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FORMATION OF CONJUGATE CURVED SURFACES THAT EXCLUDE INTERFERENCE

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The object of the study is the method of geometric design of conjugate curved surfaces. The subject of the study is the possibility of achieving the absence of interference of the designed surfaces of kinematic pairs of technical structures, for example, products in mechanical engineering (cutting tools, gearing), based on an invariant method. This problem is of significant importance for a wide class of products, but especially for gears from the point of view of the precision of manufacturing the tooth surface, load-bearing capacity, and, consequently, the reliability and durability of the transmission. When calculating the strength, each tooth of the gear can be considered as a cantilever beam, loaded at the free end by the force of interaction with the second gear wheel. In the presence of interference (for trimming gears), not only the shape of the tooth surface is disrupted, but also the cross-sectional area of the tooth base is significantly reduced. Therefore, solving such a problem is important when designing and manufacturing gears. That is why, when forming the initial curved tool surface of a cutting tool, the conjugacy condition between the points of the product and the tool is taken into account.

As a result, when designing a cutting tool using the proposed invariant method using a unified methodology, a wide class of complex curved surfaces can be formed. In particular, this applies to technological operations on numerically controlled machines. The generated conjugate curvilinear transformations of the cutting tool using an invariant method make it possible to avoid interference of mating surfaces at the design stage. At the same time, the curvilinear characteristics of surfaces are analyzed, defining a family of normals as common to two conjugate surfaces with linear contact, which is essential for the design process

Keywords: invariant method, cutting tool, mating surfaces, tooth interference, linear contact

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1. Introduction

Modern trends in the development of technology are characterized by the use of more complex curved shapes of the working surfaces of tools and related products. When processing them, cutting tools (hob cutter, disk cutter) are widely used. The most important stage in the shaping of a cutting tool (its profiling) is based on the classical theory of mating surfaces, which has been further developed in modern research.

Existing methods for designing conjugate curved surfaces lead to the need to solve a large number of equations. In addition, the design of a tool also includes the calculation of its structural elements and the development of a non-normalized secondary tool. Reducing labor intensity, improving quality and reducing design time make it necessary to automate the process of shaping the mating curved surfaces of a cutting tool. Therefore, research devoted to optimizing the design process of complex conjugate curved surfaces of kinematic pairs with guaranteed absence of interference is relevant.

2. Literature review and problem statement

Fundamental developments to eliminate interference are laid down in work [1]. The phenomenon of interference is presented as follows: «interference occurs when any part of space turns out to be inside the volumes of two bodies at the same time». If one of the bodies is a cutting tool, and the other is a part, then interference leads to undercutting and a decrease in accuracy in the manufacture of the part and makes it impossible to process it. If these bodies are gears in mesh, then the interference manifests itself as jamming, and if the manufacturing of a cutting tool – shearing of the teeth. The practical implementation of these requirements when forming curved mating surfaces is difficult due to the need to construct a large number of axoids. This circumstance significantly complicates the design process and reduces its accuracy.

The study [2] considered the problem of revealing the position of a singular point located on a surface with a return edge of a curved surface. The issue of eliminating a possible singular point remains unresolved, since the required nominal surface of the part cannot be obtained with the required accuracy.

The study [3] is of theoretical interest; an automated assessment of the manufacturability of machining processes to eliminate interference is considered; the work is devoted to the operation of grinding products. It is possible to take into account the production assessment of axisymmetric parts manufactured by cylindrical and internal grinding methods, which significantly complicates the production process.

In [4], the possibility of eliminating interference when creating silent and compact gearboxes with high torque was explored. The proposed condition for the appearance of interference during the operation of kinematic pairs is of a general nature and does not offer specific measures to eliminate it at the design stage.

The assumption in the study [5] is worthy of attention, the essence of which is that the condition for identifying interference is the determination of the limiting points at which the radius of curvature of the working surface of a wheel tooth manufactured by the rolling method is equal to zero. Objective difficulties appear with analytical calculations in the region of parameter space intended to test the validity of theoretical conclusions.

The study [6] proposed analytically obtained regions of the appearance of interference in the parameter space for control excitation. The accuracy of the theoretical predictions, as well as the effectiveness of the proposed control system, are verified by comparison with numerical simulations. Approximate results are given, but there are many contradictions and limitations.

Research [7] on interference elimination in non-circular gear machining mainly focuses on gear hobbing. However, this method has many limitations, including the inability to machine internal gears or non-round external gears with a concave pitch curve, and it may result in undercutting.

Study [8] provides an estimate of surface profile deflection due to wear, presented as a function of error. A spur gear is taken as an example system to demonstrate the interaction between surface wear and dynamic behavior. Simulations have shown that surface wear and gear dynamics are closely related.

The work [9] proposed a geometric method for determining interference based on the shape of the contact spatial surface of kinematic pairs and a cutting tool. The disadvantage is the following: for each intersection line it is necessary to calculate the coordinates using formulas specific to them; it is necessary to create an additional algorithm to determine which formula to use when determining the intersection line.

In [10], studies were carried out on conjugate curved surfaces with a decrease in contact stresses to eliminate interference during gear operation. Modernized methods of circular and helical transformation made it possible to model the original curved surface to obtain the required curved conjugate shapes of the product. In this case, the parameters of the curved surface can be fixed, except for one, and this parameter can be arbitrarily changed and adjusted, but in practical work this causes some inconvenience.

It should be noted that the accurate prediction of identifying gear interference depends on the shape of the curved mating surfaces. The peculiarity of the proposed research is as follows: the shaping of a curved conjugate surface in a section perpendicular to its axis of rotation is specified in the moving coordinate system X, Y. The choice of projection curve shape depends on the shaping surface contact lines, which helps to extend the life and improve the performance of gears.

3. The aim and objectives of the study

The aim of the study is to improve the accuracy in the formation of conjugate curved surfaces, eliminating interference at the design stage. This will make it possible to design products in mechanical engineering with complex curved surfaces, which will improve the accuracy and productivity of the process.

– study the formation of a conjugate curved surface with a linear contact;

– generate a profile of conjugate curved surfaces using the proposed method with predetermined parameters.

4. Materials and methods of the study

The object of the study is the method of geometric design of conjugate curved surfaces.

The subject of the study is the possibility of achieving the absence of interference of the designed surfaces of kinematic pairs of technical structures based on the invariant method. This approach will provide the basis for creating computer routines in the MATLAB system. Hypothetically, this will open up prospects for improving the manufacturing process. The study is based on geometric and mathematical descriptions of surfaces.

An invariant method of designing products of the required shape and manufacturing accuracy, at the design stage with predetermined parameters, will make it possible, using a unified methodology, to form a wide class of complex curved surfaces with linear contact.

5. Results of forming conjugate curved surfaces using an invariant method

5. 1. Formation of conjugate curved surfaces

In curvilinear projection, curved lines are used to obtain projections of a geometric image. In this case, the projection of a spatial point is defined as the point of intersection of the projecting line passing through the point with the projection plane. Curvilinear projection can be successfully used to solve problems of determining the meeting points of a line with a surface, as well as to find the section of a surface by a plane and the line of intersection of two surfaces.

Theorem: «If each of the conjugate surfaces Σ_A and Σ_B is considered as an envelope of families of pairwise conjugate axoids Φ_A and Φ_A, then each point of contact of surfaces Σ_A and Σ_B is defined as the point of contact of the characteristics of the axoids Φ_A and Φ_A with surface Σ_A» [1] (Fig. 1).

The initial curvilinear surface Φ is specified by the radius vector $\overline{r}(\Sigma, \tau)$ and the curvilinear axis $m(u)$, which is determined by the radius vector $\overline{m}(u)$. The surface Σ is formed by rotation around the corresponding points belonging to the $\overline{m}(u)$ axis. Each point on the surface is set to an angle $\varphi(\Sigma, \tau)$, which depends, in the general case, on the position of the point on the original surface Φ.

On the generatrix $l(\tau)$ of the surface Φ, a certain point M is selected. A horizontal plane Γ is drawn through it. The point M_0, at which the plane Γ intersects the m axis, is the projection of the point M onto the m axis. Point M rotates around point M_0 through a certain angle $\varphi(\Sigma, \tau)$. The described circular transformation of point M is determined by the position of point M^*. Carrying out similar transformations of all other points of the generatrix $l(\tau)$ of the surface Φ, let's obtain points of the helical line $l^*(\tau)$ on the surface Σ.

To form a curved contact surface Σ, it is considered that point M is specified by a radius vector (Σ, τ), where Σ and τ are the curvilinear coordinates of point M on the surface Φ.

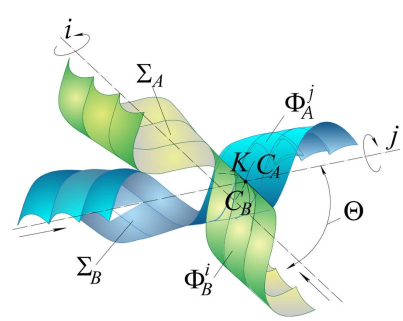

Fig. 1. Geometric model of conjugate curvilinear axoids

The transformation described above is applied to each selected point M_{ij} on each generating curved line $l_i(\tau)$ of the original surface Φ (Fig. 2).

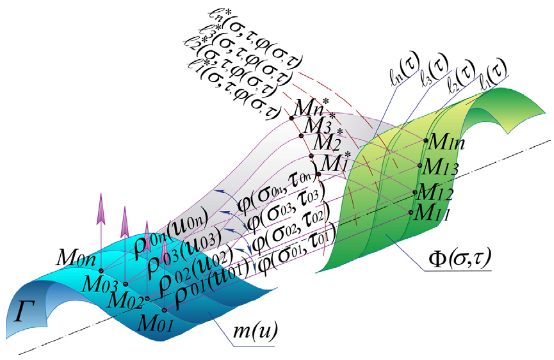

Fig. 2. Linear contact of two curved surfaces

The result is a family of curved lines (1) and linear contact of two surfaces:

$$\ell_i^*(\sigma,\tau,\varphi(\sigma,\varphi)). \tag{1}$$

The resulting system of equations (2) determines the linear contact of the curved conjugate surface Σ:

$$\left.\begin{array}{l} \ell_1^*(\sigma,\tau,\varphi(\sigma,\varphi)), \\ \ell_2^*(\sigma,\tau,\varphi(\sigma,\varphi)), \\ \ell_3^*(\sigma,\tau,\varphi(\sigma,\varphi)), \\ \dots\dots\dots\dots\dots\dots \\ \ell_n^*(\sigma,\tau,\varphi(\sigma,\varphi)). \end{array}\right\} \tag{2}$$

The theory presented above is discussed below using a developed computer routine in MATLAB:

```
% Setting cone parameters
a_cone=3;
b_cone=2;
h_cone=5;
angle_cone=–pi/12;
x0_cone=–25;
y0_cone=15;
z0_cone=–10;

% Create cone coordinates
u_cone=(0:0.05:5)';
vp_cone=[pi/2:0.05*pi:3*pi/2];
[U_cone, V_cone]=meshgrid(u_cone, vp_cone);

X_cone=x0_cone+a_cone*(1–U_cone/h_cone).
*cos(V_cone);
Y_cone=y0_cone+b_cone*(1–U_cone/h_cone).
*sin(V_cone);
Z_cone=z0_cone+0.5*U_cone.^2;

figure('Color', 'w');
Cone1=mesh(X_cone, Y_cone, Z_cone);
xlabel('x'); ylabel('y'); zlabel('z');

% Rotate cones -15°
Angle=-pi/12;
X_cone_rotated=(X_cone–x0_cone)*cos(Angle)–
(Y_cone–y0_cone)*sin(Angle)+x0_cone;
Y_cone_rotated=(X_cone–x0_cone)*sin(Angle)+
(Y_cone–y0_cone)*cos(Angle)+y0_cone;
Z_cone_rotated=Z_cone;

% Create a line intersecting a cone
VperX_cone=atan((a_cone*(cos(Angle)–1))/(b_
cone*sin(Angle)));
Xper_cone=x0_cone–a_cone*u_cone.*cos(VperX_cone);
Yper_cone=y0_cone+b_cone*u_cone.*sin(VperX_cone);
Zper_cone=z0_cone+0.5*u_cone.^2;

hold on;
Cone2=mesh(X_cone_rotated, Y_cone_rotated,
Z_cone_rotated);
LinePer_cone=plot3(Xper_cone, Yper_cone, Zper_cone).
```

The result of constructing curved surfaces is presented in Fig. 3.

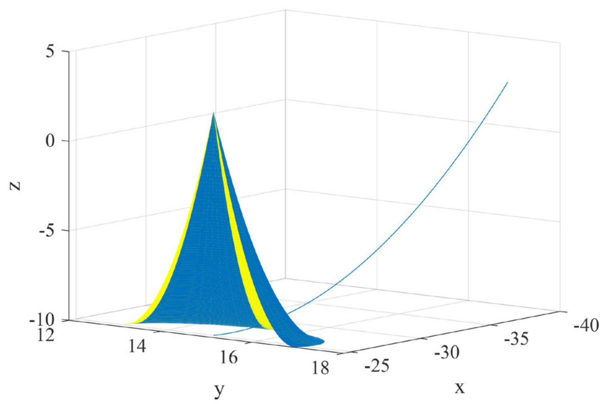

Fig. 3. Conjugate curved surfaces

In the invariant method of transforming a curved surface, in addition to the rotation angle, the shift step is specified. Thus, first the original curved surface is moved by a given step, and then a translational transformation of the resulting curved surface is performed. In the described sequence of actions, the curved axis must also be moved one step. Let the curved surface be rotated by an angle of 30° and shifted by 5 mm:

```
% Setting cone parameters
a_cone=3;
b_cone=2;
h_cone=5;
angle_cone=pi/6;
x0_cone=–25;
y0_cone=15;
z0_cone=–10;

% Create cone coordinates
u_cone=(0:0.05:5)';
vp_cone=[0:0.05*pi:2*pi];
[U_cone, V_cone]=meshgrid(u_cone, vp_cone);

X_cone1=x0_cone+a_cone*(1–U_cone/h_cone).
*cos(V_cone);
Y_cone1=y0_cone+b_cone*(1–U_cone/h_cone).
*sin(V_cone);
Z_cone1=z0_cone+0.5*U_cone.^2;
```

```
X_cone2=x0_cone–a_cone*(1–U_cone/h_cone).
*cos(V_cone);
Y_cone2=y0_cone–b_cone*(1–U_cone/h_cone).
*sin(V_cone);
Z_cone2=z0_cone+0.5*U_cone.^2;

figure('Color', 'w');
Cone1=mesh(X_cone1, Y_cone1, Z_cone1);
xlabel('x'); ylabel('y'); zlabel('z');

hold on;
Cone2=mesh(X_cone2, Y_cone2, Z_cone2);

% Rotate cones -30°
Angle=-pi/6;
X_cone_rotated1=(X_cone1–x0_cone)*cos(Angle)–
(Y_cone1–y0_cone)*sin(Angle)+x0_cone;
Y_cone_rotated1=(X_cone1–x0_cone)*sin(Angle)+
(Y_cone1–y0_cone)*cos(Angle)+y0_cone;
Z_cone_rotated1=Z_cone1;

X_cone_rotated2=(X_cone2–x0_cone)*cos(Angle)–
(Y_cone2–y0_cone)*sin(Angle)+x0_cone;
Y_cone_rotated2=(X_cone2–x0_cone)*sin(Angle)+
(Y_cone2–y0_cone)*cos(Angle)+y0_cone;
Z_cone_rotechedol=mesh(X_cone_rotated2,
Y_cone_rotated2, Z_cone_rotated2);
hold on;
Z_cone_rotated1=Z_cone1;
hold on
LinePer=plot3(X_cone_rotated1, Y_cone_rotated1,
Z_cone_rotated1);
hold on
LinePer=plot3(X_cone_rotated2, Y_cone_rotated2,
Z_cone_rotated2).
```

The result of constructing curved surfaces is presented in Fig. 4.

The studied curvilinear transformation of conjugate curved surfaces allows one to work with the values of its array (parameters) at the design stage and vary them, which will eliminate interference. The initial curved surface can be specified by any other parametric equation.

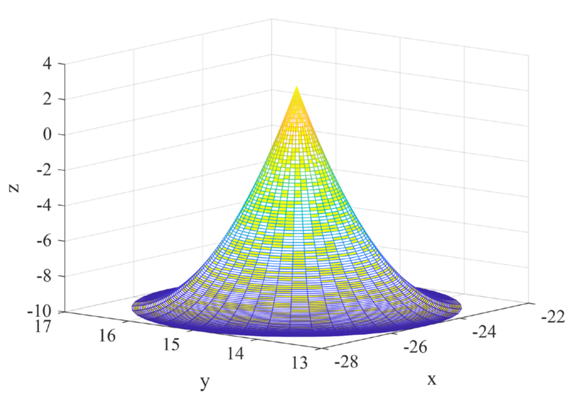

Fig. 4. Curvilinear transformation of surfaces is presented

5. 2. Formation of a profile of conjugate curved surfaces with predetermined parameters

When forming a profile of conjugate curved surfaces, surface Σ_A, conjugate with a given curved surface Σ_B, is formed according to the following method (Fig. 5).

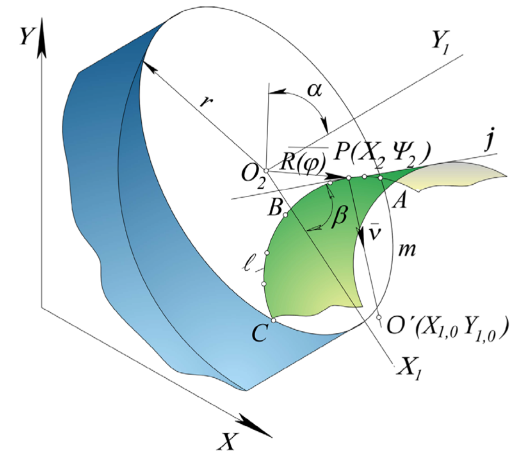

Fig. 5. Profile of a curved surface in a section perpendicular to the axis of rotation

The system of equations for the formation of the profile of conjugate curved surfaces in a fixed XY coordinate system when rotated by an angle α has the form (3):

$$\begin{cases} x = r\cdot\alpha + x_i\cos\alpha + y_i\sin\alpha, \\ e = r\cdot x_i\sin\alpha + y_i\cos\alpha, \end{cases} \tag{3}$$

where $x_i = f_i(\varphi)$, if:

$$\varphi \in [\varphi_{i-1}, \varphi_i],\ i = 1,2,\ldots,k;\ y_i = g_i(\varphi).$$

From the condition of collinearity of the tangent vector to the profile of conjugate curved surfaces in the line of tangency:

$$\frac{dx}{d\alpha}\cdot\frac{dy}{d\varphi} - \frac{dx}{d\varphi}\cdot\frac{dy}{d\alpha} = 0. \tag{4}$$

Solving equation (4), let's obtain the coupling equation with the parameters φ and α:

$$\varphi = \bar{h}_i(\alpha),$$

if

$$\varphi \in [\varphi_{i-1}, \varphi_i],\ i = 1,2,\ldots,k. \tag{5}$$

Solving equations (5) and (3) together, let's obtain the profile equations for the initial formation of the conjugate curved surface in the moving XY coordinate system:

$$\begin{cases} x = r\alpha + f_i\left(\bar{h}_i(\alpha)\right)\cdot\cos\alpha + g_i\left(\bar{h}_i(\alpha)\right)\cdot\sin\alpha, \\ y = r - f_i\left(\bar{h}_i(\alpha)\right)\cdot\sin\alpha + g_i\left(\bar{h}_i(\alpha)\right)\cdot\cos\alpha. \end{cases} \tag{6}$$

When forming the profile of conjugate curved surfaces, it is necessary to set in the developed subroutine in the MATLAB system: n – the number of control points; t – module of the tangent vector; t_1, t_2 – projections of a unit tangent vector on the $O_1\,X_1$ and $O_1\,Y_1$; k – the number of curved line segments. As a result, the result shown in Fig. 4.

The data obtained for constructing curved surfaces made it possible to transform the system of equations (2) to the form of the system of equations (7):

$$\begin{cases} x = x_0 + t\sin t, \\ y = y_0 + t\cos t, \\ z = z_0 + ct, \end{cases} \quad (7)$$

where $c=H/2p$; $0 \le t \le 2p$, $H=5$, $x_0=5$, $y_0=-25$, $z_0=-2$.

The resulting parametric system of equations for a curved surface with linear contact has the general form (8):

$$\begin{cases} x = f_x(u,v), \\ y = f_y(u,v), \\ z = f_z(u,v), \end{cases} \quad (8)$$

where $u_{min} \le u \le u_{max}$, $\nu_{min} \le \nu \le \nu_{max}$.

Curvilinear transformation of surfaces allows to work with the values of profile parameters at the design stage and vary geometric parameters.

6. Discussion of the results of the study of an invariant method of shaping conjugate curved surfaces that excludes interference

Based on the analysis of existing methods for designing and manufacturing complex mating surfaces of technical structures and their noted shortcomings in comparison with the proposed invariant design method, it is possible to conclude that the practical use of the proposed method increases the accuracy and productivity of the manufacturing process.

The feasibility of using the proposed invariant method for creating conjugate curved surfaces that interact during the operation of mechanisms is based on a geometric approach, with the help of which the coordinates (arrays) of points of conjugate curved surfaces with linear contact are determined. Elimination of a possible singular point [2] is necessary to determine the nominal surface of the part. In this case, as a result of rotation synthesis, the initial point of mutual contact of two curved surfaces is obtained.

As a result, the *x*, *y* and *z* coordinates represent a curved surface, which allows to determine the contact lines of intersection of the curved axis with the horizontal plane. This is the mathematical justification for rotating the coordinates of a given parametric surface (Fig. 3), which was more convenient [10] for obtaining the required curvilinear conjugate shapes of the product. The intersection of the axis of the curvilinear generatrix with horizontal planes is carried out according to formula (3). The result is a curved line of contact between two curved surfaces (Fig. 4), which is a one-dimensional array of values for each coordinate. From the coordinate arrays of the original and resulting curved surfaces, the coordinates of the intersection line are determined. The intersection line arrays will contain those values that are equal for the original and curved surfaces [9].

In this case, when rotating around a straight line (coordinate axis, the surface's own axis of rotation), the parameter v will be constant for any value of u. From the equality of the X coordinate values of the original and curved surfaces, the value of the parameter v is determined.

Thus, having obtained a geometric model of a curved surface and determined its curvilinear profile, it is possible to perform a theoretical interpretation – a linear trajectory of the contact of mating surfaces, described by equation (8). Modeling of conjugate curved surfaces and the algorithm for their formation allows the formation of a wide range of complex curved surfaces using a unified methodology. The versatility of the method is achieved by using the movement of the contact lines of the original surfaces relative to the *x*, *y* and *z* coordinates, which represent a curved surface, which will make it possible to determine the contact lines of intersection of the curved axis with the horizontal plane. Thus, the contact line is determined from the coordinate arrays of the original and resulting curved surfaces, and this does not require the construction of a large number of axoids [1]. It is possible to avoid profile deviation from surface wear [8], obtaining equation (6) of the profile of the initial formation of the conjugate curved surface in the moving *XY* coordinate system.

This invariant method has a number of disadvantages:

– for different types of curved surfaces, it is necessary to calculate your parameters v, and to determine the coordinates of the intersection line;

– formulas for determining the coordinates of intersection lines must be specified explicitly.

However, from the experiments performed, it follows that the contact line of intersection can be determined from the coordinates of the original surface corresponding to the values of the parameter v equal to ..., -2π, $-3\pi/2$, $-\pi$, $-\pi/2$, 0, $\pi/2$, π, $3\pi/2$, 2π, ..., i.e. thus, the generatrix of the original surface can coincide or be parallel to the *XZ* or *YZ* coordinate planes.

7. Conclusions

1. The proposed process of forming conjugate curved surfaces with linear contact allows to take into account the phenomenon of interference, which makes it possible to adapt it for use on machines with numerical control.

2. The methodology of the proposed invariant method makes it possible to determine the curved profile of surfaces with predetermined parameters.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, including financial, personal, authorship, or any other, that could affect the study and its results presented in this article.

Financing

The study was conducted without financial support.

Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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