This paper proposes an improved method for diagnosing errors of computer devices, which process economic data in computer systems, by increasing efficiency. The object of research is the processes of control and diagnosis of data errors, in the class of residuals (CR). The improved method of diagnosing economic data represented in the non-positional system of calculation in CR is based on the application of orthogonal bases of partial sets of bases. The use of these bases makes it possible to organize the process of parallel processing of the projections of the output number of the non-positional code structure. This makes it possible to increase the efficiency of data diagnostics by n times, depending on the length of the bit grid of computer systems. For a single-byte computer system, the efficiency of data diagnosis increases by 1.2 times compared to the existing method based on the zeroing principle. At the same time, the effectiveness of operational control and diagnostics systems for data processing in CR with the growth of bit grids has been proven.

Based on the results, it is shown that, unlike the correction codes used in the positional counting system, the arithmetic codes in CR have additional correction capabilities. An example of a specific implementation of the process of applying operational control and diagnosing errors in the processing of economic data presented in CR is given. The improved diagnostic method will make it possible (in comparison with existing diagnostic methods) to reduce the time, which increases the efficiency of the diagnostic procedure of data operating in CR. The results give grounds for asserting that, based on the improved method and the developed algorithm for the implementation of data diagnostics, it is possible to synthesize a device for reliable and operational control and diagnostics of economic data operating in CR

Keywords: error diagnosis, processing of economic data, class of residuals, non-positional code structure

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IMPROVING THE EFFICIENCY OF DIAGNOSING ERRORS IN COMPUTER DEVICES FOR PROCESSING ECONOMIC DATA FUNCTIONING IN THE CLASS OF RESIDUALS

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1. Introduction

Development trends of the world economy contribute to the irreversible process of digitization of this sector. Of course, the development and competitiveness of a country is possible only if the latest information technologies of data processing and promising forms of information management mechanisms in the economic sector are combined [1–3]. At the same time, under the conditions of deepening digitalization of the economy, along with traditional threats to the security of economic data (theft, disclosure, destruction, etc.), a number of additional threats have emerged. Countermeasures against existing threats are based on methods of data control and diagnostics, which to a large extent do not fulfill their functions [4].

The evolution of information technologies is based on the development and implementation of digital technologies for data processing and transmission over communication networks. This makes it possible to combine secondary networks of economic structures into a digital network of integrated services, where information (language, data, images, etc.) is transmitted in a single digital form [5]. The further promising way of improving the process of digitization of the economy will be aimed at globalization and provision of multi-service (providing a large number of new additional services) communication. Solving the above-described prospects is not possible without increasing the efficiency of the functioning of computer economic data processing devices (CEDPDs) of computer systems (CSs). After all, the intensification of the processes of digitalization of economic activity created the prerequisites for the growth of risks and threats to the integral, fault-tolerant, protected circulation of information resources, which leads to huge financial losses at the level of business entities and the state [6, 7]. Accordingly, there is a need for reliable processing and protection of economic data. It is expected to be solved primarily by increasing the authenticity and reliability of processing large amounts of information in a short time, which are growing exponentially in the economic sector, as well as by intelligent networks [8].

Based on the above, one of the main effective ways to achieve high efficiency of the operation of CEDPDs is to improve their main characteristics. Namely, the productivity and reliability of processing large arrays of information in real time, as well as the reliability of their functioning. The development of high-performance and reliable real-time CEDPDs belongs to the category of strategically important and urgent problems of scientific and technical development of prospective CSs, which confirms the relevance and importance of research in this area [9].

It should be noted that along with the widely used methods for increasing the speed and reliability of CS CEDPDs, which function in the usual positional calculation system, great prospects are opened due to the development and implementation of new ones. Namely, non-traditional methods of representing and processing data in a non-positional numbering system (NNS). In particular, options for data coding based on mathematical methods are considered, arising from a special section of mathematics – number theory, as well as the Chinese theorem on residuals within the so-called class of residuals (CR), which is the NNS.

One way to increase the efficiency of the work of CS is to increase the speed and reliability of CEDPD due to the transition to data processing in non-traditional machine arithmetic. There is a growing interest in the use of CR as an alternative calculation system in related fields of science and technology. One of these areas is the economy, where along with fast information processing, there is also an increase in the reliability of the result of solving computational problems. Therefore, the issue of developing and improving tools and methods for controlling and diagnosing errors of CS CEDPD by increasing the efficiency of data processing is important and urgent. On the basis of the developed methods, reduced to algorithms, in practice it is possible to create a class of patent-eligible devices for the implementation of the process of operational control and diagnosis of data in CEDPD.

2. Literature review and problem statement

The issues of control and error diagnosis, as one of the tools for protecting economic data, are actively studied by scientists all over the world. For example, work [10] considers the issue of forming a preventive mechanism to prevent threats to economic data in cyberspace. The model for increasing the level of information protection developed in the study is based on five components: strategy, standardization of processes, compliance with requirements, control, and resource availability. But the authors in no way take into account the possibility of using interference-resistant data coding, at the expense of alternative counting systems, such as NNS in CR.

Article [11] pays attention to the protection of information resources of enterprises from DDoS attacks as the main threat to economic data. The importance of the protection of economic data is justified in the work, based on analytical data on direct and indirect losses of economic entities in the event of the implementation of risks and threats to the functioning of information and communication technologies. At the same time, the main drawback is that only methods and means of protection based on the existing positional binary numbering system are considered. After all, existing methods of unauthorized access, hacker attacks, viruses, and other types of hacking and violation of the integrity of information are built using binary positional code. This circumstance reduces the level of data security.

The review of the literature [10–13] gives a significant insight into various methods of data diagnostics of computer data processing devices. However, the application of non-positional counting systems, especially those based on the Chinese Remainder Theorem, as a data diagnosis mechanism for computer data processing devices is a relatively unexplored area. In article [12], a discrete data controller is proposed, which guarantees an asymptotically stable resulting closed system. The paper also presents simulation examples to demonstrate the effectiveness of the proposed controller and to validate the results but does not consider the possibility of data encoding based on NNS, which could improve the processing and control of discrete data.

Article [13] investigates the synchronization of sampling data of switching neural networks with time-varying discrete and distributed delays. Given that system switching can occur during the sampling interval, the phenomenon of asynchrony is taken into account for discrete data management. Using several Lyapunov functions and the average delay time property, sufficient conditions are derived that guarantee asymptotic (exponential) stability of the error between the master and the slave neural networks. Based on this, the discrete data controller is additionally designed in such a way that the master and slave systems are synchronized with a given performance index. But there are results of research on the effectiveness of using NNS in the field of neural networks, which were not taken into account in the article.

A review of the literature [10–13] revealed a gap in existing knowledge about the use of NNS, namely CR in the methods of data diagnostics of computer data processing devices. Despite the popularity of the topic of the process of data diagnostics of computer data processing devices based on the application of positional counting systems, the potential of CR application remains quite unexplored.

There is a large arsenal of scientific publications in the field of NNS application as an alternative method for increasing the efficiency and reliability of data processing in CS:

– article [14] proposes a new method of monitoring data in a non-positional system of residue classes. For the code in the system of residual classes, the test bases are included in the general code data structure containing a set of information bases. At the same time, the balances, which represent operations on information and control bases, simultaneously and independently participate in the process of processing information. The result of information processing can be controlled either step by step or after all calculations have been completed since the error that occurred in some residue does not spread (does not «propagate») to other residues;

– in article [15], a method of increasing the reliability of control of the data submitted in CR is proposed. Procedures for obtaining a positional feature of a non-positional code, forming a single-line code, which is the basis of the proposed method of control of CR data, are considered. The method of data control in CR is presented and described. An example of the implementation of the control method for a specific CR is considered. Since the developed method of operational control of data in CR and the device for its implementation have a very low reliability of control, a method of increasing the reliability of control is proposed, based on the known method of operational control of information in CR. The results of calculations and comparative analysis of the reliability of data control in CR showed that with the growth of the bit grid of processed data, the efficiency of non-positional coding in CR increases significantly;

– paper [16] analyzes the best-known practical methods of determining the alternative set of numbers (AS) in NNS in the system of residual classes (SRC). Definition of AS is most often required for error checking, diagnosis, and correction of data in SRC, which was introduced with minimal information redundancy in the dynamics of the computational process. This indicates the occurrence of only one error in the number. The main disadvantage of the considered methods is the considerable time required to determine AC;

– in article [17], the error correction properties of redundant SRCs are investigated using a more natural approach than was previously known. A necessary and sufficient condition for the correction of this error concerning one remaining digit of any valid SRC number is determined. The minimal redundancy that allows correcting the entire class of errors with one residue is derived, and an efficient error correction procedure is given. In addition, it is shown that lower redundancy and one redundant module can allow the correction of some important subclasses of single-residue errors, such as the set of errors that distort a single bit in the code;

– in paper [18] it is shown that the use of parallel computing in the field of digital signal processing is connected with the continuous growth of requirements for the productivity of computing devices. The use of non-positional modular codes is due to the fact that these algebraic systems have the property of parallelism. These codes provide real-time arithmetic operations, including addition, subtraction, and multiplication. This is due to the fact that the information is processed independently by computer channels. In addition to increasing the speed of data processing, non-positional modular codes are able to detect and correct errors that may occur during the operation of special processors. To perform error correction procedures, the article suggests using the projection system expansion operation.

Study [19] reports a new family of low-density parity check (LDPC) codes with irregular re-accumulation (IRA) with a characteristic random structure, a common feature of practical LDPC codes adopted in most industrial communication standards. The authors introduce the notion of a residue class to create a pair of residue classes from which a compact representation of the matrix H^{d} is constructed. The advantages of the family of developed IRA-LDPC codes based on RCP are shown. Namely, the ability to provide better performance, higher structure level, and less memory consumption than practical LDPC codes in some communication standards such as IEEE 802.16e and DVB-SII.

However, with the proven fact of the effective application of CR to create high-performance and reliable CS data processing, there are still unsolved problems, namely data control and diagnostics. The time of implementation of methods and algorithms of control, diagnosis, and correction of data errors (relative to the time of information processing) is very significant in terms of execution time, which reduces the overall efficiency of using non-positional code structures (NCS). This is due to the fact that during the operation of control, diagnosis, and correction of data in CR, they are treated as non-positional operations, that is, the most difficult (high time and hardware costs) implemented in this NNS [20]. At the same time, increasing the efficiency of control in CR is accompanied by a decrease in the reliability of obtaining the true result of data processing. The lack of formulation and results of solving this problem restrains the wide potential possibilities of fast data processing inherent in the properties of CR. This circumstance creates a contradiction between the high speed of implementation of integer arithmetic operations and the low efficiency of data control and diagnostics. The results of such studies will significantly expand the area of effective application of CR.

3. The aim and objectives of the study

The purpose of the study is to increase the reliability and efficiency of control and diagnosis of errors in the processing of economic data in CSs operating in CR. This will make it possible in practice to create highly operational and highly reliable computer devices for processing economic data of computer systems that function in the non-positional system of calculation in CR.

To achieve the goal, the following tasks were set:

– to investigate the methods of diagnosis and correction of data errors in CR;

– to investigate the process of data control and diagnostics in CR;

– to improve the method of operative diagnosis of errors of CS CEDPD operating in CR.

4. The study materials and methods

The object of research is the processes of control and diagnosis of data errors represented in the class of residuals.

The subject of research is methods and means of operational control and diagnostics of economic data of computer system components functioning in the class of residuals.

The main hypothesis of the study assumes that high indicators of insecurity, low efficiency of processing, control and diagnostics of large sets of economic data reduce economic growth in general.

The principles of system analysis, the theory of numbers, the theory of computing processes and systems, as well as the theory of coding in CR were the basis of the research carried out in our work. When solving the research problems, the basics of building computer data processing devices, the theory of interference-resistant coding in the non-positional counting system in CR, as well as the sections on the theory of divisibility and the theory of comparisons of number theory were used.

The study is based on the application of the principles of data comparison and methods of increasing the efficiency (speed) of control and diagnostics based on the use of non-traditional machine arithmetic in CR. A set of properties of the non-positional system in CR was used, namely: independence, equality and low-bitness of the residuals. The use of these properties determines the data structure of the non-positional CR code, which ensures high efficiency of the implementation of computational algorithms consisting of a set of arithmetic (modular) operations in CEDPD CS.

5. Results of investigating the improved method of operative diagnosis of economic data errors

5. 1. Study of methods of diagnosis and correction of data errors in the class of residuals

In general, the process of correction (detection and correction) of errors in the informational code structure $\tilde{P}$ of data [21], represented in CR, consists of the following main stages:

– data control (the process of detecting the fact of the presence of an error in a non-positional code structure

$\tilde{P}=\left(p_1 \| p_2 \| \ldots \| p_{i-1} \| \tilde{p}_i \| p_{i+1} \| \ldots \| p_n \| p_{n+1}\right)$, where p_k $\left(k=\overline{1,n+1}\right)$ is the remainder by arbitrary modulo q_k of the number P represented in CR; $\tilde{p}_i$ distorted remainders of the number P forming an incorrect number $\tilde{P}$; $\|$ – concatenation operation (gluing operation, joining operation);

– data diagnostics (localization of the location of errors with a given depth of diagnostics) [22];

– correction of errors in the code structure of the data (restoration of distorted residues $\tilde{p}_j$ $\left(j=\overline{1,n}\right)$ of an incorrect number $\tilde{P}$ and obtaining the correct number P).

A number of scientific statements have been studied, the results of the proofs of which can be used as a basis for methods of control and diagnosis of data errors represented in the non-positional counting system in CR. Within the scope of the study, only a one-time error is assumed (in one residue p_i $\left(i=\overline{1,n+1}\right)$, of the number $P=(p_1||p_2||\ldots||p_{i-1}||p_i||p_{i+1}||\ldots||p_n||p_{n+1})$, represented in CR).

Statement 1. Let an ordered $\left(q_i<q_{i+1},\ i=\overline{1,n}\right)$ system of CR bases (modules) with n informative and one control bases $q_k=q_{n+1}$ be given, and let the number $P=(p_1\|p_2\|\ldots\|p_{i-1}\|p_i\|p_{i+1}\|\ldots\|p_n\|p_{n+1})$ is undistorted (correct), i.e. $P<G_0/q_{n+1}$, where $G_0=G\cdot q_{n+1}$, $G=\prod_{i=1}^{n} q_i$. Then the value of P will not change if this number is represented in a CR from which one base q_i is removed, i.e. if the remainder p_i is removed from the representation of the number P. The number $P_i=(p_1||p_2||\ldots||p_{i-1}||p_{i+1}||\ldots||p_n||p_{n+1})$ obtained in this way will be called the projection of the number P modulo q_i.

Statement 2. If the correct number P is specified in the ordered system of CR bases, then the projections P_i $\left(i=\overline{1,n+1}\right)$ of this number are equal to each other, i.e., $P=P_1=P_2=\ldots=P_i=\ldots=P_n=P_{n+1}<G_0/q_{n+1}$.

Indeed, for a real number P, the relation $P<G_0/q_{n+1}<$ $<G_0/q_k<\ldots<G_0/q_i<G_0/q_1$. holds According to the results of statement 2, we have $P=P_i$.

Statement 3. Let for an ordered system of CR bases $\left(q_i<q_{i+1},\ i=\overline{1,n}\right)$ all possible projections $P_i=(p_1||p_2||\ldots||p_{i-1}||p_{i+1}||\ldots||p_n||p_{n+1})$, P_i $\left(i=\overline{1,n+1}\right)$ of number $P=(p_1||p_2||\ldots||p_{i-1}||p_i||p_{i+1}||\ldots||p_n||p_{n+1})$ coincide. In this case, the number is correct.

Let the number $P=(p_1||p_2||\ldots||p_{i-1}||p_i||p_{i+1}||\ldots||p_n||p_{n+1})$ be incorrect due to the distortion of the remainder p_i modulo q_i. Let's replace the distorted remainder p_i in the number P with the correct one $\tilde{p}_i$. In this case, the correct number $\tilde{P}=\left(p_1 \| p_2 \| \ldots \| p_{i-1} \| \tilde{p}_i \| p_{i+1} \| \ldots \| p_n \| p_{n+1}\right)$. is obtained. Then, according to the result of Statement 2, we have $\tilde{P}_1=\tilde{P}_2=\ldots=\tilde{P}_i=\ldots=\tilde{P}_n=\tilde{P}_{n+1}$.

On the other hand, $P_i=(p_1||p_2||\ldots||p_{i-1}||p_{i+1}||\ldots||p_n||p_{n+1})$, and at the same time $\tilde{P}_i=\left(p_1 \| p_2 \| \ldots \| p_{i-1} \| p_{i+1} \| \ldots \| p_n \| p_{n+1}\right)$, i.e. $P_i=\tilde{P}_i$. In this case, the relation $P=P_1=\tilde{P}_1=P_2=\tilde{P}_2=\ldots$ $=P_n=\tilde{P}_n=P_{n+1}=\tilde{P}_{n+1}$ must be fulfilled. However, under the condition of Statement 3, the projection P_j $(i\neq j)$ of the number P differs from the projection P_i by the value of the remainder p_i modulo q_i. As a result, $P_i\neq P_j$, which contradicts the condition that the number P is correct.

Statement 4. If in the ordered system of CR bases the projection $P_i=(p_1||p_2||\ldots||p_{i-1}||p_{i+1}||\ldots||p_n||p_{n+1})$ of the number $P=(p_1||p_2||\ldots||p_{i-1}||p_i||p_{i+1}||\ldots||p_n||p_{n+1})$ satisfies the condition $P_i\geq G_0/q_{n+1}$, then in this case it is considered that the remainder p_i of P modulo q_i is not distorted. We note once again that only a one-time error is assumed.

Indeed, if the remainder p_i of the number P modulo q_i is distorted, then the projection of P_i, composed of the undistorted remainders $p_j\left(j=\overline{1,n+1},\ i\neq j\right)$, must be a correct number. However, according to the condition $P_i\geq G_0/q_{n+1}$, it is an incorrect number, which contradicts the condition of statement 4. In addition, note that if all values $P_i\geq G_0/q_{n+1}$ $\left(i=\overline{1,n}\right)$, are present, then the false (distorted) remainder will be p_{n+1}.

Statement 5. Let the ordered system of CR bases $\left(q_i<q_{i+1},\ i=\overline{1,n}\right)$ with n informative and one control bases $q_k=q_{n+1}$ be the number $P=(p_1||p_2||\ldots||p_{i-1}||p_i||p_{i+1}||\ldots||p_n||p_{n+1})$ satisfies the following condition $G=G_0/q_{n+1}=G_{n+1}<P<G_i$, where $G=\prod_{i=1}^{n} q_i$, $G_0=G\cdot q_{n+1}$, $G_i=\prod_{\substack{k=1;\\ k\neq i.}}^{n+1} q_k$. Then the remainders $p_z\left(z=\overline{1,i}\right)$ of the number P are not distorted (correct), if only a single error is possible (in one remainder p_i).

First, by the method of the opposite, we establish that the remainder p_i is correct. Let the remainder p_i be distorted and the remainder $\tilde{p}_i$ be correct. In this case, the correct number $\tilde{P}$ can be represented in the form $\tilde{P}=\left(p_1 \| p_2 \| \ldots \| p_{i-1} \| \tilde{p}_i \| p_{i+1} \| \ldots \| p_n \| p_{n+1}\right)$. Taking into account that the distorted remainder p_i of the distorted number P can be represented in the form $p_i=\tilde{p}_i+\Delta p_i$ (it is assumed that the error Δp_i has an additive nature, that is, $P=\tilde{P}+\Delta P$, the number $\tilde{P}$ has the form $\tilde{P}=P-\Delta P$, and since the error can be calculated as $\Delta p_i=p_i-\tilde{p}_i$, then we can write down the following mathematical relationship using the orthogonal basis B_i:

$$\tilde{P}=P-\left(p_i-\tilde{p}_i\right)\cdot B_i. \tag{1}$$

Using orthogonal bases for a given CR (where e_i is the weight of the i-th orthogonal base $B_i\left(i=\overline{1,n+1}\right)$, determined from the following ratio $B_i=e_i\cdot G_i\equiv 1(\mathrm{mod}\ q_i)$ by formula (1), we determine the numerical value of the number $\tilde{P}$ as follows:

$$\tilde{P}=P-\Delta P,$$

$$\tilde{P}=P+\left(\tilde{p}_i-p_i\right)\cdot B_i,$$

$$\tilde{P}=P+\left(\tilde{p}_i-p_i\right)\cdot e_i\cdot G_i,$$

$$\tilde{P}=P+\left(\tilde{p}_i-p_i\right)\cdot e_i\cdot\frac{G_0}{q_i}. \tag{2}$$

Since the value $\left(\tilde{p}_i-p_i\right)\mathrm{mod}\,q_i$ can vary from 0 to the value (q_i-1), the maximum possible value of the value $\left[\left(\tilde{p}_i-p_i\right)\cdot q_i\cdot G_0/q_i\right]\mathrm{mod}\,G_0$ will be equal to $\left(q_i-1\right)\cdot G_0/q_i$. In this case, the maximum value of expression (2) has the form:

$$\left\{\left[\left(\tilde{p}_i-p_i\right)\cdot e_i\cdot\frac{G_0}{q_i}\right]\mathrm{mod}\,\mathrm{G}_0\right\}=\left(q_i-1\right)\cdot\frac{G_0}{q_i}. \tag{3}$$

On the basis of expression (3) and also taking into account the condition of the statement that the number under consideration lies within the limits $G<\tilde{P}<G_i$ it is possible to write that:

$$\tilde{P}<\frac{G_0}{q_i}+\left(q_i-1\right)\cdot\frac{G_0}{q_i}=G_0. \tag{4}$$

The number $\tilde{P}$ will be correct (which lies in the informational numerical interval [0, G)) only if the addition of the quantity $\left(\tilde{p}_i-p_i\right)\cdot B_i$ (2) would make it greater than the quantity G_0. However, based on expression (4), this cannot be achieved by correcting the skewed remainder p_i of P. In this case, the remainder p_i is correct, that is, the original assumption that it is a skewed remainder is incorrect. Since the inequality $P<G_i$ is satisfied by the condition of the statement, the following inequalities are especially satisfied:

$$P<G_i<G_{i-1}<\ldots<G_2<G_1. \qquad (5)$$

Inequalities (5) confirm the conditions for the statement that the remainders $p_z\left(z=\overline{1,i}\right)$ of the number $P=(p_1\|p_2\|\ldots\|p_{i-1}\|p_i\|p_{i+1}\|\ldots\|p_n\|p_{n+1})$ are correct.

Statement 6. Let in the ordered system of CR bases with n informative and one control bases $q_k=q_{n+1}$ the number $P=(p_1\|p_2\|\ldots\|p_{i-1}\|p_i\|p_{i+1}\|\ldots\|p_n\|p_{n+1})$ satisfies the following condition $G_{n+1}<P<G_i$, where $G=G_{n+1}=\prod_{i=1}^{n}q_i$, $G_0=\prod_{i=1}^{n+1}q_i=G\cdot q_{n+1}$, $G_i=G_0/q_i$. In this case, the residues $p_1, p_2, \ldots p_i$ are considered correct (undistorted) if only a single error is possible (in one residue of the number P).

By the method of the opposite, it is proved that the remainder p_i of the number P is not distorted. Let the remainder p_i be distorted. In this case, the number P will be incorrect, that is, $P\geq G$. The undistorted remainder is denoted as $\tilde{p}_i$, and the correct number as $\tilde{P}<G$. The error ΔP is additive in nature, that is, the inequality holds:

$$P=\tilde{P}+\Delta P. \qquad (6)$$

With this in mind, let's write that:

$$P=\left(p_1\|p_2\|\ldots\|\tilde{p}_i\|\ldots\|p_n\|p_{n+1}\right)+$$
$$+\left(0\|0\|\ldots\|\Delta\tilde{p}_i\|\ldots\|0\|0\right)=\left(p_1\|p_2\|\ldots\|p_i\|\ldots\|p_n\|p_{n+1}\right). \qquad (7)$$

Formula (7) can be written as follows:

$$\Delta p_i=\left(p_i-\tilde{p}_i\right)\bmod q_i. \qquad (8)$$

Based on ratios (6) and (8), we estimate the quantitative value of the number $\tilde{P}$ as follows:

$$\tilde{P}=P-\Delta P,$$

$$\tilde{P}=P-\Delta p_i\cdot B_i,$$

$$\tilde{P}=P-\left[\left(p_i-\tilde{p}_i\right)\bmod q_i\right]\cdot e_i\cdot G_i,$$

$$\tilde{P}=P+\left[\left(\tilde{p}_i-p_i\right)\bmod q_i\right]\cdot e_i\cdot G_i. \qquad (9)$$

The maximum possible value of the quantity:

$$\left\{\left[\left(\tilde{p}_i-p_i\right)\bmod q_i\right]\cdot e_i\cdot G\right\}\bmod G_0$$

in expression (9) is determined as follows:

$$\max\left(\left\{\left[\left(\tilde{p}_i-p_i\right)\bmod q_i\right]\cdot e_i\cdot G\right\}\bmod G_0\right)=$$
$$=\left[\frac{(q_i-1)}{q_i}\right]G_0. \qquad (10)$$

Indeed, the expression $\left(\tilde{p}_i-p_i\right)\bmod q_i$ can only take values from 0 to (q_i-1). In this case, $\max\left[\left(\tilde{p}_i-p_i\right)\bmod q_i\right]=q_i-1$, and $G_i=G_0/q_i$. Ratio (9) will be represented in the form:

$$\tilde{P}=P+\frac{(q_i-1)}{q_i}\cdot G_0. \qquad (11)$$

According to the condition of the statement, we have $P<G_i=G_0/q_i$. In this case, expression (11) will take the form:

$$\tilde{P}<\frac{G_0}{q_i}+\left[\frac{(q_i-1)}{q_i}\right]\cdot G_0,$$

$$\tilde{P}<G_0\cdot\left[\frac{1}{q_i}+\frac{(q_i-1)}{q_i}\right]\rightarrow\tilde{P}<G_0. \qquad (12)$$

The number $\tilde{P}$ can be correct $\left(\tilde{P}<G\right)$, if, from the addition of the value Δp_i (see expression (8)), it would exceed the value of G_0. But, as can be seen from expression (12), this cannot be achieved by any possible correction of the incorrect remainder p_i, which contradicts the assumption that the remainder p_i of P is incorrect. Therefore, the remainder p_i is not distorted, and the number P is correct. Since $P<G_i$, the more is $P<G_i<G_{i-1}<\ldots<G_2<G_1$, from which it follows that the remainders $p_{i-1},\ldots, p_2, p_1$ are correct. Note that when $i=n$, i.e., $G_{n+1}<P<G_n$, the false (distorted) remainder will be p_{n+1}.

It can be shown that the interference-resistant R-code in CR can detect and correct the number N_{det} and N_{cor} of errors of a higher multiplicity than that determined by the general theory of coding, i.e., the value $S_{\min}$ of the minimum code distance (MCD) [23].

Indeed, let the MCD be determined by the value $S_{\min}$ for a given CR. Suppose that in this CR there are l bases for which the condition $S_{\min}<l$ is fulfilled, and in this case $X(l)=\prod_{j=1}^{l}q_{z_j}<R<\frac{G_0}{G}$. Errors in the residuals of the number P on these bases can be reliably detected.

For the original data under consideration, the error vector:

$$\Delta P=\left(\tilde{P}-P\right)\bmod \mathrm{G}=$$
$$=\left(0\|0\|\ldots\|0\|\Delta p_{z_i}\|0\|\ldots\|0\|\Delta p_{z_i}\|\ldots\|0\|0\right)$$

must have at least $(n-l)$ zero residuals. Let's determine the numerical value of the error $\Delta P=\left(B_{z_1}\cdot\Delta p_{z_1}+\ldots+B_{z_l}\cdot\Delta p_{z_l}\right)\bmod G_0$. Taking into account the fact that an arbitrary orthogonal basis in CR is represented in the form $B_{z_i}=e_{z_i}\cdot G_0/q_{z_i}$, the value of ΔP will be determined as follows:

$$\Delta P=\left(\frac{e_{z_1}\cdot G_0}{q_{z_1}}\cdot\Delta p_{z_1}+\ldots+\frac{e_{z_l}\cdot G_0}{q_{z_l}}\cdot\Delta p_{z_l}\right)\bmod G_0=$$
$$=\begin{pmatrix}\frac{e_{z_1}\cdot G_0\cdot X_1(l)}{X(l)}\cdot\Delta p_{z_1}+\ldots+\\+\frac{e_{z_l}\cdot G_0\cdot X_l(l)}{X(l)}\cdot\Delta p_{z_l}\end{pmatrix}\bmod G_0, \qquad (13)$$

where $X_i(l)=\prod_{i=1;\ i\neq j.}^{l}q_{z_j}$.

Thus, we have:

$$\Delta P=\frac{G_0}{X(l)}\cdot\left(e_{z_1}\cdot X_1(l)\cdot\Delta p_{z_1}+\ldots+e_{z_l}\cdot X_l(l)\cdot\Delta p_{z_l}\right)\bmod G_0$$

or

$$\Delta P=\left(\frac{G_0}{X(l)}\cdot\sum_{i=1}^{l}(e_{z_i}\cdot X_i(l)\cdot\Delta p_{z_i})\right)\bmod G_0.$$

Since we have $X(l)=\frac{G_0}{G}$ and $\sum_{i=1}^{l}\left(e_{z_i}\cdot X_i(l)\cdot\Delta p_{z_i}\right)\neq 0$, then $\Delta P\geq G$.

It is obvious that the sum $P+\Delta P$ of any correct ($P<G$) number P and the number corresponding to the error value ΔP cannot belong to the set of values $[0,\ G)$ of correct numbers, i.e. $\tilde{P}=(P+\Delta P)\bmod G_0 \geq G$. In this case, in the process of data control, a similar error can be detected (it is possible to detect the distorted remainder of the number P by one of the CR bases). Based on the above, it can be stated that the R-code in CR makes it possible to reliably detect all multiplicity errors from $N_{det}=1$ to $N_{det}=S_{\min}-1$. In addition to the proven reliability of corrective R-codes in CR, these codes make it possible to detxect and correct most of the errors of a higher multiplicity, which is allowed by the general theory of coding [24–26].

5. 2. Investigation of the control process and diagnostics of data in the class of residuals

Based on the results of the above considerations, the process of control and diagnosis of the data submitted in CR was examined.

Let the number $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$ be given in CR, which is checked with n informative $q_i\left(i=\overline{1,n}\right)$ and one control $q_k=q_{n+1}$ bases.

It is necessary, first of all, to control (determine the correctness) of the number $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$ in CR, secondly, to diagnose the residuals $p_i\left(i=\overline{1,n+1}\right)$ of the number P, that is, to determine the distorted (or not distorted) residuals.

The control of digital data implies ensuring the accuracy and reliability of primary information [27, 28], i.e., numbers (digits), since a certain numbering system is used in CS (the research uses a non-positional one in CR), and all operations (commands) are reduced to the execution of certain arithmetic operations on these numbers. Data control and diagnostics are interrelated processes since diagnostics is a tool of control systems to detect and prevent intrusions [29–31]. Control and diagnostics of data are carried out sequentially in two stages.

The first stage. A method of data control in CR, represented in the form of NCS $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$:

1. Determine the value of orthogonal bases (basis consisting of pairwise orthogonal vectors [32]) $B_i\left(i=\overline{1,n+1}\right)$ for the complete system of CR bases $\{q_i\}$.

2. Using the system of orthogonal bases B_i, convert the original number P in CR to the positional number system (PNS) according to the known formula:

$$P_{PNS}=\left(\sum_{i=1}^{n+1} p_i\cdot B_i\right)\bmod G_0. \qquad (14)$$

3. Perform positional comparison operations of P_{PNS} and G values. If the result of the comparison showed that $P_{PNS}<G$, then the number P is correct. If $P_{PNS}\geq G$, then it is considered that the number $\tilde{P}$ is incorrect, if only one of the residues p_i of the number P is distorted, since the occurrence of one-time errors is considered, that is, the detection and correction of one-time errors in NCS in CR.

The second stage. The method of data diagnosis in CR involves the diagnosis of NCR residues $p_i\left(i=\overline{1,n+1}\right)\ \tilde{P}$, based on the use of the obtained results of Statements 3–6.

5. 3. Improvement of the method of data diagnostics in the class of residuals

The main task of data diagnostics in CR is to determine the set $\{p_i\}$ of distorted (incorrect) residuals of a distorted (incorrect) number $\tilde{P}$. In the case of one-time errors (it is this version of the correction of one-time errors that is considered in the present study), the result of the diagnosis process is the determination of one distorted remainder p_i of the number $\tilde{P}$ modulo q_i.

The essence of diagnosing the NCS $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$ in CR is to detect distorted residues $p_{z_i}\left(i=\overline{1,w}\right)$, The diagnostic depth D in CR shall be understood as the degree of detailing of the location of the error in NCS, which consists of a set of residues $\{p_i\}$, $\left(i=\overline{1,n+1}\right)$, that is, NCS of the form $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$. As mentioned earlier, a one-time error (in one NCS residue) is assumed.

The essence of diagnostics in CR is to identify residues $p_{z_i}\left(i=\overline{1,w}\right)$, in which errors are possible (detection of distorted residues of the NCS number P). Quantitatively, the depth of data diagnosis D in CR will be estimated by the ratio: $D=1/w$, where w is the number p_{z_i} of residues $p_{z_1}, p_{z_2}, ..., p_{z_w}$ in which an error is possible. The maximum value of the diagnostic depth $D_{\max}$ is reached in the case when the error in NCS P is detected with an accuracy of one residue [33]. In this case, the maximum depth of diagnosis $D_{\max}$ shall be understood as the detection of one ($w=1$) residue of the NKC P, which contains an error, i.e., $D_{\max}=1/w=1$.

On the basis of the results of proofs (3) to (6), a method of operational data diagnosis in CR was developed, which is represented in the form of an algorithm in Fig. 1, 2.

1	Determine G_i working bases of CR $G_1=q_2\cdot q_3\cdot...\cdot q_{i-1}\cdot q_i\cdot q_{i+1}\cdot...\cdot q_n\cdot q_{n+1}$, $G_2=q_1\cdot q_3\cdot...\cdot q_{i-1}\cdot q_i\cdot q_{i+1}\cdot...\cdot q_n\cdot q_{n+1}$, ... $G_i=q_1\cdot q_2\cdot...\cdot q_{i-1}\cdot q_{i+1}\cdot...\cdot q_n\cdot q_{n+1}$, ... $G_n=q_1\cdot q_2\cdot...\cdot q_{i-1}\cdot q_i\cdot q_{i+1}\cdot...\cdot q_{n-1}\cdot q_{n+1}$, $G_{n+1}=G=q_1\cdot q_2\cdot...\cdot q_{i-1}\cdot q_i\cdot q_{i+1}\cdot...\cdot q_{n-1}\cdot q_n$.
2	Determine $B_{ij}=e_{ij}\dfrac{G_i}{q_i}\equiv 1(\bmod q_i)$ of partial orthogonal bases for this CR $G=G_{n+1}=\prod_{i=1}^{n} q_i$, $G_0=\prod_{i=1}^{n+1} q_i=G\cdot q_{n+1}$, $G_i=\prod_{k=1;\ k\neq i.}^{n+1} q_k$. $e_{ij}=\overline{1,q_i-1}$; $j=\overline{1,n+1}$ – the number of bases of the original CR; $i=\overline{1,n}$ – the number of bases in the i-th set of partial working bases of CR ($j=i+1$). $B_{11}\ \ B_{21}\ \ B_{31}\ \ ...\ \ B_{(n-1)1}\ \ B_{n1}$ $B_{12}\ \ B_{22}\ \ B_{32}\ \ ...\ \ B_{(n-1)2}\ \ B_{n2}$ ... $B_{1(n+1)}\ \ B_{2(n+1)}\ \ B_{3(n+1)}\ \ ...\ \ B_{(n-1)(n+1)}\ \ B_{n(n+1)}$
3	Determine projections $\tilde{P}_j$ of number $\tilde{P}=(p_1\,\|\,p_2\,\|...\|\,p_{i-1}\,\|\,p_i\,\|\,p_{i+1}\,\|...\|\,p_n\,\|\,p_{n+1})$ $\tilde{P}_1=(p_2\,\|\,p_3\,\|...\|\,p_{i-1}\,\|\,p_i\,\|\,p_{i+1}\,\|...\|\,p_n\,\|\,p_{n+1})$, $\tilde{P}_2=(p_1\,\|\,p_3\,\|...\|\,p_{i-1}\,\|\,p_i\,\|\,p_{i+1}\,\|...\|\,p_n\,\|\,p_{n+1})$, ... $\tilde{P}_i=(p_1\,\|\,p_2\,\|...\|\,p_{i-1}\,\|\,p_{i+1}\,\|...\|\,p_n\,\|\,p_{n+1})$, ... $\tilde{P}_n=(p_1\,\|\,p_2\,\|...\|\,p_{i-1}\,\|\,p_i\,\|\,p_{i+1}\,\|...\|\,p_{n-1}\,\|\,p_{n+1})$, $\tilde{P}_{n+1}=(p_1\,\|\,p_2\,\|...\|\,p_{i-1}\,\|\,p_i\,\|\,p_{i+1}\,\|...\|\,p_{n-1}\,\|\,p_n)$.

Fig. 1. Algorithm for implementation of the method of operational data diagnostics in the class of residuals

4	Calculation of projection values $\tilde{P}_j$ in PNS of the form: $$\tilde{P}_{\substack{j\,PNS\\ j=const}} = \left(\sum_{\substack{i=1;\\ j=1,n+1.}}^{n} p_i \cdot B_{ij}\right) \bmod G_j$$	
	$$\tilde{P}_{\substack{1\ PNS\\ j=const}} = \left(\sum_{i=1}^{n} p_i \cdot B_{i1}\right) \bmod G_1 = \left(p_1 \cdot B_{11} + p_2 \cdot B_{21} + \ldots + p_n \cdot B_{n1}\right) \bmod G_1,$$ $$\tilde{P}_{\substack{2\ PNS\\ j=const}} = \left(\sum_{i=1}^{n} p_i \cdot B_{i2}\right) \bmod G_2 = \left(p_1 \cdot B_{12} + p_2 \cdot B_{22} + \ldots + p_n \cdot B_{n2}\right) \bmod G_2,$$ $$\ldots$$ $$\tilde{P}_{\substack{k\ PNS\\ j=const}} = \left(\sum_{i=1}^{n} p_i \cdot B_{ik}\right) \bmod G_k = \left(p_1 \cdot B_{1k} + p_2 \cdot B_{2k} + \ldots + p_n \cdot B_{nk}\right) \bmod G_k,$$ $$\ldots$$ $$\tilde{P}_{\substack{n\ PNS\\ j=const}} = \left(\sum_{i=1}^{n} p_i \cdot B_{in}\right) \bmod G_n = \left(p_1 \cdot B_{1n} + p_2 \cdot B_{2n} + \ldots + p_n \cdot B_{nn}\right) \bmod G_n,$$ $$\tilde{P}_{\substack{n+1\ PNS\\ j=const}} = \left(\sum_{i=1}^{n} p_i \cdot B_{i(n+1)}\right) \bmod G_{n+1} = \left(p_1 \cdot B_{1(n+1)} + p_2 \cdot B_{2(n+1)} + \ldots + p_n \cdot B_{n(n+1)}\right) \bmod G_{n+1}.$$	
5	Compare values of $\tilde{P}_{\substack{j\,PNS\\ j=const}} = \left(\sum_{\substack{i=1;\\ j=1,n+1.}}^{n} p_i \cdot B_{ij}\right) \bmod G_j$ with module $G = \dfrac{G_0}{q_{n+1}}$.	
6	Determination of reliably undistorted $\left\{p_{z_j}\right\}$ and possibly distorted remainders $\left\{\tilde{p}_{z_j}\right\}$ of number $\left\{\tilde{p}_{z_j}\right\}$	
	$\left\{p_{z_j}\right\},\ j=\overline{l+1,n+1},$ $\tilde{P}_{1\,PNS} \geq G \rightarrow p_1,$ $\tilde{P}_{2\,PNS} \geq G \rightarrow p_2,$ $\ldots$ $\tilde{P}_{n\,PNS} \geq G \rightarrow p_n.$	$\left\{\tilde{p}_{z_j}\right\},\ j=\overline{1,l},$ $\tilde{P}_{1\,PNS} < G \rightarrow \tilde{p}_1,$ $\tilde{P}_{2\,PNS} < G \rightarrow \tilde{p}_2,$ $\ldots$ $\tilde{P}_{n\,PNS} < G \rightarrow \tilde{p}_n,$ $\tilde{P}_{(n+1)\,PNS} < G \rightarrow \tilde{p}_{n+1}.$

Fig. 2. Continuation of the algorithm for implementing the method of operational data diagnostics in the class of residuals

Based on the improved method, an algorithm (Fig. 1, 2) was developed for the implementation of reliable and operational data diagnostics in CEDPD, which function in CR. On the basis of the proposed algorithm, it is possible to synthesize patentable devices for their implementation of the process of operational data diagnostics in CEDPD.

An example of the process of operational control and diagnostics and the application of the improved method for a one-byte (l=1) data structure (8 binary digits) of CEDPD operating in CR was considered. A complete CR with one control base is given by information q_1=3, q_2=4, q_3=5, q_4=7 and control q_k=q_{n+1}=11 bases. At the same time, the requirements for the unambiguity of the representation of code words in the given information numerical range [0, G) are met.

For this CR we have:

$$G_0 = \prod_{i=1}^{n+1} q_i = q_1 \cdot q_2 \cdot q_3 \cdot q_4 \cdot q_{n+1} = 3 \cdot 4 \cdot 5 \cdot 7 \cdot 11 = 4620$$

– the total number of code words;

$$G = \prod_{i=1}^{n} q_i = q_1 \cdot q_2 \cdot q_3 \cdot q_4 = 3 \cdot 4 \cdot 5 \cdot 7 = 420$$

– the number of informational code words. In this case, the full (working) [0, G_0) and informational [0, G) numerical ranges of numbers are defined, respectively, as [0, 4620) and [0, 420). All possible sets of CR bases (modules) are given in Table 1, where $G_j = G_0 / q_j$.

Table 1

A set of partial working bases (modules) CR (l = 1)

j \ i	q_1	q_2	q_3	q_4	G_j
1	4	5	7	11	1540
2	3	5	7	11	1155
3	3	4	7	11	924
4	3	4	5	11	660
5	3	4	5	7	420

Table 2 gives all possible numerical ranges [j·G, (j+1)·G) of capturing an incorrect number $\tilde{P}$ for one-byte data (l=1); Table 3 gives values of the possible ranks r of the number P for the range [0, G_0).

Table 2

Possible numeric ranges

j	[j·G, (j+1)·G)	$\delta_{n+1}^j = \delta_5^j$
0	[0, 420)	$\delta_5^0 = 0$
1	[420, 840)	$\delta_5^1 = 2$
2	[840, 1260)	$\delta_5^2 = 4$
3	[1260, 1680)	$\delta_5^3 = 6$
4	[1680, 2100)	$\delta_5^4 = 8$
5	[2100, 2520)	$\delta_5^5 = 10$
6	[2520, 2940)	$\delta_5^6 = 1$
7	[2940, 3360)	$\delta_5^7 = 3$
8	[3360, 3780)	$\delta_5^8 = 5$
9	[3780, 4200)	$\delta_5^9 = 7$
10	[4200, 4620)	$\delta_5^{10} = 9$

Table 3

Values of possible ranks r of the number P for the range [0, G_0)

r	r·G_0=r·4620
1	4620
2	9240
3	13860
4	18480
5	23100
6	27720
7	32340
8	36960
9	41580
10	46200

Example. Suppose that in the process of data transmission or processing, instead of the correct P=(1‖0‖0‖2‖1) result of the operation 100<G=420, a number of the type $\tilde{P} = \left(0 \,\|\, 0 \,\|\, 0 \,\|\, 2 \,\|\, 1\right)$, where $\tilde{P}_{PNS}=3180>G=420$ is obtained. It is necessary to check the correctness of the number $\tilde{P}$ and diagnose its residuals $p_i\left(i=\overline{1,5}\right)$:

1. The first stage.

1. 1. All values $B_i\left(i=\overline{1,5}\right)$ of orthogonal bases for the complete system of CR bases $q_1=3$, $q_2=4$, $q_3=5$, $q_4=7$ and $q_5=11$ are determined.

1. 2. Using the values of the orthogonal bases for the complete system of CR bases, the value is determined:

$$\tilde{P}_{PNS}=\begin{pmatrix}0\cdot 1540+0\cdot 3465+\\+0\cdot 3696+2\cdot 2640+1\cdot 2520\end{pmatrix}\operatorname{mod}4620=$$
$$=3180(\operatorname{mod}4620).$$

1. 3. The obtained number $\tilde{P}_{PNS}$ and the value of $G=420$ were compared. As $\tilde{P}_{PNS}>G=420$, it is concluded that the obtained result $\tilde{P}$ is distorted by some one of the residues p_i of the correct number $P=(1\|0\|0\|2\|1)$.

2. The second stage.

2. 1. The value $B_{ij}\left(i=\overline{1,n}=\overline{1,4},\ j=\overline{1,n+1}=\overline{1,5}\right)$ of the partial orthogonal bases for each of the 5 sets of CR bases are determined (Table 4):

$$\begin{cases}B_{1j}=(1,0,0,0),\\B_{2j}=(0,1,0,0),\\B_{3j}=(0,0,1,0),\\B_{4j}=(0,0,0,1).\end{cases}$$

In the general case, the value B_{ij} of partial orthogonal bases is determined based on the following formula:

$$B_{ij}=e_{ij}\frac{G_i}{q_i}\equiv 1\left(\operatorname{mod}q_i\right),\tag{15}$$

where $e_{ij}=\overline{1,q_i-1}$ is the weight of the orthogonal basis B_{ij} [34].

Table 4

Partial orthogonal bases B_{ij} in CR for $l=1$

B_{ij} \ i / j	1	2	3	4
1	385	616	1100	980
2	385	231	330	210
3	616	693	792	672
4	220	165	396	540
5	280	105	336	120

The values of orthogonal B_i bases for CR are given: $B_1=(1,0,0,0,0)=1540$, $\overline{q}_1=1$; $B_2=(0,1,0,0,0)=3465$, $\overline{q}_2=3$; $B_3=(0,0,1,0,0)=3696$, $\overline{q}_3=4$; $B_4=(0,0,0,1,0)=2640$, $\overline{q}_4=4$; $B_5=(0,0,0,0,1)=2502$, $\overline{q}_5=6$.

2. 2. The correctness of the remainders of the number $\tilde{P}$ is determined. First, all possible projections $\tilde{P}_j$ of the number $\tilde{P}=(0\|0\|0\|2\|1)$: $\tilde{P}_1=(0,0,2,1)$, $\tilde{P}_2=(0,0,2,1)$, $\tilde{P}_3=(0,0,2,1)$, $\tilde{P}_4=(0,0,0,1)$, $\tilde{P}_5=(0,0,0,2)$ are built. Using the data in Table 4, we give the value of projections $\tilde{P}_j$ in PNS:

$$\tilde{P}_{1\text{ PNS}}=\begin{pmatrix}p_1\cdot B_{11}+p_2\cdot B_{21}+\\+p_3\cdot B_{31}+p_4\cdot B_{41}\end{pmatrix}\operatorname{mod}G_1=$$
$$=\begin{pmatrix}0\cdot 385+0\cdot 616+\\+2\cdot 1100+1\cdot 980\end{pmatrix}\operatorname{mod}1540=100<420,$$

$$\tilde{P}_{2\text{ PNS}}=\begin{pmatrix}p_1\cdot B_{12}+p_2\cdot B_{22}+\\+p_3\cdot B_{32}+p_4\cdot B_{42}\end{pmatrix}\operatorname{mod}G_2=$$
$$=\begin{pmatrix}0\cdot 385+0\cdot 231+\\+2\cdot 330+1\cdot 210\end{pmatrix}\operatorname{mod}1155=870>420,$$

$$\tilde{P}_{3\text{ PNS}}=\begin{pmatrix}p_1\cdot B_{13}+p_2\cdot B_{23}+\\+p_3\cdot B_{33}+p_4\cdot B_{43}\end{pmatrix}\operatorname{mod}G_3=$$
$$=\begin{pmatrix}0\cdot 616+0\cdot 693+\\+2\cdot 792+1\cdot 672\end{pmatrix}\operatorname{mod}924=418<420,$$

$$\tilde{P}_{4\text{ PNS}}=\begin{pmatrix}p_1\cdot B_{14}+p_2\cdot B_{24}+\\+p_3\cdot B_{34}+p_4\cdot B_{44}\end{pmatrix}\operatorname{mod}G_4=$$
$$=\begin{pmatrix}0\cdot 220+0\cdot 165+\\+2\cdot 396+1\cdot 540\end{pmatrix}\operatorname{mod}660=540>420,$$

$$\tilde{P}_{5\text{ PNS}}=\begin{pmatrix}p_1\cdot B_{15}+p_2\cdot B_{25}+\\+p_3\cdot B_{35}+p_4\cdot B_{45}\end{pmatrix}\operatorname{mod}G_5=$$
$$=\begin{pmatrix}0\cdot 280+0\cdot 105+\\+0\cdot 336+2\cdot 120\end{pmatrix}\operatorname{mod}420=240<420.$$

Thus, among all obtained projections $\tilde{P}_j$ of the number $\tilde{P}$, projections $\tilde{P}_1$, $\tilde{P}_3$ and $\tilde{P}_5$ are less than $G=420$, and projections $\tilde{P}_2$ and $\tilde{P}_4$ are greater than $G=420$. Therefore, the results of the diagnosis of an incorrect $\tilde{P}$ number state that among all the five residues of the number, it is the residues p_1, p_3 and p_5 that may be erroneous, and the residues p_2 and p_4 are definitely not distorted.

For the above example of diagnosing a number $\tilde{P}=$ $=(0\|0\|0\|2\|1)$, we have that the number p_i of residues in which a possible error is equal to 3, i.e., $w=3$, and the depth of diagnosing data in CR is $D=1/w=1/3\approx 0.3$.

Combination in time of the process of determination and analysis (comparison of the projections $\tilde{P}_{j\text{ PNS}}$ obtained in the PNS with module G) of the values $\tilde{P}_{\substack{jPNS\\ j=const}}=\left(\sum_{\substack{i=1;\\ j=\overline{1,n+1}.}}^{n}p_i\cdot B_{ij}\right)\operatorname{mod}G_j$ of the projections $\tilde{P}_j$ of the diagnosed number $P=(p_1\|p_2\|\ldots\|p_{i-1}\|p_i\|p_{i+1}\|\ldots\|p_n\|p_{n+1})$ makes it possible to increase the efficiency of the procedure for diagnosing data errors in CS CEDPD by n times.

6. Discussion of results of investigating the improved method of operative diagnosis of economic data errors

On the basis of our results (1) to (14) of investigating the structure and features of the functioning of data processing devices in CR, it is shown that the operation of CEDPD CS operating in CR is impossible without an effective control system for data represented by a non-positional code. On the basis of the principles of building non-positional code structures in CR, based on the main properties of CR, the effectiveness of applying codes in CR has been unequivocally proven. Application of codes in CR makes it possible to develop highly fault-tolerant systems and data processing devices with parallel processing of large arrays of information. Moreover, the combination of the combined use of CR and binary PNS in the construction of specialized CEDPD can lead to an increase in the productivity of CS as a whole. That is, the management of the entire CS can be carried out in binary code, and the processing of economic data will be performed based on the representation of numbers in the CR code.

Methods of diagnosis and correction of data errors in CR are considered, 6 main statements are given and confirmed by analytical relations (1) to (13). The process of monitoring and diagnosing data is carried out sequentially in two stages. The first stage (control of data in CR) implies representing the data in the form of NCS $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$. Determination of values of orthogonal bases B_i $(i=\overline{1,n+1})$ for the complete system of bases $\{q_i\}$ CR. Using the system of orthogonal bases B_i, the original number P in CR is transformed into PNS according to formula (14). After that, a positional comparison of the values of P_{PNS} and G is performed. If the result of the comparison showed that $P_{PNS}<G$, then the number P is correct. If $P_{PNS}\geq G$, then it is considered that the number $\tilde{P}$ is incorrect, if only one of the residues p_i of the number P is distorted, since the occurrence of one-time errors is considered, that is, the detection and correction of one-time errors in NCS in CR. The second stage (diagnostics of data in CR) involves the diagnosis of NCS residuals $p_i\left(i=\overline{1,n+1}\right)$ $\tilde{P}$, based on the use of the obtained results of Statements 3–6. That is, at the second stage, the developed method of operational diagnostics of data in CR is used, which is represented in the form of an algorithm in Fig. 1, 2. The essence of the developed method is to determine the working bases G_i in CR, partial orthogonal bases B_{ij} of this CR (15), and the number $\tilde{P}$ projections $\tilde{P}_j$ (Fig. 1). After that, the values of projections $\tilde{P}_{\substack{j\,PNS\\ j=\text{const}}}=\left(\sum\limits_{\substack{i=1;\\ j=\overline{1,n+1}.}}^{n} p_i\cdot B_{ij}\right)\bmod G_j$ in PNS are calculated, one compares $\tilde{P}_{j\,PNS}$ with the modulus $G=G_0/q_{n+1}$, on the basis of which the reliably undistorted $\{p_{z_j}\}$ and possibly distorted residuals $\{\tilde{p}_{z_j}\}$ of the number $\tilde{P}$ are determined (Fig. 2).

That is, the improved method of data diagnosis in CR, based on the determination of partial orthogonal bases, involves combining in time the processes of analyzing the projections of the diagnosed number. The use of partial orthogonal bases makes it possible to increase the efficiency of diagnosing data of computer devices processing economic data of computer systems. The application of this method makes it possible to reduce the time of diagnosis by n times, which increases the efficiency of diagnostic operations for data errors in CR. The improved operational method of diagnostics, namely the set of analytical ratios (1) to (14), is a mathematical model for creating effective control and error diagnosis systems of CEDPD CS in CR.

In the theory of non-positional interference-resistant coding, various methods of data control are used. A feature of the proposed solution is the parallel processing of the projections of the initial number of the CR code structure. The operations of control (determining the correctness of the number P) and diagnostics (analysis of the correctness or incorrectness of the residues) are carried out one after the other in sequence without significant delays. In contrast to existing methods of direct comparison and the principle of zeroing, the proposed data control and diagnosis algorithm is operational without loss of reliability. This becomes possible due to the operation of positional comparison of number values, which increases the reliability of data control and diagnostics (first stage, item 5. 2).

The results of studies of methods of diagnosis and correction of data errors in CR showed that, unlike the corrective codes used in PNS, arithmetic codes in CR have additional corrective capabilities. Thus, the presence of simultaneously primary and secondary information redundancy in NCS can in some cases provide the possibility of correcting single errors in CR when the MCD is equal to one. However, additional data processing procedures are required to correct single errors. In particular, this is achieved by applying temporary redundancy in addition to information redundancy. This circumstance necessitates the use of additional time for data correction, which reduces the overall effectiveness of using correction codes in CR.

In the general case, the task of data diagnostics in CR is to determine the set $\{p_i\}$ of distorted (incorrect) residuals of a distorted (incorrect) number $\tilde{P}$ modulo q_i. The degree of detailing of the location of the error in PNS is determined by the diagnosis depth D. Since our article considers a single error (in one PNS residue), the maximum diagnostic depth $D_{\max}$ is the detection of one ($w=1$) PNS residue P in which there is an error, i.e., $D_{\max}=1/w=1$.

Existing methods of control and diagnostics, as a rule, are not very effective due to the complexity of implementation. For example, the sample data control method [12, 13], which guarantees the asymptotic stability of the obtained results. Others are built using PNS, which have the main drawback in data processing – the presence of inter-bit connections [22, 24]. Inter-bit connections affect the methods of implementation of arithmetic operations, limit the speed of the data control and diagnostics process. In the improved method, there are no data connections, so the result of data processing can be controlled either step by step or at the end of all calculations since the error that occurred in any one residue does not spread to the rest of the residues. Methods of determining an alternative set of numbers (AS) are also used to perform the operation of diagnosing data in CR when one-time errors occur. At the same time, the main drawback of the considered methods is a significant time for determining AS [20], which reduces the overall efficiency of diagnosing data in CR.

The main limitation of the study is the use of the method of operational diagnostics of integer economic data represented in CR. In the case of non-integer data representation, the speed and reliability of the processing implementation is lost, but this limitation is not critical since CEDPD operate precisely with digital integer data.

The shortcoming of the study includes the time lost for converting economic data from binary code (since most CEDPD CS are represented in the form of a combination of zeros and ones) into NCS of CR. This shortcoming can be solved in the creation of new or improvement of existing algorithms for the conversion of binary code in NCS CR.

Based on the improved method of operational data diagnostics, it is possible to develop a method of correcting data errors represented in CR. The development of research consists in creating an effective system for detecting and preventing intrusions. This will make it possible to improve the efficiency of error correction of CEDPD data, by organizing the process of parallel correction of errors in the group of residues of the controlled number. But it is necessary to take into account that the data correction method must be based on the value (single, double error) and the location of the error in the remainder of the number.

7. Conclusions

1. The methods of diagnosis and correction of data errors in CR were studied. The process of correction (detection and correction) of errors in the information code data structure represented in CR, which consists of the following main stages: control, diagnosis, and correction of errors in the data code structure, was analyzed in detail. It was concluded that the diagnostic process is an integral part of error correction in the processing of CEDPD data. We considered a number

of statements and proved the statements by the method of the opposite. Based on these statements, it was concluded that the sum $P+\Delta P$ of any correct ($P<G$) number P and the number corresponding to the error value ΔP cannot belong to the set of values $[0, G)$ of correct numbers, i.e. $\tilde{P}=(P+\Delta P)\bmod G_0 \geq G$. In the process of data control, a similar the error can be detected, i.e., detect the distorted remainder of the number P by one of the CR bases. Therefore, the correcting CR codes make it possible to reliably detect all multiplicity errors from $N_{det}=1$ to $N_{det}=S_{\min}-1$. The results of the proofs of these statements were the basis of an improved method of operative diagnosis of data errors represented in the non-positional numbering system in CR.

2. On the basis of the results obtained from the studies of the process of control and diagnosis of data errors represented in CR, it is shown that, unlike the correction codes used in PNS, the arithmetic codes in CR have additional corrective capabilities. Thus, the presence of simultaneously primary and secondary information redundancy in PNS, in some cases, can provide the possibility of correcting one-time errors in CR with the MCD equal to $S_{\min}=1$. However, additional economic data processing procedures are required to correct one-time errors. In particular, this is achieved by applying temporary redundancy in addition to information redundancy. This circumstance necessitates the use of additional time for control, diagnosis, and correction of data, which reduces the overall effectiveness of the use of correction codes in CR. This shortcoming was solved by positional comparison of number values, which does not require additional time for conversion from binary code to NCS, and also increases the reliability of data control and diagnostics. In contrast to existing methods of direct comparison and the principle of zeroing, the proposed data control and diagnosis algorithm is operational without loss of reliability.

3. A method of diagnostics in CR based on the application of orthogonal bases B_{ij} of partial sets of bases (modules) has been improved. Partial orthogonal bases are formed from the original full system of bases (modules) $q_i\left(i=\overline{1,n+1}\right)$. Their use allows organizing the process of parallel processing of projections $P_i=(p_1||p_2||...||p_{i-1}||p_{i+1}||...||p_n||p_{n+1})$ of the initial number $P=(p_1||p_2||...||p_{i-1}||p_i||p_{i+1}||...||p_n||p_{n+1})$ of the CR code structure, which makes it possible to increase by n times the efficiency of diagnosing data in CEDPD. An example of a concrete implementation of the process of applying the method of operational control and error diagnosis in the processing of economic data represented in CR is given. It is shown that the use of the improved diagnostic method will make it possible to increase the efficiency of diagnostics and correction of errors in CS CEDPD operating in CR. On the basis of the improved method, an algorithm was developed for the implementation of reliable and operational data diagnostics in CEDPD, which operate in CR. On the basis of the proposed algorithm, it is possible to synthesize patentable devices for their implementation of the process of operational data diagnostics in CEDPD.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

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