The object of research is the process of determining the coordinates of aerial objects by small-sized radars. The main hypothesis of the study assumes that combining two small-sized radars into a network could improve the accuracy of determining the coordinates of aerial objects.

It was established that when determining the coordinates of an aerial object by a small-sized radars, the accuracy of determining the range is much better than the accuracy of determining the angular coordinate. To eliminate this shortcoming, a two-position network of small-sized radars and their error ellipses was considered. It is proposed to use the range-finding method in each small-sized radar station of the two-position network.

A method for determining the coordinates of aerial objects in a two-position network of small-sized radars has been improved, which, unlike known ones:
- enables a synchronous survey of the airspace by small-sized radars;
- measures the range to the aerial object by two small-sized radars;
- determines the coordinates of the aerial object by the joint processing of radar information from two small-sized radars.

The accuracy of determining the coordinates of aerial objects in a two-position network of small-sized radars was evaluated. The experimental evaluation was carried out by means of simulation using the method of Monte Carlo statistical tests. Working areas of the two-position network of small-sized radars were calculated. It was established that the two-position network of small-radars works only in the area formed by the intersections of the viewing areas of small-sized radars during their autonomous operation.

Keywords: small-sized radar, coordinate determination accuracy, two-position network, error ellipse

1. Introduction

Under modern conditions, the detection and determination of the coordinates of aerial objects is carried out by small-sized radars (RSs) [1, 2]. Such radars have low weight, electronic scanning of the airspace, etc. At the same time, small-sized RSs, as a rule, have a wide antenna pattern. So, for example, the width of the AN/TPQ-48 (49) (United States of America (USA)) RS antenna's directional pattern along the angular coordinate (azimuth) is 19 degrees [3].
The width of the directional diagram of the AN/TPQ-64 RS antenna (USA) in angular coordinates is several units of degrees [2].

The wide directional pattern of the antenna of a small-sized radar leads to a deterioration in the accuracy of determining the coordinates of an aerial object. This is especially critical when issuing combat information regarding damage to air objects such as a cruise missile [4] or an unmanned aerial vehicle [5, 6].

Known methods for increasing the accuracy of determining the coordinates of aerial objects in small-sized radars involve the introduction of structural or software changes in the algorithms for processing radar information. This, in turn, leads to an increase in the weight of the radar and the complication of radar information algorithms.

At the same time, it is possible to use system effects when combining two small-sized RSs into a two-element network. The use of known methods for determining the coordinates of aerial objects in a two-position network of small-sized radars will make it possible, in comparison with a single radar, to increase the accuracy of determining the coordinates of aerial objects such as cruise missiles and unmanned aerial vehicles.

Therefore, improving the accuracy of determining the coordinates of an aerial object in a two-position network of small-sized RSs is an urgent task.

2. Literature review and problem statement

In [7], the use of broadband signals is proposed to increase the accuracy of determining the coordinates of objects. The disadvantage of [7] is the need to make structural changes to the small-sized radar.

In [8], a method of coherent accumulation was devised to increase the accuracy of determining the coordinates of an aerial object. The disadvantage of [8] is the time limitation of coherent accumulation. This is due to the limitation of the coherence interval of signals from an aerial object.

The gradient method and the particle swarm method are proposed in [9]. The particle swarm method improves the quality of the method. The disadvantage of [9] is ambiguity when determining the coordinates of an aerial object.

In [10], a cross-section is proposed to increase the accuracy of determining the coordinates of an aerial object. The disadvantage of [10] is the high-quality operation of the method only in the case of representing the object by the Sverlig model, which is unlikely in small-sized radars.

In [11], a special metric for determining coordinates is proposed to improve the accuracy of determining the coordinates of an aerial object. This metric is based on the methods of the Dempster-Shafer theory. The disadvantage of [11] is the limited use of a special metric in small-sized radars.

In [12], to increase the accuracy of determining the coordinates of an aerial object, the use of several radars operating autonomously is proposed. The disadvantage of [12] is significant economic costs associated with an increased number of RSs.

In [13], the use of several autonomous radars emitting a sounding signal at different frequencies is proposed. An increase in accuracy is proposed due to an increased signal-to-noise value from several autonomous radars. The disadvantage of [13] is the significant complication of building a radar system and radar information processing algorithms.

In [14], it is proposed to improve the accuracy of determining the coordinates of aerial objects by using complex sounding signals to detect inconspicuous aerial objects. The disadvantage of [14] is the complication of algorithms for processing signals reflected from aerial objects.

In [15], the difference-rangefinder and total-difference rangefinder methods for determining the coordinates of an aerial object are proposed. It was established that the accuracy of determining the coordinates of an aerial object depends on the width of the antenna pattern and the signal/noise ratio. The disadvantage of [15] is the impossibility of implementing the specified methods in small-sized RSs.

In [16], a method of signal processing in a network of surveillance two-coordinate radars is proposed. The disadvantage of [16] is the lack of synchronization of the operation of such radars.

Paper [17] proposed methods of coherent processing of signals from two radars that are connected in a network. The disadvantage of [17] is the impossibility of ensuring coherence in the operation of two radars, which, in turn, negates the advantages of such a network.

In [18], it is proposed to improve the accuracy of determining the coordinates of an aerial object due to the transfer of processing to the spectral domain. The disadvantage of [18] is the need for information on the parameters of an unknown signal reflected from an aerial object.

Study [19] proposed methods for increasing the accuracy of determining the coordinates of an aerial object based on the Helstrom and Petrov-Galerkin strategy. The disadvantage of [19] is the need for information on the parameters of an unknown signal reflected from an aerial object.

Paper [20] proposed methods for improving the signal/noise ratio by adding cellular communication energy. This will, of course, theoretically lead to an increase in the accuracy of determining the coordinates of an aerial object. But there is a difficulty in synchronizing the radar and cellular communication stations.

Methods of increasing the signal-to-noise ratio by adding cellular communication energy are proposed in [21], but two radars are used. This will, of course, theoretically lead to an increase in the accuracy of determining the coordinates of an aerial object. But there is a difficulty in synchronizing the radar and cellular communication stations.

Methods of increasing the signal/noise ratio by adding the energy of navigation signals are proposed in [22]. This will, of course, theoretically lead to an increase in the accuracy of determining the coordinates of an aerial object. But there is a difficulty in synchronizing radar and sources of navigation signals.

In [23] it is proposed to introduce an additional receiving channel (for example, a television signal) in the radar. This will increase the overall signal-to-noise ratio. But, in turn, this requires a structural reconstruction of RS, which is problematic.

In [24], the radar detection zone with an additional reception channel was calculated. This theoretically increases the accuracy of determining the coordinates of an aerial object by improving the total signal-to-noise ratio. But there is a problem of compensating the penetrating signal.

In [25], the use of information from transponders onboard an aerial object is proposed. The disadvantage of [25] is the lack of theoretical calculations and experimental research.
In [26], the use of Automatic Dependent Surveillance-Broadcast (ADS-B) transponders is proposed. The disadvantage of [26] is that the method works only if there are ADS-B transponders on board the air object.

In [27], navigation methods for increasing the accuracy of determining the coordinates of aerial objects (methods of the Loran-C navigation system (USA)) are proposed. The disadvantage of [27] is the use of navigation methods only for solving navigation tasks.


In [29], the use of the Wide Area Multilateration (WAM) system is proposed. The disadvantage of [29] is the need to ensure a significant power of reflected signals due to the large distances between WAM receivers.

In [30], a theoretical method for improving accuracy is proposed, which is based on the maximum likelihood method. The disadvantage of [30] is the calculation of complex objective multidimensional functions.

In [31], the use of only quadratic objective functions is proposed. This reduces the search space for such functions. But as a result of using the methods from [31], the estimates of the coordinates of the aerial object are biased and suboptimal.

In [32], the MLAT technology is additionally used when determining the coordinates of an aerial object. The disadvantage of [32] is that the method works only if there are ADS-B transponders on board the air object.

In [33] it is proposed to suppress the penetrating signal in the additional channel of radar reception. The disadvantage of [33] is the destruction of both a radar and suppressing the useful signal reflected from an aerial object.

In [34], the use of a genetic algorithm for building a RS network is proposed. The disadvantage of [34] is the mandatory knowledge of a priori information about the flight trajectories of aerial objects.

In [35], the integration of information from radar and sound sources of information is proposed. The disadvantage of [35] is the complexity of processing and complexing heterogeneous signals.

In [36], a sound method for detecting unmanned aerial vehicles of the Shahed type (Iran) is proposed [37]. The disadvantage of [36] is the small detection range of an unmanned aerial vehicle.

In [37], a method for detecting a multirotor drone is proposed. The method involves the use of active location methods. The disadvantage of [37] is ignoring the issue of determining the coordinates of the drone.

In [38], the use of a frequency-dispersed antenna array was proposed to ensure a narrow beam of the antenna’s directional pattern. Calculations are given for determining the range and angle of an aerial object. The disadvantage of [38] is the need to make structural changes to the radar antenna system.

Thus, the known methods for improving the accuracy of determining the coordinates of aerial objects in small-sized radars are aimed at:

- making structural changes to the radar itself;
- complicating radar information processing algorithms.

The main drawback of the known methods for determining the coordinates of aerial objects in small-sized radars is the low accuracy of determining the coordinates of aerial objects.

Requirements for the accuracy of determining the coordinates of aerial objects by small-sized radars should be tens of meters in range and tenths of degrees in angular coordinates [39]. Such accuracy is necessary to meet the requirements of means of destroying aerial objects [2, 40].

The above requirement can be met by combining two small-sized RSs into a network. Therefore, it is necessary to solve the problem of improving the accuracy of determining the coordinates of an aerial object in a two-position network of small-sized radars.

3. The aim and objectives of the study

The purpose of this study is to improve the accuracy of determining the coordinates of an aerial object by combining two small-sized RSs into a network. This will make it possible to meet the requirements for the accuracy of determining the coordinates of aerial objects by small-sized radars.

To achieve the goal, it is necessary to solve the following tasks:

- define the main stages of the method for determining the coordinates of aerial objects in a two-position network of small-sized radars;
- to evaluate the accuracy of determining the coordinates of aerial objects in a two-position network of small-sized RSs.

4. The study materials and methods

The object of our study is the process of determining the coordinates of aerial objects by a small-sized RS.

The main hypothesis of the research assumes that combining two small-sized radars into a network could increase the accuracy of determining the coordinates of aerial objects.

The following research methods were used during the study:

- radar location methods;
- methods of multi-position radar location;
- methods of digital signal processing;
- mathematical apparatus of matrix theory;
- methods of system analysis;
- methods of statistical theory of detection and measurement of parameters of radar signals;
- differential calculus methods;
- methods of probability theory and mathematical statistics;
- iterative methods;
- methods of mathematical modeling.

The following limitations and assumptions were adopted during the research:

- small-sized RSs with digital signal processing are considered;
- the range of operation of small-sized radars – X-band (centimeter wavelength from 2.5 cm to 3.75 cm);
- air objects considered are cruise missiles and unmanned aerial vehicles;
- the trajectory of cruise missiles and unmanned aerial vehicles is considered piecewise linear;
- the average effective scattering surface of cruise missiles and unmanned aerial vehicles is taken from (0.4–0.6) m² to (1–1.5) m², depending on the location of air objects relative to small-sized radars;
– it is considered that reception of a reflected signal from an aerial object is ensured in a small-sized radar;
– a synchronous inspection of the airspace by small-sized radars is ensured;
– there are no artificial and natural obstacles;
– modeling is carried out by the method of Monte Carlo statistical tests.

### 5. Results of research on improving the accuracy of determining the coordinates of an aerial object

#### 5.1. Main stages of the method for determining the coordinates of aerial objects in a two-position network of small-sized radars

As mentioned above, small-sized RSs have a wide directional pattern in the angular coordinate. This causes the elongated error ellipse along the angular coordinate and its flattening along the range (Fig. 1). In Fig. 1, we marked $\delta_R$ – the width of the directional pattern of the small-sized radar antenna by range, $\delta_\varepsilon$ – the width of the directional pattern of the small-sized radar antenna by angular coordinate.

**Fig. 1.** Ellipse of range and angular coordinate errors of a small-sized RS operating under autonomous mode

Therefore, when determining the coordinates of an aerial object with a small-sized radar, the accuracy of determining the range is much better than the accuracy of determining the angular coordinate. To eliminate this shortcoming, let’s consider two small-sized radars and their error ellipses. The total error ellipse of two small-sized radars is shown in Fig. 2 and represents the intersection of the error ellipses of each small-sized radar operating under an autonomous mode.

**Fig. 2.** Ellipse of range and angular coordinate errors of two small-sized radars

Fig. 2 demonstrates that the error ellipse of two small-sized RSs is formed as a result of the intersection of the error ellipses of each of the small-sized radars that work autonomously. This fact is also illustrated in Fig. 3. In Fig. 3, the letter B indicates the distance between two small-sized RSs (network base).

Therefore, owing to the intersection of the error ellipses of two small-sized radars, the accuracy of determining the coordinates of aerial objects increases significantly, especially in terms of angular coordinates.

**Note:** Each small-sized RS works autonomously while the range-finding method is used to determine the coordinates of an aerial object. The advantage of the distance measuring method is its simplicity.

The basic stages of the method for determining the coordinates of aerial objects in a two-position network of small-sized RSs are shown in Fig. 4.

**Fig. 4.** Main stages of the method for determining the coordinates of aerial objects in a two-position network of small-sized radars

These stages are:
1. Input of initial data: coordinates of small radars; mean square error (MSE) of the range determination of each
small-sized radar \((\sigma_{R1}, \sigma_{R2})\); issuance of preliminary target instructions regarding the coordinates of the aerial object (if possible and necessary).

2. Provision of a synchronous survey of the airspace by small-sized RSs.

3. Calculation of distances to an aerial object from each small-sized radar using the range-finding method. Ranges to an aerial object are calculated according to expressions (1), (2):

\[
R_1 = 0.5ct_1, \quad (1)
\]

where \(R_1\) is the range from RS 1 to the aerial object; \(c\) – speed of light; \(t_1\) is the delay time of the probing signal from RS 1:

\[
R_2 = 0.5ct_2, \quad (2)
\]

where \(R_2\) is the range from RS 2 to the aerial object; \(t_2\) is the delay time of the probing signal from RS 2.

4. Determination of the coordinates of the aerial object \(x_{AO}, y_{AO}\).

The coordinates of the aerial object \(x_{AO}, y_{AO}\) are determined by expressions (3), (4) [41]:

\[
x_{AO} = \frac{1}{2B} R_1^2 - R_2^2, \quad (3)
\]

\[
y_{AO} = \frac{1}{2B} \sqrt{(2R_1^2 B^2 + 2R_2^2 B^2 + 2R_1^2 - R_2^2 - B^2)}. \quad (4)
\]

5. Checking the conditions for tracking an aerial object by small-sized RSs.

6. Determining the trajectory of an aerial object through joint processing of radar information from two small-sized RSs.

7. Issuance of targets for means of destruction (if necessary).

Thus, a method for determining the coordinates of aerial objects in a two-position network of small-sized RSs has been improved, which, unlike the known ones:

– enables a synchronous survey of the airspace by small-sized RSs;

– measures the range to the air object by two small-sized RSs;

– determines the coordinates of the aerial object by joint processing of radar information from two small-sized RSs.

5. 2. Assessment of the accuracy of determining the coordinates of aerial objects in a two-position network of small-sized radars

To assess the accuracy of determining the angular coordinate of aerial objects in a two-position network of small-sized radars, we shall calculate MSE according to expression (5):

\[
\sigma_\sigma = \frac{\sigma_x \sqrt{2}}{B \sin(\epsilon)}, \quad (5)
\]

where \(\sigma_x\) is the MSE in determining the angular coordinate of an aerial object; \(\sigma_R\) is the MSE in determining the range of an air object in each small-sized RS (it is assumed that \(\sigma_{R1} = \sigma_{R2} = \sigma_R\)).

To assess the accuracy of determining the plane coordinates of aerial objects in a two-position network of small-sized RSs, we shall calculate MSE according to expression (6):

\[
\sigma_y = \sigma_x \frac{\sqrt{2}}{\sin(\varphi)}, \quad (6)
\]

where \(\sigma_y\) is the MSE in determining the plane coordinates of an aerial object; \(\varphi\) is the angle (Fig. 5), which is determined by expression (7):

\[
\varphi = \arctg \left\{ \frac{0.5B - R_1}{0.5B + R_1} \right\} + \arctg \left\{ \frac{0.5B - R_2}{0.5B + R_2} \right\}, \quad (7)
\]

Values \(B, R_0, \gamma\) are specified in Fig. 5.

Fig. 5. A network of two small-sized radars for determining the coordinates of an aerial object

Experimental assessment will be carried out by simulation. In this case, we shall use the Monte Carlo statistical test method [42–44]. The following limitations and assumptions were adopted during the simulation:

– two small-sized RSs have a digital receiver and implementation of digital signal processing;

– the Kalibr cruise missile (Russian Federation) is considered as an air object;

– there is only one aerial object in the viewing areas of small-sized RSs;

– it is considered that reception and optimal processing of the reflected signal from an aerial object is ensured in two small-sized RSs;

– there are no artificial and natural obstacles;

– software: high-level programming language and interactive environment for programming, numerical calculations, and visualization of results MATLAB R2017b [45];


Fig. 6 shows the MSE in determining the coordinates of an aerial object by two small-sized RSs.

Fig. 7 schematically shows the working zones of the two-position network of small-sized RSs. It should be noted that the two-position network works only in the area formed by the intersections of the viewing areas of small-sized RSs during their autonomous operation (shaded areas in Fig. 7).
Fig. 6. Mean squared errors in determining the coordinates of an aerial object by two small-sized radars

Fig. 7. Schematic representation of the working zones of a two-position network of small-sized radars

It is proposed to use the range-finding method in each small-sized radar of a two-position network. The main stages of the method for determining the coordinates of aerial objects in a two-position network of small-sized RSs are shown in Fig. 4.

Thus, a method for determining the coordinates of aerial objects in a two-position network of small-sized RSs has been improved, which, unlike the known ones (for example, [13, 14]):

- enables a synchronous survey of the airspace by small-sized RSs;
- measures the range to the air object by two small-sized RSs;
- determines the coordinates of the aerial object by joint processing of radar information from two small-sized RSs.

A feature of the method is the use of a two-position network of small-sized RSs to determine the coordinates of an aerial object.

The accuracy of determining the coordinates of aerial objects in a two-position network of small-sized RSs was evaluated. We derived expressions for evaluating the MSE coordinates of aerial objects (expressions (5), (6)). Experimental evaluation was carried out by means of simulation using the method of Monte Carlo statistical tests. We formulated limitations and assumptions adopted during the simulation. We calculated working zones of a two-position network of small-sized RSs (Fig. 7). It was established that the two-position network of small-sized RSs works only in the area formed by the intersections of the observation zones of small-sized RSs during their autonomous operation (shaded areas in Fig. 7). It is shown that, if the detection range of a small-sized radar under an autonomous mode is B km (the distance between small-sized RSs (base)), then the operation of a two-position network is possible in area 1 in Fig. 7. If the detection range of a small-sized radar under an autonomous mode is 2B km, then the operation of a two-position network is possible in area 2 in Fig. 7. The size of region 2 in Fig. 7 is significantly larger than the size of region 1 but the MSE in region 2 is also higher.

Fig. 6, 7 demonstrate that a two-position network of small-sized RSs provides determination of the coordinates of an aerial object only in the sector (shaded area in Fig. 7). Enlarging this sector is the subject of further research.

Fig. 7 shows that if the detection range of a small-sized radar under an autonomous mode is B km (the distance between small-sized RSs (base)), then the operation of a two-position network is possible in area 1 in Fig. 7. If the detection range of a small-sized radar under an autonomous mode is 2B km, then the operation of a two-position network is possible in area 2 in Fig. 7. The size of area 2 in Fig. 7 significantly exceeds the size of area 1 but MSE in area 2 is also higher.

Fig. 6, 7 demonstrate that a two-position network of small-sized RSs provides determination of the coordinates of an aerial object only in the sector (shaded area in Fig. 7). Enlarging this sector is the subject of further research.

6. Discussion of results of the study on improving the accuracy of determining the coordinates of an aerial object

It was established that when determining the coordinates of an aerial object with a small-sized radar, the accuracy of determining the range is much better than the accuracy of determining the angular coordinate. To eliminate this shortcoming, a two-position network of small-sized RSs and their error ellipses were considered (Fig. 2). Owing to the intersection of the error ellipses of two small-sized RSs, the accuracy of determining the coordinates of aerial objects is significantly increased, especially in terms of angular coordinates (Fig. 3).

7. Conclusions

1. A method for determining the coordinates of aerial objects in a two-position network of small-sized RSs has been improved, which, unlike the known ones:

- enables a synchronous survey of the airspace by small-sized RSs;
- measures the range to the air object by two small-sized RSs;
determines the coordinates of the aerial object by joint processing of radar
information from two small-sized RSs.
A feature of the method is the use of a two-position network of small-sized RSs to determine the coordinates of an aerial object.

2. An assessment of the accuracy of determining the coordinates of aerial objects in a two-position network of small-sized RSs was carried out. We derived expressions for estimating the MSE coordinates of aerial objects. The experimental evaluation was carried out by means of simulation using the method of Monte Carlo statistical tests. We calculated working zones of the two-position network of small-sized RSs. It was established that the two-position network of small-sized RSs works only in the area formed by the intersections of the viewing areas of small-sized RSs during their autonomous operation.

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### Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

### Funding

The study was conducted without financial support.

### Data availability

All data are available in the main text of the manuscript.
Information and controlling system

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