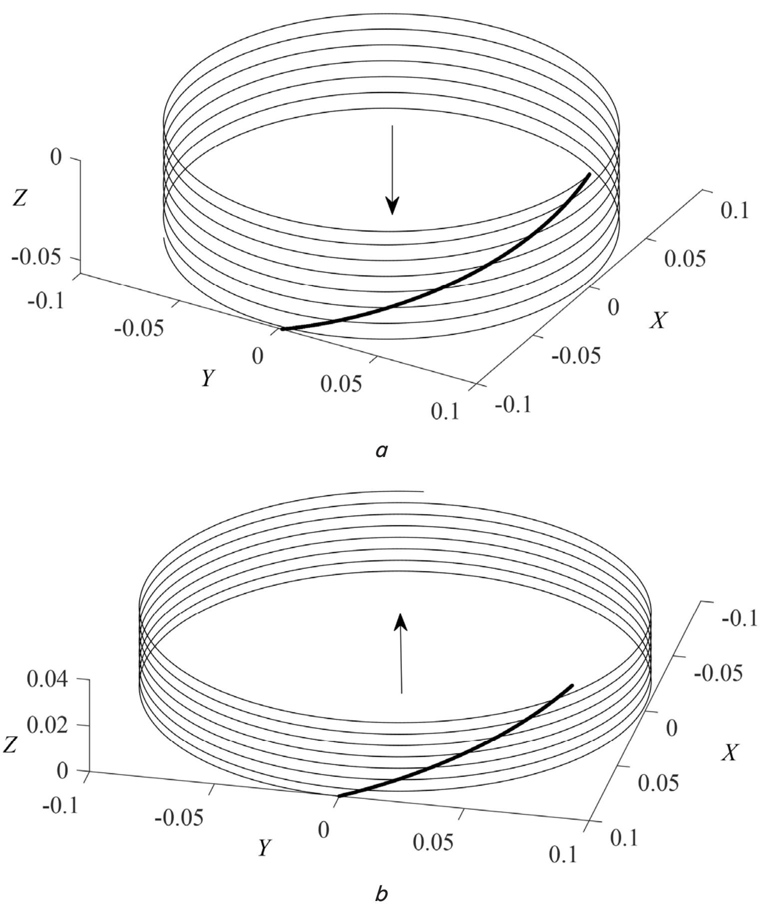


Fig. 2. Relative (thick line) and absolute (thin line) trajectories of particle motion at $f=f_c=0.3$, $R=0.1$ m, $\beta=20°$ for 3 s: *a* – $\omega=15$ s^{-1} (particle descends); *b* – $\omega=16$ s^{-1} (the particle rises)

Once the motion is stabilized, both trajectories (relative and absolute) become helical lines of a steady pitch.

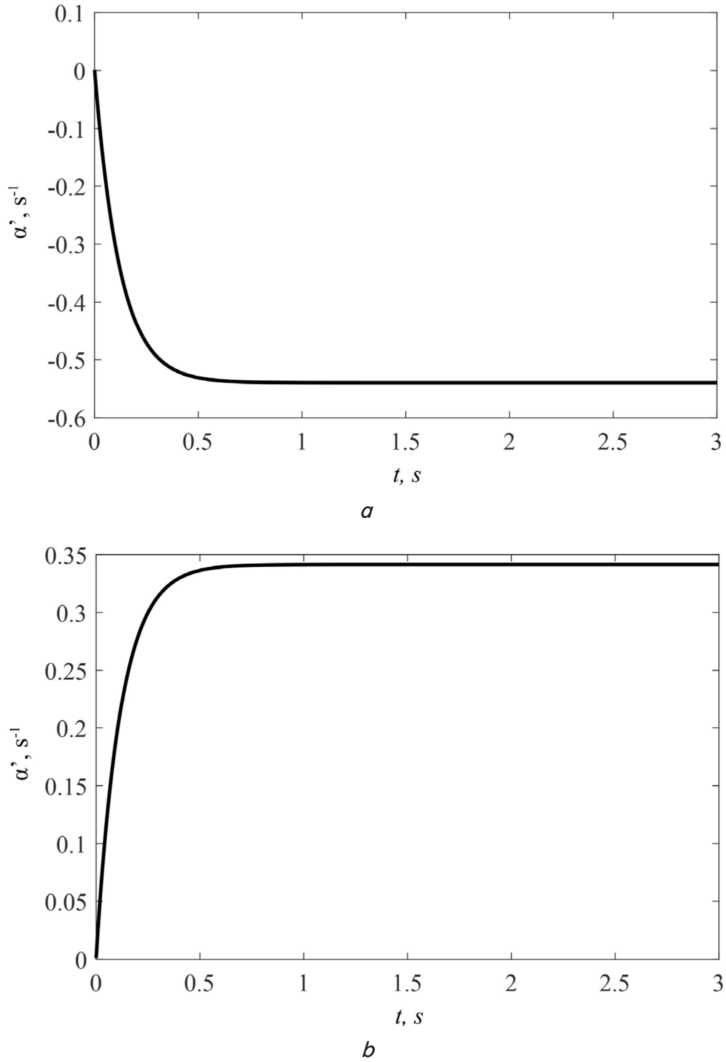


Fig. 3. Plots of changes in angular velocity α': *a* – when the particle is lowered; *b* – when the particle rises

5. 3. Regularities of particle displacement at the horizontal axis of the screw

At φ=90°, the resulting equations (14) to (16) will describe the process of moving a particle through a horizontal screw. As in the previous case, the solution of only one equation by numerical methods makes it possible to find the dependence $\alpha=\alpha(t)$:

$$\alpha''=\frac{g\cos\beta}{R}(\cos\beta-f\sin\beta)\sin(\omega t-\alpha)+$$
$$+\frac{f_R\cos\beta(\omega-2\alpha'+\omega\cos2\beta-f\omega\sin2\beta)}{2R\sqrt{\omega(\omega-2\alpha')\cos^2\beta+\alpha'^2}}\times$$
$$\times\left[R(\omega-\alpha')^2+g\cos(\omega t-\alpha)\right]. \qquad (19)$$

With a horizontal axis, the particle moves in the lower part of the casing at any angular speed of rotation of the screw. The relative trajectory is a spiral line, and the absolute trajectory is a line that, after stabilization, coincides with the rectilinear generatrix of the cylinder (Fig. 4, *a*).

In Fig. 4, *b*, an arrow shows the direction of movement of the particle, and the arrow coincides with the lower generatrix of the cylinder. It can be seen that the absolute trajectory over time approaches a straight line, which is deviated from the lower generatrix by a certain angle relative to the axis of rotation. From Fig. 4, *c*, it can be seen that this deviation after stabilizing the movement is 18°. At f_c=0, there will be no deviation and the particle will move along the lower generatrix.

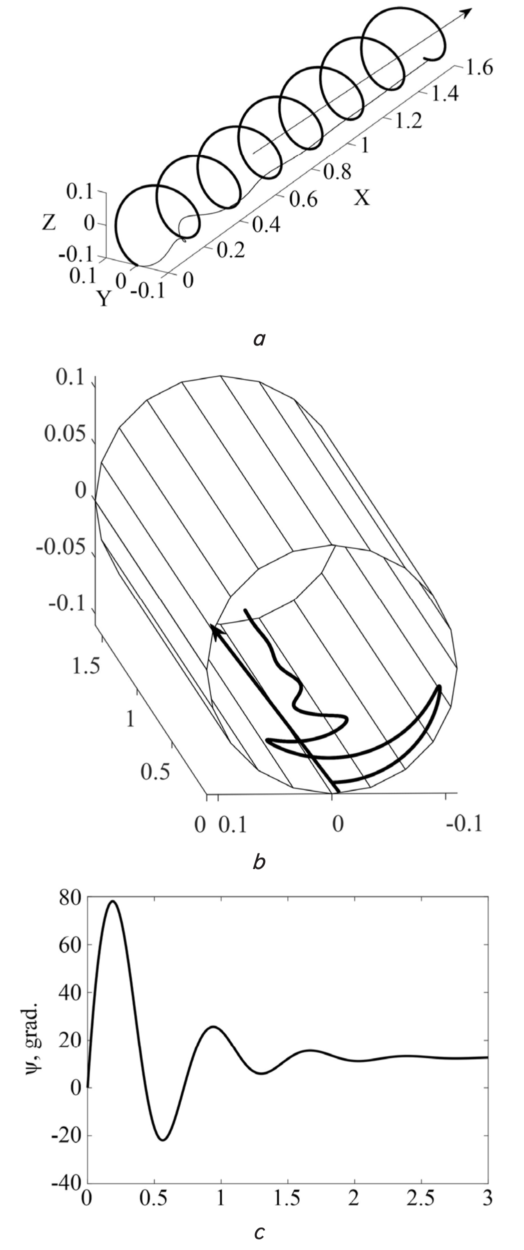
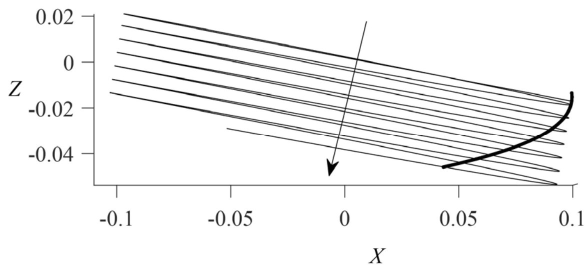


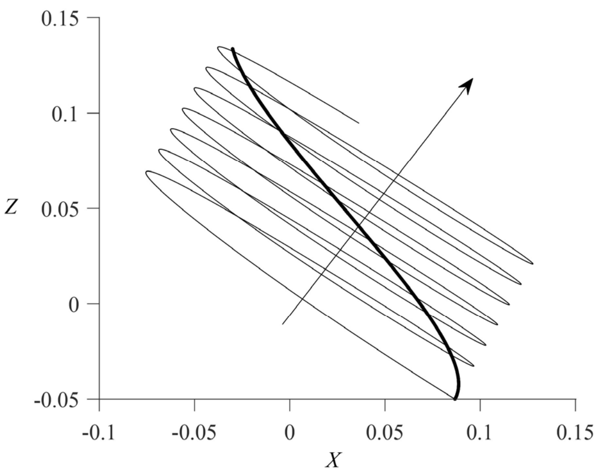
Fig. 4. Graphical illustrations of the movement of a particle with the horizontal axis of the screw location at β=20°, $f=f_c$=0.3, R=0.1 m, ω=15 s^{-1}: *a* – relative and absolute trajectories; *b* – absolute trajectory on the casing surface; *c* – plot of the deviation of the absolute trajectory from the lower generatrix of cylindrical casing in the angular dimension

5. 4. Movement of the particle by the screw in the process of changing the position of its axis

If we compare the movement of a single particle by vertical and horizontal screws, then there is a significant difference between these processes. First, in order to move the particle upwards, the vertical screw must have the proper angular velocity of its rotation, while there is no such requirement for the horizontal screw. Second, the absolute trajectory of a particle moving through a vertical screw is a helical line, while for a horizontal screw it is a straight line. It is obvious that these different processes must somehow transition into each other as the inclination angle φ changes from 0° (vertical auger) to 90° (horizontal auger). In Fig. 2, *a*, it is shown that at ω=15 s^{-1} the particle falls down. With the same angular speed and other unchanged parameters, we shall tilt the screw. Fig. 5 shows the relative and absolute trajectories of the particle movement for different angles φ of the screw inclination.



a

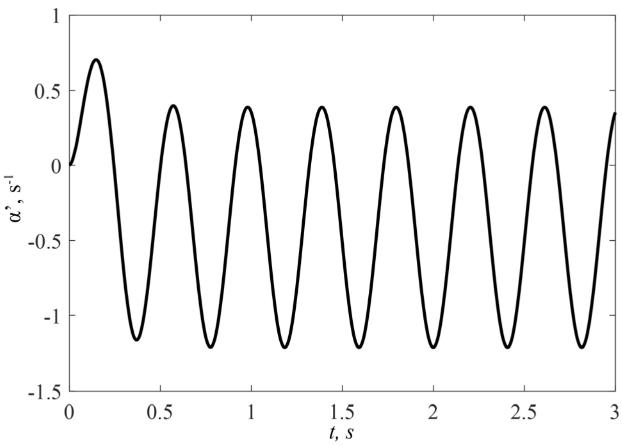


b

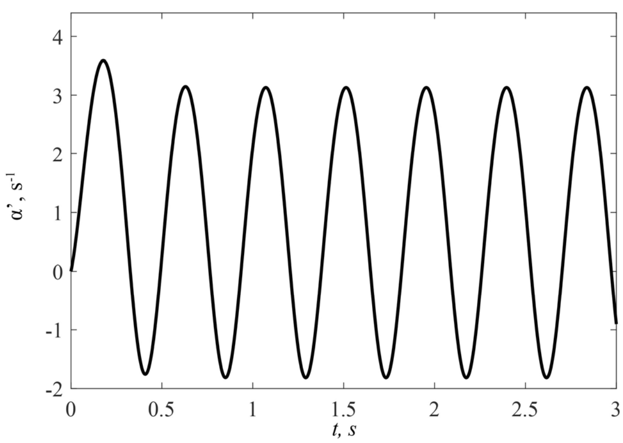
Fig. 5. Absolute (thin line) and relative (thick line) trajectories of particle movement at β=20°, $f=f_c=0.3$, $R=0.1$ m, ω=15 s^{-1} and different angles of inclination of the screw: *a* – angle of inclination φ=10°; *b* – angle of inclination φ=30°

In the first case (Fig. 5, *a*), the particle moves down, and in the second case (Fig. 5, *b*) – it rises up. Fig. 6 shows the built plots of the angular velocities of particle sliding along the periphery of the screw for these cases.

For these two cases, the characteristic difference in comparison with the vertical screw is the plots of angular velocity of the particle sliding. With a vertical auger, the angular speed of sliding after stabilization of the movement becomes constant both during the downward and upward movement of the particle (Fig. 3). This means that it slides along the surface of the screw with a constant speed down or up and becomes equal to zero when the angular speed becomes equal to ω_o according to (18). This does not happen with an inclined auger. The angular speed of particle sliding changes its direction, that is, it oscillates on the surface of the screw in opposite directions (Fig. 6). When the particle moves down, these oscillations occur in such a way that the negative part prevails (Fig. 6, *a*), and when the particle moves up, the opposite occurs (Fig. 6, *b*). It is obvious that there is such an angle of inclination φ at which the particle does not move in either direction in the direction of the axis. It is impossible to find it analytically precisely because of these fluctuations. It was found by visualizing the trajectories by selecting the appropriate value of φ between φ=10° and φ=30°. This value corresponds to φ=19.5°. Then the absolute trajectory is practically a circle on the surface of the casing, and the relative trajectory is a small segment of the helical line on which the particle performs the same oscillations in opposite directions (Fig. 7, *a*). Fig. 7, *b* shows the plot of fluctuations of the angular speed of sliding, from which it can be seen that the positive and negative parts are equal.



a



b

Fig. 6. Plots of changes in the angular velocity of particle sliding during 3 s at β=20°, $f=f_c=0.3$, $R=0.1$ m, ω=15 s^{-1} and different angles of inclination of the screw: *a* – angle of inclination φ=10°; *b* – angle of inclination φ=30°

In the same way, owing to the visualization of the trajectories, it is possible to find the value of angle φ at which the absolute trajectory goes from a helical line to a straight line. Studies have shown that for the previously indicated parameters of the screw, friction coefficients, and angular speed of rotation of the screw, this occurs at φ=57.5°. Fig. 8 shows trajectories at angles φ, the values of which differ little from φ=57.5° but are on different sides from it. It can be seen from this figure that there is not only a qualitative change in the shape of the absolute trajectory but also the speed of transportation. Fig. 8 corresponds to the process that took place within 3 seconds. When the absolute trajectory transitions to a straight line (Fig. 8, *b*), the distance covered by the particle when moving up the rise indicated by the arrow has more than doubled.

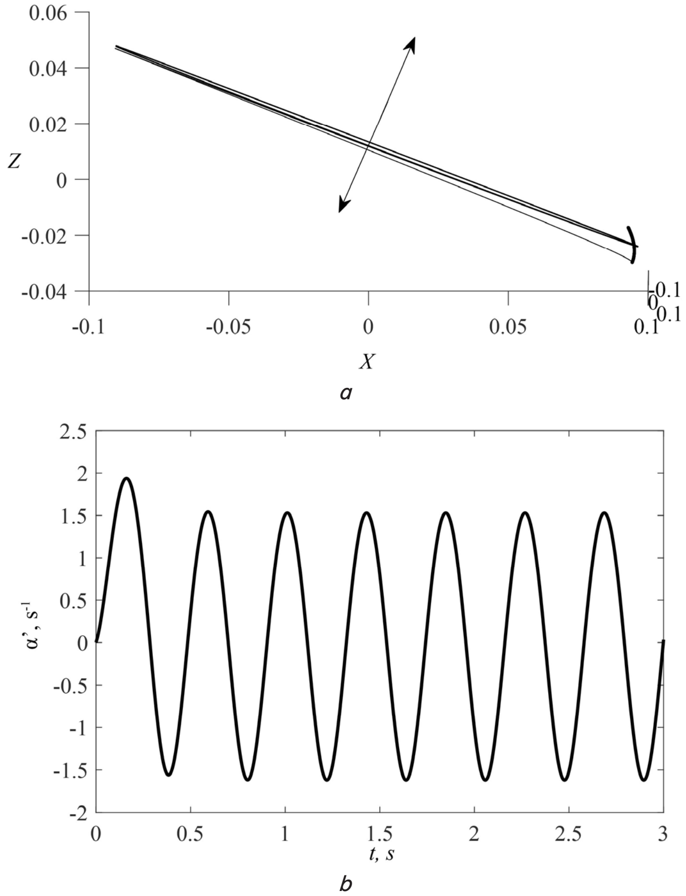


Fig. 7. Graphic illustrations for the case when the particle does not move along the axis and performs small oscillations at β=20°, f=f_c=0.3, R=0.1 m, ω=15 s^{-1} and the angle of inclination of the screw φ=19.5°: *a* – absolute and relative trajectories; *b* – plot of change in the angular velocity of the particle sliding

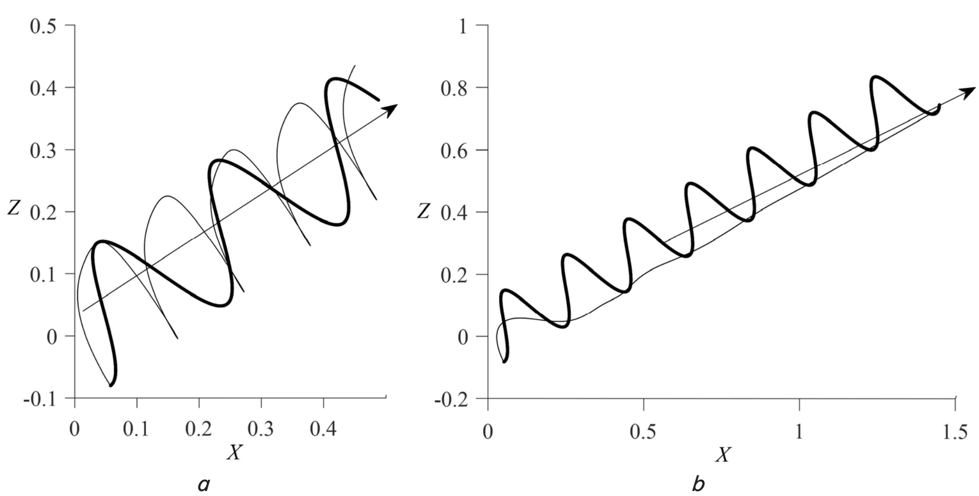


Fig. 8. Absolute (thin line) and relative (thick line) trajectories of particle movement at β=20°, f=f_c=0.3, R=0.1 m, ω=15 s^{-1} and different angles of inclination of the screw: *a* – angle of inclination φ=55°; *b* – angle of inclination φ=60°

It should be noted that other factors influence the process of particle movement. In particular, a decrease in the coefficient of friction f on the surface of the screw and an increase in the coefficient of friction f_c on the surface of the casing have a positive effect on its upward transportation. On the other hand, at f_c=0, i.e., at a completely smooth surface of the casing, the upward transport of the particle becomes impossible at all values of the angle φ, except for the angle φ=90°, when the transport occurs in the horizontal direction.

6. Discussion of results of investigating the movement of a particle by a screw

Our solutions close the problematic part of the research regarding the transportation of a particle by a screw that rotates in a stationary casing. After all, this process has been studied in detail for two positions of the screw with a vertical and horizontal position of its axis. The rest of the processes with different angles of its inclination required research, which was done as a result of building a generalized model of particle movement. An unsolved question was the issue of analytical description of the process of transporting a particle by an inclined screw. This process is radically different from the processes for vertical and horizontal screws. However, as the screw axis tilts from a horizontal to a vertical position, the particle transport process must somehow change. In the course of the research, the main difference in particle transport by an inclined screw was clarified. When a particle is transported by a horizontal or vertical screw, the process stabilizes over time and the particle slides along the surface of the screw with a constant angular velocity in both cases. At the same time, it rises up along the vertical axis of the screw and the absolute trajectory of its movement is a helical line (Fig. 2). At the same time, the angular speed of particle sliding becomes constant after the transition period (Fig. 3). With the horizontal axis of the screw, the absolute trajectory after the transition period is a straight line (Fig. 4). In both cases, the angular velocity of the particle sliding along the surface of the screw is constant. For an inclined screw, stabilization of the angular speed of particle sliding on the surface of the screw does not occur. It can either rise up or fall down, and at the same time the angular speed of sliding has an oscillatory character. In one case, there is a larger component in the oscillations that causes the particle to rise up, in the other – to fall down. The rise or fall of the particle depends on the angle of inclination of the screw, with other parameters unchanged. It is possible to choose such an angle of inclination when the particle does not fall and does not rise but performs an oscillating movement, while the components of the oscillations of the angular speed of the particle slide become the same in one and the opposite direction. The transition from a helical line to a straight line in absolute motion also occurs at a certain value of the angle of inclination of the screw.

The disadvantage of the model built is the lack of consideration of the interaction of particles among themselves.

The further direction of research is to consider the influence of structural and technological parameters on the nature of the movement of the particle (friction coefficients on the screw and on the casing, angular speed of rotation of the screw, diameter of the casing).

7. Conclusions

1. A generalized mathematical model of the movement of a particle by an inclined screw rotating inside a fixed casing was constructed, and a system of corresponding differential equations of its motion was built. A screw with a vertical axis was taken as a basis. All the forces applied to the particle, except the force of gravity, are bound to the surfaces of the screw and casing, as well as to the relative and absolute trajectories, and turn with them when they deviate from the vertical direction. Taking this into account, a system of equations of motion of the particle in projections on the axis of the fixed coordinate system was built. Partial cases can be obtained from it: at a zero value of the angle of inclination, the system of equations describes the movement of the particle when the axis of the screw is vertical, and at a value of $90°$ – when it is horizontal.

2. The regularity of the particle lift with the vertical location of the screw axis has been clarified. The critical value of the angular velocity of rotation of the screw was found, at which the lifting of the particle becomes impossible for the given design parameters of the surfaces of the screw and casing and coefficients of friction. After stabilization of the movement, the particle sliding trajectories on the surface of the screw and on the surface of the casing are helical lines of different constant steps. At the same time, the angular speed of rotation of the particle around the axis of the screw during its sliding is constant. If the surface of the casing is completely smooth, then the lifting of the particle becomes impossible.

3. The regularity of the movement of the particle when the axis of the screw is horizontal is clarified. It occurs regardless of the value of the angular speed of rotation of the screw. After the motion is stabilized, the trajectories of particle sliding on the surface of the screw are a helical line of constant pitch, and on the surface of the cylindrical casing – a straight line coinciding with its generating line. At the same time, the deviation of this generatrix from the lowest generatrix of the cylindrical casing was found. If the surface of the casing is absolutely smooth, then the movement of the particle takes place along its lowest generatrix. As a result of the research, the transient processes of the movement of the particle during the gradual deviation of the axis of the screw from the vertical position to its horizontal position were clarified.

4. It is shown that the nature of the movement of a particle through an inclined screw is significantly different from its movement through both a vertical and a horizontal screw. For horizontal and vertical screws, after the transition period, the movement of the particle stabilizes, and it moves along a fixed trajectory. This trajectory is a helical line in the vertical direction or a straight line in the horizontal direction. Such stabilization does not occur for an inclined screw. With an inclined screw, the movement of the particle can occur both up and down, depending on the angle of inclination of the axis of the screw. However, in both cases, the angular speed of particle sliding changes its direction, that is, it oscillates on the surface of the screw in opposite directions in such a way that the absolute movement occurs in one direction or in the opposite direction of the axis of the screw. It is possible to find such an angle of inclination of the screw when the particle does not move in either direction in the direction of the axis, but even at the same time it oscillates on the surface of the screw in opposite directions.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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