

MULTIPLEX TECHNIQUE OF DATA TRANSMISSION IN RESIDUAL CLASS SYSTEMS

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The research object: data transmission in optical communication lines. The subject of research is algorithms for the construction of digital data and methods of their transmission over buses in optical computer systems and in backbone fiber-optic systems.

The problem to be solved is the need to devise new methods that ensure increased reliability and cryptographic stability of optical transmission systems. To solve the task, the issue of expanding the theory of timer signal structures and the system of residual classes for the organization of multifactorial multiplex data transmission through the channels of modern information transmission systems was investigated. The factor space is defined (as an example) for fiber optic transmission systems where different multiplexing options are used or may be used.

The possibility of adapting algorithms for the construction of digital signal structures for their further transmission in the system of residual classes by various methods of multiplexing has been substantiated. The main principles of transmission were considered: the principle of independence of multiplexing the transmission of residues on each module and the principle of logical dependence and physical independence of the system of channels for transmission of residue values for a specific module of a specific system of residue classes.

The basic principle is that at each specific point in time in the multivariate binary space, only one of the possible values of each factor can be equal to unity. A comparison with existing transmission systems shows that the proposed technique could provide data transmission at a speed of up to 16 Tbit/s in a transmission bandwidth of 200 THz. At the same time, the capacity of the alphabet of transmitted characters will be 39468 different characters. It also provides a significant increase in the reliability of the entire transmission system

Keywords: timer signal structures, algorithms, systems of residual classes, multiplexing, coding, optical transmission systems

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1. Introduction

It can be assumed that the next stage of the development of computer technology will differ qualitatively in the use of ultra-fast optical technologies. Currently, the implementation of all-optical computers is held back only by the problem of data storage in the optical range.

Optical systems are not necessarily built on the use of the usual binary logic. If we supplement the fast performance of operations in the optical range with fast algorithms for data preparation and transformation, we obtain the general characteristics of the computing systems of the future.

In this sense, the use of modular arithmetic is absolutely logical. However, aspects of code transmission in systems of residual classes (SRC) have not yet found a worthy place in the scientific and technical literature, despite the history of their application.

Therefore, the development of theoretical foundations, methods, and algorithms for the construction of digital code structures in SRC and their transmission in optical computing systems, as well as in modern and promising fiber-optic transmission systems (FOTS) is a relevant area of research.

The results of research in this field are important from both a theoretical and a practical point of view. The scientific significance of the research is the development of theoretical foundations of the adaptation of existing algorithms for the construction of digital code structures and the design of the latest technologies and algorithms for data transmission and processing in the optical range. The practical significance is that the transmission of data in SRC codes makes it possible to significantly increase the reliability and cryptographic protection of transmission systems due to the use of significant information redundancy, as well as due to the possibility of transmitting the remains of different modules in separate channels.

2. Literature review and problem statement

The efficiency of data transmission and processing in the optical range is not only in ultra-high speeds but also in the possibility of applying various calculation logics. Thus, there are known studies of the application of, for example, ternary (triple) logic in optical computers [1, 2]. But the application of SRC is not considered in those works.

It is known that representing data in non-positional counting systems in some cases gives a significant advantage in the speed of data processing. Thus, modular SRC arithmetic makes it possible (in a certain bit grid) to avoid the operation of transferring a bit unit as a result of summation or subtraction. There are known studies in the direction of improving the forms of SRC [3] but they do not directly relate to the problems of data transmission in optical systems, for example, in FOTS.

Many papers consider the principles of information processing in modular arithmetic. For example, methods of solving polynomial equations using modular arithmetic are investigated in [4]. A significant number of publications tackle the areas of application of SRC. For example, in [5], the possibility of using SRC for pattern recognition is investigated. In [6], methods of encrypting messages with the remainder of the modules are investigated.

Among the small number of works specifically addressing the transmission of SRC codes, we note study [7]. In this work, it is proposed to transmit information in wireless sensor networks using SRC. At the same time, different residues are transmitted in different channels, which significantly increases the reliability of the system. However, the cited work does not consider the possibility of organizing transmission channels with different multiplexing techniques.

In [8], the main provisions of the theory of timer signal structures (TSS) [8] were chosen as the basis of research into the problem of transmission of SRC codes. It is shown that the organization of information flows in TSS structures gives certain advantages in solving the tasks of increasing the reliability of data transmission and correcting possible errors. Works [9, 10] report studies on the applicability of TSS for the transmission of SRC codes. In those works, it is assumed that the values of the residues for different modules of SRC are transmitted by separate fragments of the code package. Essentially, in a time-division transmission system, multiple time-spaced channels are created. In each of these subchannels, the value of only one residue of a specific SRC module is transmitted.

At present, the highest speed of information transmission is provided by the mainline FOTS at the regional, state, and international levels. At the same time, these transmission systems currently or in the near future provide the possibility of compression and ensuring an increase in the speed of its transmission by various multiplexing techniques, in particular:

- spatial multiplexing (distribution of data flows in different fibers or, in the future, in different cores of one optical fiber [11]);
- frequency (spectral) multiplexing (for example, in dense spectral compression systems – DWDM [12]);
- time multiplexing (distribution of the data flow into elementary consecutive time fragments, each of which corresponds to a separate channel [13]);
- polarization multiplexing (distribution of channels according to the system of polarization axes of the optical signal [14]);

- signal or logic multiplexing (phase modulation, amplitude modulation, forming pulses with different low-frequency envelope shapes, linear-frequency modulation [15], etc.);
- complex multiplexing (for example, phase-code manipulation).

However, available studies do not take into account that it is precisely in optical transmission systems that there is a fundamental possibility of using a set of multiplexing techniques. Therefore, the partially solved problem is spreading the theory and methods of TSS for promising FOTSS, the purpose of which is the transfer of codes in SRC.

3. The aim and objectives of the study

The purpose of this research is to devise fundamentally new, multi-factorial multiplex methods of digital data transmission, formed in SRC based on the principles of the TSS theory, the provisions of which apply to a significant class of multiplexing techniques in relation to transmission over fiber optic bus systems. This will make it possible, firstly, to separate the transmission of the remaining values of SOK modules by channels using different multiplexing techniques, and secondly, to significantly increase the performance of data transmission systems thanks to the application of algorithms of the TSS theory, to significantly increase their resistance to failures, as well as to improve the reliability and cryptographic protection of data that are transmitted.

To achieve the goal, the following tasks were set:

- on a specific example of complex residue channel (CRC) system, solve the problem of formulating and substantiating the principle of independence of multiplexing of the transmission of residuals of SRC modules;
- to state and justify the principle of logical dependence and physical independence of sub-channels of CRC;
- to determine the main dependences for the calculation of intersymbol protection intervals;
- to give estimates of the potential speed of transmission when using the multiplex technique of data transmission in systems of residual classes.

4. The study materials and methods

The object of our research is data transmission in optical lines. The subject of research is algorithms for the construction of digital data and methods of their transmission over buses in optical computer systems and in backbone fiber-optic systems.

The main hypothesis of the study assumes that the values of the residual values of the SRC modules can be transmitted in separate channels of different physical nature. In order to confirm the productivity and adequacy of this hypothesis, specific models of CRC systems are considered.

Models of CRC systems are based on parametric estimates of modern DWDM-type fiber optic transmission systems. However, to expand the number of multiplexing options, it is assumed that in the near future, in addition to frequency and time multiplexing, other techniques will also be used, namely:

- polarization multiplexing;
- spatial multiplexing in many cores of a microstructured optical fiber.

It is also important to assume that the distribution property of signal recorders on the receiving side in terms of

frequency and time significantly exceeds the effective width of optical pulses in the frequency domain and the effective duration in the time domain.

Research methods used in the work are:

- methods of theories of positional and non-positional calculation systems [3];
- methods of the theory of timer signal structures [8];
- methods of the theory of signal transmission in fiber optic systems [16].

5. Results of research into the multiplexing technique of data transmission in residual class systems

5.1. An example of the implementation of a system of complex residual channels and justification of the principle of independence of multiplexing residual transmission

A specific version of SRC is defined by a system of modules [9], which can be conveniently represented in the form of a tuple:

$$RNS = [M_1, M_2, \dots, M_N], \tag{1}$$

where M is the value of whole mutually prime modules. Then the capacity (number of different symbols) of the alphabet $A(RNS)$ is determined by the expression:

$$V(RNS) = \prod_{n=1}^N M_n. \tag{2}$$

Variant of the transmitted symbol:

$$a_k \in A(RNS), k = 0, 1, \dots, V(RNS) - 1 \tag{3}$$

is also determined by the tuple of the values of the remainders from division by modulo RNS (1):

$$a_k = [R(k, 1), R(k, 2), \dots, R(k, N)], \tag{4}$$

where $R(k, n)$ is the value of the remainder (Remainder) modulo n for the variant of the transmitted symbol (3).

An example of the technical implementation of CRC system is given in Table 1, where only four multiplexing techniques are provided.

Based on the data in Table 1, it is possible to set the capacity of the transmitted alphabet $V(RNS) = 2 \times 3 \times 5 \times 7 = 210$ different symbols. Each symbol is transmitted in one cycle of the transmission system.

Table 1
An example of the technical implementation of CRC system for $RNS = [2, 3, 5, 7]$

SRC module	CRC organization technique
2	Polarizing: 2 polarization axes
3	Spatial: 3 optical fibers
5	Temporary: 5 slots in the code link
7	Frequency: 7 carriers

The proposed principle of multiplexing independence boils down to the fact that for each SRC module (1) with number n , a separate logical, or better – physical CRC is organized, which can be in one of M_n states: $0, 1, \dots, M_n - 1$.

Certain arguments can be given in favor of this multiplexing technique.

Let's consider a specific practical situation: one of the three fibers (Table 1) turned out to be broken. For example, during the period of fiber switching in the intermediate optical coupling. On the receiving side, a signal will be received almost instantly: a parametric failure of CRC modulo 3.

This signal is broadcast to the transmitting side. On the transmission side, encoders are rearranged: module 3 is excluded from processing. The capacity of the SRC alphabet is reduced by 3 times. Accordingly, the speed of information transmission decreases by 3 times. But at the same time, the most undesirable "denial of service" state of the system does not occur since the remaining 2 fibers transmit the remnants of all other SRC modules. Moreover, one undamaged fiber is enough.

Similarly, failure of a laser diode (transmit side) or a photodiode (receive side) will only disable the remainder modulo 7 transmissions, the transmission rate will be reduced by a factor of 7 but no denial-of-service event will occur.

Thus, the proposed principles of organization of CRC systems make it possible to significantly improve the fault tolerance of transmission systems.

5.2. Formulation and substantiation of the principle of logical dependence and physical independence of sub-channels of CRC

On the receiving side, the CRC status is converted into SRC codes. If necessary, these codes are processed and converted, for example, into a binary code by decoders. Table 2 gives the truth table for the CRC decoder according to module 3, where the symbols $B0$ and $B1$ denote the bits of a binary number, and the symbol Error is the signal of a false (emergency) state of CRC.

Table 2

The truth table of the decoder of CRC states modulo 3

Condition of CRC channels			Status of decoder outputs		
Channel 2	Channel 1	Channel 0	$B1$	$B0$	Error
0	0	0	0	0	1
0	0	1	0	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

The operation logic of such a decoder is elementary: the usable code of the remainder (0, 1, or 2) is received only when one and only one of CRC channels is active, modulo 3. In other cases, an error signal is received. False states can be interpreted in different ways. For example, the state of the input (0, 0, 0) can be interpreted as a failure of CRC, and a state of the type (0, 1, 1) or (1, 1, 0) as a parametric failure: the detectors will not be able to distinguish between the states of neighboring channels.

The example of Table 2 immediately shows the advantages and obvious disadvantages of the principle of organizing CRC as a system of logically dependent channels. In general, both follow from the obvious information loss.

Advantages:

- the possibility of transferring the residues of SRC in a separate CRC;
- high reliability of the transmission system;
- a high level of cryptographic protection of transmission channels.

Disadvantages:

- excessive information loss;
- a significant reduction in transmission speed compared to known multiplexing techniques.

The noted disadvantages practically negate all the advantages. It is especially bad that the information loss rate is different for CRC in different modules. Thus, for CRC of module 7, in the case of logically independent transmission channels, it is possible to transmit $2^7=128$ different message options in one time slot. For logically dependent channels, the number of options is only 7.

However, the proposed principles of the organization of CRC systems make it possible to bring the transmission speed closer to the theoretical one.

A specific example of CRC system is considered below: $RNS=[5,7]$, where the CRC modulo 5 is organized in the time domain, and the CRC modulo 7 is organized in the frequency domain (Table 1). We also use one of the possible grids of DWDM frequency channels with a step of 100 GHz [12]. Based on the concept of the Nyquist interval (essentially, the Nyquist-Kotelnikov theorems), we define the concept of a pulse base with an effective spectrum width W and an effective duration T :

$$B = W \cdot T = \text{const}, \tag{5}$$

where the const value depends on the shape of the pulse envelope. Usually, we can take $B \approx 1$. Then, in the proposed example, the pulse duration will be 10 ps.

Thus, this transmission system can be in 128 states (by frequency) at any moment of time and on a certain section of the line length. During a packet of 5 pulses (50 ps), there can be only $128 \times 5 = 640$ different states of the transmission channel. In this case, the division of channels in different areas (frequency and time) does not allow for inter-channel (frequency) and inter-pulse (time) interference. The time and frequency channels are thus logically independent but physically dependent: the pulses are separated from each other by both frequency and tracking time.

In the proposed version of logical dependence in each domain – frequency, time, polarization, etc. – the concept of interference becomes absent. The logical dependence of CRC channels (only one of the channels is active at any given time) leads to physical independence, which can be considered a paradox. In this case, there are no contradictions with the concept of the Nyquist interval: pulses both had a base $B \approx 1$ and will have the same base. The difference is that now in time the pulses will not necessarily follow each other with an interval T based on the effective duration of pulses. Now completely different parameters of the transmission system are important: the resolution of the synchronization system, the accuracy of the gating system. Similarly, in the frequency range, the width of the pulse spectrum W is not as important as the resolution of the frequency detectors for separating the carrier (center) frequencies of the channels.

The noted paradox for the proposed example in the time domain is illustrated in Fig. 1.

The upper plot of Fig. 1 shows the transmission of a code packet 1, 1, 0, 1, 1 in the classical sense of

the Nyquist interval. A value of “1” is transmitted by a high level of the signal, and a value of “0” by a low level that exceeds the noise threshold. At the same time, there is no interpulse interference. In the lower plot of Fig. 1, in the same time interval of 50 ps, there are already 25 pulses, which are well separated in time but have overlapping zones. The difference is that now only one of the 25 time channels can be active during the code packet. That is, there will be no interpulse interference.

Another effect of the logical dependence of channels is characteristic of FOTS (Fig. 2). A time interval of 50 ps duration is considered here. But due to dispersion, the effective duration of pulses on the receiving side may differ several times from the duration on the transmitting side [16].

It is clear that this effect is taken into account when designing FOTS. The upper plot of Fig. 2 shows how the envelope of a bundle of 5 pulses on the receiving side would look without taking dispersion into account. As you can see, it is impossible to separate the pulses from each other on the receiving side.

The lower plot of Fig. 2 shows that in the case of logical dependence, all 5 pulses are well separated since only one of them propagates along the optical path during 50 ps.

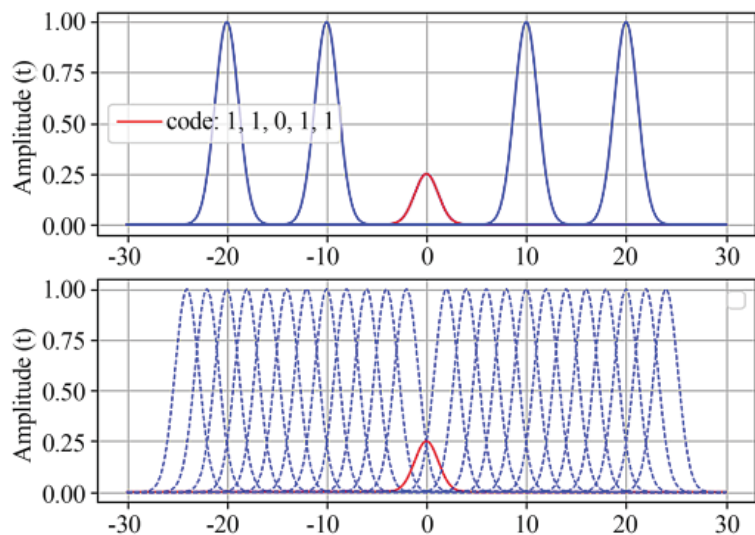


Fig. 1. Illustration of the paradox of logically dependent and independent channels

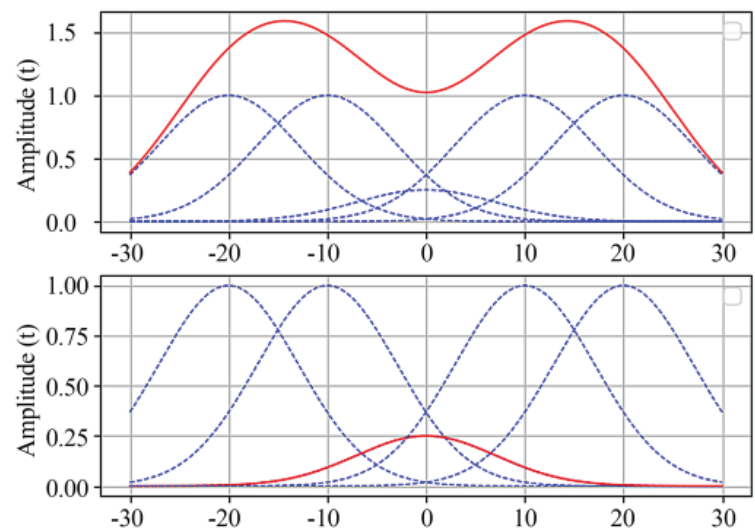


Fig. 2. Resistance to dispersion effects

Similar diagrams can be given for the frequency domain. But let's return to the technical characteristics according to Table 1. Thus, there is a total frequency band of $7 \times 100 = 700$ GHz (for all frequency channels) and a time interval of 50 ps (for all time channels).

Today, DWDM systems with a frequency grid step of 50 and 25 GHz are already in operation [12]. In the recommendations of the International Telecommunication Union, frequency plans with a step of 12.5 and even 6.25 GHz are planned in the future. But while the width of the channels is determined in steps of 6.25 GHz, the resolution of frequency detectors for selecting carrier frequencies should be at least half of the bandwidth of bandpass filters.

In this example, each band of 100 GHz can be divided into $2^4 = 16$ logically dependent sub-channels with a "width" of $w_1 = 6.25$ GHz each, even with the current state of technology. Similarly, the time interval $T = 10$ ps can be divided into sub-cycles with a duration of $t_1 = 2$ ps each, which is shown in Fig. 1. Then, in the common band of 700 GHz, it will be possible to transmit already 400 symbols in logically dependent channels at a time interval of a code packet of 50 ps. This number is quite close to the previously determined value of 640 options for logically independent channels. However, at the current level of development of FOTS technology, even a value of 400 is not the limit.

Intermediate conclusion: the organization of CRC system according to the principle of logical dependence and physical independence makes it possible to organize data transmission at approximately the same speed as in the classic case of logically independent, but physically connected channels.

Additional arguments can be given in favor of the proposed principle of logical dependence and physical independence of channels.

During the time of the code package (one character of the SRC alphabet), only one of CRC channels for this module is in an active state. Thus, the prerequisites are created for transmission compression due to the convergence of channels based on different physical principles. In particular, the research considered an example of only polarization [16] and spatial [11] multiplexing (Fig. 3).

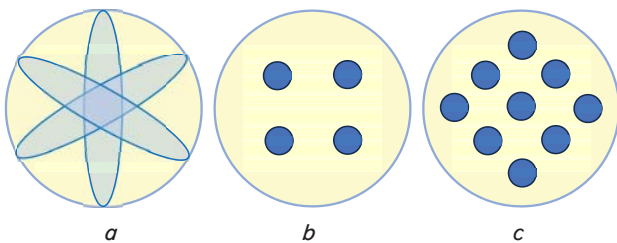


Fig. 3. Polarization and spatial multiplexing: *a* – polarization multiplexing; *b, c* – spatial multiplexing in many cores of a microstructured optical fiber

In Fig. 3, *a*, a variant of polarization multiplexing on 3 channels is conditionally shown. The channels differ in the arrangement of the main axes of elliptical polarization with a step of 60° . Fig. 3, *a* demonstrates the following:

- the energy potential is used extremely inefficiently: the total overlapping area of the horizontal and vertical components of the radiation fields hardly reaches half the area of the total figure;
- there is a significant area of intersection of the ellipses, which "ensures" a high level of inter-channel interference

during the simultaneous passage of signals in different polarization channels.

In the case of logical dependence (only one of the channels is active), it is possible to significantly increase the number of polarization axes and use elliptical polarization with a significantly lower eccentricity.

Fig. 3, *a, b* shows 2 variants of the location of the microstructured fiber cores. It is obvious that the distance between the cores depends, among other things, on the energy level of the output modes, if different signals are propagated through different cores at the same time. But if these signal parcels are clearly separated in time according to the principle of logical dependence, then, again, interference effects can be almost neglected. This approach makes it possible either to significantly increase the number of cores in the fiber, or to reduce the diameter of the fiber.

As can be seen, the logical dependence of the channels makes it possible in some cases to focus not on the parameters of the mutual influence between the channels but on the possibilities of technological processes, accuracy, and sensitivity of detectors, etc.

To illustrate the principle of logical dependence and physical independence of CRC subchannels, we shall give a simple example for the alphabet $RNS = [2, 3]$. The capacity of such an alphabet is $V(RNS) = 2 \times 3 = 6$ different symbols. Correspondence of data (codes) in positional systems and residue codes for this alphabet are given in Table 3, where the active states (1 – "active state") of the corresponding channels for CRC 2 and CRC 3 are also shown.

Table 3

Correspondence table of codes in positional systems and codes in SRC [2, 3]

Codes in positional systems		Residue codes and the corresponding condition of CRC channels				
Decimal	Binary	Modulo 2 residuals (CRC 2)		Modulo 3 residuals (CRC 3)		
		0	1	0	1	2
1	001	–	1		1	
2	010	1	–	–	–	1
3	011		1	1	–	–
4	100	1	–	–	1	–
5	101	–	1	–	–	1
6	110	1	–	1	–	–

Possible states of CRC channels for two cases according to Table 3 are shown in Fig. 4, where T_0 and T_1 are clock intervals (time slots) of successive code packets.

According to the data in Table 3, in the time slot T_0 , the remainder 0 is transmitted on the zero channel of CRC 2 and the remainder 2 on the second channel of CRC 3. The state of these channels is a logical unit. Logical zeros are transmitted on all other channels. According to the data in Table 3 states of the channels in the time slot T_0 uniquely correspond to the number 2 in the decimal code, or the number 010 in the binary code.

A completely different case is modeled in Fig. 4 in time slot T_1 : values of a logical unit are transmitted simultaneously on two channels of CRC 3 (zero and second), i.e., two channels of one CRC are simultaneously active. None of the options in Table 3 corresponds to such a set of residual values. Therefore, in the time slot T_1 , the erroneous state of the transfer of residues by module 3 in CRC 3 is simulated,

that is, the state “ERROR” will be recorded on the receiving side: an accident on this channel. Further, according to the operation protocol of the entire system, this emergency signal will be transmitted via feedback to the transmission side. Coder of residuals according to Fig. 4 will adaptively switch from encoding in the *RNS* [2, 3] alphabet to encoding in the *RNS* [2] alphabet. That is, the capacity of the alphabet will decrease by three times, and the bit rate of transmission will also decrease by three times. But at the same time, the worst situation “denial of service” will not arise. It is clear that CRC according to Fig. 4 can be of different physical nature. For example, CRC 2 is organized as two channels by the technique of polarization multiplexing (two mutually perpendicular axes of elliptical polarization), and the channels of CRC 3 are organized by the technique of spectral multiplexing with a small difference in carrier frequencies.

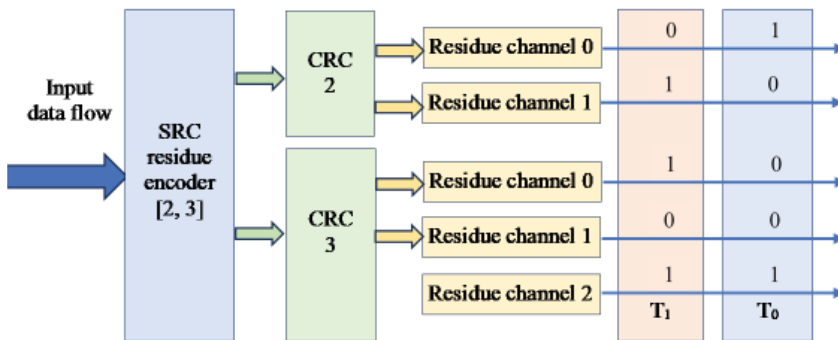


Fig. 4. States of CRC channels for transmission of SRC codes with the alphabet $RNS=[2, 3]$

5. 3. Solving the problem of defining intersymbol protection intervals

The stated principles of construction of CRC systems make it possible to practically exclude the phenomenon of inter-channel and inter-pulse interference during the code package of one symbol. The duration of such a parcel will be determined by the technique of time multiplexing. According to the example of the parameters in Table 1 time multiplexing is responsible for the transmission of residues modulo 5, and in the general case – residues modulo M_t .

At the same time, the transfer of residual values across all modules of SRC is carried out in time:

$$T_{RNS} = M_t \cdot t_1. \tag{6}$$

In the considered example, the code package of one SRC symbol will take 10 ps. But symbols must be transmitted one by one along the FOTS path. At the same time, the conditions of physical independence may be violated, and the logical dependence applied only to 5 time channels. For two sequences of symbols, there will be 10 such time channels, for three 15, etc. Then there are almost inevitable edge effects between code packages of symbol sequences. And all the above logic refers exclusively to the transmission of one SRC symbol. The negative effect of violating the principle of logical dependence and physical independence is illustrated in Fig. 5.

The upper plot in Fig. 5 shows a typical edge effect: in the packet of the previous symbol, the last time channel was active, and in the packet of the next symbol – the first. The presence of interpulse interference is obvious. But at the same time, it is possible that both frequency and polarization channels will also be in the same or similar states. That is, the phenomenon of inter-channel interference can also be observed [16]. At the same time, the failure of the transmission system will occur immediately on the CRC of a large part of the modules.

A simple way to eliminate intersymbol interference is shown in the lower plot in Fig. 5: a protective interval equal to the Nyquist interval is organized between the code parcels of individual SRC symbols. In this case, the duration is 10 ps.

It is obvious that the inclusion of protection intervals reduces the speed of transmission of SRC alphabet characters. For certainty, it was assumed that the Nyquist interval is divided by an integer k subacts: $T=k \cdot t_1$. Then it follows from expression (6) that the estimate of the relative slowing down of symbol transmission over time T_{RNS}^* , taking into account the protection interval, is:

$$dT = \frac{T_{RNS}^*}{T_{RNS}} = \frac{k \cdot t_1 + M_t \cdot t_1}{M_t \cdot t_1} = 1 + \frac{k}{M_t}. \tag{7}$$

It follows from dependence (7):

- in the considered example, $dT=2$, i. e., the inclusion of a protective interval reduces the symbol transmission rate by half;
- a reduction in the k/M_t ratio is effective; hence, the rule follows: if the

modules of SRC (1) are located in the order of growth $M_1 < M_2 \dots < M_N$, then $M_N \equiv M_t$, i. e., the values of the residuals for the largest module should be transferred in the temporary CRC;

– parameters of the synchronization system and its reliability are the most important for the functioning of the CRC system.

The last conclusion applies in general to all TSS transmission systems.

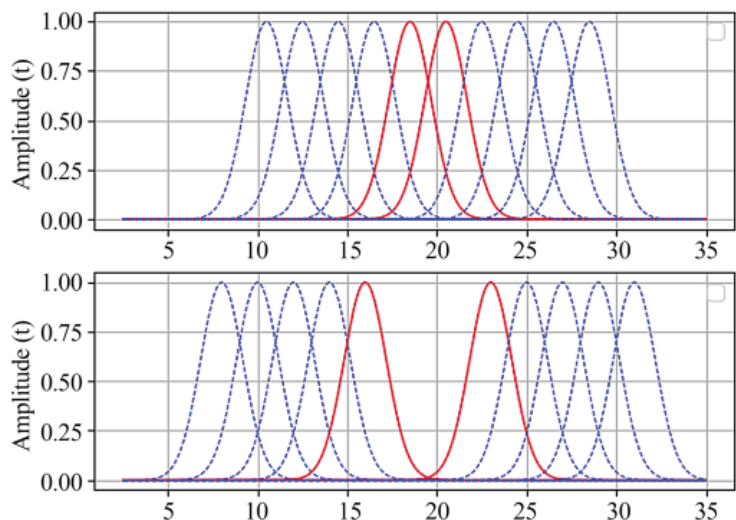


Fig. 5. Edge effects of intersymbol interference

5. 4. Evaluation of the potential transmission speed when using the multiplexing technique of data transmission in residual class systems

Modern DWDM backbone multiplexers organize up to 96 transmission channels with a bandwidth of up to 10 Gbit/s in each channel. That is, the total transmission rate BR (Bit Rate) is up to 0.96 Tbit/s along one optical fiber in one direction. At the same time, the total DWDM frequency band has a width of approximately 200 THz [12].

The studies consider an example of calculations of the proposed technique of transmission of SRC codes, where the parameter values either correspond to the current state of technology development or can be achieved in the near future. The values of the parameters of conditional CRC system are given in Table 4.

According to the data in Table 4, the capacity of the alphabet will be $V(RNS)=3 \times 4 \times 11 \times 13 \times 23=39468$ different symbols. Accordingly, taking into account the intersymbol protection intervals, the following amount of information will be transmitted in a time period of 33 ps:

$$I = \log_2 V(RNS) \approx 15.27 \text{ bit.}$$

Then, for one CRC system, the transmission speed will be:

$$BR_{KKZ} = I / (T + t_1) \approx 0.4627 \text{ Tbit / s.} \tag{8}$$

Table 4

An example of the technical implementation of CRC system for $RNS=[3,4,11,13,23]$

SRC module	Multiplexing technique	Parameter
$M_f=3$	Spatial	3 fibers
$M_p=4$	Polarizing	4 polarization axes
$M_w=11$	Frequency	11 carriers in increments $\omega_1=3$ GHz
$M_c=13$	Spatial	13 cores in each fiber
$M_t=23$	Temporal	23 time channels with step $t_1=1$ ps, protective intercharacter interval $T=10$ ps

The numerical value in expression (8) may seem fanciful. However, the conditions of comparison with the value of 10 GHz per one frequency channel are not completely correct: Table 4 provides for the use of 3 fibers with 13 cores in each. Therefore, the relative transmission speed per fiber and per core is:

$$BR^* = \frac{BR_{KKZ}}{M_f \cdot M_c} \approx 0.0119 \text{ Tbit/s} = 11.9 \text{ Gbit/s.} \tag{9}$$

The resulting value in expression (9) does not take into account that between the frequency bands with a width of 33 GHz, it is also necessary to organize protective intervals with a width of 100 GHz. Therefore, to obtain comparable estimates, the obtained value must be divided by 1.33. Then, compared with existing DWDM systems, based on a conventional channel with a width of 100 GHz, $BR^* \approx 8.95$ Gbit/s. That is, the achievable transmission speed in the proposed CRC system is only 10 % less than what modern DWDM systems can theoretically provide.

Thus, in the case of parametric characteristics of CRC system according to Table 4, the proposed technique will make it possible in the 200 THz band to transmit data with a bit rate of approximately 16 Tbit/s, which can be consid-

ered acceptable not only for existing but also for prospective transmission systems.

6. Discussion of results of investigating the multiplexing technique of data transmission in residual class systems

Our solutions partially solve the problematic issues of research.

The effectiveness of the application of the principle of multiplexing independence is shown, which consists in the fact that the fault tolerance of transmission systems is increased. This provision is confirmed by real examples of the functioning of transmission systems (Table 1), where emergencies may occur. At the same time, the capacity of the alphabet of symbols transmitted in clock intervals is limited only by the number of CRC and the number of subchannels of residual values. For an example of the option in Table 1 capacity of the alphabet will consist of 210 different characters.

The following proposed principle of logical dependence and physical independence is illustrated in Fig. 1, 2. The application of this principle makes it possible to approximate the transmission speed in CRC systems to the parameters of existing transmission systems. It is also shown that data transmission in a separate residual channel leads to significant information redundancy (Table 2). The application of this principle makes it possible to increase the fault tolerance of transmission systems since the emergency state of one of the residual transmission channels leads only to a decrease in the transmission speed but does not lead to the failure of the entire system as a whole.

The analysis of possible intersymbol interference in CRC systems makes it possible to determine the parameters of intersymbol protection intervals in time space according to formula (7), which was also derived as a result of our research. It is shown that the duration of the protective intervals should be equal to the duration of the Nyquist interval.

The analysis of the bandwidth of CRC systems makes it possible to conclude that the transmission speed may decrease slightly compared to the existing DWDM systems [12]. On the basis of data on the characteristics of modern DWDM systems, which allow data to be transmitted at the highest speed when using known coding techniques and the potentially achievable characteristics of promising transmission systems, it is established that the data transmission speed of the proposed technique in a frequency band with a width of 200 THz can reach 16 Tbit/s, which can be considered very close to the theoretically achievable value. However, the advantages of the proposed technique of transmission are the presence of information redundancy [9, 10], which can be used to increase the cryptographic protection of transmission systems.

Thus, this paper partially covers the problematic issues of extending the theory of timer signal structures to the case of multifactor multiplexing.

It is clear that in order to solve practical tasks using the proposed technique of transmission, it is necessary to take into account the specific parametric characteristics of the systems being designed. This is what imposes restrictions on the conditions of applicability of the considered technique. The strictest restrictions, as follows from formula (7), are imposed on CRC in time space. Currently, for a frequency grid of 100 GHz with a pulse duration of approximately 10 ps, it is already possible to use synchronization systems

with clock pulses with a duration of up to 2 ps. Accordingly, when the step of the frequency grid is reduced to 50 or 25 GHz, these restrictions will be less strict – 4 and 8 ps, respectively. The number of CRC for other techniques of multiplexing is limited only by the capabilities of the technological processes of equipment production and cable data transmission infrastructure.

Certain shortcomings of the study are that the work does not consider the effects of various physical phenomena on the range and speed of transmission. Among such phenomena, dispersion and nonlinear distortions of optical signals should be considered first of all. However, for short distances, our solutions can be considered quite correct.

The development of this research will involve solving a number of problems. In particular, the unsolved problem is the synthesis of such SRC alphabets, the number of elements of which is very close to the power of 2. Such alphabets will make it possible to use both the advantages of the proposed technique of transmission and to minimize information redundancy. Another meaningful task is the development of crypto-protection algorithms using the proposed technique of transmitting SRC codes.

7. Conclusions

1. The principle of independence of multiplexing of SRC codes makes it possible to transmit values of module residues in different ways. This provides increased fault tolerance of fiber optic transmission systems.

2. The principle of logical dependence and physical independence makes it possible to consolidate transmission channels in different spaces. Compared to the duration of pulses according to Nyquist-Kotelnikov theorems, this principle makes it possible to transmit pulses 8–16 times more often.

3. The determined parameters of protective intersymbol intervals make it possible to calculate the characteristics of

promising transmission systems. In the time space, the duration of the protective intervals is equal to the duration of the Nyquist pulses for the corresponding width of transmission channel in the frequency space.

4. The achievable speed of SRC code transmission using the proposed technique is slightly lower (about 10 %) than theoretically achievable in existing transmission systems. Studies show that the achievable transmission speed with the proposed technique can be up to 16 Tbit/s in the total frequency band with a width of 200 THz. At the same time, a significant advantage of the proposed technique is the presence of certain information redundancy, which can be effectively applied using cryptographic methods of information protection.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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