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REVEALING PATTERNS IN THE INFLUENCE OF VARIABLE PERMEABILITY OF WELL BOTTOMHOLE ZONES ON THE OPERATIONAL MODES OF UNDERGROUND GAS STORAGE FACILITIES

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The object of this study is underground gas storage facilities (UGSF). The main problem being solved is to ensure effective management of the operation process of underground gas storage facilities (UGSF) at operational and forecast time intervals. One of the main factors that affect the operating modes of UGSF is significantly non-stationary filtration processes that take place in the bottomhole zones of wells. The complexity of assessing the multifactorial impact on depression/repression around the wells affects both the speed and accuracy of calculating the mode parameters of UGSF operation. Analysis of the results of well studies revealed a significant area of uncertainty in the calculation of the filtration resistance coefficients of their bottomhole zones. A satisfactory accuracy of the result in the expected time was achieved by building a model of integrated consideration of the influence of the parameters of all the bottomhole zones of the wells on the mode of UGSF operation. It turned out that the integrated consideration of the impact on the parameters of the bottomhole zones of the wells neutralized the effect of significant changes in the filtration resistance coefficients of the wells and ensured a sufficient speed of calculation of UGSF operation modes. Simultaneous simulation of ten operating UGSFs under the peak mode of withdrawal the entire available volume of active gas takes no more than six minutes. The speed of simulation of filtration processes in the bottomhole zones of wells ensured finding the best of them according to one or another criterion of operation mode quality.

As a result of the research, a model was built and implemented by software, which was tested under real operating conditions and provides optimal planning of UGSF operating modes for given time intervals. Its use is an effective tool for the operational calculation of current modes and technical capacity of UGSF for a given pressure distribution in the system of main gas pipelines. The performance of the constructed mathematical methods has been confirmed by the results of numerical experiments

Keywords: underground gas storage facility, filtration resistance, Darcy's law, skin factor, Forchheimer coefficient

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1. Introduction

Underground gas storage facility is a complex technological object. In the research object, as a complex dynamic system, distributed gas-dynamic and filtration processes take place at significant spatial and temporal time intervals. Transient processes of continuous and discrete types in system objects vary in time from seconds to several months. Features of the research object affect the complexity of models and methods of their investigation. In the process of UGSF operation, there is often a change in the states of the isolation and control valves, which affects the change in its topology and, accordingly, the change in the structural model and gas flow models in the technological sites of UGSF. Topology changes also occur during reconstruction and repair works.

The complexity of UGSF modeling as a single hydraulic object is influenced by the variety of objects in terms of technological purpose and the mathematical representation of their gas flow models with concentrated and distributed parameters. The variety of gas flow models at objects complicates the construction of an integrated model of UGSF and methods of their implementation. A particular difficulty is the influence of various factors on the modeling of nonlinear processes. These factors include predictable, weakly predictable, random, concentrated, and distributed, slow and fast, internal, and external.

The dimensions of gas-dynamic, filtration, and thermal processes in objects vary by up to six orders of magnitude. Other parameters – time of simulation, transition modes and planning of operation modes of UGSF – change on an interval from seconds to several seasons of gas withdrawal/injection.

All objects that have a gas flow model exert an influence on the operating mode of UGSF. The study of the model of each facility of UGSF must be carried out in connection with the system of models of other objects. Often, the properties of individual objects and the composition of the system are different – synergistic effects are manifested.

In the process of researching complex objects, it is often necessary to present them through components in such a way that, upon completion of the study of the components, it is possible to «stitch» them. It turns out that such a representation cannot be arbitrary. For example, the process of «stitching» models of hundreds of wells with a complex gas collection system is an incorrect mathematical problem.

The most difficult to study are the processes in the underground part of UGSF. For this, geophysical, geological, hydrodynamic, and other methods are used. The interpretation of the results obtained by different methods in the process of researching reservoirs is often contradictory and ambiguous. This makes it difficult to identify the factors influencing the filtration processes, which are necessary to build the most adequate models.

Modeling packages are an effective tool for ensuring optimal management of the operation process of a complex system – UGSF. Their development is based on the results of fundamental research in many fields of science, in particular in the field of underground hydrodynamics.

Therefore, it is a relevant problem to construct an integrated model that takes into account the influence of the parameters of all the bottomhole zones of wells on the mode of UGSF operation.

2. Literature review and problem statement

The beginning of the study of fluid filtration in rocks can be attributed to 1927, when Kozeny solved the Navier-Stokes equation for fluid movement in a porous medium, representing it as a set of pores of the same size [1]. He established a relationship between the permeability and porosity of the rock and the surface area of voids.

The physical and reservoir properties of rocks are much more complex. They are different in the number of particles of different sizes in the reservoir rocks and in the concavity and fissuring, etc. Such properties affect both the open and effective porosity of reservoirs, as well as their effective permeability.

Subsequently, the Hagen-Poiseuille equation [2] was derived, which describes the flow in a single straight capillary tube and is the simplest flow equation. A tortuosity coefficient was introduced into it, which made it possible to use the size distribution of pores to calculate the permeability of sand rocks.

Pore size distribution can be estimated from core samples, if available. Practice shows that the permeability of rocks according to the results of core research conducted by existing methods is ten times, and sometimes more than times, lower than the permeability obtained according to the processing data of industrial studies.

The general expression for fluid flow in porous media was built by Darcy in 1856 based on the interpretation of various parameters affecting the flow of water through sand filters; it was named Darcy's law [3].

Gas permeability of a porous medium under the conditions of a linear resistance law (Darcy's law) is proportional to the gas flow rate. Gas flows into the well in violation of Darcy's law – the linearity of the relationship between the permeability of the porous medium and gas flow is violated.

Works [1–3] do not provide for the calculation of the throughput capacity of the bottomhole zones of wells in the quadratic and transitional regions of resistance, and also do not take into account additional resistances – the skin factor and resistance generated by the nature of the opening of the bottomhole zones of wells.

Studies [1–3] were further developed. For example, in [4], separate problems that arise in the process of extracting residual gas from depleted gas deposits with a deteriorated condition of the bottomhole zone of the formation are investigated. One of the methods of reducing the impact of the contaminated bottomhole zone of the reservoir on the productive characteristics of wells is the creation of perforation channels in the bottomhole zone.

The influence of the number and sizes of perforation channels on the flow rate of a gas well, depending on the permeability of the reservoir at constant wellhead pressure, was studied. An unsolved issue is the study of the effectiveness of additional perforation under the conditions of various options for contamination of the bottomhole zone of the reservoir. The separate identification of factors affecting the permeability of the formation is also an issue. Carrying out additional perforation of the column is an expensive procedure, and therefore it is necessary to be sure that it is necessary, and not the cleaning of the bottomhole zone. Transient non-stationary operation modes – under variable mouth pressure – have not been studied.

In works [5, 6], the causes and nature of contamination of the bottomhole zone of gas wells are given. The structure of the skin factor is revealed. The characteristics of the types of well completions and the main parameters necessary to determine the skin factor are given. Studies of the influence of the permeability and thickness of the contaminated bottomhole zone of the reservoir on the value of the skin factor have been carried out. Optimal values of permeability and thickness of the contaminated zone were obtained, above which the skin factor changes little.

The research was carried out using a well-known software tool. The results of the study have theoretical significance. Under the conditions of UGSF operation, such detailed studies of wells are not carried out because they require turning the wells into non-working mode. This reduces the productivity of UGSF and increases the consumption of fuel gas per unit volume of extracted gas.

Methods of single-point (one-time) testing of wells using dimensionless gas inflow characteristics are proposed in [7] to calculate the current productivity of gas wells with a wide range of reservoir properties.

It is shown that the average absolute value of errors between the method of multi-point analysis and the single-point one proposed in the cited work is 11.6 %. But the questions related to the substantiation of the universality of the method remained unresolved. All factors influencing the permeability of the bottomhole zone act in a non-linear manner. Moreover, the nonlinearity is different for each well. Such methods should be expected, it may be appropriate to use them in gas fields, provided Darcy's law is fulfilled and the reservoir system is homogeneous. In the general case, it is shown that this method contradicts the physical nonlinear model of gas movement in the bottomhole zone to the wells on UGSF.

Works [8, 9] report the study of the coefficient β, which occurs in the nonlinear model of gas inflow to wells.

The main issue is the representation of this coefficient through the parameters of the structure of the porous medium – geological and gas dynamic. The main parameters – porosity, structure of the porous medium, penetration, etc. in the near and far zones of the wells are known approximately enough that it is problematic to find a representation of the coefficient β through effective parameters. This coefficient is often called eddy drag or non-Darcy.

In [8], it was established that there is a direct relationship between porosity and grain size – the larger the grain size, the greater the porosity, and vice versa. It was established that the coefficient β is inversely proportional to the porosity and permeability, and also that the coefficient β is correlated with the permeability, porosity, and tortuosity of reservoir layers.

The results of the work can be obtained purely theoretically. For this, there is no need to conduct natural experiments. There are no studies of the influence of the β coefficient on the dynamics of the change in the amount of depression/repression in the field of bottomhole failure under non-stationary modes of their operation.

The authors of paper [9] propose a new mathematical model that takes into account the behavior of real gas to assess the efficiency of gas flow to production wells. The results showed that more than 90 % of the energy of the gas filtration flow is consumed by inertial forces. The permeability of the formation, the initial formation pressure, the skin factor have a great influence on the productivity of the gas flow to the wells. The proposed model is claimed to predict well performance curves with high accuracy.

The main result of the work can be obtained from the following considerations. In the SI system, the coefficient of hydraulic resistance to movement in the pipe is approximately 10^{-2}, and the coefficient of filtration resistance in the reservoir layers is close to 10^{-12}. The proposed gas inflow model is stationary. Representations of the coefficient β are taken from the literature. It is incorrect to use it to conduct research on other wells, as such a presentation is specific to a specific well with specific factors influencing its productivity. The results are partial. Such a stationary model cannot predict the productivity of wells under variable conditions of its operation.

In [10], a radial flow experiment was performed, and it was found that non-Darcy flow exists even at very low flow velocities. In addition, the non-Darcy effect was found to arise not only from turbulence but also from inertial effects. The existence of non-Darcy flow was confirmed for all investigated samples.

In the cited work, the conclusion is important, which was confirmed not theoretically but in the process of conducting natural experiments under laboratory conditions. Different filtration flow rates with different porosity of artificial samples were considered. And in all cases, the non-Darcy coefficient was calculated.

Most authors of the above works study the impact on the productivity of the wells under known factors of influence on the permeability of the bottomhole zones under stationary conditions of their operation. Under the real conditions of operation of wells, it is impossible to accurately identify the factors affecting the productivity of wells. Wells, as an object, under real conditions are not operated separately but are integrated into a complex thermo-hydraulic system that simultaneously affects the productivity of each well separately and all wells in general. In such cases, the problem of well research is systemic.

There are no studies on the influence of various factors on the dynamics of changes in the amount of depression/repression at monthly or seasonal time intervals depending on the variable productivity of wells.

The above works confirm that Darcy's law is true in limited intervals of filtration flow velocities. A common approach has been to use the Forchheimer equation and its inertial flow parameter (β) as an extension of Darcy's law beyond the linear flow domain. This is particularly important for fracture conductivity calculations where flow rates are much higher than in the surrounding reservoir. New experimental data under conditions of very high flow rates have convincingly shown that Forchheimer's equation, like Darcy's law, has a limited range of application. At high flow velocity gradients, it is not possible to predict the parameters of filtration flows using the Darcy or Forchheimer equations. Analysis of well test results also show that β is not a unique function of permeability as expected but is the same function of Reynolds number as Darcy permeability. This leads to different values of β for the same porous medium depending on the flow range used for the measurement.

The results of works [8–10] confirm the problematic nature of building universal models of gas inflow to well blowouts that would work reliably in the entire range of operational data changes at significant time intervals.

More complex problems arise in the process of researching wells that are integrated into a single hydraulic complex – UGSF. There are no such studies in open sources. There are problems with the study of factors affecting the flow rate of each of the hundreds of wells on UGSF, given the incompleteness of the existing information support. Moreover, there is a significant area of uncertainty in the influence of poorly predicted factors on the permeability of the bottomhole zones.

In order to neutralize the influence of the mentioned ill-defined factors on the stability of the process of calculating the mode parameters of UGSF operation, it is necessary to devise models of the integrated influence of variable depressions/repressions of wells on the pressure at the gas collection point. It should be expected that such an approach will ensure guaranteed accuracy and speed of obtaining the result – calculation of UGSF operating modes.

3. The aim and objectives of the study

The purpose of this study is to determine the factors affecting the amount of depression/repression of non-stationary gas filtration in the reservoir-well system under the conditions of UGSF operation in gas injection and gas withdrawal modes. This will make it possible to construct robust methods and algorithms of minimum complexity for operational and predictive dispatching control of UGSF, taking into account the main factors influencing non-stationary filtration processes.

To achieve the goal, the following problems were set:

– to analyze models of gas inflow to the well under conditions of stationary and non-stationary filtration processes;

– to build a model of the integrated influence of variable volumes of well gas withdrawal/injection on depression/repression in the bottomhole zones of the wells and on the operating parameters of UGSF.

4. The study materials and methods

The object of our research is underground gas storage facilities as part of the gas transmission system. The subject of the study is the influence of the coefficients of filtration parameters and non-stationary filtration processes on the depression/repression of pressure in the bottomhole zones of wells during the operation of underground gas storage facilities.

The research hypothesis was as follows. Underground gas storage wells are operated with varying amounts of gas injection/withdrawal throughout the year. In contrast to gas storage facilities, the process of extracting gas from deposits takes place over ten years or more. And that is why filtration processes in the bottomhole zones of UGSF take place an order of magnitude more intensively than they take place in the bottomhole zones of gas fields. The operation process of UGSF is variable both in terms of gas volumes and the number of wells involved, which affects the change in the speed of filtration processes in reservoirs and, especially, in the bottomhole zones. The existing permeability anisotropy of the reservoir layers and the bottomhole zones of the wells and their possible unpredicted contamination cause the variability of filtration flows in terms of direction and intensity, and this, in turn, affects the variability of the filtration resistance coefficients of the bottomhole zones. That causes problems of calculating the coefficients of filtration resistances of the bottomhole zones of wells for variable flow rates and for predicted time intervals.

Assumptions were formed in the process of analyzing the measured data – formation pressure, pressure at the gas collection point, and variable volumes of gas withdrawal. It consists in the following. The technique for solving the inverse problem – gas collection point – formation-collector can be used to find the amount of depression/repression in the bottomhole zones of wells as a function of flow rate and time. The results of the analysis revealed that it is possible to build a model of the relationship between these measured data. Repeated numerical experiments made it possible to work out the coefficients of the model, which ensured the calculation of the operation modes, with the necessary accuracy, of UGSF operation during several seasons. The proposed integrated model took into account the influence of variable permeability of the bottomhole zones of wells with variable volumes of gas withdrawal/injection without conducting well research.

This will make it possible to calculate the technological chain reservoir-collector – gas collection point. This hypothesis is based on the fact that the variable factors influencing the filtration resistance coefficients of the bottomhole zones of hundreds of wells can be considered, to a certain extent, random. Their conditionally averaged value is stable and creates a predictable effect on the value of pressure on GCP. The construction of a model of the influence of the integrated effect of depression/repression on the pressure on GSP is built by solving inverse problems.

Input data for the research:

– the results of wells investigation under gas injection and withdrawal modes over ten years (reports of geological and technological exploitation of underground gas storages);

– information bases of measured data (accounting and analytical system of the gas transmission enterprise).

Additional input data are obtained as a result of numerical experiments using GIMS and GTS Calculation PC software packages [11–15], which are involved in the process of solving the mode-technological problems of planning operational modes of UGSF operation. The realized models of filtration and gas dynamic processes are given in [13]. They provide the solution of a set of direct and inverse problems of calculation and analysis of UGSF operating modes.

5. Results of the construction of models and methods of their implementation for calculating depression/repression in the bottomhole zones

5. 1. Analysis of the main stationary and non-stationary models of gas flow to wells

5. 1. 1. Analysis of general formulas for estimating the coefficients of filtration resistance of the bottomhole zones of wells

Darcy's law [16]:

$$-grad p=\frac{\mu\nu}{k}, \tag{1}$$

where p is gas pressure, Pa; μ – coefficient of dynamic viscosity, Pa·s; ν is the filtration rate. m/s; k – gas permeability of the filtration area, m^2.

Darcy's law is not universal and has its limits of application. In paper [17], to reduce the errors of Darcy's law at high gas filtration speeds, a correction factor $f(\mathrm{Re})$ was introduced, with the help of which the fluid filtration law for high speeds is represented in the form:

$$-grad p=f(\mathrm{Re})\frac{\mu\nu}{k}=\frac{\mu\nu}{k_{ef}}, \tag{2}$$

where Re is the Reynolds number.

Later, Forchheimer proposed a binomial filtering law [17]:

$$-grad p=\frac{\mu\nu}{k}+\beta\frac{\rho\nu}{\sqrt{k}}, \tag{3}$$

where β is the vortex drag coefficient.

For large pressure gradients, Forchheimer's binomial law also gives an error. In this connection, the authors of [17] proposed a new model containing, in addition to parameters β, k, two additional parameters α, k_{mr}:

$$\frac{k}{k_{ef}}=f(\mathrm{Re})=\frac{(1+\beta\,\mathrm{Re})^{\alpha}}{k_{mr}(1+\beta\,\mathrm{Re})^{\alpha}+1-k_{mr}}. \tag{4}$$

In the same work [17], the use of polynomial approximations of the Barry-Conway law is recommended in the form:

$$-gradp=\left[\sum_{i=1}^{n}C_i\left(\text{Re}\right)^i\right]\frac{\mu\nu}{k}, \qquad (5)$$

where C_i are approximation coefficients.

It is problematic to use the given formulas under the real conditions of operation of wells on UGSF. Firstly, the wells work under variable operation modes in terms of intensity, the influencing factors are unstable, there are also factors that generate the skin factor, etc. Secondly, simultaneous polynomial approximation and identification of several poorly defined parameters under conditions of insufficient measurement accuracy of many parameters may turn out to be an incorrect task.

5. 1. 2. Analysis of the stationary gas filtration model under the condition of fulfillment of Darcy's law

Steady-state gas filtration under the condition of fulfillment of Darcy's law in a formation that is homogeneous in terms of collector properties, with flat-radial gas inflow in the formation is described by the Laplace equation [12]:

$$\frac{1}{r}\frac{\partial p^2}{\partial r}+\frac{\partial^2 p^2}{\partial r^2}=0. \qquad (6)$$

If constant withdrawal is set on the internal circuit, and constant gas pressure is set on the external circuit, we get:

$$p_k^2-p^2\left(r\right)=\frac{q\mu z p_a T}{\pi k h T_s}\ln\frac{R_k}{r}, \qquad (7)$$

and in the other case, if a constant gas pressure is set on the internal and external circuits, we obtain:

$$p^2\left(r\right)=p_k^2+\frac{p_k^2-p_c^2}{\ln\left(R_k/R_c\right)}\ln\frac{r}{R_k}. \qquad (8)$$

Establishing the conditions under which Darcy's law is satisfied or not fulfilled depends on the flow rate of the well and the permeability of the bottomhole zone under which the given formulas (7), (8) hold within the accuracy of the measurements. There are no conditions that would unambiguously identify the applicability of Darcy's law under the conditions of operation of wells with variable withdrawal/injection volumes.

5. 1. 3. Analysis of stationary gas filtration under the condition that Darcy's law is not fulfilled

According to the spherical law of gas underflow, which is different from Darcy's law with Forchheimer's β coefficient, the pressure distribution in the bottomhole zone of the well satisfies the following equation [18]:

$$-d\left(\frac{p}{p_0}\right)^2=\frac{\mu}{\pi h k p_0}\frac{q_0}{F}\,dF+\beta\frac{\rho_0}{\pi p_0 dh}\frac{q_0^2}{F^2}\,dF. \qquad (9)$$

Different representations of the vortex drag coefficient are given in the literature. They are obtained in the process of field experiments and depend on many parameters [6]:

$$\beta=\beta\left(d,m,k,q,s\right),$$

where p_0, q_0, ρ_0 are values of pressure, well flow, and gas density under normal conditions; d is the diameter of the rock grain. μm; m – layer porosity, 1/m; q – flow rate of the well under reservoir conditions, m^3/s; s – the technique of opening the well hole; F – filtration surface area, m^2; h is layer thickness, m.

For dense rocks, the Ergun model is used, which includes the following ratios [19]:

$$\beta=\frac{1.75}{d}\frac{1-m}{m^3},\ \text{and}\ k=\frac{d^2}{150}\frac{m^3}{\left(1-m\right)^2}. \qquad (10)$$

The coefficient β is derived by approximating the results of hydraulic studies of bottomhole zones.

For flat-radial underflow of gas to the bottomhole zone [18]:

$$\frac{dp^2}{dr}=\frac{\mu}{k}\frac{p_a}{2\pi r h p}q+\beta_*\rho_a\frac{p_a}{2\pi^2 r^2 h^2}q^2.$$

Hence:

$$p_k^2-p_c^2=\frac{\mu}{\pi k h}\ln\frac{R_k}{r_c}q+\beta_*\frac{\rho_a p_a}{2\pi^2 h^2}\left(\frac{1}{r_c}-\frac{1}{R_k}\right)q^2.$$

The equation of gas flow to the gas well blowout is written in the form of a binomial formula that characterizes the dependence of reservoir energy losses on flow rate:

$$p_k^2-p_c^2=Aq+Bq^2, \qquad (11)$$

$$A=\frac{\mu}{\pi k h}\ln\frac{R_k}{r_c},$$

$$B=\beta_*\frac{\rho_a p_a}{2\pi^2 h^2}\left(\frac{1}{r_c}-\frac{1}{R_k}\right),$$

where Aq corresponds to pressure losses caused by viscous forces; Bq^2 corresponds to pressure losses caused by inertial forces.

In the case of isotropic formations for perfect wells, the filtration coefficients will be:

$$A=\frac{\mu z p_a T_{reservoir}}{\pi k h T_a}\left(\ln\frac{R_k}{r_c}+\frac{h}{nR_0}\right),$$

$$B=\frac{\rho_a z p_a T_{reservoir}}{2\pi^2 l h^2 T_a}\left(\frac{1}{r_c}-\frac{1}{R_k}+\frac{h^2}{3n^2R_0^3}\right),$$

In the case when the circular filtration area of the bottomhole zone of the wells is conditionally divided into two zones:

– reservoir zone – permeability of the near zone of the reservoir;

– bottomhole zone – permeability of the area of perforation channels.

In this case [12]:

$$p_{reservoir}^2-p_b^2=A_1\frac{q_0}{k_{reservoir}}+A_2\frac{q_0}{k_b}+B_1\frac{q_0^2}{k_{reservoir}^{3/2}}+B_2\frac{q_0^2}{k_b^{3/2}},$$

or

$$p_{reservoir}^2-p_b^2=Aq+Bq^2,$$

where the following notation is entered:

$$A_1=\frac{1}{h\pi}\mu p_0\ln\frac{R_k}{R_c},$$

$$A_2=\frac{\mu p_0}{\pi h_x}\ln\frac{2R_c h}{2r_k l_k n_0 h_x+\Theta\left(n_0\right)\left(r_1^2-r_2^2\right)},$$

$$B_1 = 12\cdot10^{-5}\,\frac{\rho_0 p_0 d^2}{2\pi^2 h^2 m}\left(\frac{1}{R_c}-\frac{1}{R_k}\right),$$

$$B_2 = \frac{\rho_0 p_0 d^2}{\pi^2 h_x m}\left(\frac{1}{2r_k l_k n_0 h_x + \Theta(n_0)\left(r_1^2 - r_2^2\right)} - \frac{1}{2R_c h}\right),$$

$$A = \frac{A_1}{k_{reservoir}} + \frac{A_2}{k_b},\; B = \frac{B_1}{k_{reservoir}^{3/2}} + \frac{B_2}{k_b^{3/2}},$$

ρ_0, p_0, q_0 – values of density, pressure, and volumetric transport under normal conditions; h_x – height of the perforated column; n_0 – perforation density; r_1, r_2 – radii of the casing column (inner) and tubing ring (outer), respectively; r_k, l_k – radius and length of the perforation channel.

In this case, the feeding area is taken as the surface of a cylinder of radius R_k and height h_x. The function $\Theta(n_0)$ is determined experimentally and is equal to zero in the case of $n_0=0$.

There are the following problems of calculating the filtering resistance coefficients:

– filtering resistances should be calculated based on the permeability coefficient of the reservoir, its macro-roughness coefficient, or the vortex resistance coefficient;

– the vortex resistance coefficient β partially takes into account the structure of the porous medium in the near zone of wells and it is different for different wells [17];

– the vortex resistance coefficient β can be established in the process of conducting numerical experiments on real data obtained as a result of well research;

– for different wells, the radius of the bottomhole zone will be different, and it can be variable and depend both on the flow rates of the wells and on the nature of the operation mode at a certain time interval;

– it is problematic to estimate the quality of the bottomhole zone, as well as the skin factor, of wells with satisfactory accuracy, because there are many other factors, the influence of which on the flow rate of wells cannot be separately identified;

– it is necessary to establish the area of uncertainty of the calculation of the filtering resistance according to the accuracy of the measured data;

– it should be expected that the filtration resistances are affected by the constant change in the filtration flows in the working area and spatially oriented anisotropy in permeability, including its vertical component.

The movement of gas from the reservoir to the gas collection point is influenced by the filtering resistances of the wells, the hydraulic resistances of the wells, the wellheads and gas collection systems, etc. The dynamics of changes in the amount of depression or repression in the vicinity of wells are influenced by the filtration resistances of the near zone of gas inflow to the wells and the amount of their flow rates. The magnitude of the pressure change around the well over time should be divided into a fast and a slow component. The first component, and it is the main one, is formed in seconds (jump-like) due to debit changes. The second, slow one, which is formed during the entire time of operation of the wells with a stable gas flow rate. Different mathematical models are used to calculate the components of pressure changes in the vicinity of wells – stationary and non-stationary gas underflow to wells. The difficulty of using a non-stationary model is its sensitivity to the value of the empirical parameters of the model and, accordingly, to the accuracy of measurements of formation and gas pressure at the bottomhole zone. In the process of UGSF operation, the mentioned pressures are influenced by many poorly defined factors, and depending on the mode of UGSF operation, the nature of one or another influencing factor on their value is constantly changing, which affects the parameters of the model.

To analyze the influence of filtration resistance on the flow rate of wells, the values of filtration resistance obtained in the process of researching wells for several years and resistances obtained in the process of conducting numerical experiments according to data were used.

Analysis of the results of numerical experiments demonstrated that the available procedures for calculating the filtration resistance:

– estimate only those factors that are available at the time of the analysis of the wells on the filtering resistances;

– do not provide for assessment of the influence of individual factors on filtering resistances;

– do not take into account the variability of possible modes of operation of UGSF, in particular, reservoir layers;

– the calculation process does not take into account all the main existing influencing factors;

– do not evaluate the area of uncertainty of the calculation results based on the accuracy of the input measured data;

– do not assess the impact on the calculation results of switching wells to an individual separator and separate gas flow measurement in the process of their research.

5. 1. 4. Analysis of the exponential model of well productivity

Exponential well productivity models are proposed based on the following dependence [20]:

$$q = C\left(P_e^2 - P_\omega^2\right)^n, \tag{12}$$

where C is a coefficient, n is a coefficient that varies between 0.5 and 1.

The coefficient n shows the degree of non-Darcian flow. If n is 1, this indicates Darcy's law flow. If n is equal to 0.5, then the flow is turbulent.

In work [20], for various reservoir layers, different representations of the relationship between flow rate and pressures are proposed – reservoir and bottomhole pressures, for production wells:

$$P_e^2 - P_\omega^2 = \frac{1.291\times10^{-3}T\mu z}{kh}\left(\left(\ln\frac{r_e}{r_\omega}+S\right)Q + DQ^2\right),$$

$$P_e^2 - P_\omega^2 = \frac{1.291\times10^{-3}T\mu z}{kh}\left(\ln\frac{0.472r_e}{r_\omega}+S\right)Q + $$
$$+\frac{2.828\times10^{-3}\beta\gamma zT}{r_\omega h^2}Q^2, \tag{13}$$

$$P_e^2 - P_\omega^2 = \frac{42.42zp_{sc}Q}{khT_{sc}}\times$$
$$\times\left(\lg\frac{8.085kt}{m\mu C_t r_\omega} + 0.87SQ + 0.87DQ\right),$$

the found coefficients C and n are such that the exponential models (12) with an accuracy within (2–16) % correspond to the above (13) in the case of stable modes of operation of wells.

Here, P_e is the reservoir pressure, MPa;

P_ω is the pressure at the bottom of the well, MPa;

r_e – effective radius, m;

r_ω – radius of the well, m;

S – skin factor, dimensionless;

Q – flow rate of gas wells under standard conditions;

γ is the relative density of natural gas, dimensionless;

p_{sc} – pressure under standard conditions, 0.101325 MPa;

T_{sc} – temperature under standard conditions, 293.16 K;

C_t is the compressibility factor, MPa;

D – (non-Darcy) flow coefficient different from Darcy.

An analysis of one of the methods of researching wells at the stage of gas withdrawal using the exponential (simplified) procedure was carried out (12). According to (12), the $E=Bq/A$ value is constant and does not depend on the volume flow of gas. In the process of preliminary data analysis, questions arose, the answers to which can be obtained only as a result of conducting a more detailed study of them.

The consistency of formula (11) of the measured and calculated data was also checked, in which A and B are the coefficients of the filtration resistance of the bottomhole zones of the wells.

The results of the analysis are given in Tables 1, 2, where it is indicated:

$$p^2_{reservoir} - p^2_{bottomhole} = \Delta,$$

$$E = Bq / A,$$

B, A=const – the value of the coefficient B according to the tabular value of A; A, B=const – the value of the coefficient A according to the tabular value of B; E_{roz} is the calculated ratio based on the found values of A and B; E_{tabl} – calculated ratio based on tabular values of A and B; $(Aq+Bq^2)_{tabl}$ – the value of the right-hand side of formula (11) according to tabular data; $(Aq+Bq^2)_{rozr}$ is the value of the right-hand side of formula (11) according to the calculated data.

Similar values from 2014–2015 for the same wells.

Table 1

Results of well research for 2018–2019

$p_{reservoir}$	$p_{bottomhole}$	q	A	B	Δ	B, A=const	A, B=const	E_{roz}	E_{tabl}	$(Aq+Bq^2)_{tabl}$	$(Aq+Bq^2)_{rozr}$
33.6	30.8	85	0.638	0.018	180.3	0.017	0.634	2.34	2.33	180.67	179.97
33.6	31.8	89	0.381	0.010	117.7	0.011	0.433	2.18	2.34	113.12	122.32
33.5	30.8	42	1.239	0.069	173.6	0.069	1.244	2.33	2.33	173.40	173.82
33.3	31.0	77	0.596	0.018	147.9	0.017	0.527	2.51	2.34	153.21	142.57
33.3	31.2	142	0.294	0.005	135.5	0.005	0.272	2.42	2.32	138.54	132.37
33.2	30.8	79	0.603	0.018	153.6	0.017	0.538	2.49	2.33	158.73	148.47
31.9	30.0	247	0.147	0.001	117.6	0.001	0.130	2.53	2.35	121.72	113.50
35.1	33.2	52	0.725	0.033	129.8	0.034	0.806	2.20	2.33	125.58	133.96

Table 2

Results of well research for 2014–2015

$p_{reservoir}$	$p_{bottomhole}$	q	A	B	E_{tabl}	A, B=const	B, A=const	Δ	$(Aq+Bq^2)_{tabl}$	$(Aq+Bq^2)_{rozr}$
27.60	22.70	80.00	0.914	0.027	2.337	0.945	0.027	246.470	244.000	2.293
27.60	23.50	293.00	0.214	0.002	2.328	0.217	0.002	209.510	208.645	2.310
32.10	30.60	26.00	1.091	0.098	2.333	1.072	0.097	94.050	94.546	2.357
27.50	23.60	303.00	0.196	0.002	2.319	0.203	0.002	199.290	197.101	2.272
32.10	31.50	117.00	0.105	0.002	2.340	0.080	0.002	38.160	41.032	2.749
32.20	31.60	125.00	0.099	0.002	2.273	0.081	0.002	38.280	40.500	2.551
26.40	24.20	305.00	0.107	0.001	2.280	0.121	0.001	111.320	107.055	2.132
27.40	22.10	51.00	1.535	0.070	2.332	1.564	0.071	262.350	260.875	2.308
26.80	23.60	254.00	0.192	0.002	2.381	0.178	0.002	161.280	164.897	2.492

Analysis of our data given in Tables 1, 2 revealed that for the values of parameters A and B specified in the tables, equality (11) is not fulfilled. Further, even if you take one of the tabular parameters, the values of the second do not match. If we take the measured data on one of the wells for two years (data from Tables 1, 2), then the values of parameters A and B, calculated on the basis of formula (11), take negative values, which does not correspond to actual data.

Further, according to tabular data (Tables 1, 2), parameter E is constant, i.e.:

$$E = Bq / A \equiv \text{const}.$$

Hence $Bq=AE$. Then:

$$p^2_{reservoir} - p^2_{bottomhole} = Aq + Bq^2 = Aq(1+E). \qquad (14)$$

On the other hand:

$$A = Bq / E.$$

And:

$$p^2_{reservoir} - p^2_{bottomhole} = Aq + Bq^2 = Bq^2(1+1/E). \qquad (15)$$

Thus, based on relations (14) and (15), we obtained that the difference $p^2_{reservoir} - p^2_{bottomhole}$ has both linear and quadratic behavior with respect to the flow rate of the well q, which is impossible with constant A and B. It follows that these coefficients of filtration resistance are dependent on the flow rate of the well.

Since, as follows from the analysis, the coefficients must be constant, they cannot be determined from the system of linear equations. Next, constant A and B should characterize the parameters of each well bottomhole zone. It follows from the tabular data that there is no such characteristic since the constants are significantly different in different years. It is obvious that during the year, the parameters of the well bottomhole zone could not change so much.

The flow of gas to the well in the assumption of the spherical law of gas inflow is described by the equation that models the process of gas movement to the bottomhole and relates the bottomhole pressure to the formation pressure:

$$-d\left(\frac{p}{p_0}\right)^2 = \frac{\mu}{\pi h k p_0}\frac{q_0}{F}dF + \beta\frac{\rho_0}{\pi p_0 d h}\frac{q_0^2}{F^2}dF, \qquad (16)$$

where p_0, q_0, ρ_0 are values of pressure, well flow, and gas density under normal conditions; F is the area of the filtration surface.

The solution to equation (16) for a particular β takes the form:

$$p^2_{reservoir} - p^2_{bottomhole} = Aq + Bq^2 = \frac{\mu z p_0 T_r}{\pi k h T_0}\left(\ln\frac{R_k}{r_c}\right)q +$$
$$+\frac{12\cdot 10^{-5} d^3}{mk^{3/2}}\frac{\rho_0 z p_0 T_r}{2\pi^2 h^2 T_0}\left(\frac{1}{r_c}-\frac{1}{R_k}\right)q^2.$$

From the last equation, we obtained:

$$E = \frac{12\cdot 10^{-5} d^3}{mk^{1/2}}\frac{\rho_0}{2\pi h}\left(\frac{1}{r_c}-\frac{1}{R_k}\right)\frac{1}{\mu}\frac{q}{l\,\mathrm{n}\frac{R_k}{r_c}} \simeq \frac{q}{k^{1/2} l}.$$

It follows that the ratio E is a function of the flow rate of the wells.

5. 1. 5. Analysis of models of non-stationary gas filtration in the bottomhole zone of wells

The mathematical problem of calculating the change in pressure in the area of the well bottomhole zone, during time t and under the condition of a constant flow rate, came down to solving the equation of the theory of the elastic mode of filtration [18]:

$$\frac{1}{r}\frac{\partial p^2}{\partial r} + \frac{\partial^2 p^2}{\partial^2 r^2} = \frac{1}{\chi}\frac{\partial p^2}{\partial t}. \qquad (17)$$

Here $\chi = kp_0 / m\mu$,

$$\frac{\pi k h}{\mu p_a}\left(r\frac{\partial p^2}{\partial r}\right)_{r\to 0} = q = \text{const}.$$

Subject to $t=0$, $p = p_0 = \text{const}$,

$r\to\infty$, $p = p_0 = \text{const}$,

where p_a is atmospheric pressure.

The solution to equation (17) will be written in the form:

$$p_0^2 - p^2(r,t) = -\frac{q\mu p_a}{2\pi k h} Ei\left(-\frac{r^2}{4\chi t}\right), \qquad (18)$$

where $Ei(x) = -\int_{-x}^{\infty}\frac{e^t}{t}\mathrm{d}t$ is the integral exponential function.

According to Ramanujan's formula:

$$Ei(x) = \gamma + \ln x + \exp\left(\frac{x}{2}\right)\sum_{n=1}^{\infty}\frac{(-1)^{n-1} x^n}{n! 2^{n-1}}\sum_{k=0}^{[(n-1)/2]}\frac{1}{2k+1}, \qquad (19)$$

where γ=0.5772 is the Euler-Mascheroni constant.

Taking into account $\chi = kp_0/m\mu$ for small values of the arguments, we get:

$$p_A^2(t) = p_0^2 + \frac{q\mu p_a}{2\pi k h} Ei\left(-\frac{R_c^2}{4\chi t}\right) \approx p_0^2 - \frac{q\mu p_a}{2\pi k h}\ln\frac{2.25 k p_0 t}{R_c^2 m\mu}. \qquad (20)$$

Hence:

$$p_c^2(t) = p_0^2 - q(ab + a\ln p_0 t),$$

where $a = \frac{\mu p_a}{2\pi k h}$, $b = \ln\frac{2.25k}{R_c^2 m\mu}$.

The convergence of formula (19) largely depends on the value of the argument. For small times, that is, under the conditions of starting modes, due to the large argument, it is necessary to take a large number of terms in the series. The way out of this situation is the use of approximation formulas for calculating the integral indicator function.

There is a relation [21]:

$$Ei(-x) = -E_1(x).$$

Approximations by polynomials and rational functions depending on the argument are constructed for the function $E_1(x)$.

For $0\le x\le 1$ with an accuracy of $2\cdot 10^{-7}$:

$$E_1(x) = -\ln x + a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5, \qquad (21)$$

$a_0=-0.57721566$, $a_1=0.99999193$, $a_2=-0.24991055$, $a_3=$ $=0.05519968$, $a_4=-0.00976004$, $a_5=0.00107857$.

For $1\le x<\infty$ with accuracy $2{*}10^{-8}$:

$$E_1(x) = \frac{1}{x}e^{-x}\frac{x^4 + aa_1 x^3 + aa_2 x^2 + aa_3 x + aa_4}{x^4 + bb_1 x^3 + bb_2 x^2 + bb_3 x + bb_4}. \qquad (22)$$

Here:

$aa_1=8.5733287401$, $bb_1=9.5733223454$,

$aa_2=18.0590169730$, $bb_2=25.6329561486$,

$aa_3=8.6347608925$, $bb_3=21.0996530827$,

$aa_4=0.2677737343$, $bb_4=3.9584969227$.

Taking into account the accuracy of the input digital information, the given formulas provide sufficient accuracy for the practice of performing the necessary calculations. The calculation results demonstrate a high convergence of the calculation results according to various calculation formulas – the results coincide with the accuracy of four significant figures within the limits of the change of $r\in[0.01–400]$ m, $t\in[1–1600]$ h.

Studies have shown that the asymptotic formula gives less accurate results for large radii and small times. The times of transient modes are short. Therefore, in such cases, it is necessary to use approximation formulas. It should be noted that since the time for calculating the pressure using the approximation formulas is small and is not associated with a loss of accuracy, so it makes no sense to use asymptotic expressions to calculate the integral exponential function.

5. 1. 6. Estimation of formation pressure recovery time in the bottomhole zone of wells

To analyze the pressure distribution in the well region, it is advisable to write the filtration equation in cylindrical coordinates:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{D}{p_0}\frac{\partial p}{\partial \tau}, \qquad (23)$$

where r is a radius vector drawn from the center of the well. Considering that the area of gas inflow to the well compared to the entire reservoir is small, the parameters included in the last equation can be considered constant in the coordinate for some time interval.

Let the radius of the outer circle S_0 be equal to a, the radius of the inner circle concentric to it $\tilde{s}$ equal to b, in the center of which the well is located. At the outer boundary S_0–$\partial P/\partial r=0$; on the inner border – $P=P_2=\text{const}$. Here, $P=p^2$, $P_2=p_2^2$, $P_0=p_0^2$. The initial pressure distribution is given by the function $f(r)$. Under such conditions, the reservoir pressure in the vicinity of the well, subject to Darcy's law, changes as follows [22]:

$$p^2(r,t)=\frac{\pi^2}{2}\sum_{n=1}^{\infty}\frac{\alpha_n^2 J_0^2(a\alpha_n)}{J_0^2(a\alpha_n)-J_0^2(b\alpha_n)}\times$$
$$\times e^{-\kappa_1\alpha_n^2 t}U_0(r\alpha_n)\int_a^b rf(r)U_0(r\alpha_n)\,\mathrm{d}r-$$
$$-\pi\sum_{n=1}^{\infty}\frac{\{p_2J_0(a\alpha_n)-p_1J_0(b\alpha_n)\}J_0(a\alpha_n)U_0(r\alpha_n)}{J_0^2(a\alpha_n)-J_0^2(b\alpha_n)}e^{-\kappa_1\alpha_n^2 t}+$$
$$+\frac{p_1 ln(b/r)+p_2\ln(r/a)}{\ln(b/a)}.$$

In the last equations, it is noted:

$$U_0(\alpha_n r)=J_0(\alpha_n r)+A_n N_0(\alpha_n r),$$
$$A_m=-\frac{J_0(\alpha_n b)}{N_0(\alpha_n b)}=-\frac{J_1(\alpha_n a)}{N_1(\alpha_n a)},$$
$$D=\frac{m\mu}{k},$$
$$\tau=\frac{p_2}{p_0}t+\left(1-\frac{p_2}{p_0}\right)\frac{1-e^{-\beta t}}{\beta},$$
$$\beta=\frac{p_0 k\alpha_n^2}{2m\mu},$$

p_0, p_2 – the initial pressure value and the pressure value at the boundary of the area, $J_i(\alpha_n r)$ – the Bessel function of the real argument of order i, $N_i(\alpha_n r)$ – the Neumann function of order i, α_n – roots of the equation:

$$J_0(\mu x)N_1(x)-J_1(x)N_1(\mu x)=0,$$
$$\mu=b/a,\ a\alpha_n=x,\ b\alpha_n=\mu x.$$

Reservoir pressure is related to the bottomhole one by ratio:

$$p_{reservoir}^2-p_{bottomhole}^2=A_{12}q_0+B_{12}q_0^2.$$

It follows from the last two formulas that the formation pressure distribution depends on the time of well operation. Since the Fourier-Bessel series is rapidly converging and the first term, which depends on time, makes the main contribution to the sum of the series, and the coefficient:

$$\frac{\{p_2J_0(a\alpha_1)-p_1J_0(b\alpha_1)\}J_0(a\alpha_1)U_0(r\alpha_1)}{J_0^2(a\alpha_1)-J_0^2(b\alpha_1)},$$

is bounded from above by some value H and does not depend on time, then the transition time is determined from the inequality:

$$e^{-\kappa_1\alpha_1^2 t}<\varepsilon/H.$$

Here, ε is a value that evaluates the proximity of the non-stationary to the stationary operation mode. To estimate the exponent $e^{-\kappa\alpha_n^2 t}$, the following inequalities are obtained:

$$e^{-\kappa_1\alpha_n^2 t}=\exp\left(-\frac{kp_0}{m\mu}\alpha_n^2 t_\varepsilon\right)<\varepsilon/H,$$
$$-\frac{kp_0}{m\mu}\alpha_n^2 t_\varepsilon<\ln\frac{\varepsilon}{H},$$
$$t_\varepsilon>-\frac{m\mu}{kp_0\alpha_n^2}\ln\frac{\varepsilon}{H}. \quad (24)$$

If $\ln\frac{\varepsilon}{H}=-4, m=0.3, \mu=1.2\cdot10^{-6}, k=1.2\cdot10^{-12}, p_0=40\cdot10^5$, $\alpha_1^2=0.49$, then:

$$t_\varepsilon>0.06. \quad (25)$$

Since time is in days, the last inequality holds when the time exceeds one and a half hours. The time depends significantly on the permeability coefficient. The permeability of the formation of the bottomhole zones of individual wells varies hundreds of times. The time needed to restore the pressure in the well's bottomhole zone will also change by as many times.

5. 1. 7. Method for calculating the pressure in the bottomhole zone with a variable flow rate

In the process, at a certain intensity of UGSF operation, the gas pressure at the gas collection point grows faster than the reservoir average in the area of gas withdrawal. At monthly time intervals, the difference between the formation pressure and the pressure on GCP can increase to five atmospheres. And therefore, it should be expected that the main share of change in the pressure difference occurs due to changes in the depression in the bottomhole zone.

Under the condition of Darcy's law and for variable flow rates of wells, the relationship between formation and surface pressures can be calculated as follows. Let the forecasted time series t_i, q_i ($i=1,...,n$) be known – the flow rate of wells q_i ($i=1,...,n$) at time points t_i ($i=1,...,n$). Then, according to the superposition method, we obtained:

$$p_0^2-p_A^2(t)\simeq\frac{\mu p_a}{2\pi kh}\left(\begin{aligned}&q_1\ln\frac{2.25\chi t}{R_c^2}+\\&+\sum_{i=1}^{n}(q_{i+1}-q_i)\ln\frac{2.25\chi(t-t_i)}{R_c^2}\end{aligned}\right). \quad (26)$$

If you enter the designation $\varphi=2.25kp_0/m\mu R_c^2$, then:

$$p_0^2-p_A^2(t)\simeq\frac{\mu p_a}{2\pi kh}\left(\begin{aligned}&q_1(\ln\varphi+\ln t)+\\&+\sum_{i=1}^{n}(q_{i+1}-q_i)(\ln\varphi+\ln(t-t_i))\end{aligned}\right).$$

The given formula (26) allows one to calculate the depression/repression in the bottomhole zones of the wells for variable flow rates of the wells, provided that Darcy's law is fulfilled. During the high-speed filtration process in the vicinity of the wells, it is possible to create a vortex resistance, and in such cases, the given formulas provide the calculation of the pressure distribution in the zones outside the bottomhole zone.

Measuring the flow rate of wells in certain cases is complicated due to the complexity of the gas collection system. There are several gas collection systems – loop and loop-collector. Gas flow measurements in the case of the first type of gas collection are carried out directly at the gas collection point.

In the second case, the measurement should be carried out on the loop to the gas collection collector. In this case, gas flow rate can be estimated by measuring the buffer (mouth) and annular pressures.

Mouth pressure is calculated according to the following formula [18]:

$$p_g = \sqrt{p_v^2 \; e^{-b} - \lambda \, z \frac{RT}{D} \left(\frac{M}{S} \right)^2 \frac{1-e^{-b}}{b} L}, \qquad (27)$$

$$S = \frac{\pi D^2}{4}, b = \frac{2gL}{zRT}, \; M = \rho_0 \, q_0,$$

$$p_g^2 = p_v^2 e^{-b} - \lambda \, z \frac{RT}{D} \left(\frac{M}{S} \right)^2 \frac{1-e^{-b}}{b} L.$$

The relationship between the annular pressure and the hole pressure:

$$p_z^2 = p_v^2 e^{-2b}.$$

Then:

$$p_g^2 = p_z^2 e^{b} - \lambda \, z \frac{RT}{D} \left(\frac{\rho_0 q_0}{S} \right)^2 \frac{1-e^{-b}}{b} L,$$

$$U = \lambda \, z \frac{RT}{D} \left(\frac{\rho_0}{S} \right)^2 \frac{1-e^{-b}}{b} L,$$

$$p_g^2 = p_z^2 e^{b} - U q_0^2.$$

And the flow rate under normal conditions is equal to:

$$q_0 = \sqrt{\frac{p_z^2 e^{b} - p_g^2}{U}}.$$

Let ε_z and ε_g be the measurement errors of the annular and mouth pressures, i.e.:

$$p_z = p_{zz} + \varepsilon_z, \; p_g = p_{gg} + \varepsilon_g.$$

Then:

$$q_0 = \sqrt{\frac{\left(p_{zz} + \varepsilon_z\right)^2 e^{b} - \left(p_{gg} + \varepsilon_g\right)^2}{U}}.$$

Using the above ratios, we obtained:

$$q_0 \approx \frac{1}{\sqrt{U}} \sqrt{\left(p_{zz}^2 e^{b} - p_{gg}^2\right)} + \frac{1}{\sqrt{U}} \frac{p_{zz}\varepsilon_z e^{b} - p_{gg}\varepsilon_g}{\sqrt{\left(p_{zz}^2 e^{b} - p_{gg}^2\right)}}.$$

From which it follows that the error in calculating the volumetric flow rate in the presence of errors in the measurements of the annular and buffer pressures will be:

$$\sqrt{U}\Delta q_0 = \frac{p_{zz}\varepsilon_z e^{b} - p_{gg}\varepsilon_g}{\sqrt{\left(p_{zz}^2 e^{b} - p_{gg}^2\right)}}.$$

The results of the numerical experiment are given in Table 3.

The error of calculating the volumetric flow rate of wells (Table 3) is within the limits $\varepsilon_z \le \sqrt{U}\Delta q_0 \le \varepsilon_g$.

Table 3

Error in calculating the volumetric flow rate of wells

Well number	p_{gg}	p_{zz}	ε_z	ε_g	$\sqrt{U}\Delta q_0$
109	29.8330	34.5330	0.0200	0.0150	0.0154
111	41.1330	42.2300	0.0200	0.0150	0.0189
112	30.6300	31.3300	0.0200	0.0150	0.0192
113	42.0300	43.7300	0.0200	0.0150	0.0180
114	30.2300	31.0300	0.0200	0.0150	0.0189
115	29.5300	31.9300	0.0200	0.0150	0.0165

5. 1. 8. Evaluating the area of uncertainty of filtration resistance and its influence on the modes of operation of underground gas storage facilities

The evaluation of the area of uncertainty of the filtration resistance and its influence on the performance of UGSF and fuel and energy costs was carried out on real operating data. Numerical experiments were carried out using the software package [14]. The calculation was carried out under the formation pressure in the area of gas bottomhole equal to 30 atm and the pressure in the gas pipeline-outlet equal to 45 atm. Two options were considered, which differ in different pressures at the gas collection point – the option of 1–20 atm and the option of 2–25 atm. The simulation results are given in Table 4. The average values of the filtration resistance coefficients, which are used to calculate the volumes of withdrawal and fuel gases (first and second lines), obtained as a result of the study of the wells at the Bilche-Volytsko-Uherske UGSF under stationary modes in different seasons of gas withdrawal. The results of the third line are obtained by the filtration resistance coefficients obtained in the process of adapting the reservoir model to the measured data during ten years of UGSF operation.

Table 4

Productivity of the underground gas storage facility for the filtration resistance coefficients obtained as a result of our studies of wells

Seasons of operation	Gas withdrawal volumes million m^3/d		Fuel gas volumes million m^3/d	
	Variant 1 P=20	Variant 2 P=25	Variant 1 P=20	Variant 2 P=25
Season 1	29.89	17.06	0.489	0.188
Season 2	23.35	13.08	0.409	0.179
Averaged over 10 seasons of data	35.75	18.60	0.587	0.198

Our analysis of the simulation results revealed that it is problematic to use the results of the well research to calculate the operating modes in the entire working area of productivity due to the significant difference between the measured and calculated data. The established coefficients of filtration resistance of the bottomhole zones of the wells give a discrepancy when calculating the volumes of gas withdrawal in both options of approximately 30 %, and of fuel gas by 18 and 5 % in the first and second variants, respectively.

5. 2. Construction of models of the integrated influence of variable flow rates of wells on the depression/repression near bottomhole zones

The main goal of researching wells is to identify factors affecting their productivity and to find parameters of the model of gas inflow to wells in the entire range of changes in the intensity of its operation. It is the gas pressure near bottomhole zone that is most sensitive to changes in the flow rate of wells. This affects the modes of operation of booster compressor stations (BCS), in particular significantly, under the conditions of peak loads on UGSF. For dispatching services, it is not only important to calculate the accuracy of UGSF operating modes but also to quickly obtain the result.

Taking into account the specificity of each well, and there are at least a hundred of them in separate storages, with frequent changes in gas withdrawal/injection volumes, leads to an increase in calculation time with unguaranteed accuracy of the result. As shown by the results of the study of wells on UGSF, the change in the mode of UGSF operation has a poorly predicted effect on the filtration resistances of the bottomhole zones of the wells. It should be expected that the integrated consideration of the impact on the parameters of the wells' bottomhole zones, non-stationary transient modes of UGSF operation, will level out significant changes in the filtration resistance of individual wells and will ensure a sufficient speed of obtaining simulation results. The speed of simulation will ensure that, in an acceptable time, an analysis of a larger number of options is carried out and the best one is selected according to one or another quality criterion. And therefore, the accuracy and speed of obtaining the result require taking into account the integrated influence of the parameters of the bottomhole zones of wells on the mode of UGSF operation.

Integrated dynamic multi-parameter models were built for gas storage facilities based on measured data, which take into account the influence of changes in gas withdrawal/injection volumes over time based on operational data during several seasons of operation of the gas storage facility. Approbation of the constructed adaptive model was carried out, which clarifies the influence of non-stationary (dynamic) processes on the hydraulic processes that occur in reservoir layers and in the bottomhole areas of wells. Separate results of the quality of using such models are shown in Fig. 1. The given results are intermediate. The complexity of adapting models of centrifugal superchargers characterizes both the complexity of the processes and the insufficiency of reliable input data. In the presence of operational data on the structure of implemented technological schemes, the state of isolation valves, gas preparation facilities, etc., it would be possible to simplify the construction of adaptive models of dynamic gas-dynamic and filtration processes.

According to operational measured and calculated data, it is not always possible to correctly interpret the discrepancy between their values, especially in cases of significant transient non-stationary reservoir operation modes. One of the explanations for such a situation can be a gas flow break (under the gas withdrawal mode) or a hydraulic shock (under the gas injection mode). In open sources, such dynamic effects are studied exclusively for liquid flows.

Each well i (i=1, 2, ..., n) under the gas withdrawal mode at time t is characterized by the following set of parameters (P_{ip}, P_{iw}, q_i, t_j) – reservoir pressure, bottomhole pressure, flow rate, and measurement time, respectively. Time t_j ($m \le j \le k$) changes hourly and is given discretely by $t_m, t_{m+1}, ..., t_0, t_1, ..., t_k$. During the operating season for each well i, we have $PQ_{ij}(P_{ip}, P_{iw}, q_i, t_j)$ for only one value of j. The $PQ_{ij}(P_{ip}, P_{iw}, q_i, t_j)$ data allow one to calculate one value of the stationary coefficients of the filtering resistance. There is no guarantee that these coefficients will be the same for another set of measurements. P_{ip} is measured for each well number i from the set of wells $\{m_k\}$ ($k \le n$) daily. For a set of $\{P_{ip}\}$ measurements, their average value $\overline{P_p}$, is calculated, which is considered the average formation pressure in the working zone of wells.

To calculate the pressure at the gas gathering point P_{ggs} in the time interval $t \in [t_0, t_k]$ for the given pressure distribution in the reservoir at the time t_k, it is necessary to calculate the technological chain [14] – formation-bottomhole zone-wells-gas collection system-gas collection point for the specified hourly gas withdrawal volumes $q(t)$, $t \in [t_k, t] \left(q((t) = \sum_i q_i(t) \right)$. The result of the calculation will be:

– distribution of formation pressure, at the given time interval $[t_m, t_0]$, as well as distribution of formation pressure in the working zone of the wells;

– hydraulic losses $\Delta P_{ij}(q_i, t_j) = P_{i_p} - P_{i_w}$ in the bottomhole zone of each i-th well;

– hydraulic losses at each well and in the gas collection system $\Delta_i = P_{i_w} - P_{gcp}$.

In order to calculate $\Delta P_{ij}(q_i, t_j)$, the characteristics of the formation of the bottomhole zone of each well $C_i(k_i, \beta_i, S_i, t)$ are required – permeability, vortex resistance, skin factor as a function of the structure of the porous medium, variable flow rate over time, respectively, and also $P_{i_w}(t) = f(P_{i_p}, q_i, C_i, t)$. For operational calculations – daily modes of UGSF operation, it is enough to know the average values $\overline{C}_i(k_i, \beta_i, S_i)$ for each well. In practice, for each well, there are known $\overline{C}_i(k_i, \beta_i, S_i)$ only in the case of one measurement during the season of well operation. For the vortex coefficient β, it is often taken as its approximate representation found for the reservoir layer due to its permeability, porosity, grain size of the reservoir rock, etc. [13]. There are no studies of the dependence of parameters on variable flow rates in the non-stationary case $C_i(k_i, \beta_i, S_i, t)$ and rather it is not expected in the near future. During the operation of wells, the factors affecting these coefficients constantly and unpredictably change, and the share of their influence also changes. The forecast operating modes of UGSF at significant time intervals, calculated according to $P_{i_w} = f(P_{i_p}, q_i, \overline{C}_i)$ will differ slightly from the real ones. And in order to find this difference, a functional relationship was built between the total flow rate of the wells at the time interval $t \in [t_m, t_0]$ and the measured pressure on the gas well, the average coefficients of filtration resistance of the wells, the total flow rate of the wells at time t_0. To calculate the operating modes of UGSF at the time intervals $t \in [t_0, t_k]$, the found dependence, with sufficient accuracy, ensures the calculation of pressure on the gas station.

The construction of a model of the integrated influence of variable well flow rates on the depression/repression of the wells' bottomhole zones was preceded by the calculation of the gas pressure at GCP during the time interval $t \in [t_0, t_k]$ with a time step of one hour. For the initial data, the following were taken:

– reservoir pressure distribution $P(x_i, y_i, t_0)$, where (x_i, y_i) are the coordinates of the peaks of the triangulation of the reservoir layer surface for the finite element method;

– filtration resistance coefficients (a_j, b_j), where j – numbers of working wells;

– number of working wells $n(t)$ and hourly gas consumption $Q(t)$ at time t.

As a result of the calculation, we obtained:

– $P(x_i, y_i, t)$ – reservoir pressure distribution;

– $\underline{\Delta P}_{j1}(q_j(t))$ – depression in the bottomhole zone of wells j;

– $\overline{P}_p$ – average reservoir pressure in the area of working wells;

– $P_{gcp1}(t)$ – the gas pressure at GCP;

– $q_j(t)$ – flow rate of wells.

As expected, the calculated pressures at GCP $P_{gcp1}(t)$ differed from the measured $P_{gcp2}(t)$ over the entire time interval.

In order for the calculated $P_{gcp1}(t)$ and measured $P_{gcp2}(t)$ data to be close with the required accuracy, it was proposed to build such a function (model) $\Delta P(t) = f(Q, \overline{P}_p, t)$ on the time interval $t \in [t_0, t_k]$, which would ensure the calculation of $P_{gcp2}(t)$ under the condition $\overline{P}_p + f(Q, \overline{P}_p, t)$ of necessary accuracy.

The construction of the function $\Delta P(t) = f(Q, \overline{P}_p, t)$ was carried out by the technique of approximation of the data obtained as a result of solving the inverse problem – calculation of the formation pressure in the area of wells based on the pressure at GCP $P_{gcp2}(t)$ and the gas flow rate $Q(t)$.

The test results of the developed models for the Dashavske UGSF are shown in Fig. 1. The legend to these figures is as follows:

– PRsrM – calculated average reservoir pressure of gas in the area of operation of wells, ata;

– PGgsClc – calculated gas pressure at the gas collection point, ata;

– PGgs – measured gas pressure at the gas collection point, ata;

– Q – volumes of gas injection/withdrawal, million m^3/day, million m^3/d;

– PRsrClc – approximate measured average reservoir pressure in the working zone of wells;

– AktiveGas – volume of active gas, million m$^3 \cdot 10^2$.

Fig. 1 shows the results of the pressure calculation at the gas collection point (GCP) of Dashavske UGSF. Modeling of UGSF operation modes was carried out during the pumping season. The number of working wells and hourly volumes of gas injection are taken as input data. For a comparative analysis of the accuracy of the calculation, the formation pressure in the injection area and the pressure at the gas collection point were taken. The coincidence of the mentioned calculated values with the measured ones is close to the accuracy of their measurement.

Analysis of results:

– the amount of pressure at GCP significantly depends on the volume of gas withdrawal;

– significant inertia of filtration process parameters is observed;

– the change in pressure at GCP, due to a sudden change in the volume of gas withdrawal, can be divided into instantaneous and slowly variable;

– for a certain volume of gas withdrawal, relative stability of the pressure at GCP is observed.

Somebody also calculated the daily operating mode of UGSF using the developed web application (Fig. 2).

Fig. 2 shows the calculated operation modes, among which it is possible to single out operation modes that are optimal according to certain criteria of optimality.

The same Fig. 2 demonstrates the interpretation of the abbreviated representation of the three-stage gas compression by the booster compressor station. As you can see, three gas pumping units are offered for each gas compression stage.

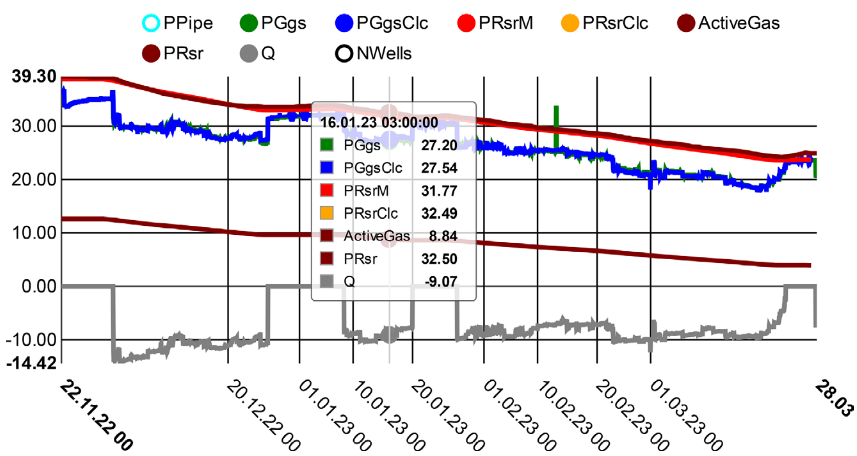
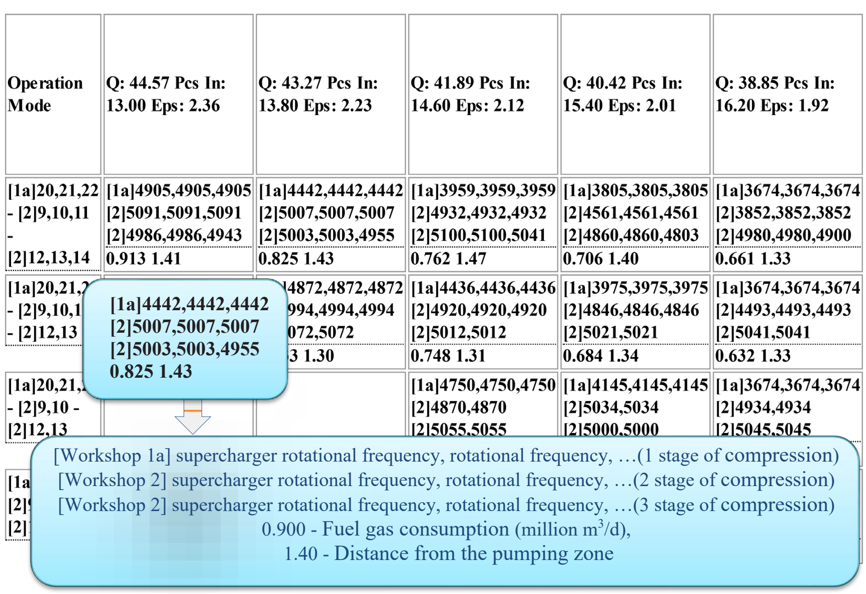


Fig. 1. Dashavske underground gas storage facility.
Gas withdrawal during 2022–2023



Operation Mode	Q: 44.57 Pcs In: 13.00 Eps: 2.36	Q: 43.27 Pcs In: 13.80 Eps: 2.23	Q: 41.89 Pcs In: 14.60 Eps: 2.12	Q: 40.42 Pcs In: 15.40 Eps: 2.01	Q: 38.85 Pcs In: 16.20 Eps: 1.92
[1a]20,21,22 - [2]9,10,11 - [2]12,13,14	[1a]4905,4905,4905 [2]5091,5091,5091 [2]4986,4986,4943 0.913 1.41	[1a]4442,4442,4442 [2]5007,5007,5007 [2]5003,5003,4955 0.825 1.43	[1a]3959,3959,3959 [2]4932,4932,4932 [2]5100,5100,5041 0.762 1.47	[1a]3805,3805,3805 [2]4561,4561,4561 [2]4860,4860,4803 0.706 1.40	[1a]3674,3674,3674 [2]3852,3852,3852 [2]4980,4980,4900 0.661 1.33
[1a]20,21,… - [2]9,10,1… - [2]12,13	[illegible]	…4872,4872,4872 …994,4994,4994 …072,5072 …3 1.30	[1a]4436,4436,4436 [2]4920,4920,4920 [2]5012,5012 0.748 1.31	[1a]3975,3975,3975 [2]4846,4846,4846 [2]5021,5021 0.684 1.34	[1a]3674,3674,3674 [2]4493,4493,4493 [2]5041,5041 0.632 1.33
[1a]20,21,… - [2]9,10 - [2]12,13			[1a]4750,4750,4750 [2]4870,4870 [2]5055,5055	[1a]4145,4145,4145 [2]5034,5034 [2]5000,5000	[1a]3674,3674,3674 [2]4934,4934 [2]5045,5045
[1a… [2]… [2]…	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Fig. 2. Results of the calculation of options for possible daily modes of operation of the Bilche-Volytsko-Uherske underground gas storage facility

In the first row of table in Fig. 2 for the specified daily volumes of gas withdrawal, the calculated pressures at GCP, as well as the gas compression coefficients, are given. The calculated possible variants of modes for a given daily volume of gas withdrawal are shown vertically.

6. Discussion of results of investigating models of the integrated influence of well flow rates on the bottomhole pressure

Inflow of gas to the well, in which there is a linear relationship between the speed of movement and the pressure gradient, is characteristic of gas filtration beyond the boundaries of the bottomhole zones of the wells. Therefore, formulas (7) and (8) can be used for well research only in cases of small well flow rates. In all other cases, the results of the study will have a significant error.

Formula (9) is most often used to build a nonlinear model of gas flow to the well. The main problem that arises in this case is to find a functional representation of the Forchheimer coefficient β through the parameters of the reservoir layer. Known representations (10) of the coefficient β are with constant permeability, porosity, and diameter of the rock grains. The structure of the porous medium is established during the study of core rock in under laboratory conditions. Today, such studies are absent due to the absence of the cores themselves. Moreover, the comparative analysis of the results of the core research, according to the filtration parameters obtained under laboratory and natural conditions by hydraulic methods, differ by an order of magnitude. Formula (10) is constructed for a homogeneous porous medium. The reservoir layers are, in the vast majority, anisotropic in spatial coordinates. The inhomogeneous structure of the porous medium is also characterized by the tortuosity coefficient of the capillaries, for which there is no evaluation procedure.

A large number of wells are used for gas storages, which are located in areas beyond the reach of modern communication systems. This does not allow obtaining the necessary measured data in real time for their processing and analysis. And that is why simplified methods of well research are often used – «one-point». It is shown that it follows from (14) and (15) that «one-point» methods cannot be used to study wells at UGSF due to the non-linear nature of gas movement in their bottomhole zones.

The analytical solution (20) of the non-stationary model of gas inflow to wells (17) can be found only in the case of its linearization. And therefore, such solutions can be used both for the analysis of the gas movement process in areas somewhat distant from the bottomhole zones, and for estimating the time of reservoir pressure recovery (24). The recovery time of the formation pressure depends significantly on the permeability of the «feeding» area of wells and it can be significant. And therefore, establishing its value during a short time of well research is problematic.

The use of the superposition method (26) allows us, approximately, to investigate filtration flows of a more complex nature – with an arbitrary change in the flow rate of the well and with the simultaneous operation of wells with different flow rates. Flow measurements of wells in the case of plume gas gathering must be obtained directly at the well, which is a problem. And that is why a method is proposed that requires measurement of the annular and gas pressure on the well buffer. The study of the error of this method demonstrates its sufficient accuracy (Table 3).

The comparative analysis of the results of well research for different years was carried out by calculating the operation modes of UGSF based on fixed input data (Table 4). The results of the research of the wells for different seasons of UGSF operation significantly affect the calculated modes of UGSF operation. Therefore, the results of the research of wells conducted during one time interval cannot be used to calculate operation modes at other time intervals.

And what's more, the research of wells is carried out by separating them from the system and connecting them to a separate separator and gas flow meter. As demonstrated by field and numerical experiments, the operating parameters of wells that work separately from the system and as part of the system sometimes differ by 30 %.

The analysis of models of gas inflow to wells under conditions of stationary and non-stationary filtration processes showed that the existing models of gas inflow to wells are unsuitable for solving operation mode problems of UGSF operation for significant time intervals. They are obtained in the process of researching wells for a short period of time and do not make it possible to identify all factors, especially the magnitude of the influence of each of them, on the magnitude of depression and repression in the bottomhole zones. Moreover, the factors and their magnitude change over time, unpredictably, constantly. Real measured data show that the daily change of depression/repression, which affects the mode of operation of wells, and accordingly UGSF, for a month or more, on average, varies beyond the accuracy of its measurement. Therefore, building a model that would provide such a high sensitivity to the influence of many poorly defined factors is problematic.

In the process of UGSF operation, pressure and gas flow are constantly measured at a gas collection point (GCP). The problem of calculating reservoir pressure in the reservoir layer and, in particular, in the working zone was already solved [11]. The idea arose to combine the models of the bottomhole zone, wells, and the gas collection system into a single integrated model. Based on the measured data at GCP, calculated reservoir pressures, and the measured withdrawal/injection volumes, it was possible to build a model of the integrated effect of the change in depression/repression of all wells in general on the pressure at GCP. This was enough so that this pressure, the known gas flow rate, and the pressure in the gas pipeline allowed us to calculate the mode of operation of compressor stations. The results of this approach are shown in Fig. 1, 2.

The idea of building a model of the integrated effect of the change in the depression/repression of all wells in general on the pressure at GCP arose as a result of numerical studies. UGSF was simulated for a given gas flow rate Q under the withdrawal mode and for different distribution of gas between n wells. It turned out that the value of pressure at GCP was quite stable. The value of pressure at GCP was quite stable even in the case of a change in the formation gas pressure on the contours of the gas inflow to the wells according to a uniform distribution within the limits of two or more atmospheres. Hypothetically, the stability of pressure in such cases is explained by the network technique of gas collection, the single GCP and the redistribution of flows in it according to the laws, in the stationary case – Kirchhoff's laws. The total hydraulic losses in such networks change slightly because the nonlinear relationship between the pressures in the network sections and the amount of gas flow is weakly manifested over short distances. And therefore, it should be expected that the Q value itself has a more significant effect on the pressure at GCP, and not its distribution and the wells, as well as the average pressure, and not the deviation from the average value on the contours of the bottomhole zones.

The constructed model of the integrated influence (MII) of change in the depression/repression of all wells in general on pressure at GCP ensured the stability and satisfactory accuracy of the calculation of the modes of operation of GCP throughout the entire forecast season of its operation.

It is impossible to conduct a comparative analysis of the proposed approach with existing ones in the absence of any results in open sources related to detailed calculations of UGSF operating modes at significant time intervals. Oral information is known – only the coefficients of filtration resistances, which are obtained for stationary modes of operation of UGSF, are used. From the results of our work, it follows that this is not enough, especially for calculating the forecast operating modes of UGSF (Table 4).

The obtained solutions, as part of the software modeling package, provided the calculation of various options for the forecast operation modes of all UGSFs integrated with GTS during the 2023–2024 gas withdrawal season. Moreover, it is ensured that not only design but also non-design variants of UGSF operation are worked out.

Regarding the limits and conditions of applicability of the results. One should consider the following:

– a two-dimensional reservoir layer model is used, which is sufficiently accurate for modeling filtration processes in reservoirs in which the dimensions of the surface differ by an order of magnitude or more from the vertical component of its capacity;

– the permeability of the reservoir layers significantly depends not only on the structural properties of the porous medium of the reservoir layers but also on the composition of natural gas, especially on changes in the degree of gas humidity and the concentration of heavy hydrocarbons. In the case of gas injection with a changed composition, the MII model should be refined;

– if the MII model was built based on the data obtained as a result of UGSF operation at the maximum volume of active gas, then it should be expected that it can be applicable in all other cases as well.

Each change of depression/repression has conditionally two components – fast and slow. The component of rapid change is estimated by the coefficients of filtration resistances of the bottomhole zones of wells, which are obtained in the process of researching wells under stationary modes of their operation. The influence of the slow change of depression/repression in the bottomhole zones on the calculation of pressure at GCP is integrally taken into account by the MII model. For changes in the gas composition, not only the MII model should be refined but also the coefficients of the filtration resistances of near bottomhole zones.

The development of this research may proceed in the following directions:

– maximum automation of the process of specifying the parameters of the model of the integrated influence of change in the depression/repression of bottomhole zones on the mode of UGSF operation according to the discrepancies between the predicted and measured data;

– identification of possible influencing factors on changes in the parameters of the model of the integrated effect of changes in the depression/repression of bottomhole zones on the mode of UGSF operation.

7. Conclusions

1. It has been established that almost all models of gas flow to wells, which are also used at UGSF, were mainly developed and tested first at gas production fields. The nature of gas field exploitation is significantly different from UGSF exploitation. Even for gas fields, where the exploitation of wells takes place with an order of magnitude less intensity, there is no unified approach to the study of the factors and the nature of each factor influencing the productivity of their wells. Each reservoir layer as well as the bottomhole zone of wells are unique and require constant study. This is due to the fact that in the course of intensive operation, often under peak conditions, the filtration properties of the areas of gas inflow to the wells change. And such a change is poorly predicted. The analysis of the results of the assessment of the area of uncertainty of the filtration resistance and their impact on the performance of UGSF and fuel and energy costs, demonstrated that the existing procedures cannot predict the behavior of coefficients of the filtration resistance for significant time intervals.

2. A study of the influence of variable flow rates of wells on the depression/repression of bottomhole zones and, accordingly, on the pressure at a gas collection point was carried out. The problem of the complexity of calculating the depression/repression of all wells near bottomhole zone is replaced by the problem of building a model that integrally takes into account the influence of the depression/repression of all wells in general on the dynamic mode data of UGSF. This has made it possible to calculate with the required accuracy the pressure at a gas collection point and, accordingly, ensured the calculation of the operation modes of compressor stations in an acceptable time and with the required accuracy. This technique ensured the calculation of optimal modes of operation, as well as the planning of the optimal operation of all UGSFs as a single thermo-hydraulic complex integrated with GTS for significant time intervals – gas injection/withdrawal seasons. Simultaneous simulation of ten operating UGSFs under the peak mode of withdrawal of the entire available volume of active gas (15.7 billion m^3) takes no more than six minutes.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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