This paper considers a thin-walled steel beam-wall with broken edges, which is part of many structures. The wall of this beam consists of two prismatic parts with a straight transition from a lower wall height to a higher one, forming a broken upper edge together with the edges of the prismatic parts. The bottom straight edge of the wall is attached to the cladding.

The beam-wall is affected by static and cyclic nominal loads, which can cause the appearance of elastic-plastic deformations in the stress concentrator. This causes non-fulfillment of static strength and the appearance and growth of fatigue cracks.

In the current work, procedures of design and verification calculation of a steel beam-wall with fractured edges at elastic static and cyclic elastic-plastic deformation in the stress concentrator are proposed. The material of the beam is ideal elastic-plastic.

Features of the procedures are the possibility of optimal design under conditions of elastic and elastic-plastic deformation, using dependences only for optimal elastic design. A distinctive signature of the procedures is that, through Neiber's formula, elastic-plastic characteristics are not determined by known elastic ones, as usual, but vice versa. According to the established dependences for cyclic elastic-plastic deformations in the concentrator, the theoretical concentration coefficient is determined, which, in turn, is involved in determining the optimal geometric parameters.

The procedures give reliable results with nominal symmetrical cyclic loads up to 0.6 of the yield strength. This is because Naber's formula always yields conservative results, causing excess strength.

The procedures can be applied separately for stretching-compression and bending, and with their combined action

Keywords: beam-wall, broken edge, optimal design, low-cycle fatigue life, cyclic elastic-plastic deformations

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DEVISING PROCEDURES FOR DESIGN AND VERIFICATION CALCULATIONS OF THE BEAM-WALL WITH EDGE BREAKS UNDER STATIC AND CYCLIC LOADS

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1. Introduction

In civil construction and in the composition of a vessel's hull, there are still structural elements of the design methodology that are absent or have a narrow focus. One of these structural elements is a beam-wall (Fig. 1), which is attached to the cladding I by the lower straight edge 2, and the upper broken edge is free.

Beam-wall in Fig. 1 is subjected to axial loads p_1, which can be caused, for example, by the general bending of a vessel's hull and bending loads p_2 from the local bending of the floor.

Beams of this type can work both under elastic and elastic-plastic deformation conditions, under static and cyclic loads. The appearance of plastic deformations in the stress concentrator 6 (Fig. 1) leads to non-fulfillment of static strength, and cyclic loads cause the appearance and growth of fatigue cracks in the concentrator. This forces us to consider the calculation and design of such a beam from the point of view of fatigue life, and the presence of elastic-plastic deformation complicates this problem. Moreover, loss of stability is possible. For the considered beam, there are modes of operation when elastic-plastic deformation cannot always be avoided, which must be taken into account. For example, if such a beam-wall is part of the ship's hull or floating structures, then the elastic-plastic deformation of the beam-wall is ensured during the following operations of the vessel: unloading-loading, docking, extreme loads during waves, etc.

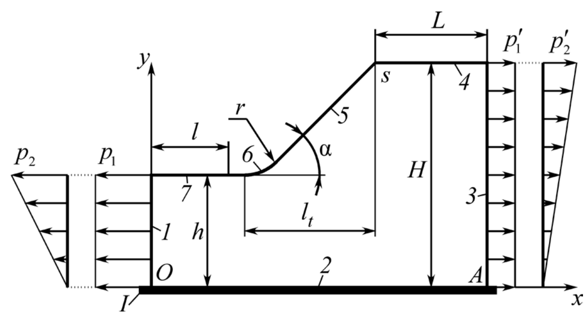

Fig. 1. General view of the beam-wall

The appearance of plastic deformations in concentrator 6 (Fig. 1) formally makes it impossible to use the beam-wall as part of civil structures because the standards do not allow it. The growth of fatigue cracks in concentrator 6 (Fig. 1) can cause the failure of complex assemblies, which include this beam. These are mostly ship structural assemblies. There is a need to calculate such a beam-wall under the action of static and cyclic loads. Currently, there are no systematic procedures for assessing the strength and durability of such beam-walls (Fig. 1), and, accordingly, design recommendations for them. As a result, software packages have to be used every time for calculating beam-walls with broken edges, which leads to an increase in the complexity of designing structures in general at the initial stages.

Therefore, studies that consider a beam-wall with broken edges (Fig. 1), with the aim of devising appropriate procedures, are relevant.

2. Literature review and problem statement

The beam-wall operates in the flat stress-strain state (SSS). Work [1] gives the dependences for the theoretical concentration coefficients in concentrator 6 (Fig. 1). In paper [1], the sizes and shapes of the plastic zones and the field of intensity of elastic-plastic deformations (in the form of contour lines) are established, depending on the geometric parameters and the applied load. Works [1, 2] present dependences for the intensity of elastic-plastic deformations in concentrator 6 of a steel beam-wall (Fig. 1) for an ideal elastic-plastic diagram. In [1], with static load, and in [2] – with symmetrical cyclic nominal load. Work [2] is the development of study [1]. In [1, 2], the influence of technological factors, such as the weld, was not taken into account. Under real conditions, the weld, as a rule, is far away from concentrator 6 (at least 50 mm, in the most unfavorable case 25 mm for gas welding), to avoid thermal effects. The material of works [1, 2] will be used for devising procedures within the framework of the current study.

Our review of the literature was aimed at considering general optimization algorithms for structures that could be used to devise procedures of design and verification calculations for the beam-wall in Fig. 1. This is due to the fact that there are no systematic procedures for calculating static and cyclic strength for the beam-wall under study (Fig. 1), and therefore it is necessary to look for a tool for devising these procedures.

Work [3] follows Prager's approach in optimization search, which has its well-known advantages and disadvantages. The search for the development of methods of optimal design of structures in book [3] followed two slightly different paths. Optimality conditions in [3] usually have an analytical nature, and therefore their strict satisfaction is limited to relatively simple and not always practical structures, which calls into question in some cases the approaches of work [3].

Paper [4] reports new ways of formulating construction optimization problems and techniques for solving them in comparison with study [3]. However, work [4] mostly offers topological optimization based on the analysis of the direct stress-strain state (SSS) in the elastic region. This will force us to impose restrictions on edges 4, 5, 7 by keeping them straight, and edge 6 in the form of a circle (Fig. 1), which will complicate the procedure for building an optimization model even for an elastic region. Paragraph 4.4 "Optimization of Stress Concentration for Elastic Plates with Holes" [4] is interesting, where the optimization of the concentration factor is discussed. But under the conditions of the optimization study of the beam-wall (Fig. 1), the concentration coefficient is constant, in order to be able to link the elastic and elastic-plastic problems.

The optimization methods in [5], as in study [4], work directly with SSS and topology, which creates the same problems that were considered in the analysis of work [4]. Paper [6] mostly discusses the implementation features of numerical optimization procedures already known, and partially discussed in papers [3–5], which can be useful only for software implementation of algorithms in the future.

Many new authentic algorithms are proposed in [7], which, in contrast to works [3–6], are more practical. However, the algorithms in [7] also work directly with the geometric parameters and SSS of structures, and do not provide an opportunity to combine, for example, elastic and elastic-plastic deformation, as for a beam-wall (Fig. 1).

A general feature of works [3–7] is that optimization procedures are discussed separately for each type/kind of deformation, and their combination requires discussion. Works [3–7] contain provisions and authentic procedures and understanding of the problems of optimal design, which cannot cover all the features of the operating conditions of structures.

This makes it necessary to carry out optimization studies of the beam-wall separately under elastic, plastic, and even cyclic deformation, which will increase the complexity of the study, although it will give more accurate results.

Work [8], which reports a method of optimal design of structures taking into account the requirements of fatigue life, is devoid of the indicated shortcomings of studies [3–7]. The basic idea is to use load history data in combination with finite element stresses and material fatigue properties to calculate fatigue life in an optimization process. Durability requirements are taken into account as lateral constraints, and the weight of the structure as an objective function. Deformation criteria are used in the optimization process, and the connection between the nominal stresses and the characteristics of elastic-plastic deformation in the concentrator is carried out using Neiber's formula. The prototype of this procedure [8] (with some modifications) was used in the development of design dependences for the investigated beam-wall under cyclic loading for the calculation of fatigue life.

Some applications of the new procedure of structural optimization of shape to maximize fatigue life or inspect gaps for damage-resistant structures are presented in [9]. As part of the reported procedure, a new simple method called FAST (Failure Analysis of Structures) was used to estimate the stress intensity factor (SIF) for cracks in the notch. The FAST method, which is an extension of the biological algorithm, was applied to study the problem of fatigue life optimization as a design goal. In this method, fatigue life is considered through crack parameters. For elastic deformation, the method from [9] promises to be acceptable by the authors. However, for engineering use, the approach from [9] remains complicated because in work [9] the SIF and crack growth parameters are involved. This requires additional theoretical and experimental studies for concentrator 6 (Fig. 1), which negates the ready-made developments in [1, 2]. The reliability of the proposed approach [9] in the case of elastic-plastic deformation in concentrator 6 (Fig. 1) is uncertain.

Paper [10] considers structural topological optimization with restrictions that take into account fatigue life in the multi-cycle region, which already makes it impossible to apply [10] for low-cycle fatigue life in concentrator 6 (Fig. 1). However, in the future, the findings of [10] may be useful in the development of universal procedures of fatigue life, both for multi- and low-cycle areas.

Topological optimization is applied in [11] to discuss the problem of fatigue load resistance. Fatigue life is maximized by optimizing the shape of the structure under cyclic plasticity in conjunction with Lemaitre's failure law. The topological optimization algorithm is detailed. However, in order to develop regular dependences of beam-wall design under conditions of elastic-plastic deformation, it is necessary to carry out serial calculations again using this method.

Work [12], where research is conducted using neural networks, can be attributed to the original methods for optimizing structures taking into account low-cycle fatigue. This type of research is quite complex and as a result not very common to be used for routine research.

Plots of the optimal geometric parameters of the beam-wall in Fig. 1, taking into account technological factors, are given in [13]. A more detailed discussion of paper [13] will be given below.

Our review of the literature revealed that for the development of optimal design procedures, it is necessary to re-build the objective functions and constraints for the beam-wall, which will complicate the research.

Thus, the review of the literature showed that there are no ready-made procedures of design and verification calculations for the beam-wall under any type of deformation. The use of optimization algorithms and procedures from [3–12], except for [8], will cause the fact that separate algorithms will have to be developed for devising optimal design procedures for elastic static and elastic-plastic cyclic loads. This will increase the complexity and time of development of the specified procedures for the beam-wall in Fig. 1.

One of the simplest ways out of this situation can be that on the basis of the results [1, 2] for the beam-wall, using [8], design and verification procedures can be devised. These procedures will make it possible, for two types of deformation of a completely different nature (for elastic static and elastic-plastic cyclic), to perform optimal design of the beam-wall using procedures for elastic deformation only. This convenience is contrasted with the rest of the procedures from [3] to [12], except for [8], which separately offer optimal design procedures for elastic, elastic-plastic, fatigue cyclic deformation, etc.

3. The aim and objectives of the study

The purpose of this study is to devise procedures for designing and verifying calculations of a beam-wall with edge fracture under conditions of static elastic and cyclic elastic-plastic deformation of the concentrator, which will make it possible to assess strength and fatigue life.

To achieve the goal, the following tasks must be solved:

– to build analytical dependences for the optimal design methodology for static elastic deformation;

– to compare the fatigue life results obtained on the basis of the developed dependences with the experimental-theoretical method (ETM);

– to carry out the transformation of the Neiber formula in order to relate the elastic and elastic-plastic deformation parameters in the concentrator;

– on the basis of the dependences for the optimal design procedure at static deformation and the developed dependences based on the deformation criteria, to devise procedures for calculating the beam-wall at cyclic deformation.

4. The study materials and methods

The object of our study is a beam-wall with broken edges (Fig. 1). The hypothesis is accepted, according to which the neutral layer of the beam-wall coincides with the lower edge 2 (Fig. 1). This hypothesis is justified by the fact that with a gradual increase in the width of the attached belt of the cladding I (Fig. 1), the neutral layer approaches edge 2, and the stresses in concentrator 6 increase at the same time. At the limit transition, when the width of the attached cladding belt I (Fig. 1) tends to infinity, the neutral layer of the beam-wall coincides with edge 2. In this case, edge 2 has no vertical displacements, and in concentrator 6 (Fig. 1) the greatest stress occurs, compared to other load schemes.

It was assumed that the external nominal cyclic load is symmetric, of constant amplitude, which makes it possible to take into account the most unfavorable operating condition. To simplify the study of the stress-strain state, the influence of technological factors was not taken into account.

The research method is numerical methods, which are used in the search for unknown parameters of empirical dependences by the method of least squares. This is, in particular, one of the implementations of the simplex method, the method of conjugate gradients, the method of Newton and the fastest descent, developed by us in the C++ programming language.

The subject of the study is the geometric parameters of the beam-wall (Fig. 1).

5. Results of research into optimal design and fatigue durability

5. 1. Construction of analytical dependences for the design methodology at static elastic deformation

Paper [13] gives charts of the optimal geometric parameters for the beam-wall (Fig. 1), such as the angle α of edge 5 and the relative radius of rounding r/h of edge 6 (Fig. 1). At the same time, the fixed values were the wall height ratio H/h and the concentration coefficient k_1 for stretching-compression and the concentration coefficient k_2 for bending. The objective functions ensured the minimization of the mass of wastes, time, cost, and effort of manufacturing, taking into account the constraints related to $k_{1,2}$. The values of optimal α and r/h were obtained for the upper limit $r/h \leq r_{h\,\max}=1$. There are also (very) rough approximations of these charts, which need to be replaced or clarified for the possibility of their inclusion in design procedures. The statement of the optimization problem in work [13] was formulated as follows: to find the optimal α and r/h for given H/h and $k_{1,2}$, minimizing the objective functions. To this statement, the following formulation of the problem must be added: find the optimal α for given H/h, $k_{1,2}$ and r/h, minimizing the objective functions. Because r/h may be subject to technological limitations related to hardware capabilities, which will be discussed in more detail below.

Thus, for the convenience of further explanations, we define two optimization problems:

1) find the optimal α and r/h for given H/h and $k_{1,2}$, minimizing the objective function described in [13];

2) find the optimal α for the given H/h, $k_{1,2}$ and r/h, minimizing the objective function described in [13].

Stretching-compression is a more dangerous stress state than bending, so in the following explanations attention is paid to stretching-compression. For the convenience of further explanations, plots of the optimal geometric parameters in stretching-compression, taken from work [13] for the 1st optimization problem, are shown in Fig. 2.

Shown in Fig. 2, the dependences have the following features. At values of H/h from 1.1 to 1.8, the moment of disappearance of the inclined straight edge 5 occurs at the beginning of the inclined straight lines that appear with increasing argument k_1 (Fig. 1). Analytically, it can be written as follows:

$$\left.\begin{array}{l}\text{if } H-h>r(1-\cos\alpha)\rightarrow \text{edge 5 exists,}\\ \text{if } H-h\le r(1-\cos\alpha)\rightarrow \text{edge 5 absent.}\end{array}\right\} \quad (1)$$

If there is no straight edge 5, then the angle α is defined as follows:

$$\alpha=\arccos\left(1-\frac{H-h}{r}\right),\ \text{rad}. \quad (2)$$

The curvilinear dependences of angle α on concentration coefficients k_1 correspond to the first condition (1) when the straight edge 5 exists (Fig. 1). Rectilinear dependences correspond to the second condition (1) when edge 5 does not exist, and there is only edge 6.

If, under the conditions of operation, the beam-wall is subjected to only stretching-compression load p_1, then the optimal geometric parameters α and r/h must be determined according to plots in Fig. 2, depending on the concentration coefficient k_1.

If, under the conditions of operation, the beam-wall is only loaded by bending p_2, then the optimal geometric parameters α and r/h must be determined according to the corresponding plots in figures from [13], depending on the concentration coefficient k_2.

If the operation of the beam-wall is expected under the combined action of stretching-compression and bending, then the optimal geometric parameters α and r/h must be determined according to plots in Fig. 2, depending on the concentration coefficient k_1. This is due to the fact that the value of concentration coefficient k_2 at bending is always smaller with the same geometric parameters of the beam-wall. Performing calculations depending on k_1, the optimal concentrator parameters are obtained with some margin of safety.

Empirical dependences were built for α and r/h, which approximate the plots in Fig. 2. These dependences are presented below and correspond to formulas (3) and (4). The structures of empirical dependences were established as a result of studying and analyzing the dependences of α and/or r/h on each argument k_1 and H/h separately. The unknown parameters of the indicated empirical dependences were found by the method of least squares using one of the implementations of the simplex method, as well as Newton's method, the methods of conjugate gradients and the fastest descent to refine the results obtained by the simplex method. The values of concentration coefficients k_1 and the wall height ratio H/h lie within: $k_1 \in [1.2;\ 2.98]$, and $H/h \in [1.1;\ 3.0]$. The discrete values of k_1 and H/h within the specified limits are distributed uniformly through a constant step. When deriving empirical formulas for optimal α and r/h, 990 combinations of k_1 and H/h were used.

The optimal values of inclination angle α for fixed values of concentration coefficient k_1 in stretching-compression can be determined by the following dependences:

$$\left.\begin{array}{l}\alpha=\begin{cases}(0.09\ln H_h+0.12)\times\\ \times\operatorname{atanh}\left(\sin\left(\left(\dfrac{0.33}{\ln H_h}+2.38\right)\cdot(k_1-1)\right)\right),\\ \text{if } H-h>r(1-\cos\alpha)\rightarrow\text{edge 5 exists;}\\ (1.55H_h-0.24)k_1-1{,}51H_h+\\ +0.22,\ \text{if } H-h>r(1-\cos\alpha),\\ \text{if } H-h\le r(1-\cos\alpha)\rightarrow\text{edge 5 absent;}\end{cases}\\ \text{if } \alpha>\dfrac{\pi}{2}\rightarrow\alpha=\dfrac{\pi}{2},\ \text{if } \alpha<0.1396,\ (8^\circ)\rightarrow\alpha=0.1396;\\ H_h=H/h,\ \alpha,\ \text{rad},\end{array}\right\} \quad (3)$$

where the values of angles α can be determined with an accuracy of ±14 % for curvilinear dependences (edge 5 exists) and with an accuracy of ±2 % for rectilinear dependences (edge 5 does not exist) relative to the results shown in the plots of Fig. 2, *a*.

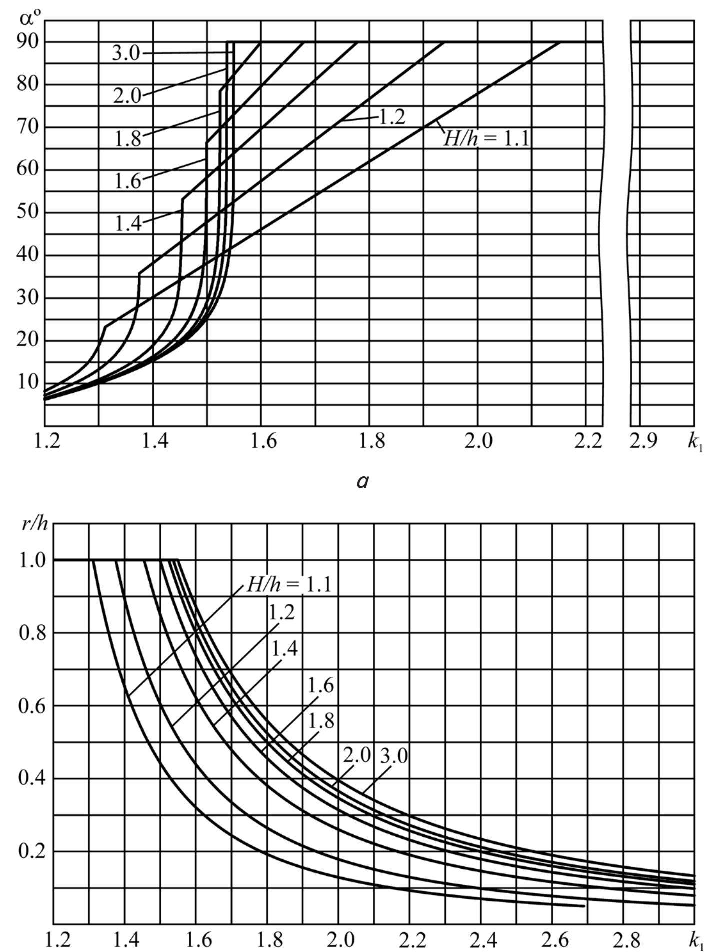

Fig. 2. Plots of optimal parameters depending on the concentration coefficient k_1 at stretching-compression and the wall height ratio H/h: *a* – dependence plot for the optimal angle α; *b* – dependence plot for the optimal relative radius r/h

The optimal values of the relative radius r/h for fixed values of coefficient k_1 can be determined by the dependences:

$$\left.\begin{aligned}&\frac{r}{h}=\frac{0.31\tanh\left(1.68H_h-1{,}47\right)}{\ln\left(0.83k_1\right)}-\\&-0.21\tanh\left(2{,}16H_h-1{,}94\right),\\&\text{if } r/h>1\rightarrow r/h=1,\ \text{if } r/h<0.05\rightarrow r/h=0.05;\\&H_h=H/h,\alpha,\text{rad},\end{aligned}\right\}\quad(4)$$

where the value of r/h can be calculated with an accuracy of ±7 % relative to the results shown in the plots of Fig. 2, *b*.

Formulas (3), (4) are valid under the following restrictions:

$$8°\leq\alpha\leq90°;\ \ 0.05\leq r/h\leq1;$$

$$1.2\leq H/h\leq3;\ \ 1.2\leq k_1\leq3.\quad(5)$$

The first expression (3) for α approximates curvilinear dependences (Fig. 2, *a*), the second expression approximates rectilinear ones. Control values calculated according to formulas (3), (4) are as follows:

1) when:

k_1=1.48; H/h=1,8; α=0.43752 rad≈25°; r/h=1,17639→1,0;

edge 5 exists:

$H-h$=0.8>r·(1–cosα)=1·(1–cos25°)=0.0937.

From the plot of Fig. 2, *a*, α=0.44528 rad; from the plot of Fig. 2, *b*, r/h=1.0;

2) when:

k_1=1.66; H/h=1.4; α=1.30980 rad≈75°; r/h=0.51739;

there is no edge 5:

$H-h$=0.4<r·(1–cosα)=1·(1–cos75°)=0.7412.

From the plot of Fig. 2, *a*, α=1.31805 rad; from the plot of Fig. 2, *b*, r/h=0.52939.

The peculiarity of formulas (3) is that for a given concentration coefficient k_1 it is not known in advance whether there is a straight edge 5 or not. Therefore, it is necessary to check the following condition for the established k_1:

$$\left.\begin{aligned}&k_1<k_{1d}\rightarrow\text{edge 5 exists},k_1\geq k_{1d}\rightarrow\text{edge 5 absent},\\&k_{1d}=0.56\tanh\left(1.98\frac{H}{h}-1.477\right).\end{aligned}\right\}\quad(6)$$

The given procedure of optimal design can be applied for static loads of a beam-wall under the action of load p, which corresponds to p_1 according to the type/kind of load (Fig. 1). Load p is the value of the nominal load determined at the level of the upper edge 7 of the smaller wall (Fig. 1). Moreover, if only stretching-compression takes place, then $p=p_1$, if bending only, $p=p_2$. If there is a combined action of stretching-compression and bending, then the nominal load p should be determined as:

$$p=p_1+p_2,\quad(7)$$

where p_1 and p_2 must be of the same sign to take into account the most unfavorable combination of these loads.

The maximum permissible concentration coefficient $k_{\max}$, which must be used when determining the optimal geometric parameters α and r/h, will be determined as:

$$k_{\max}=\frac{[\sigma]}{p\cdot n_k},\quad(8)$$

where n_k=1.2 is the reserve factor for the concentration factor; [σ] – permissible reduced stresses.

The coefficient n_k=1.2 is justified by the largest error (5 %) of the empirical formula (1) from paper [1] and the largest error (14 %) among the empirical formulas (3), (4).

The algorithm for finding the optimal geometric parameters α and r/h can be represented as follows.

1) set the maximum permissible value of the concentration coefficient k_1 according to (8) or from other conditions;

2) check condition (6) and determine whether edge 5 exists (Fig. 1);

3) with the applied H/h and the established k_1, find α according to the plot in Fig. 2, *a*, or from formula (3) based on the condition of point 2. Find r/h according to the plot in Fig. 2, *b*, or from formula (4).

There are other ways of constructing optimal research schemes related to the manufacturing technology of the structural assembly. The fact is that a free flange (Fig. 3) can be attached to the upper curved edge of the beam-wall (Fig. 1).

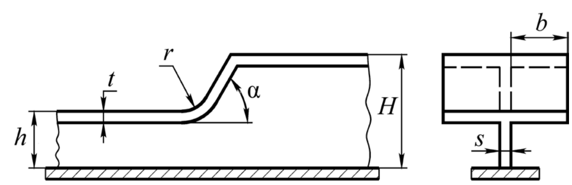

Fig. 3. General view of the investigated beam with broken edges

A technological limitation may be imposed on the radius r, which may depend on the thickness and mechanical properties of the material of the free flange or on the capabilities of the equipment. In this case, it is necessary to use any empirical dependence for $k_{1,2}$ reported in work [1]. In the selected dependence for $k_{1,2}$, it is necessary to set fixed values of $k_{1,2}$, H/h, r/h and solve the resulting nonlinear equation with respect to α.

The value of angle α at fixed k_1, H/h, r/h can be calculated using the following expression:

$$\left.\begin{aligned}&\alpha=16.7\,\text{atanh}\left(\frac{1.82\cdot\left(k_1-1\right)\cdot\left(\frac{r}{h}\right)^{\beta}}{\tanh\left(1.7\frac{H}{h}-1.2\right)}\right)\times\\&\times\tanh\left(1.2\frac{H}{h}-0.5\right)\cdot\left(\frac{r}{h}\right)^{\gamma},\\&\beta=0.7-0.16\frac{h}{H},\gamma=-0.04\frac{H}{h}-0.25,\alpha°,\end{aligned}\right\}\quad(9)$$

which was obtained from formula (1) from study [1].

In the case of a free flange, the rounding radius r, heights H, h are determined as shown in Fig. 3. This must be taken into account when making the wall. The radius of the wall without a free flange in this case will be $r_w=r+t$; small and large wall heights $h_w=h-t$, $H_w=H-t$.

As shown by numerical and field experiments, the material of stepped beams, which is above the line $\beta\approx 60^\circ$ (Fig. 4), practically does not participate in deformation during bending and/or stretching-compression.

This material is the so-called "dead zone", where the stresses are practically zero. That is, in the "dead zone" the material does not work against deformation. To reduce the mass of the beam, this material can be discarded, which, however, will increase the complexity of manufacturing and the mass of waste.

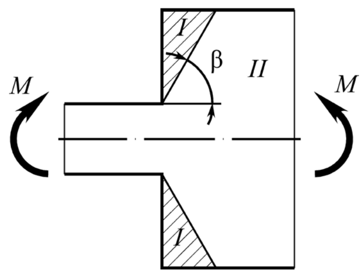

Fig. 4. Stepped prismatic beam during deformation: I – material of the beam that does not work during deformation or "dead zone"; II – the beam material is involved in deformation

5. 2. Comparison of fatigue life results

In work [2], the cyclic deformation of a steel beam-wall under the action of nominal loads p_1 was studied. Loads p_1 are cyclic, symmetrical, of constant amplitude. Under the influence of p_1, in concentrator 6 (Fig. 1), elastic-plastic deformation took place. The material of the beam-wall is ideal elastic-plastic. Empirical formulas for the amplitudes of the intensity of cyclic (full) elastic-plastic $\varepsilon_{i\ \text{cycle}}$ and plastic $\varepsilon_{ip\ \text{cycle}}$ deformations in concentrator 6 (Fig. 1) were derived for the specified conditions in work [2]. These empirical formulas are valid for the load level $p/\sigma_s\leq 0.65$ and will be used in the deformation criteria for fatigue life assessment.

The indicated amplitudes [2] of the intensity of cyclic (full) elastic-plastic $\varepsilon_{i\ \text{cycle}}$ and plastic $\varepsilon_{ip\ \text{cycle}}$ deformations take the form:

$$\left.\begin{aligned}&\varepsilon_{i\,\text{cycle}}=2\bar{\varepsilon}_i\cdot\varepsilon_{iey}\cdot 1.12\cdot 1.15,\\&\varepsilon_{ip\,\text{cycle}}=2\left(\bar{\varepsilon}_i-1\right)\cdot\varepsilon_{iey}\cdot 1.21\cdot 1.15,\end{aligned}\right\}\Rightarrow$$
$$\Rightarrow\left.\begin{aligned}&\varepsilon_{i\,\text{cycle}}=2.58\cdot\bar{\varepsilon}_i\cdot\varepsilon_{iey},\\&\varepsilon_{ip\,\text{cycle}}=2.78\cdot\left(\bar{\varepsilon}_i-1\right)\cdot\varepsilon_{iey},\end{aligned}\right\}\qquad(10)$$

where $\bar{\varepsilon}_i$ is the relative intensity of elastic-plastic (full) deformations, which depending on the applied load $p^{rel}=p/\sigma_s$ is determined by the following formula [1]:

$$\left.\begin{aligned}&\bar{\varepsilon}_i\left(p^{rel}\right)=a\cdot\exp\left(b\cdot\sin\left(p^{rel}\right)\right),\\&a=\left(\frac{1}{c}\right)^{\beta},\ b=\frac{\ln\left(\frac{1}{c}\right)}{p_{\min}^{rel}-0.9},\\&c=1.8\cdot\bar{\varepsilon}_{ip0.9}+1,\ \beta=-\frac{p_{\min}^{rel}}{p_{\min}^{rel}-0.9},\end{aligned}\right\}\qquad(11)$$

$\bar{\varepsilon}_{ip0.9}$ is the relative intensity of plastic deformations at the relative load $p/\sigma_s=0.9$, which is determined by the following empirical dependence [1]:

$$\left.\begin{aligned}&\bar{\varepsilon}_{ip0.9}=a_0 r_h^{\beta};\\&a_0=1{,}4\cdot\tanh\left(H_h\right)-0.14\cdot\alpha;\\&\beta=-b_0\cdot\tanh\left(\alpha\cdot b_1\right);\\&b_0=0.12H_h+0.65;\ b_1=-0.44H_h+2.84;\\&r_h=r/h,\ H_h=H/h,\ \alpha,\ \text{rad},\end{aligned}\right\}\qquad(12)$$

ε_{iey} is the intensity of elastic deformations at which plastic deformation begins in the concentrator, which is determined by the dependence:

$$\varepsilon_{iey}=\frac{\sigma_s}{3G},\qquad(13)$$

where σ_s is the yield strength; G – shear modulus;

$p_{\min}^{rel}$ is the minimum relative load at which elastic-plastic deformation begins in the zone of stress concentration, which is determined by the expression:

$$p_{\min}^{rel}=1/k_1.\qquad(14)$$

Before using formulas (10) to (12), it is necessary to make sure that the beam-wall is under the conditions of elastic-plastic deformation. For this, it is necessary to ensure the fulfillment of the following conditions:

$$p_{\min}^{rel}\leq p^{rel};\quad p_{\min}^{rel}=1/k_1;\quad p^{rel}=p/\sigma_s.\qquad(15)$$

Formulas (10) to (12) are valid for the range of parameters:

$$H/h\in\left[1.6;2.4\right];\quad r/h\in\left[0.05;0.2\right];\quad \alpha\in\left[20^\circ;75^\circ\right].\qquad(16)$$

Fatigue durability was evaluated for uniaxial (one type of load) cyclic symmetric load $p=p_1$ of constant amplitude (Fig. 1) under soft load conditions. The analysis of fatigue life of concentrator 6 of the investigated beam-wall was carried out in a symmetrical cycle of the cyclic diagram using deformation criteria and the experimental-theoretical method (ETM) [14, 15].

Table 1 gives a number of the most used and specific deformation criteria for assessing fatigue life during elastic-plastic deformation for symmetric cyclic diagrams. These criteria are defined for materials with a strength limit of $\sigma_u<700$ MPa and are taken from work [16], which in turn refers to the original sources.

The following notations are used in the above formulas:

ε – range of intensity of elastic-plastic deformations of the cyclic diagram;

$\varepsilon_f=\ln\dfrac{1}{1-\psi_k}$ – true relative elongation at the place of formation of the neck after rupture, or true fracture ductility;

ψ_k – relative narrowing at the place of formation of the neck after rupture;

σ_{-1} is the endurance limit for a symmetric cycle based on 10^6 cycles;

σ_u – strength limit; σ'_f, ε'_f – respectively, coefficients of fatigue strength and plasticity; b, c are constants.

Coefficients σ'_f, ε'_f, b, c are determined experimentally and depend on many parameters. Work [16] provides sources where you can find research and analysis of parameters σ'_f, ε'_f, b, c depending on various factors. In [16], it is shown (with reference to the relevant work) that for carbon and low-alloy ductile steels:

$$\left.\begin{array}{l}\sigma'_f=(0.92\ldots1.15)\sigma_f,\ \varepsilon'_f=(0.35\ldots1.0)\varepsilon_f,\\ b=-0.12,\ c=-0.6,\end{array}\right\} \tag{17}$$

where σ_f are true stresses at rupture.

Table 1

Deformation criteria for assessing fatigue durability

#	Criteria name	Formula
1	Manson's	$\varepsilon=\varepsilon_f^{0.6}\cdot N^{-0.6}+3.5\frac{\sigma_B}{E}N^{-0.12}$
2	Langer	$\varepsilon=\frac{\varepsilon_f}{2N^{0.5}}+\frac{2\sigma_{-1}}{E}$
3	Coffin-Manson	$\frac{\varepsilon}{2}=\frac{\sigma'_f}{E}(2N)^b+\varepsilon'_f(2N)^c$
4	V. Subramanya Sarma i K. A. Padmanabhan (SP)	$\varepsilon=1.17\left(\frac{\sigma_u}{E}\right)^{0.832}N^{-0.09}+0.0266\varepsilon_f^{0.155}\left(\frac{\sigma_u}{E}\right)^{-0.53}N^{-0.56}$
5	SWT (Smith-Watson-Topper)	$\frac{\sigma_{\max}\cdot\varepsilon}{2}=\frac{(\sigma'_f)^2}{E}(2N)^{2b}+\sigma'_f\varepsilon'_f(2N)^{b+c}$

For carbon and low-alloy ductile steels, the value σ_f can be calculated using the formula given in [17]:

$$\sigma_f=\sigma_u(1+1.4\psi_k). \tag{18}$$

The most conservative estimate when used (17) occurs when:

$$\sigma'_f=0.92\sigma_f,\quad \varepsilon'_f=0.35\varepsilon_f, \tag{19}$$

And the most progressive assessment will be when:

$$\sigma'_f=1.15\sigma_f,\quad \varepsilon'_f=1.0\varepsilon_f. \tag{20}$$

Next, a comparison of the results obtained by ETM with the results obtained by deformation criteria by Manson, Langer, Coffin-Manson, SP, SWT for a symmetrical cycle is given (Table 1). Explanations are made for the structural assembly with parameters H/h=2.0; α=60°; r/h=0.16, made of St3 steel. The necessary initial data for calculations are given in Table 2.

Table 2

Initial data on the calculation of fatigue strength

#	Quantity name	Parameters	
1	Name of the material	St 3	09G2
2	Modulus of elasticity at stretching E, MPa	$2\cdot10^5$	$2\cdot10^5$
3	Yield strength σ_s, MPa	235	340
4	Yield strength σ_u, MPa	420	460
5	Relative elongation after rupture δ_5	0.27	0.21
6	Transverse constriction after rupture ψ_f	0.57	0.67
7	Theoretical concentration coefficient k_1	2.62	
8	Minimum relative load at which plastic deformation begins, $p_{\min}^{rel}=1/k_1$	$\frac{1}{2.62}=0.382$	
9	Minimum absolute load at which plastic deformation begins $p_{\min}=p_{\min}^{rel}\cdot\sigma_s$, MPa	0.382·235=90	
10	Intensity of deformations at which plastic deformation begins, ε_{iey}	0.00101833	

When performing "binding" in ETM, it is necessary to take into account the probability of fatigue damage of the structural assembly, where the appearance of a fatigue crack of a certain length is accepted as fatigue damage. The lower the set probability of occurrence of fatigue damage, the fewer cycles are obtained per ETM. For "binding" in ETM, the model used is made of 09G2 steel with the following parameters (the index "0" means that the designations refer to the model): H_0=50 mm, h_0=25 mm, r_0=4 mm, α_0=60°. The theoretical coefficient of concentration during stretching is k_{10}=2.62. The ratio of the geometric parameters of the model and the investigated beam-wall are the same, which makes it possible to minimize the distortion of the results.

The parameters of the attachment point in ETM were obtained by statistical processing of the results of fatigue tests of the models. Two anchor points are applied, corresponding to the minimum and maximum number of cycles to failure of the model from the sampled results. The first anchor point has parameters: range of nominal stresses of the model σ_{nom01}=270 MPa; the corresponding number of cycles before the appearance of fatigue damage in the model (cracks ≈1 mm long) N_{01}=13700; cycle asymmetry coefficient R_0=0. The second anchor point has parameters: σ_{nom02}=238 MPa, N_{02}=17700, R_0=0. According to ETM, the S-N curve of the structure under study is constructed: the dependence of range of nominal stresses σ_{nom} on the number of cycles N, where the number of cycles varies within:

$$n\in(0,8),\quad n=\lg N. \tag{21}$$

Fig. 5 shows plots of the dependence of the number of cycles N on the range of nominal symmetrical cyclic stresses σ_{nom} applied to edge 1 (Fig. 1). In Fig. 5, the following curves are marked: $a_{1,2}$ – according to ETM, respectively for the 1st and 2nd anchor points; b – Manson's criterion; c – Langer's criterion; d – Coffin-Manson criterion; e – SP criterion; f – SWT criterion. Line g in Fig. 5 separates the elastic (below) and elastic-plastic regions (above), which is drawn at the level of $\sigma_{nom}=2\cdot p_{\min}$=180 MPa (Table 2).

Curves b–f (Fig. 5) were obtained as follows. According to the known range of nominal loads [$-p$; $+p$], applied to edge 1 (Fig. 1), the range of full cyclic elastic-plastic deformations $\varepsilon=\varepsilon_{i\ cycle}$ was determined according to the first formula (10). By substituting the obtained value of ε into the selected expressions of the deformation criteria (Table 1), the number of cycles N is determined, in which the equality of the left and right parts of the equations is ensured.

It is necessary to pay attention to the behavior of plots of the curves, which are obtained according to the deformation criteria, when using conservative (19) and progressive (20) values of the coefficients of fatigue strength and plasticity. When using progressive values of these coefficients, the specified plots are much closer to each other than when using conservative values. This means that the progressive parameters (20) provide a smaller scatter of the data according to the deformation criteria than the conservative parameters (19).

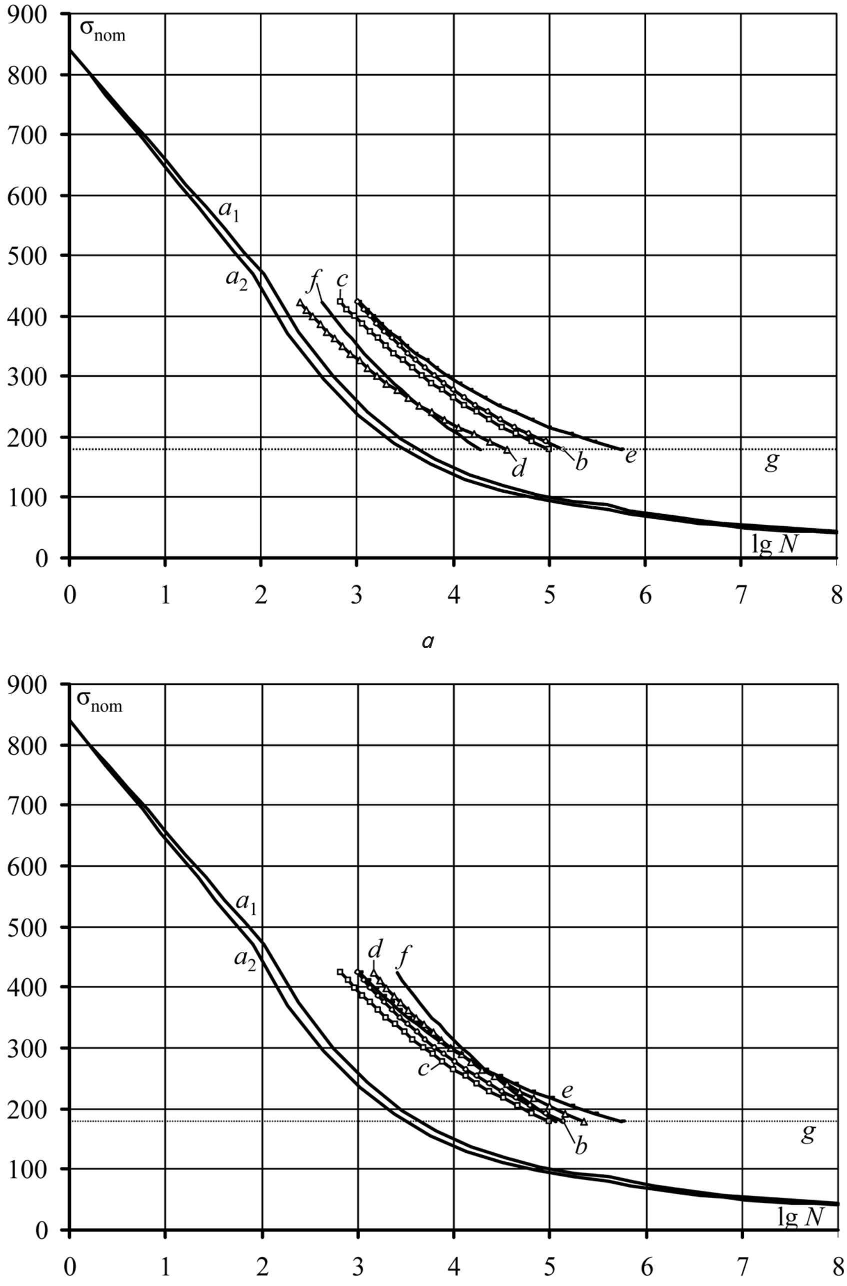

Fig. 5. Comparison of criteria for a symmetrical cycle: *a* — curves obtained by deformation criteria using (19); *b* — the same, but using (20)

5. 3. Conversion for the Neiber formula

As one knows, the classical Neiber formula looks like this:

$$\left.\begin{aligned}&\frac{K_\varepsilon\cdot K_\sigma}{K_t^2}=c,\ K_\varepsilon=\frac{\varepsilon_{i\,\max}}{\varepsilon_{i\,\mathrm{nom}}},\ K_\sigma=\frac{\sigma_{i\,\max}}{\sigma_{i\,\mathrm{nom}}},\\ &\sigma_{i\,\max}=\sigma_s,\ \sigma_{i\,\mathrm{nom}}=p,\ \varepsilon_{i\,\mathrm{nom}}=\frac{\sigma_{i\,\mathrm{nom}}}{3G},\end{aligned}\right\}\qquad(22)$$

where c=1 in the classic case, and in the general case it depends on parameters;

K_t, K_σ, K_ε – concentration coefficients, respectively, theoretical, elastic-plastic stresses, elastic-plastic deformations; $\varepsilon_{i\max}$, $\varepsilon_{i\mathrm{nom}}$ – intensity of elastic-plastic deformations in the concentrator and nominal, respectively; $\sigma_{i\max}$, $\sigma_{i\mathrm{nom}}$ – intensity of elastic-plastic stresses in the concentrator and nominal stresses, respectively.

After simple transformations from (22) the following is obtained:

$$\frac{K_\varepsilon\cdot K_\sigma}{K_t^2}=c,$$

$$c=\frac{3G\cdot\sigma_s\cdot\varepsilon_{i\,\max}}{p^2\cdot K_t^2}\Rightarrow$$

$$\Rightarrow K_t=\sqrt{\frac{3G\cdot\sigma_s\cdot\varepsilon}{(2\div 2.6)p^2\cdot c}},\quad c=1,\qquad(23)$$

where $\varepsilon_{i\,\max}$ is determined by (11), and $\varepsilon=\varepsilon_{i\,\mathrm{cycle}}$ is determined by (10).

Having the concentration coefficient K_t, determined by (23), it is possible to find the optimal geometric parameters of radius r of edge 6 and the angle of inclination α of edge 5 (Fig. 1) according to the procedures of optimal design of the beam-wall proposed above.

Additional "safety" coefficients in (23) are not applied because with c=1, it is possible to obtain a conservative estimate of the optimal geometric parameters of the beam-wall, and, as a result, increased metal capacity. In [1], it was proved that there is no point in correcting the classical Neiber formula for symmetric cycles of external load when c=1.

5. 4. Procedures for calculating the beam-wall under cyclic loading

Below there are the proposed procedures of design and verification calculations of low-cycle fatigue life of a beam-wall using deformation criteria within the framework of our study. These procedures can be used only for a monotonic cyclic load of a constant amplitude symmetrical cycle of the nominal load p.

The procedure of verification calculation of the beam-wall using deformation criteria is as follows:

1. Specify: range of nominal cyclic loads p, $[-p; +p]$ of constant amplitude; geometric parameters H, h, r, α; yield strength of the material σ_s.

2. According to the known parameters of point 1, determine the range of cyclic elastic-plastic deformations $\varepsilon=\varepsilon_{i\,cycle}$ at the dangerous point of the beam-wall stress concentrator according to (10).

3. Based on known ε, find the number of cycles N before the appearance of fatigue damage, using one of the deformation criteria (Table 1), if necessary, using the load and durability reserve factors.

A procedure of design calculation of a beam-wall using deformation criteria is as follows:

1. Set: the number of cycles N before the appearance of fatigue damage; height ratio H/h.

2. Using one of the deformation criteria (Table 1), find the range of cyclic elastic-plastic deformations ε at the dangerous point of the stress concentrator.

3. Based on known ε, calculate the theoretical concentration coefficient K_t according to (23).

4. Find the optimal values of r/h and α according to formulas (3), (4) or only α according to (9) if r/h is fixed.

The devised procedures can also be applied to bending, when only p_2 acts (Fig. 1), or when the stretching-compression p_1 and bending p_2 are combined and in phase, regardless of the p_1/p_2 ratio. In this case, the nominal load p should be determined according to (7), where p_1 and p_2 should also be of the same sign to take into account the most unfavorable combination of these loads.

6. Discussion of the results of optimal design and fatigue life

Dependences for α and r/h are shown in Fig. 1 and their corresponding approximations (3) and (4) obtained for the upper limit $r/h \leq r_{h\ max}=1$. With a gradual decrease in the value of $r_{h\ max}$, the straight lines in the plots of Fig. 1, *a* disappear. This leads to the fact that an increase in the concentration coefficient k_1 (at fixed $r_{h\ max}$) ensures the minimization of the objective function more often at $r/h=r_{h\ max}$. At the same time, the angle α is mainly varied to ensure the minimization of the objective function. In further optimization studies for the beam-wall, it is necessary to introduce an additional parameter $r_{h\ max}$, which represents the upper limit of the radius.

The presence of a free flange, which can be attached to the upper broken edge of the beam-wall, changes the geometry of the stress concentrator and the value of the concentration coefficients in it. More dangerous are concentrators at the junction of the free flange with the wall and in the weld that connects them (if any).

The larger the values of $k_{1,2}$ are taken when designing the beam-wall, the better its manufacturability and the lower the manufacturing cost, but the worse the margin of fatigue strength and crack resistance under variable loading.

If a designer can guarantee that the directions and signs of p_1 and p_2 will not change throughout the life of the beam-wall, then p_1 and p_2 in (7) can be taken with their signs. This applies only to static elastic loading.

In the design calculation of the inclined part of the beam-wall with a known static load p and the height ratio H/h, the concentration coefficient $k_{1,2}$ should be as large as possible. But at the same time, $k_{1,2}$ should be such that with a known nominal load p, the effective reduced stresses in the concentrator do not exceed the allowable ones.

In [1] it was shown that when $p/\sigma_s \leq 0.6$, the coefficient c from formula (23) is actually within [0.82; 0.97]. At $c=0.82$, the theoretical concentration factor K_t will be 1.1 times greater than the value of K_t when $c=1$. That is, the use of the conservative value $c=1$ in the last formula (23) will lead to obtaining an always lower K_t than it actually is. Low K_t improves fatigue life and crack resistance.

The presented algorithm will always provide a reliable estimate of the optimal geometric parameters separately under the action of stretching-compression p_1, separately under bending p_2, as well as under the combined action of stretching-compression and bending. In the case of joint action of stretching-compression and bending, and separately only in bending, the estimate will be conservative. However, under the action of only bending loads p_2 (Fig. 1), to obtain progressive optimal parameters α and r/h, it is possible to use plots from work [13], which are built for bending and depend on k_2.

Having an expression for the stress concentration coefficient, the beam-wall can always be designed in such a way as to provide only elastic SSS at the concentrator. In this case, it is possible to apply well-known strength criteria for fatigue life to predict fatigue life. Methods for calculating fatigue life using strength criteria are more developed, they can be found in classic works and are not considered here. Due to design features, there are cases when restrictions are imposed on angle α and radius r, in which it is no longer possible to avoid the appearance of plastic deformations in the concentrator for a given nominal load.

The deformation criteria (Table 1) used in this work are the simplest and were obtained for samples that were brought to failure. Fatigue cracks grow in thin-walled structures before their destruction. Therefore, it is the deformation criteria that relate the parameters of crack growth to the number of cycles and the range of the cyclic diagram that would be appropriate. However, at present, these criteria are in a state of formation and are developing in several directions using the same basic deformation criteria of the Coffin-Manson type, fracture mechanics, etc. So, it is still too early to argue about the possibility of their implementation in the fatigue life assessment methodology.

Analyzing the plots in Fig. 5, it can be concluded that the ETM fatigue strength assessment gives the most conservative results. This is explained by the fact that it is based on Neiber's formula, which gives inflated values of elastic-plastic deformations, which in turn leads to an underestimated number of cycles before the appearance of fatigue damage. With the same value of the range of nominal stresses, the number of cycles according to deformation criteria is always higher than according to ETM. As the range of nominal stresses decreases, the difference between the number of cycles obtained by deformation criteria and by ETM increases, and the number of cycles by ETM is always smaller. The "binding" applied provides a 50 % chance of fatigue damage. If the probability of this damage is assumed to be lower, then the ETM curves will be lower, and the fatigue assessment will accordingly be even more conservative.

When using the deformation criteria, it is necessary to understand that formulas of the Manson, Langer type are obtained for the probability of destruction of 50 %. And therefore, during practical calculations, two reserve coefficients are usually introduced: for load and for durability, which correspond to various programs of transition from given project (working) conditions to the limit state. These two coefficients are connected by a complex nonlinear relationship and can be found in the specialized literature. For different types of structures (civilian, ship), these coefficients will be different even for different places where similar beam-walls are installed. The study of these coefficients can be the subject of future research.

It should be noted that the proposed fatigue life assessment procedures contain intermediate links that provide conservative results. Therefore, the procedures of fatigue life will also be conservative, although more progressive in comparison with the procedures of S-N curves, such as ETM, because the cyclic elastic-plastic deformation in the concentrator is more accurately taken into account.

In future studies, for example, another Neiber-like formula (e.g., Hlinka, etc.) that relates elastic and elastoplastic characteristics can be applied to obtain a more advanced estimate. This issue needs additional study. Another way is to improve Neiber's formula depending on the geometric, mechanical, and force parameters.

The procedures devised for the beam-wall apply to a wide range of geometric and strength parameters and begin to close the currently almost unfilled niche of design procedures for beams with broken edges in general.

For a more progressive assessment of the optimal parameters based on (10), it is possible to build plots of the optimal values of r and α similar to the plots in Fig. 1. This is due to the fact that the deformations in (10), in turn, are expressed through geometric parameters and the nominal load p.

The features of the proposed procedures are that elastic and elastic-plastic deformation are connected in them, owing to the use of the ideas from work [8], which was discussed above. This distinguishes our procedures from those given in [3–7, 9–12], where optimal solutions for each type of deformation are considered separately. In the case of using the approaches from [3–7, 9–12], the optimization problem would have to be solved for the case of elastic and elastic-plastic deformation depending on the parameters of the rigid deformable body. In the case of the beam-wall in Fig. 1, the optimal dependences of the geometric parameters are obtained once for the elastic problem, which, based on the ideas from [8], are related to the elastic-plastic problem of cyclic deformation.

The proposed procedures of calculation and design of the beam-wall make it possible to carry out evaluations for the most unfavorable operating conditions and represent one of the boundaries of the (virtual) polygon of strength evaluation, which contains admissible solutions.

The limitations of the study are as follows:

1. The nominal load p should be symmetrical, of constant amplitude, with a value of no more than 0.6 from the yield strength of the beam-wall material. This type of loading is the most unfavorable for the beam-wall, which will give conservative results. With asymmetric p cycles, the durability assessment will be more progressive.

2. With simultaneous action of stretching-compression p_1 and bending p_2, loads p_1 and p_2 must be in phase and of the same sign.

3. The material of the beam is steel, and the material model is ideal elastic-plastic, which makes it possible to conduct calculations for materials that have a horizontal yield point. Hardening, softening, Bauschinger effect, etc. are not taken into account. That is, alloyed steels and high-strength steels cannot be considered as a material for a beam-wall with broken edges in Fig. 1.

4. Elastic-plastic deformation must be ensured in the concentrator, and for this it is necessary to always monitor the fulfillment of condition (15). That is, the assessment of fatigue life in the case of cyclic elastic deformation in concentrator 6 (Fig. 1) using these procedures is impossible. The last point can also be attributed to the shortcomings.

Disadvantages of the study are some uncertainties in the fatigue damage criterion in concentrator 6 (Fig. 1). Usually, this criterion is the length of the fatigue crack, but it can vary for different approaches. Applying the deformation criteria, failure is formally considered to be the destruction of the microvolume at the dangerous point of concentrator 6. This approach correlates with S-N curve methods. However, in fact, fatigue cracks grow in concentrators depending on the number of cycles, for which appropriate criteria have been created, which are still in the formative stage.

Disadvantages include the conservative results of the assessment of fatigue life with the combined action of stretching-compression and bending or only bending. The procedures are focused on stretching-compression as the most dangerous stress state.

The development of this research can be as follows:

1. Establishing dependences for the values of elastic-plastic cyclic deformations in concentrator 6 (Fig. 1) depending on the geometric parameters, the applied load and the asymmetry of the cycle. This will make it possible to more accurately predict fatigue life and obtain more progressive estimates.

2. Neiber's formula, which relates elastic and elastic-plastic parameters, should be clarified, or replaced with similar ones.

3. Develop procedures for determining optimal parameters not using theoretical concentration coefficients, as was done in [13], but using the range of cyclic elastic-plastic deformations. This will make it possible not to use Neiber's formula at all, as a connecting link between elastic and elastic-plastic deformation but will increase the overall complexity of research.

4. Instead of simplified deformation criteria, use criteria that take into account the speed of crack growth in the proposed procedures.

5. Investigate the cases when loads p_1 and p_2 are not in phase, of different frequency and amplitude, which will be reflected in the type of applied deformation criteria.

6. Taking into account a more accurate material model that takes into account hardening and softening, the Bauschinger effect, etc.

7. The above-mentioned area of research, in addition to steel, can be carried out for other types of materials: aluminum, titanium alloys, etc.

The proposed procedures may be further advanced in order to take into account the influence of technological factors, such as the presence of a weld seam, corrosive wear, quality of edge 6 processing (Fig. 1), etc. The specified factors can be taken into account by coefficients or functional dependences on cyclic elastic-plastic deformations, the number of cycles, the value of the nominal cyclic load or other parameters.

The procedures devised will make it possible to reduce the design time of structural elements that include a beam-wall with broken edges in the early stages of design. It will not be necessary to involve the computing power of calculation systems every time. Instead, you can immediately obtain the geometric parameters of the beam-wall under the given conditions.

Our procedures can be proposed for inclusion in the design standards of civil engineering and shipbuilding.

7. Conclusions

1. Analytical dependences in the optimal design procedure for static elastic deformation were developed for stretching-compression only, as the most dangerous type of deformation. The peculiarity of the procedures is that they provide optimal parameters α and r/h for fixed height ratios H/h and concentration coefficients k_1. As a rule, the height ratio H/h is fixed and is determined by the methods of construction mechanics, and the coefficient k_1 allows one to link elastic static and elastic-plastic cyclic deformation, which have different physical nature. The specified procedure can be used both for bending separately and for the joint action of stretching-compression and bending, which, however, in both of these cases

will provide excess strength compared to stretching-compression. In bending, the theoretical concentration coefficient is 30 % lower compared to stretching-compression in the extreme case, with the same parameters.

2. The devised procedures at conservative and progressive values of fatigue strength and plasticity coefficients always give 30–40 % more progressive results than ETM, according to the indicator lg *N*, where *N* is the number of cycles.

3. To obtain an expression for the theoretical concentration coefficient, the classical Neiber formula was transformed, which will always provide an underestimated theoretical concentration coefficient compared to the actual one. In the extreme case, it will be 1.1 times less than the actual one if the stretching-compression load does not exceed 0.6 of the yield strength. The understated theoretical coefficient improves the indicators of fatigue life and crack resistance. This, in turn, will provide excess strength both for compression and bending separately, and when they are combined.

4. Calculation procedures of the beam-wall under cyclic loading allow design and verification calculations under the condition of elastic-plastic deformation of the beam-wall stress concentrator. These procedures were devised on the basis of known deformation criteria and developed dependences of the range of cyclic deformations. The use of these procedures will provide conservative results compared to real materials. This is due to the use of an ideal elastic-plastic model of the material, the classical Neiber formula, and taking into account the most unfavorable mode of operation, when the cyclic nominal load is symmetric. The amplitude of cyclic loads should not exceed 0.6 of the yield strength.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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