A study of a model of a discrete-continuous type of impactor in the energy transfer phase during the impact of a striker and a tool is presented. The device is used to destroy rocks, in construction equipment, and in the oil industry. In the mathematical model, the tool is represented by a rod with a variable profile, and the striker is a discrete element with a consolidated mass. The presence of rigid and dissipative connections models the impact interaction. The motion of the interacting elements of the impactor is described by a system of differential equations linked by boundary and initial conditions. The model allows determining the parameters of influence on the characteristic of the shock pulse at variable resistance of the working medium. The force of impact of a discrete element and the contact end of the rod is represented as a power law dependent on the difference in displacements of the contacting elements. The finite difference method is used to solve the initial boundary value problem. The parameters of the difference scheme were determined through modelling problems and were as follows: time step (1, ..., 5)·10^{-5} s; length step – (0.1...0.3) of the tool length, and for the mixed scheme – within 0.5...0.8. It was found that the time of striker-to-tool co-impact, depending on the stiffness coefficient, was 200...300 μs. With a load of up to 90 kN in the time range of 0...4 ms, the normal stresses in the tool sections at different times were 200...250 MPa. The combination of discrete and continuous elements simplifies the calculation scheme. It allows to determine the distribution of force characteristics in the cross-sections of the tool, the force and time of impact, and the influence of the working environment on these parameters. The developed model can be used to design impactors and optimize their parameters

Keywords: impact device, discrete-continuous model, co-impact force, boundary conditions, dissipative resistance

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DEVELOPMENT OF A DISCREET-CONTINUOUS MATHEMATICAL MODEL OF A PERCUSSION DEVICE WITH PARAMETERS OF INFLUENCE ON THE CHARACTERISTICS OF AN IMPACT PULSE

Viktor Slidenko
Doctor of Technical Sciences, Associate Professor*
Oleksandr Slidenko
PhD
"Fenix-K" Limited Liability Company
Kotelnikova str., 7, Kyiv, Ukraine, 03115
Liubov Marchuk
Corresponding author
Postgraduate Student*
E-mail: marchuk@lll.kpi.ua
Viacheslav But
Postgraduate Student*
*Department of Automation of Electrical and Mechatronic Complexes
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"
Peremohy ave., 37, Kyiv, Ukraine, 03056

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1. Introduction

Percussive devices with a linear drive (pneumatic, hydraulic, electromechanical, and others) are widely used in construction, mining, and oil production equipment. The design of high-speed technical impact machines involves the use of mathematical modeling methods due to the short interaction time of the design elements of these devices. One of the urgent tasks in the design of impulse devices is the task of increasing efficiency while reducing the recoil reaction on the device body.

The use of an adaptive hydraulic impact device is associated with the stochastic variability of the properties of the treated environment, which determines the presence of control elements in the design of the impact device – the striker. The amount of stroke of the hammer is regulated by the control system according to the readings of the sensors of the movement of the tool. When the resistance of the rock increases, the amount of penetration of the tool into the rock decreases and, with the help of the control system, the working stroke of the hammer increases. Accordingly, the impact energy increases. Thus, the main design parameters affect the magnitude of the impact pulse and the coefficient of maximum energy transfer to the processed medium. Modeling of the collision process of the striker with the tool and the development of the algorithm for the application of the numerical method of solving the initial-boundary problem, taking into account the resistance of the working environment, are relevant for determining the main design parameters.

2. Literature review and problem statement

General issues of optimization and directions of development of impact devices are given in [1], which emphasizes the relevance of the study of energy components during the interaction of elements of such devices.

Designing devices requires the development of mathematical models and algorithms for their research. The work [2] analyzes the interaction of the impact type drilling tool with the working environment. A model with elements of stiffness and plasticity is built. The model consists of discrete elements, which limits its wide use in practice. In addition, only the working environment is simulated. Models of impact devices with a linear electromagnetic drive are considered in works [3, 4]. In [3], a discrete model is studied, in which the main attention is paid to the optimization of the electromagnetic drive. Moreover, only low-frequency oscillations are considered. Discrete models are also used. In [4], a striker with an electromagnetic drive is considered. To increase the efficiency, the striker holding method is used, which is possible when using an electromagnetic drive. The calculation scheme uses the interaction of discrete elements. Such a model does not allow determining the stress in the tool, which arises as a result of the impulse action of the striker on the tool.

The cited works do not solve the problem of researching the high-frequency oscillations occurring in the tool and determining the force factors (tension, deformation) in the cross-sections of the tool. It should be noted the works in which the actual problems of optimization of the working bodies of percussion devices are discussed. The work [5] analyzes the optimization of impact devices using analytical and parametric methods. For this, special software "Adams" and the energy method are used. In work [6] it is emphasized that one of the methods of solving the listed problems is the use of impact devices with several strikers. In such devices, there is a continuation (prolongation) of the pulse and a reduction in the vibration of the case due to a reduction in the recoil reaction.

To obtain the stress distribution in the cross-sections of the tool, it is necessary to consider the tool as a rod of variable cross-section. The work [7] considers the improvement of impact efficiency due to the optimization of the shape of the striker. The shape of the tool is taken as simplified, and therefore such a model does not allow determining the interaction parameters in the striker-tool system. Experimental studies of the interaction of the tool with the working environment are carried out in [8]. These experiments confirm the need to study the impact of the striker shape and tool design on the working environment. The implementation of the program for calculating the parameters of the striker is given in [9]. The use of discrete elements and graphic methods do not solve the problem of determining the force characteristics and interaction parameters in the system of strikers – tool – working environment.

In [10], a discrete-continuous model is considered, in which, in the presence of a tool-rod, the masses of the striker and the body are taken into account during the impulse load on the tool from the part of the processed medium. That is, the interaction of the elements of the impact device at a given load on the tool was considered only from the side of the working environment. The interaction of the striker with the tool is not considered in this work. The study of such a system was carried out using the finite difference method. It is important to consider a similar scheme with an impulse load on the striker and the nonlinear force of the collision between the striker and the tool when simulating the resistance of the environment.

It should be noted that shock systems can have a variety of designs, which consist of a different number of elements. Their parameters depend, in particular, on the form and duration of the shock pulse generated in the environment being processed.

The general problem is to determine the main parameters of the interaction of the striker with the tool under the action of an impulse load on the striker, taking into account the variable resistance of the machining medium. Such parameters are: characteristics of high and low frequency oscillations of the instrument; the force of interaction and the time of co-impact between the striker and the tool; distribution of power characteristics in variable sections of the tool along the length and over time.

3. The aim and objectives of the study

The aim of the study is to create a discrete-continuous mathematical model that allows performing computational experiments with variation of stiffness and dissipation parameters when determining the impulse load. The dependence of the load on the pre-impact speed and external force provides opportunities to control the impact pulse characteristic.

To achieve the aim, the following objectives were set:

– build a calculation scheme: "discrete element – rod of variable section – working environment";

– formulate an initial boundary value problem with an ordinary differential equation and a wave equation in partial derivatives under boundary conditions with nonlinear stiffness coefficients;

– formulate the approximation of the differential problem by the difference problem, develop the algorithm for solving the difference problem and its implementation in the Mathcad system (USA);

– conduct computational experiments for various parameters of the impulse load and determine the stress distribution in the cross-sections of the variable profile tool by length and time.

4. Materials and methods of the study

The object of the study is the process of interaction of the striker with the tool of the impact device under impulse loads and in the presence of the resistance force of the processing medium. Longitudinal oscillations of the tool under the action of longitudinal impulse forces are considered.

The mathematical model consists of a discrete element and a rod of variable section, which are connected by rigid and dissipative elements. The resistance of the working environment is also simulated by rigid and dissipative connections.

The main assumptions and simplifications adopted in the work:

a) the cross section of the rod remains flat (hypothesis of flat sections);

b) the strength of the impact interaction depends on the difference in the displacements of the end face and the discrete element according to the power law;

c) movements and deformations of the rod sections depend only on the coordinate x and time t;

d) transverse oscillations of the rod are not taken into account;

e) the effect of the valve is described by a short-acting force in time.

Oscillation equations make up a system of two differential equations, ordinary and partial differential equations. Connections between elements are described by boundary conditions. The initial conditions specify the initial state of the system. This is how the initial boundary value problem is formulated. The force of interaction between the striker and the end of the tool is represented as a power dependence of the difference in their movements.

To find a solution to the problem, the finite difference method is used in combination with Euler's method. For partial linearization of the boundary condition, the sweep method is used. The parameters of the finite difference method are found using a model problem, which consists of the equations of motion of two discrete elements with elastic and dissipative connections, taking into account the resistance of the working medium.

The implementation of the finite difference method for the initial-boundary value problem and the Runge-Kutta method for the model problem is performed in the Mathcad system. The following should be noted regarding the conduct of computational experiments. The common computer program contains the solutions of the main and model problems. Graphs built in one coordinate system allow to compare the results of calculations and choose rational parameters of the numerical method. This approach ensures the necessary accuracy of solutions at short and long intervals of time.

5. Results of the study of the interaction process of the impact device elements

5. 1. Construction of the calculation scheme of the impact device

Fig. 1 shows the structural and calculation schemes of the impact device.

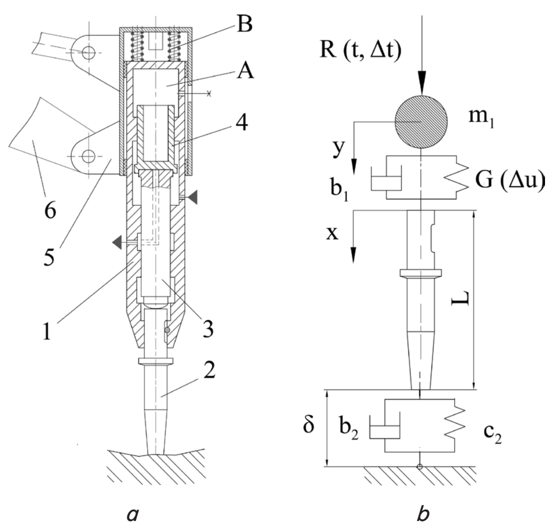


Fig. 1. Percussive device: *a* — structural diagram; *b* — calculation scheme; 1 — body; 2 — tool; 3 — fight; 4 — valve; 5 — adapter; 6 — manipulator; A — pneumatic accumulator chamber; B — depreciation block; m_1 — mass of the striker; $G(\Delta u)$ — stiffness function; c_2 — stiffness coefficient of the rock massif; b_1, b_2 — dissipation coefficients of the contact elements of the striker, tool and rock massif; L — length of the rod tool; δ — depth of the tool into the processed environment; $R(t,\Delta t)$ — external force

The calculation scheme of the impactor was built, which makes up the system "hammer – tool of variable section – processing (working) environment". The discrete element of the mass m1 was represented by the coupled striker (3) and the valve (4). It is taken into account that an external force $R(t,\Delta t)$ acts on a discrete element, which has a periodic or pulse character. The interaction between the striker and the tool was modeled by nonlinear rigid $G(\Delta u)$) and dissipative (b_1) connections. Similar connections (b_2, c_2) modeled the interaction of the tool with the working environment.

The following notations are introduced: $u(t,\ x)$ – displacement of the cross section of the rod with the x coordinate; t – time; $y(t)$ – displacement of the center of the discrete element with mass m_1; E – modulus of elasticity; ρ – density of the rod material; $a=\sqrt{E\rho^{-1}}$ – speed of sound in the rod material; $S=S(x)$ is the cross-sectional area of the tool rod.

The mathematical model that describes the collision process with the end of the tool in the presence of medium resistance is represented by an initial-boundary value problem.

5. 2. Formulation of the initial boundary value problem

The equations of motion of the cross-sections of the tool and the discrete element are presented in the form:

$$\frac{\partial^2 u(t,x)}{\partial t^2}=a^2\left[\frac{1}{S(x)}\cdot\frac{dS(x)}{dx}\frac{\partial u(t,x)}{\partial x}+\frac{\partial^2 u(t,x)}{\partial x^2}\right],$$

$$0<t\le T,\quad 0\le x\le L, \tag{1}$$

$$m_1\frac{d^2y}{dt^2}=R(t)+G(\Delta u)\cdot(u(t,0)-y)+$$

$$+b_1\frac{d}{dt}(u(t,0)-y). \tag{2}$$

The boundary conditions for the tool reflect the nature of the interaction of the ends with the discrete element and the working environment:

$$S(0)E\frac{\partial u}{\partial x}(t,0)=-G(\Delta u)(y(t)-u(t,0))-$$

$$-b_1\left(\frac{dy}{dt}-\frac{\partial u(t,0)}{\partial t}\right), \tag{3}$$

$$S(L)E\frac{\partial u}{\partial x}(t,L)=-c_2u(t,L)-b_2\frac{\partial u(t,L)}{\partial t}. \tag{4}$$

The initial conditions for the tool and the discrete element reflect the initial impulse impact of the discrete element on the end of the tool:

$$u(0,x)=0,\ \frac{\partial u}{\partial t}(0,x)=0, \tag{5}$$

$$y(0)=0,\ \frac{dy}{dt}(0)=W_1. \tag{6}$$

Conditions (6) mean that at the initial moment of time at zero displacement, the discrete element has a pre-impact velocity W_1.

The dependence of the coefficient of rigid connection of a discrete element with the end of the tool on the difference in displacements is determined by the formula:

$$G(\Delta u)=\begin{cases} c_1\cdot\Delta u^{\alpha}, \text{if } \Delta u\geq 0,\\ c_0, \text{if } \Delta u<0.\end{cases} \quad (7)$$

In formula (7) $\Delta u=y(t)-u(t,0)$, $0\leq\alpha\leq 0.5$.

Formula (7) models the contact interaction of a discrete element with the end of the tool.

Stiffness c_0 simulates the connection due to friction and is taken as a small value. In equation (2), the term R(t) simulates an external periodic force acting on a discrete element. The external force was given by various formulas:

$$R(t)=P\cdot|\sin(\omega t)|, \quad R(t)=P\cdot\sin(\omega t),$$

where ω – frequency of oscillations, which corresponds to the frequency of natural oscillations of the "hammer-valve-pneumoaccumulator" system, as a discrete element, at the moment of collision between the hammer and the tool.

The action of an external force on a discrete element during a short period of time Δt was also considered:

$$R(t,\Delta t)=\begin{cases} P, \text{if } 0\leq t\leq\Delta t,\\ 0, \text{if } t>\Delta t.\end{cases}$$

Therefore, the external force is a consequence of the action of the coupled valve and striker during discharge of the pneumatic accumulator and can affect the strengthening or weakening of the shock interaction and thus affect the increase or decrease in the amplitude of the oscillations of the tool end.

5. 3. Approximation of a differential problem by a discrete problem

The initial boundary value problem (1)–(7) was approximated by a discrete problem. The basis for choosing a mixed difference scheme was the results obtained in works [11, 12]. Equations in partial derivatives were approximated by a difference mixed scheme with weighting coefficients γ:

$$\frac{u_i^{n+1}-2u_i^n+u_i^{n-1}}{\tau^2}=$$
$$=\gamma a^2\left[\begin{array}{l}\dfrac{1}{S(x_i)}\cdot\dfrac{S(x_{i+1})-S(x_{i-1})}{2h}\times\\ \times\dfrac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2h}+\\ +\dfrac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{h^2}\end{array}\right]+$$
$$+(1-\gamma)a^2\left[\begin{array}{l}\dfrac{S(x_{i+1})-S(x_{i-1})}{S(x_i)2h}\times\\ \times\dfrac{u_{i+1}^n-u_{i-1}^n}{2h}+\dfrac{u_{i+1}^n-2u_i^n+u_{i-1}^n}{h^2}\end{array}\right], \quad (8)$$

$$i=1,\ldots,N-1, \quad n=1,\ldots,M-1.$$

To simplify the algorithm implementation, the approximation of the boundary conditions with the first order in step h was chosen:

$$S(0)E\frac{u_1^{n+1}-u_0^{n+1}}{h}=-G(\Delta u^n)\cdot(y^{n+1}-u_0^{n+1})-$$
$$-b_1\left(\frac{y^{n+1}-y^n}{\tau}-\frac{u_0^{n+1}-u_0^n}{\tau}\right), \quad (9)$$

$$S(L)E\frac{u_N^{n+1}-u_{N-1}^{n+1}}{h}=-c_2u_N^{n+1}-b_2\frac{u_N^{n+1}-u_N^n}{\tau}. \quad (10)$$

The oscillation equations of the discrete element were approximated by the implicit Euler scheme with partial linearization of the stiffness coefficient (the value Δu^n was calculated on the previous time layer):

$$m\frac{y^{n+1}-2y^n+y^{n-1}}{\tau^2}=R(t_n)+G(\Delta u^n)(u_0^{n+1}-y^{n+1})+$$
$$+b_1\left(\frac{u_0^{n+1}-u_0^n}{\tau}-\frac{y^{n+1}-y^n}{\tau}\right). \quad (11)$$

The initial conditions for the tool rod and the discrete element were approximated with the first order in τ:

$$u_i^0=0, \ (u_i^1-u_i^0)\cdot\tau^{-1}=0, \ x_i=ih, \ i=1,2,\ldots,N. \quad (12)$$

$$y^0=0, \ (y^1-y^0)\tau^{-1}=W_1. \quad (13)$$

Here $t_n=n\times\tau$, $\tau=T\times M^{-1}$, $x_i=i\times h$, $h=L\times N^{-1}$ – grid area parameters $u_i^n=u(t_n,x_i)$, $y^n=y(t_n)$ – grid functions (functions defined only at grid nodes).

The following algorithm for solving the discrete problem (8)–(13) was used.

The system of equations (8)–(11) on each time layer $t_n=n\tau$ was solved by the running method [13], adapted for a mixed system with boundary conditions. Equation (8) is reduced to the form:

$$A_iu_{i+1}-B_iu_i+C_iu_{i-1}=-F_i. \quad (14)$$

After performing the transformation of equations (8), the formulas for the coefficients A_i, B_i, C_i i F_i; $i=1,2,\ldots,N-1$:

$$A_i=-\frac{a^2\gamma\tau^2}{h^2}\left(\frac{S(x_{i+1})-S(x_{i-1})}{4S(x_i)}+1\right),$$

$$B_i=-\left(1+\frac{2a^2\tau^2\gamma}{h^2}\right),$$

$$C_i=\frac{a^2\gamma\tau^2}{h^2}\left(\frac{S(x_{i+1})-S(x_{i-1})}{4S(x_i)}-1\right),$$

$$F_i=\left[\begin{array}{l}-2u_i^n+u_i^{n-1}-(1-\gamma)\dfrac{a^2\tau^2}{h^2}\times\\ \times\left(\begin{array}{l}\dfrac{S(x_{i+1})-S(x_{i-1})}{4S(x_i)}(u_{i+1}^n-u_{i-1}^n)+\\ +u_{i+1}^n-2u_i^n+u_{i-1}^n\end{array}\right)\end{array}\right],$$

$$i=1,2,\ldots,N-1.$$

Below is the algorithm of the running method taking into account the boundary conditions and equation (11):

1) From the boundary condition (9) and the formula of the running method:

$$u_i=\alpha_iu_{i+1}+\beta_i$$

for $i=N-1$, α_{N-1} and β_{N-1} are defined:

$$\begin{cases} u_{N-1}^{n+1} = u_N^{n+1} \cdot \dfrac{S(L)E + c_2 h + b_2 h\tau^{-1}}{S(L)E} - \dfrac{b_2 h}{\tau S(L)E} u_N^n, \\ u_{N-1}^{n+1} = \alpha_{N-1} u_N^{n+1} + \beta_{N-1}. \end{cases}$$

So, the obtained values:

$$\alpha_{N-1} = \frac{S(L)E + c_2 h + b_2 h\tau^{-1}}{S(L)E}, \ \beta_{N-1} = -\frac{b_2 h}{\tau S(L)E} u_N^n.$$

2) Coefficients α_i and β_i are found by the "inverse formulas" of the running method:

$$\alpha_{i-1} = \left(B_i - \frac{A_i}{\alpha_i} \right) (C_i)^{-1}, \ \beta_{i-1} = \frac{\beta_i \cdot (B_i - C_i \alpha_{i-1}) - F_i}{C_i};$$

$$i = N-1, N-2, \dots, 2, 1.$$

3) Boundary condition (10) allows finding the relationship between u_1^{n+1} and y^{n+1}. This relationship is obtained from the system of equations:

$$\begin{cases} S(0)E \dfrac{u_1^{n+1} - u_0^{n+1}}{h} = -G(\Delta u^n)\left(y^{n+1} - u_0^{n+1}\right) - \\ -b_1 \left(\dfrac{y^{n+1} - y^n}{\tau} - \dfrac{u_0^{\,n+1} - u_0^{\,n}}{\tau} \right), \\ u_0^{n+1} = \alpha_0 u_1^{n+1} + \beta_0. \end{cases} \quad (15)$$

Values u_1^{n+1} are expressed in terms of other quantities:

$$u_1^{n+1} = \\ = \frac{h\left[\left(G(\Delta u^n) + \dfrac{b_1}{\tau}\right)\right] y^{n+1} - \dfrac{b_1 h}{\tau}\left(y^n - u_0^n\right) - \beta_0 d_0(\Delta u^n)}{\alpha_0 d_0(\Delta u^n) - S(0)E}, \quad (16)$$

where:

$$d_0(\Delta u^n) = G(\Delta u^n) h + b_1 h \tau^{-1} + S(0)E, \ \Delta u^n = y_2^n - u_0^n.$$

As a result, a system of equations with respect to u_1^{n+1}, y^{n+1} is obtained:

$$\begin{cases} u_1^{n+1} = \dfrac{y^{\,n+1}\left(hG(\Delta u^n) + \dfrac{hb_1}{\tau}\right) - \dfrac{b_1 h}{\tau}\left(u_0^n - y^n\right) - \beta_0 d_0}{\alpha_0 d_0 - S(0)E}, \\ m \dfrac{y^{n+1} - 2y^n + y^{n-1}}{\tau^2} = R(t_n) + G(\Delta u^n)\left(u_0^{n+1} - y^{n+1}\right) + \\ + \dfrac{b_1}{\tau}\left(u_0^{n+1} - u_0^n - y^{n+1} + y^n\right). \end{cases} \quad (17)$$

From the second equation of system (17), y^{n+1} is expressed in terms of other variables, including u_N^{n+1}, and the notation is introduced:

$$d_1(\Delta u^n) = \tau\left(\tau G(\Delta u^n) + b_2\right) m^{-1}.$$

As a result, the formula is obtained:

$$y^{n+1} = (R_1 + R_2) \cdot R_3^{-1}, \quad (18)$$

where:

$$R_1 = \frac{\alpha_0 d_1 \left[\dfrac{hb_1}{\tau}\left(u_0^n - y^n\right) - \beta_0 d_0 \right]}{\alpha_0 d_0 - S(0)E} + \beta_0 d_1,$$

$$R_2 = \left(y^n - u_0^n\right)\frac{b_1 \tau}{m} + 2y^n - y^{n-1} + \frac{\tau^2}{m} R(t_n),$$

$$R_3 = 1 + d_1 - \frac{d_1 \alpha_0 h \cdot \left(G(\Delta u^n) + \dfrac{b_1}{\tau} \right)}{\alpha_0 d_0 + S(0)E}.$$

Substituting the value of y^{n+1} into the first formula of system (17) led to the following equations u_1^{n+1} for u_0^{n+1}:

$$u_1^{n+1} = \frac{h\left[\left(G(\Delta u^n) + \dfrac{b_1}{\tau}\right)\right] y^{n+1} - \dfrac{hb_1}{\tau}\left(y^n - u_0^n\right) - \beta_0 d_0}{\alpha_0 d_0 - S(0)E},$$

$$u_0^{n+1} = \alpha_0 u_1^{n+1} + \beta_0, \quad (19)$$

and further defined:

$$u_i^{n+1} = \left(u_{i-1}^{n+1} - \beta_{i-1}\right) \cdot \alpha_{i-1}^{-1}, \ i = 2, 3, \dots, N. \quad (20)$$

In this way, the solution of the system of equations (8)–(11) is obtained at each time layer.

The general algorithm of mathematical model research is presented in Fig. 2 in the form of a scheme. The Mathcad system (USA) was chosen to implement the algorithm. In this system, functional autonomous blocks that solve special problems have been developed.

Below are the main functional blocks and their purpose.

The function $G(\Delta U)$ determines the dependence of the stiffness coefficient on the difference in the displacements of the discrete element and the end tool. The block $Pu(x_1, x_2)$ determines the force of the collision of discrete elements of the model problem. $DN(N, T, M)$ is a general control block in which the coefficients of the system of difference equations are calculated, communication with the sweep block and transition to the next time layer is carried out. The function $trdag(N, T, M)$ implements the sweep method taking into account the conditions of the relationship of the equations, the function $S(x, X, Y)$ determines the profile of the tool at the given coordinate vectors X, Y.

The calculation scheme forming the model problem is presented in Fig. 3, *a*. The system of equations of motion of two discrete elements connected by rigid and dissipative connections is taken in the form:

$$m_1 \frac{d^2 x_1}{dt^2} = R(t, \Delta t) - Pu(x_1, x_2) + b_1\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right), \quad (21)$$

$$m_2 \frac{d^2 x_2}{dt^2} = Pu(x_1, x_2) + \\ + b_1\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) - c_2 x_2 - b_2 \frac{dx_2}{dt}, \ t \in [0, T]. \quad (22)$$

Initial conditions:

$$\frac{dx_1}{dt}(0) = W_1, \ \frac{dx_2}{dt}(0) = 0, \quad (23)$$

$$x_1(0) = 0, \ x_2(0) = 0. \quad (24)$$

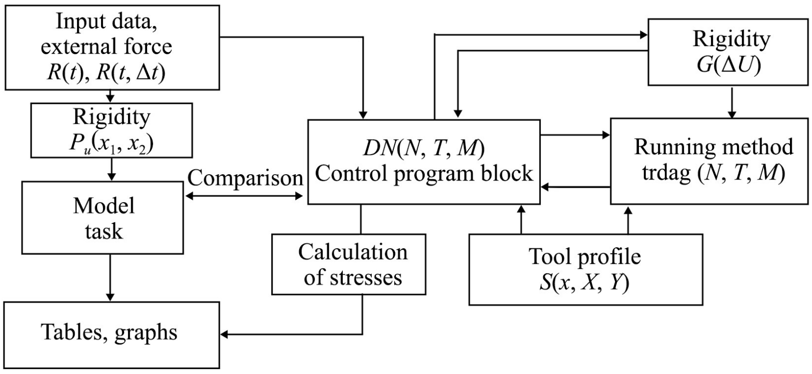


Fig. 2. General algorithm of mathematical model research

The force of the contact collision is determined by a formula similar to formulas (8):

$$Pu(x_1,x_2)=\begin{cases} K_0(x_1-x_2)^{\alpha}, \text{if } x_1 \geq x_2, \\ 0, \text{if } x_1 < x_2. \end{cases} \quad (25)$$

System (21)–(25) simulates the case when the tool rod is replaced by a discrete element. Comparison of solutions (low-frequency oscillations) serves as the basis for the correct choice of the algorithm for solving the difference problem and the parameters of the difference scheme.

5. 4. Results of computational experiments

During the calculations, the main attention was paid to determining the parameters of the collision process of the discrete element with the end of the tool. After a short time of the co-hitting process, a fine mesh was applied. The reliability of the obtained results is of significant importance. To estimate the parameters of the difference method, which ensure acceptable accuracy of the solution, a model problem was used. The solution of the model problem (21)–(25) was obtained by the Runge-Kutta method in the Mathcad system. A comparison of the obtained solution with the solution of problem (8)–(13) with the mass of the second discrete element, which is equal to the mass of the tool, is presented in Fig. 3, *b*.

Main parameters: L=1.2 m; m_1=20 kg; m_2=133 kg; W_1=5 m/s; K_0=10^{11} N/m$^{1.5}$; c_1=10^{11} N/m$^{1.5}$; c_0=3×10^4 N/m; c_2=2×10^8 N/m; b_1=1.2×10^3 N/m; b_2=10^3 Ns/m, γ=0.5; h=0.012 m; τ=8.3×10^{-7} s.

Curves (4) and (5) in Fig. 3, *b* is the solution of the initial-boundary value problem and differs from the solution of the model problem, that is, curve (1), mainly by the presence of a high-frequency component. This component demonstrates the spread of oscillations along the length of the tool (the result of the impulse interaction of a discrete element with the end of the rod). The practical coincidence of the low-frequency component solutions of the model problem and the initial boundary value problem confirms the correctness of the chosen algorithm.

Solving on a fine grid under the same parameters made it possible to determine the difference in the displacements of the ends of the rod after the impact (propagation of the wave of displacements of the cross-sections of the rod-tool). The results of the calculations are presented in Fig. 4. Comparing the displacements of the discrete element and the contact end of the tool made it possible to determine the collision time T_i from the condition $y \geq U_0$ under the given input conditions. It should be noted that the co-impact time depends on many factors (pre-impact speed, stiffness characteristics, geometric dimensions of the tool, mass of the discrete element and the tool). The influence of these factors was not investigated, only the ability of the algorithm to conduct such studies is demonstrated here.

Fig. 5 shows repeated collisions of a discrete element with the end of the tool for a short period of time (Fig. 5, *a*) and a long period of time (Fig. 5, *b*) in the absence of an external force, i.e., the striker impact is simulated only by the pre-impact speed.

Main parameters: m_1=13 kg; m_2=36.8 kg; L=0.6 m; c_1=8.0×10^{11} N/m$^{1.5}$; c_2=2×10^8 N/m; c_0=3×10^6 N/m; W_1=5 m/s; b_1=0, b_2=10 Ns/m; L=0.6 m; h=0.12 m; τ=2 10^{-5} s; γ=0.8.

The presence of an external periodic force acting on a discrete element significantly changes the pattern of oscillations. Let's consider the case when the external force changes according to the law $R(t)=P\cdot|\sin(\omega t)|$, where P=90 kN, ω=800 1/s (Fig. 6, *a*). The action of the external force only in the positive direction leads to the stabilization of joint vibrations of the discrete element and the ends of the tool (Fig. 6, *b*). The results of the calculations showed that changing the parameters of the external force (frequency ω and amplitude P) can lead to both the amplification of oscillations and their damping.

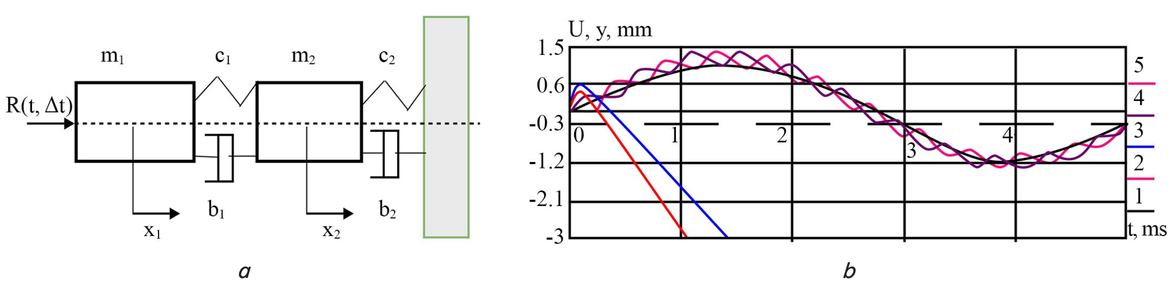


Fig. 3. Model problem and comparison of solutions: *a* – calculation scheme of the model problem; *b* – comparison of solutions: 1 – x_2; 2 – x_1 – solution of the initial problem (Runge-Kutta method in Mathcad); 3 – y; 4 – U_0; 5 – U_N –solution of the initial boundary value problem

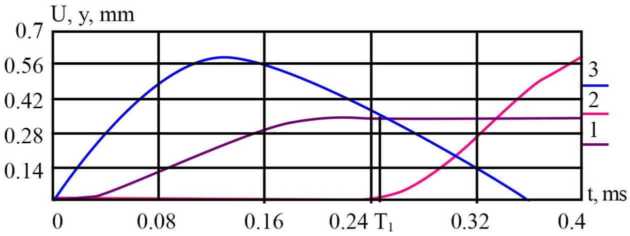


Fig. 4. Movement of the extreme ends of the rod-tool and discrete element: 1 – U_0: 2 – U_N; 3 – y, $T_1 \approx 0.25$ ms

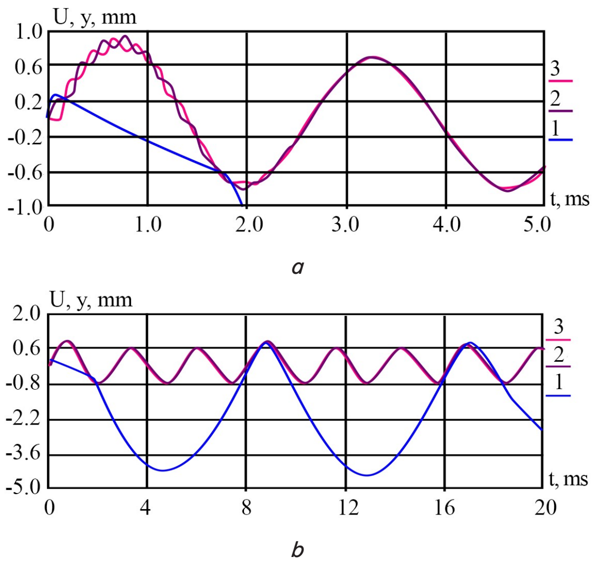


Fig. 5. Movement of elements in the process of interaction: *a* – a short period of time; *b* – a long period of time; 1 – y; 2 – U_0; 3 – U_N

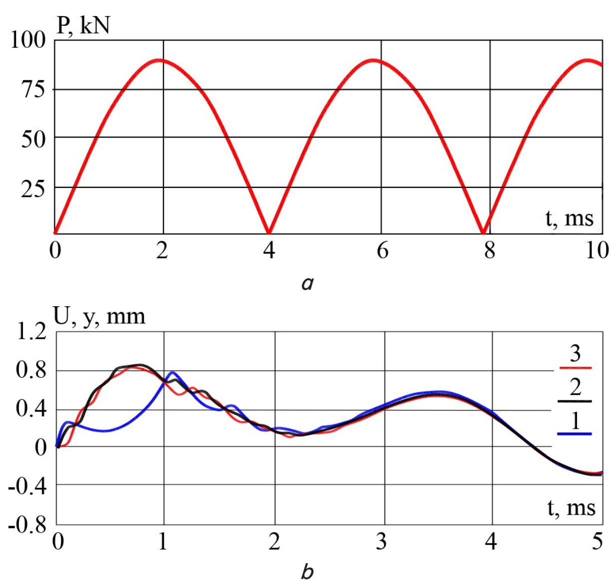


Fig. 6. Oscillations in the presence of an external force: *a* – external periodic force; *b* – oscillations of the ends of the tool; 1 – y; 2 – U_0; 3 – U_N

Fig. 7, *b* shows the displacement of the discrete element and the ends of the tool during the short-term action of an external force (Δt=δ=0.1 ms) on the discrete element (Fig. 7, *a*).

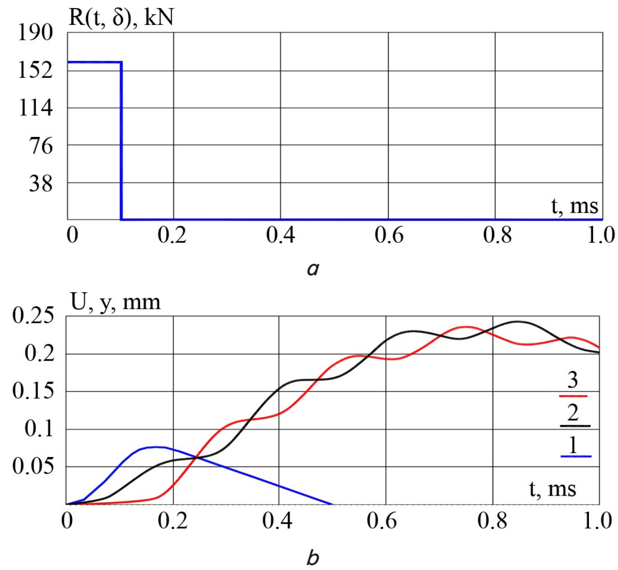


Fig. 7. Imitation of impulse action using an external force: *a* – external force; *b* – oscillations of a discrete element and rod ends; 1 – y; 2 – U_0; 3 – U_N

The short-term action of an external force on a discrete element leads to a similar effect of the occurrence of high-frequency oscillations of the cross-sections of the tool, characteristic of impact interaction. Let's note that impact simulation using an external force allows to vary the moment of impact during the operation of the device. For this, it is enough, for example, to define the force in the form:

$$R(t,t_1,\Delta t)=\begin{cases}P, \text{if } t_1 \le t \le t_1+\Delta t,\\ 0, \text{if } t > t_1+\Delta t.\end{cases}$$

Fig. 8, *b* shows the dependence of normal stresses on time in sections close to the ends of the tool.

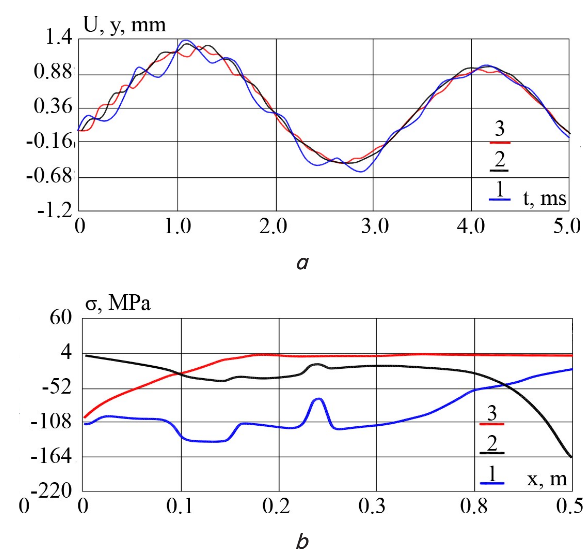


Fig. 8. Movement of elements and stress distribution: *a* – Movement of elements in the process of interaction: 1 – y; 2 – U_0; 3 – U_N; *b* – stress distribution along the length of the tool at different moments of time: 1 – t=31.25 μs; 2 – t=125 μs; 3 – t=250 μs

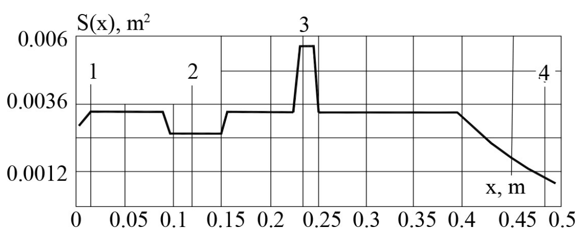
Main parameters: m_1=13 kg; L=0.5 m; c_1=8.0×10^{11} N/m$^{1.5}$; c_2=2×10^8 N/m; c_0=3×10^6 N/m; W_1=5 m/s; b_1=3.0×10^3 N/m; b_2=1.0×10^3 N/m.

Calculations were carried out with a variable cross-section of the rod-tool with a conical working part. The change in the cross-sectional area of the tool along the length is shown in Fig. 8, *a*. The dependence of stresses on time for different cross-sections of the tool is shown in Fig. 9, *b*. The values of stresses in sections 1–4 correspond to the profile of the tool.

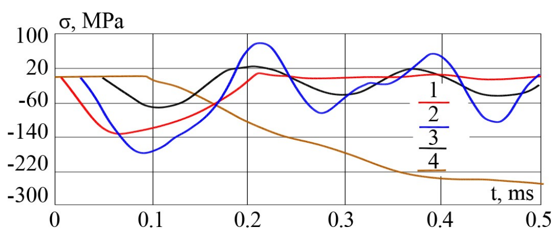
Stresses were determined by the formula:

$$\sigma_i^n = E\frac{u_{i+1}^n - u_{i-1}^n}{2h}, \quad n = 0,..,M; \quad i = 1,..,N-1. \tag{26}$$

Initial data: m_1=10 kg; W_1=5 m/s, E=2.1×10^{11} Pa.



a



b

Fig. 9. Tool profile and stress change: *a* — distribution of the cross-sectional area of the tool along the length; *b* — dependence of stresses on time in different sections of the tool: x_1=0.01 m; x_2=0.12 m; x_3=0.235 m; x_4=0.485 m

The highest compressive stress was obtained in the conical part of the tool (250 MPa). The distribution of stress along the length of the tool made it possible to estimate the most loaded cross-sections of the tool at different points in time. The obtained stresses do not exceed the permissible ones and depend on impulse loads.

6. Discussion of the results of modeling the process of oscillations of the elements of the impact device

The discrete-continuous model of the impact device turned out to be an effective tool for the study of oscillatory processes in the "impactor – tool – working environment" system.

The use of a combination of discrete and continuous elements makes it possible to simplify the algorithm for solving the initial-boundary problem (1)–(6) and at the same time preserve the peculiarities of the impulse interaction in the system "hammer - tool - working environment" (Fig. 1).

This model makes it possible to evaluate the main factors affecting the impact process and its effectiveness. The following factors can be distinguished: the previous speed of the striker, the time of action of the external force, the mass of the tool and the striker, the stiffness of the contact interaction. It should be noted that a small correction of the direction of the acting forces and a change of rigid and dissipative connections will allow to study the process of the reverse influence of the working environment on the tool and through it on the striker [10]. Estimation of the stress in the cross-sections of the tool of variable cross-section will allow designing a rational profile of the tool of the impact device (Fig. 8, 9).

The mathematical model research algorithm (Fig. 2), unlike the algorithm in [9], consists of separate program blocks in the Mathcad system. The autonomous operation of each block will allow comparing the results of computational experiments with different input data in automatic mode.

It turned out to be useful to simulate the impact of the striker with the tool by introducing an external force, which is the result of the systemic action of the striker, valve and pneumatic accumulator on the tool (Fig. 6, *a*). The introduction of such a force will allow to establish feedback of the pre-impact speed with the resistance of the rock being processed and, if necessary, to change the speed, and therefore the energy of the impact by changing the charging energy of the pneumatic accumulator. In addition, during the operation of the technical device, random forces arise that can affect the process of transmitting the impulse to the medium being processed. With the help of an external force, it becomes possible to simulate such an action (Fig. 7). In contrast to the models [7–9], which are studied by analytical methods, the proposed model is more complex, and the numerical method (8)–(13), the parameters of which are determined using model problems, turned out to be effective for its study.

It turned out that the representation of the force of the impact interaction as a power dependence on the difference in the displacements of the discrete element and the contact end of the tool (7) allows to study the process parameters and their dependence on various factors (previous speed, masses of the striker and the tool, stiffness and dissipation coefficients, external forces, Fig. 5). In particular, the impact time was estimated based on the difference between the displacements of the discrete element and the contact end of the tool (Fig. 4). The degree of plasticity of impact interaction was modeled with the help of a dissipative element. Here, it is interested in determining the dependence of the interaction duration on the difference in speed of the discrete element and the end of the tool. With such an interpretation, the duration of the impact turns out to be shorter, the developed algorithm and program allows to determine this duration.

Control of the process of impulse transfer to the processed medium can be implemented with the help of an external force of a periodic and pulse nature. To implement such a problem, it is necessary to have a numerical algorithm that is stable in a wide range of parameters. A strict mathematical justification of such a range can be considered the most reliable. Such a goal is partially achieved with the help of specially constructed model problems, for example problems (21)–(24). Model problems can be of discrete or continuous type and are solved using standard methods. From continuous model problems, it is possible to single out the problem of oscillations of a rod of constant cross-section in the presence of elastic connections. The analytical solution of such a problem is found by the Fourier method [11]. The application of model problems made it possible to conclude that the difference scheme with weighting coefficients (8) is the most rational.

Thus, the use of a discrete element connected to a continuous one (a rod with a variable cross-section), which in turn has a connection with the processing environment, made it possible to investigate the process of oscillations in the system. The use of a discrete element allows to determine the impact force and the parameters of the impact with the tool. Using the finite difference method allows to determine the internal force factors in the cross-sections of the tool, their distribution and change over time.

The combination of discrete and continuous elements with a nonlinear connection between them made it possible to simplify the calculation scheme and at the same time take into account the presence of force factors in the cross sections of the tool

It should be noted that when solving practical problems, a significant change in the input data requires a new setting of the parameters of the numerical method (γ, τ, h). For this, model problem (21)–(24) was used. In addition, in some cases, the Fourier method can be used to compare solutions [11].

The presented work has the following limitations:

– the model takes into account only the longitudinal vibrations of the tool in the presence of a longitudinal load, in practice there are also transverse vibrations that must be taken into account;

– the resistance of the processing medium may have a hysteretic character, which must be taken into account.

Disadvantages inherent in the presented work:

– the work does not consider a strict mathematical justification of the numerical method with the determination of the region of stability and the size of the error;

– the identification of the model relative to the experimental data, which would allow evaluating the ranges of changes in the main parameters of the model, has not been carried out.

The selection of high and low frequencies of instrument oscillations is of particular importance, since these frequencies are characteristics of the wave and inertial components of the impact (Fig. 3). Such an allocation will allow to evaluate the relevant energy components of the shock pulse and identify the factors affecting these components. At the same time, the effectiveness of high-frequency oscillations of the impact end of the tool is of particular interest and should be investigated in further works of the authors.

The resistance of the working environment is represented by a combination of rigid and dissipative resistances (4). The issue of the level of influence of the character of this resistance on the duration of interaction between the striker and the instrument and on other parameters of the wave process is debatable. It can be assumed that this influence depends on the ratio of the stiffness parameters of the working environment and the stiffness parameters of the contact between the striker and the end of the tool. If their values are proportional, such an influence can be significant. In addition, the length of the tool is also important here, which determines the time of passage of the shock wave from the contact end to the working end and back.

The obtained results can be applied in designing and optimizing the parameters of impact devices (hydro hammers, construction machines, jackhammers, devices of the oil production industry).

The following can be considered possible directions of further research development:

– construction of a model of an impact device with two strikers;

– taking into account in the model the hysteretic nature of the resistance of the processing environment;

– consideration of transverse loads on the tool and transverse vibrations;

– model identification with respect to experimental data;

– determination of energy components that are transferred from the tool to the processing environment.

7. Conclusions

1. The calculation scheme of the impactor has been built with the setting of influencing parameters on the characteristics of the impact impulse in the system “hammer – tool of variable cross-section – working environment”. The coupled striker and valve are represented by a discrete element, which is acted upon by an external force of a periodic or pulse nature. The interaction of system elements is modeled by nonlinear rigid and dissipative connections.

2. A mixed initial-boundary value problem with an ordinary differential equation and a wave equation in partial derivatives has been formulated. The impact force of a discrete element and the contact end of the tool is represented as a power-law dependence on the difference in the displacements of the contacting elements. The initial velocities of the discrete element and the end of the tool or a fast-acting external force determine the short impact period. The external force allows changing the nature of the collision depending on the direction, amplitude and frequency of oscillations. Boundary conditions for a tool describe the interaction of its ends, respectively, with a discrete element and the working environment. The mathematical model allows to estimate the time of collision between the striker and the tool, which depends on the stiffness coefficient and is 200...300 μs.

3. For the approximation of the difference problem, the mixed difference scheme with weighting coefficients turned out to be the most rational. The algorithm for solving the difference problem is built on the basis of the method of running on each time layer with partial linearization of nonlinear rigid connections. The rational parameters of the difference scheme were determined using model problems and were: time step (1, ..., 5)$\cdot 10^{-5}$ s; step length – (0.1...0.3) from the length of the tool, and for a mixed scheme – within 0.5...0.8. The algorithm of the numerical method is implemented in the Mathcad system.

4. Computational experiments have been carried out taking into account the impulse load with a force of up to 90 kN in the time range of 0...4 ms, which made it possible to estimate the normal stresses in the cross-sections of the tool at different moments of time. The maximum of them was 200...250 MPa, which can be used when designing impact mechanisms and choosing rational modes of their operation.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, including financial, personal, authorship, or any other, that could affect the study and its results presented in this article.

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Data availability

The manuscript has no associated data.

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