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CONSTRUCTION OF A MATHEMATICAL MODEL FOR APPROXIMATING THE SPHERE BY STRIPS OF UNFOLDING SURFACES

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Approximating non-expandable surfaces by compartments of expandable ones makes it possible to simplify the process of obtaining the required shape without loss of operational properties. There is a known approximation of a sphere using the example of a ball when its surface can consist of polygons. However, this list does not exhaust the possible options for approximating the sphere. Its approximation by truncated cones tangent to parallels or by congruent cylindrical petals tangent to meridians is known.

Any line on the surface of a sphere is a line of curvature. This means that the common line of contact of the expanded surface with the sphere will be a line of curvature for the expanded surface as well (rectilinear generatrices of the expanded surface will cross this line at a right angle). When building a sweep of such a surface, the line of contact will be transformed but the rectilinear generatrices will remain perpendicular to it, which simplifies the construction of the sweep.

The approximation of the sphere by congruent strips, the number of which can be different, starting from one, is considered. A necessary condition for such an approximation is a common line of contact of adjacent strips. To this end, the line of contact on the sphere or the guide curve must have an appropriate shape. Such a curve is taken as a slope line (a curve whose tangents form a constant angle of inclination to the horizontal plane). The study results are the parametric equations of the strip touching the sphere and its corresponding equations on the sweep. The construction of the strip on the sweep is explained by the invariance of the geodesic curvature of the guide curve when the strip is bent until it aligns with the plane. This explains the difference between the proposed approach and conventional methods of sphere approximation.

Approximating the sphere by strips of unfolding surfaces has a practical application in architecture with spherical elements, as well as in religious buildings with domes in the form of a part of the sphere

Keywords: sweep of the sphere, line of contact, guide curve, geodesic curve, parametric equations

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1. Introduction

In the modern scientific world, sphere approximation is one of the important problems that finds its application in many fields of science and engineering. This problem consists in approximating a geometric object – a sphere, known for its perfect smoothness and symmetry, to simpler mathematical models, such as conic surfaces, polygons, or other geometric figures.

Sphere approximation is an actual task in numerical methods of calculations, computer graphics, geodesy, astronomy, topography, and many other fields. This topic arouses considerable interest among researchers due to the need to solve complex tasks related to the measurement and analysis of objects in three-dimensional space.

Therefore, the development of various approaches to sphere approximation, the analysis of their advantages and disadvantages, and the practical application of these methods are an urgent issue. In addition, modern technologies and software tools that allow solving sphere approximation tasks with high accuracy and speed deserve attention.

In the geometric sense, a sphere is a special surface. It has a Gaussian curvature, an average curvature, all lines on its surface are lines of curvature. Any piece of its surface can slide on its own without the deformation used in spherical joints. This property is also characteristic of the plane. There are other properties that somehow make these surfaces similar. As the radius of the sphere increases to infinity, its limited area turns into a plane, and all spherical figures turn into flat ones. Such a transition makes it possible to design spherical mechanisms on the basis of flat mechanisms.

The sphere is an important element of architectural forms. It has the largest volume with the smallest surface area. This means that heat loss in a house of this shape will be the smallest compared to other houses. Given this, approximation of the sphere by elements of the same type is an urgent task.

2. Literature review and problem statement

Parts of machines that create friction pairs are of interest to many researchers. The quality of the surface layers applied to the steel elements of the machine parts and the geometry of the contact surfaces have a significant impact on the wear resistance and durability of the parts. Research into the influence of the quality of the surface treatment of parts on microgeometry [1] confirms the close relationship between these parameters.

Scientists consider in detail the influence of surface treatment methods on the quality parameters of resulting coatings [2]. This is a technological approach to solving the issue of increasing the durability and wear resistance of parts. However, ensuring the necessary operational properties of surfaces can be achieved geometrically. Mathematical models are used for numerical simulations of many processes [3].

The properties of the surface of the sphere are used for the design of spherical mechanisms, in particular, to design bevel gears. For example, in work [4] the use of a spherical ellipse for the design of a bevel transmission, which is an analog of a transmission between parallel axes, is considered. In the cited work, ellipses act as non-circular wheels. In [5], the formation of an isometric grid on the surface of a sphere is considered.

Paper [6] shows an interesting application of spherical surface approximation for the estimation of lighting in 3D visualization. At the same time, the approximation is carried out by numerous rectangular blocks, which leads to a significant distortion of the image.

Scientists considered the approximation of the sphere by various elements. Thus, the visualization of a spherical image by a set of locally flat grids tangent to the icosahedron is described in [7]. The authors claim that changing the resolution of these meshes independently of the resolution level allows for efficient representation of high-resolution spherical images. At the same time, it is possible to take advantage of the icosahedral spherical approximation with low distortion. Additional research [8] was aimed at cartographic analysis of such statements. It confirmed a 12.6 % improvement in semantic segmentation results. However, this indicator can be increased by applying to the sphere approximation of other planar elements. The most accurate is the approximation of the sphere with ribbons or stripes.

Approximation of a sphere by a continuous tape is considered in work [9]. To this end, the sphere is assigned to the isometric grid of coordinate lines. A spherical line is taken as a guide curve – an analog of Archimedes' spiral on a plane. In [10], the trajectory of the unit radius vector of the helical line, which turns it into a spherical curve, is taken as a guide curve. The approximation result obtained for this case is similar to the previous one. However, this is not the end of the problem of approximation of the sphere by strips of expanding surfaces. In particular, existing approaches do not use slope lines. The use of a spherical slope line as a guide curve makes it possible to diversify the possibilities of approximation, which also affects the aesthetic appearance of the approximated sphere. In addition, a significant increase in the accuracy of the approximation is expected.

3. The aim and objectives of the study

The purpose of this study is to design a geometric model of the approximation of the sphere by strips of expanding surfaces. This will make it possible to expand the methods of approximation of the sphere, which can be applied when parqueting spherical shells of architectural structures.

To achieve the goal, the following tasks must be solved:

– to derive parametric equations of the strip of the unfolding surface, tangent to the sphere along the specified curve in such a way that the adjacent strips have a common line of contact;

– to construct a sweep of the strip tangent to the sphere, based on the theory of surface bending;

– to visualize variants of the approximated sphere and sweep of a separate strip.

4. The study materials and methods

The study of the properties of the spherical slope line led to the hypothesis that it can be used as a guide curve for the approximation of the sphere by the strips of the unfolding surfaces, which is the object of our study. To this end, the apparatus of analytical and differential geometry was used, as well as means of computer visualization of the results.

The construction of the approximated sphere with strips was carried out in the environment of the software package «MATLAB», and the construction of the sweep of the strip was carried out using the graphic modeling system «Simulink» of the same environment, in which the numerical integration was carried out. The adequacy of the model built was checked using analytical calculations. It is the graphic images of the approximated sphere that confirm the reliability of our results as they clearly show the coverage of the sphere by strips of unfolding surfaces without gaps and overlays.

5. Results of investigating the approximation of a sphere of unit radius by strips of expanding surfaces

5. 1. Deriving parametric equations of the strip that approximates the surface of the sphere

The parametric equations of a sphere of unit radius take the form:

$$X = \cos\varepsilon\cos\gamma;$$
$$Y = \cos\varepsilon\sin\gamma;$$
$$Z = \sin\varepsilon, \quad (1)$$

where γ and ε are independent surface variables, and γ is numerically equal to the angle of rotation of a surface point when it moves along a parallel, and ε is the second angle, which is numerically equal to the angle of rotation of a surface point when it moves along a meridian.

If two independent variables γ and ε are connected by a certain dependence, i.e., if we proceed to one variable, then a curved line will be described on the surface of the sphere. The slope line belonging to the sphere is described by the following parametric equations:

$$x = a\cos a\gamma\sin\gamma - \cos\gamma\sin a\gamma;$$
$$y = -a\cos a\gamma\cos\gamma - \sin\gamma\sin a\gamma;$$
$$z = \sqrt{1-a^2}\cos a\gamma, \quad (2)$$

where γ is an independent variable, the physical essence of which is the same as that of the surface, however, the point moves along the surface in a defined way, namely along the curve (2); a is a constant value less than one (a<1), which depends on the angle β of the rise of the line (2). In Fig. 1, *a*, according to equations (1), a sphere is constructed, and according to equations (2) – a curve (2) on its surface.

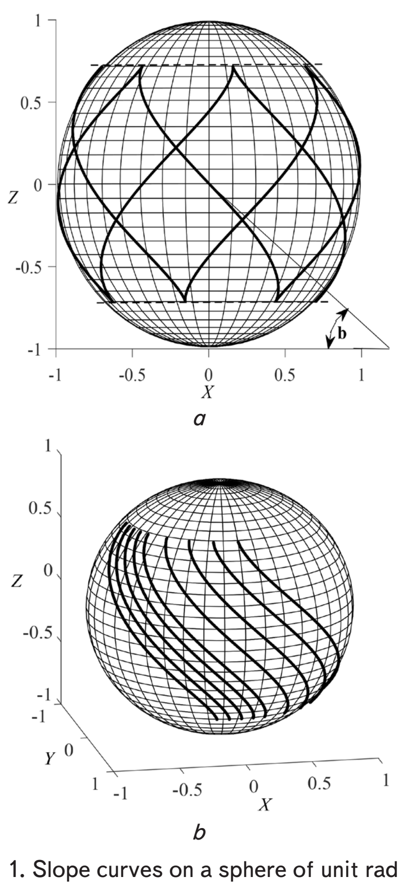


Fig. 1. Slope curves on a sphere of unit radius: *a* – a=0.7, β=44.4°, the curve has turning points at the limit position of the parallel; *b* – rotation of a part of the slope curve within the turning points by a given angle around the *OZ* axis

For further calculations, one needs to have the first and second derivatives of equations (2):

$$x' = \left(1-a^2\right)\sin\gamma\sin a\gamma;$$
$$y' = -\left(1-a^2\right)\cos\gamma\sin a\gamma;$$
$$z' = -a\sqrt{1-a^2}\sin a\gamma. \quad (3)$$

$$x'' = \left(1-a^2\right)\left(a\sin\gamma\cos a\gamma + \cos\gamma\sin a\gamma\right);$$
$$y'' = \left(1-a^2\right)\left(a\cos\gamma\cos a\gamma - \sin\gamma\sin a\gamma\right);$$
$$z'' = -a^2\sqrt{1-a^2}\cos a\gamma. \quad (4)$$

The angle of elevation β of line (2) can be found by the formula:

$$\mathrm{tg}\beta = \frac{z'}{\sqrt{x'^2 + y'^2}} = \frac{a}{\sqrt{1-a^2}}. \quad (5)$$

According to (5), the angle β is constant. For the value a=0.7, it is equal to 44.4° (Fig. 1, *a*). The value of such an elevation angle is possible only on a limited area of the sphere between symmetrical parallels, which are marked by dashed lines. The turning points of the curve are located on these parallels. In Fig. 1, *b*, the section of the curve between the indicated parallels is plotted with the rotation of this curve by 7.5° around the *OZ* axis (higher density between the curves) and 15° (lower density between the curves). It can be seen from the drawings that the distance between the curves is constant. Based on this, it is possible to make an assumption about the possibility of approximating the sphere along the slope lines by strips of unfolding surfaces of constant width.

The size of the area of the sphere between the symmetrical parallels, which in Fig. 1, *a* are indicated by dashed lines, depends on the value of angle β of the guide line elevation. At β=90°, such a section is absent at all, as it degenerates into the equator of the sphere. As the angle β decreases, the size of this area increases. In order to construct the section of the curve between the limit parallels, it is necessary to set the limits of the change of the parameter (angle) γ. For example, in Fig. 2, spherical slope curves are constructed with an indication of the value of the angle β and the limits of change of parameter γ.

We shall look for a strip of the unfolding surface along the direction line of the slope on the sphere as the circumscribing surface of the one-parameter set of planes tangent to the sphere along this line. The normal vector for each plane of this set is the normal to the sphere. Its projections onto the coordinate axis coincide with the parametric equations of the guide curve (2). The rectilinear generatrix of the unfolding surface (torse) is the result of the intersection of two infinitely close tangent planes. Being in two planes at the same time, the generatrix torse is perpendicular to the normal vectors of these planes. So, the directional vector of the generatrix can be defined as the vector product of two adjacent normals:

$$\bar{I} = \bar{N}\times\left(\bar{N}+d\bar{N}\right) = \bar{N}\times d\bar{N}. \quad (6)$$

The direction of the vector product will not change if the vector $d\bar{N}$ is replaced by a vector parallel to it $d\bar{N}/d\gamma$. The coordinates of the vector $d\bar{N}/d\gamma$ are derivatives of equations (2) and are given in (3). As a result of vector multiplication of expressions (2) and (3), we shall get the direction vector of the rectilinear generatrix of torse, which, after reduction to unity, will be written:

$$I_x = \cos\gamma\cos a\gamma + a\sin\gamma\sin a\gamma;$$

$$I_y = \sin\gamma\cos a\gamma - a\cos\gamma\sin a\gamma;$$

$$I_z = \sqrt{1-a^2}\sin a\gamma. \quad (7)$$

The unfolding surface (torse) is constructed as a linear surface, the rectilinear generatrices of which pass through each point of the guide curve (2) parallel to the guide vector (7), i.e.:

$$X = x(\gamma) + uI_x;$$

$$Y = y(\gamma) + uI_y;$$

$$Z = z(\gamma) + uI_z, \quad (8)$$

where $x(\gamma)$, $y(\gamma)$, $z(\gamma)$ are parametric equations (2) of the guide curve; u – the second independent variable of the surface – the length of the rectilinear generatrix of torse; I_x, I_y, I_z are the coordinates of the direction vector (7) of the rectilinear generatrix of torse.

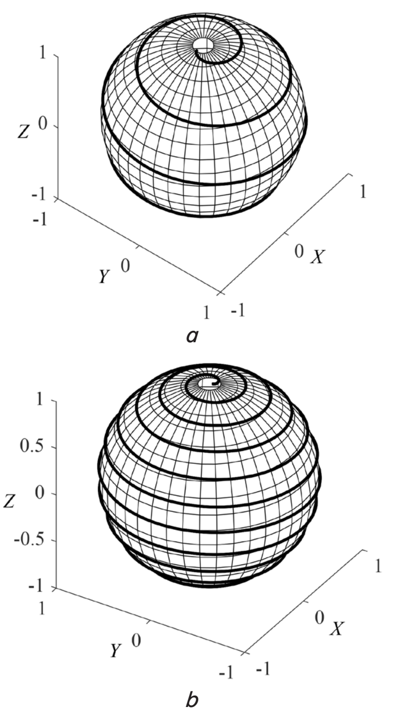


Fig. 2. Slope curves on a sphere of unit radius with different elevation angles: *a* – a=0.1 (β=5.7°), γ=0...10π; *b* – a=0.04 (β=2.3°), γ=0...25π

By substituting the equation of the curve (2) and the coordinates of the vector (7) in (8), the parametric equations of the unfolding surface are finally built:

$$X = \cos a\gamma(u\cos\gamma + a\sin\gamma) + \sin a\gamma(ua\sin\gamma - \cos\gamma);$$

$$Y = \cos a\gamma(u\sin\gamma - a\cos\gamma) - \sin a\gamma(ua\cos\gamma + \sin\gamma);$$

$$Z = \sqrt{1-a^2}(\cos a\gamma + u\sin a\gamma). \quad (9)$$

When constructing the surface according to equations (9), the length of the line of contact of the strip with the sphere depends on the limits of the change of the parameter γ, and the width of this strip depends on the limits of the change of the parameter u.

5. 2. Deriving parametric equations of the strip sweep and constructing its contours

Construction of the sweep is carried out by means of differential geometry. The basis can be taken from the fact that the geodesic curvature of the directional tangent curve, which is common to the sphere and the torse, does not change when the strip is stretched on a plane. If the parametric equations of the curve on the surface are known, then the geodesic curvature can be found from the determinant:

$$k_g = \left(\frac{d\gamma}{ds}\right)^3 \begin{vmatrix} x & y & z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix}, \quad (10)$$

where $d\gamma/ds = 1:(ds/d\gamma)$. Expression $ds/d\gamma$ is the derivative of the arc s of the directional curve (2). It is determined through derivatives (3) of the curve according to the formula:

$$\frac{ds}{d\gamma} = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{1-a^2}\sin a\gamma. \quad (11)$$

The first line in the determinant (10) is the coordinates of the unit vector normal to the surface along the curve, which for the sphere coincides with the equations of the curve. After substituting equations (2), first (3) and second (4) derivatives and inverse dependence (11) and simplifications into (10), we obtained:

$$k_g = \mathrm{ctg}\, a\gamma. \quad (12)$$

If the k_g dependence of the spatial curve on the surface of the torse is known, then the parametric equations of the curve on the sweep will be written according to the known formulas of differential geometry:

$$x = \int \cos\left(\int k_g \mathrm{d}s\right)\mathrm{d}s;$$

$$y = \int \sin\left(\int k_g \mathrm{d}s\right)\mathrm{d}s. \quad (13)$$

The expression in parentheses, which is the angle of rotation of the tangent to the curve on the sweep, can be integrated, taking into account the expression ds from (11):

$$\int k_g \mathrm{d}s = \int \mathrm{ctg}\, a\gamma\sqrt{1-a^2}\sin a\gamma \mathrm{d}\gamma = \frac{\sqrt{1-a^2}}{a}\sin a\gamma. \quad (14)$$

After substituting (14) into (13), further integration becomes impossible. To construct a guide curve on the sweep, you need to apply numerical integration of the resulting equations:

$$x = \sqrt{1-a^2}\int \cos\left(\frac{\sqrt{1-a^2}}{a}\sin a\gamma\right)\sin a\gamma \,\mathrm{d}\gamma;$$

$$y = \sqrt{1-a^2}\int \sin\left(\frac{\sqrt{1-a^2}}{a}\sin a\gamma\right)\sin a\gamma \,\mathrm{d}\gamma. \quad (15)$$

Through each point of the curve (15) on the sweep perpendicular to it a rectilinear generatrix of the torse passes. Taking this into account, the parametric sweep equations take the form:

$$X=\sqrt{1-a^2}\int\cos\left(\frac{\sqrt{1-a^2}}{a}\sin a\gamma\right)\sin a\gamma\,\mathrm{d}\gamma-$$

$$-u\sin\left(\frac{\sqrt{1-a^2}}{a}\sin a\gamma\right);$$

$$Y=\sqrt{1-a^2}\int\sin\left(\frac{\sqrt{1-a^2}}{a}\sin a\gamma\right)\sin a\gamma\,\mathrm{d}\gamma+$$

$$+u\cos\left(\frac{\sqrt{1-a^2}}{a}\sin a\gamma\right), \tag{16}$$

where u is the second independent variable – the length of the rectilinear generatrix on the sweep.

5. 3. Construction of strips based on the built parametric equations approximating the sphere and sweep of a separate strip

Parametric equations (9) make it possible to construct a strip of the sweep surface tangent to the sphere, and equation (16) – its contours on the plane, i.e., sweep. At the same time, the limits of change of independent parameters γ and u should be the same for the strip on the sphere and its sweep on the plane. For example, in Fig. 3, *a*, the strips tangent to the sphere are plotted along the lines at a=0.7, shown in Fig. 1, *b*.

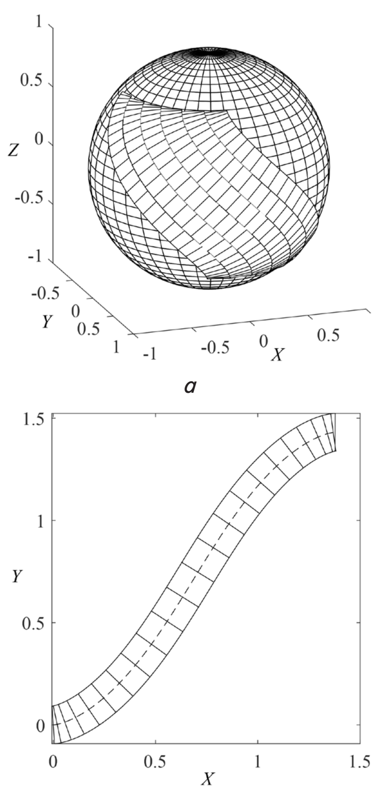


Fig. 3. Images constructed according to the built equations (9) and (16): *a* – strips of surfaces tangent to the sphere and rotated around the *OZ* axis by 15°; *b* – sweep of a separate band

The parameters change limits are as follows: γ=0...1.5π, u=–0.092...0.092. With the same values of the parameters, using numerical methods, a sweep of the strip is constructed according to equations (16) in Fig. 3, *b*.

In Fig. 4, *a*, we constructed a strip at a=0.1, which is tangent to the curve on the sphere shown in Fig. 2, *a*. The parameters change limits are as follows: γ=0...10π, u=–0.105...0.105.

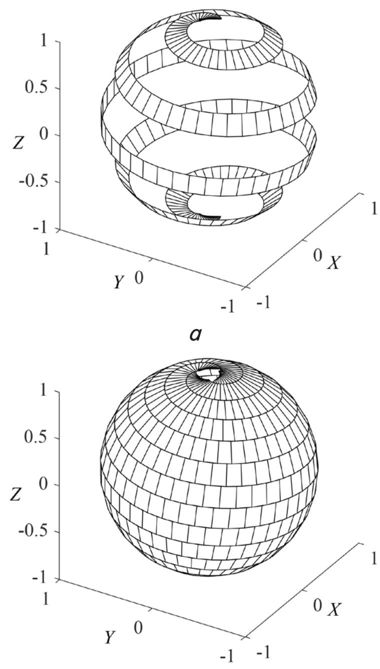


Fig. 4. Image of the strips tangent to the sphere, constructed according to the derived equations (9): *a* – a separate band; *b* – three identical strips rotated around the *OZ* axis by an angle of 120°

It would be possible to approximate the sphere with one strip. To this end, it would be necessary to take a larger width, but the approximation would be rough. A fairly accurate approximation is achieved with three strips three times smaller in width (Fig. 4, *b*). In Fig. 5, *a*, a sweep of a separate band is constructed.

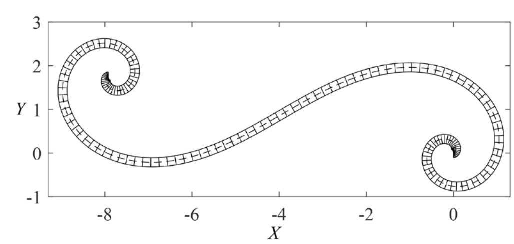
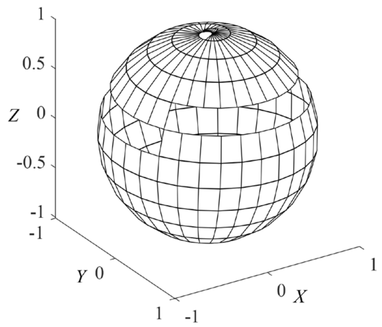


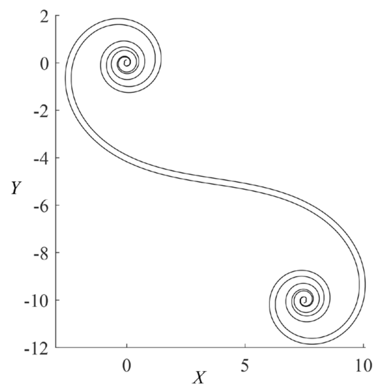
Fig. 5. The sweep of the strip that approximates the sphere in Fig. 4

Finally, the sphere can be approximated by a single strip along the curve shown in Fig. 2, *b*. The approximated sphere and the sweep of the strip are shown in Fig. 6. The parameters change limits are as follows: γ=0...25π, u=–0.125...0.125.

For clarity, part of the strip in Fig. 6, *a* is not shown within the limits of the change of the parameter γ from 8π to 10π. The inner surface of the sphere is visible through the hole.



a



b

Fig. 6. Graphic images of the approximation of the sphere by one strip: *a* — approximated sphere; *b* — strip sweep

6. Discussion of the procedure for approximating the sphere by strips of unfolding surfaces and the construction of sweeps of these strips

The resulting images of the approximated spheres confirm the reliability of the mathematical statements. A question may arise regarding the reliability of the built sweeps of strips approximating the sphere. If necessary, this can be verified by finding the first quadratic shape of the surface (9) and its sweep (16). The peculiarity of the proposed method is that for the approximation of the sphere slope lines are used for the first time, the elevation angle of which can be specified. Works [7, 8] argue about the approximation of the sphere by compartments of surfaces of the same or different shape. In works [6, 10], the sphere is approximated by strips of unfolding surfaces, but the lines of contact are other curves, different from the slope line.

The figures of the approximated spheres (Fig. 3, *a*, 4, *b*, 5, *a*) show that adjacent strips have a common line of contact. This is ensured by a correctly defined strip width, that is, the limits of the change of the *u* parameter. It should be noted that the theoretical finding of these limits causes difficulty, which is a drawback of the study, however, in this case, one can use the visualization capabilities of modern computer graphics software products. If the strip width is narrower than required, the computer image will show a gap between adjacent strips. This makes it possible to select the required width of the strip very quickly.

There are certain limitations for the proposed sphere approximation method, which are the impossibility of approximating the sphere near its poles. However, these limitations are not essential. As shown in Fig. 1, the guide line is located between two symmetrical parallels. This means that the sphere can be approximated not completely, namely in the specified area. However, this applies to lines with relatively large elevation angles. If the elevation angle is reduced, the area of the sphere that cannot be approximated will also decrease accordingly. This area can be reduced to almost zero, but at the same time there will be only one lane. Evidence of this is the sphere approximated by one strip in Fig. 6, *a* at the elevation angle of the guide curve $\beta=2.3°$. The disadvantage of this approximation is that in the region of the pole, the width of the strip should decrease, which is not predicted by the derived equations, in which the strip has a constant width. Further research can eliminate this shortcoming, but it is more expedient to approximate the area around the pole, for example, by a flat area outlined by a circle or a cone that is close to this area.

An important characteristic of a guide curve is the length of its arc. When finding the sweep, it plays a key role since the length of the arc on the strip approximating the sphere and on its sweep is constant. The length of the arc is determined by integrating expression (11), which in general rarely succeeds in integrating. In the considered case, it is not only possible but the expression itself after integration has a simple form. Owing to this, the parametric equations of the sweep (13) are greatly simplified since one of the integrals can be replaced by the corresponding expression. As a result, the parametric equations (16) include only one integral each, which require the use of numerical methods when constructing the strip sweep.

Thanks to the application of the theory of differential geometry and the devised technique, it became possible to approximate the sphere by several identical, including one, continuous strips. Parametric equations were derived for constructing sweeps of these strips. This provides an advantage in comparison with known methods of approximating the sphere and thereby closes the problematic part of the transition from graphical methods of approximation and construction of sweeps to an analytical description of both the approximated sphere and strip sweeps.

Further development of our research may address the approximation of architectural objects, which include domes in the form of a sphere or its part. If such domes are covered with sheet material, then this material can only be an unfolding surface when wrapping the sphere. By changing the angle of elevation of the guide curve, the width of the strip, you can give the roof an original shape and geometric expressiveness.

7. Conclusions

1. In the proposed technique, the slope curve on the surface of the sphere serves as a guide curve, which allows the sphere to be approximated by both several and one strip of unfolding surface of a constant width. To implement such an approximation, a mathematical description was developed in the form of parametric equations both tangent to the strip surface and its sweep.

2. The proposed approximation technique involves the use of a differential geometry apparatus to construct a sweep. A feature of the resulting parametric sweep equations is that they require numerical calculation methods. The reliability of the obtained sweep equations is confirmed by the coefficients of the first quadratic form of the strip equations on the sphere and on the sweep.

3. The reliability of the resulting mathematical statements is confirmed by the constructed images of various variants of the approximated spheres. Visualization of the

approximated sphere is carried out by multiplying a separate strip based on its parametric equations. Visualization of the strip sweep was also carried out, which makes it possible to fabricate flat blanks from sheet material to approximate the sphere or to cover spherical structures with sheet material in construction practice.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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