UDC 0004.652
DOI: 10.15587/1729-4061.2023.292850

MODIFICATION OF CHAOTIC INTERLEAVER FOR TURBO CODES WITH A CHANGE TO THE DUFFING EQUATION AND ACCOUNTING FOR THE DISTANCE SPECTRUM OF THE CODE

Vladislav Topalov
Corresponding author
PhD, Associate Professor*
E-mail: logosvlad@gmail.com
Iryna Tregubova
PhD, Associate Professor, Head of Department*
Mykola Severyn
Senior Lecturer*
Iryna Hurklis
Senior Lecturer*
*Department of Computer Science
State University of Intelligent Technologies and Telecommunications
Kuznechna str., 1, Odesa, Ukraine, 65023

Various types of interleavers in turbo codes and their parameters affecting the efficiency of turbo codes are considered. It is noted that the type of interleaver directly affects the efficiency and error correcting of turbo codes. Also, the efficiency of turbo codes is influenced by the parameters of the minimum distance, the length of interleaver, and the distance spectrum of the code.

A modification of the chaotic interleaver of turbo codes is proposed with the change of the equation to the Duffing and with examining the code's distance spectrum with the condition of increasing the code's distance between the code words with a small weight. The algorithm for modifying the chaotic interleaver with the Duffing equation and with examining the code's distance spectrum of turbo codes is presented. The characteristics of the modified chaotic interleaver with the Duffing equation and with examining the code's distance spectrum of turbo codes according to various parameters of turbo codes are given. This modification of the interleaver of turbo codes increased the minimum distance between elements for different lengths of the interleaver and polynomials of the turbo code by 10 % ...33 %. Given this, there was an increase in the energy efficiency of the turbo codes by 0,05, ..., 0,25 dB in comparison with a chaotic interleaver without modification at the same value of the bit error probability. When increasing the length of the modified chaotic interleaver with the Duffing equation and applying distance spectrum of the code the increasing the energy efficiency of the turbo code slows down compared to the chaotic interleaver without modification.

The application scope of the modified chaotic interleaving with the Duffing equation and with examining the code's distance spectrum of turbo codes is the infocommunication channels for mobile, wired, and satellite communications

Keywords: chaotic interleaver modification, Duffing equation, Turbo codes, weight distribution, energy efficiency

Received date 08.09.2023
Accepted date 29.11.2023
Published date 21.12.2023

How to Cite: *Topalov, V., Tregubova, I., Severyn, M., Hurklis, I. (2023). Modification of chaotic interleaver for turbo codes with a change to the Duffing equation and accounting for the distance spectrum of the code. Eastern-European Journal of Enterprise Technologies, 6 (9 (126)), 32–38. doi: https://doi.org/10.15587/1729-4061.2023.292850*

1. Introduction

The main problem in information communication networks is the transmission of information without errors. This issue is solved by using integrated measures:

– increasing the power of the transmitter;

– using interference-resistant codes;

– improving the characteristics of communication channels;

– using interleavers close to the random distribution of elements;

– improving the characteristics of the reception equipment;

– reservation of communication channels;

– re-transmission of those elements that were accepted by mistake;

– other.

Turbo codes are one of the new error correcting codes that use an interleaver in their composition to obtain a random distribution of elements.

Turbo codes were presented for the first time in [1] by a group of French scientists in 1993 as a parallel connection of recursive systematic convolutional codes. Turbo codes made it possible to approach the Shannon limit by a distance of 1 dB owing to the near-random formation of code combinations. The scheme of the cascade connection of recursive systematic convolutional codes (RSCC) in the turbo code, which was proposed by a group of French scientists [1], presents a turbo code with a speed of R=1/2 and a polynomial G=15/13, which was then approved in the third generation CDMA2000/LTE standard [2]. But as it was noted in works [3, 4], when recursive systematic convolutional codes are connected in parallel in the turbo code, the phenomenon of the error floor (error floor) occurs in turbo codes – at values of the probability of a bit error from 10^{-5} to 10^{-6} the decrease in the probability of an error is slowed down. As further studies have shown, the phenomenon of the error threshold is affected by the type and parameters of the interleaver [4]. Also, as shown in [4], the design of the interleaver and the type of iterative algorithm for decoding turbo codes make it possible to increase the energy efficiency of the turbo code, as well as to reduce the error threshold phenomenon.

A turbo code interleaver is a device that rearranges symbols within a block according to a certain law. The law can act as a tabular, matrix, or mathematical description of the construction of an interleaver. Owing to the interleaver with a long length in the turbo code, this code can be characterized as a code with a random structure of code combinations, and according to Shannon's theorem, such codes make it possible to approach the maximum bandwidth of the channel.

The search for the most successful interleavers for various parameters of the turbo code and iterative decoding algorithms, which made it possible to increase the energy efficiency of the turbo code and reduce the computational complexity, continues. Therefore, studies that improve the characteristics of the interleaver, and thereby the Turbo code, are relevant.

2. Literature review and problem statement

In work [1], a block interleaver was used. The block interleaver performs operations on the input sequence of elements, which is written into the matrix in rows, and read in columns [5]. With such interleaving, the maximum distance $d_{\max}$ between information elements is found using the expression $d_{\max}=(m\cdot n-1)$ and the maximum delay in the formation of the output sequence – $D=2(m\cdot n-1)$. The block interleaver has also become widely used in wired communication standards. For example, in ADSL (Asymmetric Digital Subscriber Line – ITU-T G.992.1, G.992.2) and ATM (Asynchronous Transfer Mode) protocols, the use of a block interleaver allows the use of less redundant code for error correction. But the disadvantages of the interleaver: the sequence after the standard algorithm of the block interleaver is difficult to make random and the length of the block interleaver in practice does not exceed 1024 bits. Some of the shortcomings were solved by a diagonal interleaver.

With a diagonal interleaver, the output sequence is also written in lines, but read diagonally [5], its sequence is more random than that of a block sequence. The maximum delay in such an interleaver is similar to the delay in a block interleaver and is determined by $D=2(m\cdot n-1)$. Diagonal interleavers are used in some cellular communication standards. For example, a diagonal interleaver (8×57) bits is used in cellular communication according to the GSM standard. But the formation of elements is still far from random, so the use of combined interleavers was proposed [2] – block-cyclic interleavers. In the LTE standards [2], a block-cyclic interleaver with a duration of 40–5114 bits was used, and in CDMA2000: 250–4090 bits.

In a block-cyclic interleaver, the input sequence i:

– is written in lines;

– cyclic shifting of elements in rows is carried out;

– columns can also be cyclically shifted;

– then the output sequence is read column by column.

Operations of cyclic shifts in the rows of a block interleaver make it possible to get an output sequence closer to the random distribution of elements compared to a block interleaver.

The main delay in the formation of the output sequence in the block-cyclic interleaver is the sum of the delays during filling into the matrix: delays during the cyclic shift of each row, the cyclic shift of columns, and the reading of elements from the matrix. The efficiency of using this interleaver as part of a cascade construction of a turbo code is higher than in block or cyclic interleavers due to the formation of an output sequence that is closer to a random distribution. But random interleavers have a better sequence of elements. They form an initial sequence of elements close to a random distribution using mathematical laws that describe pseudo-random sequences or using pseudo-random generators.

The most common random interleavers are:

– pseudo-random interleaver;

– modified S-type pseudo-random interleaver;

– chaotic interleaver.

A pseudo-random interleaver permutes elements according to a random law [6, 7]. The random law is given by means of random or pseudo-random generators with a repetition period approaching infinity. The main steps in forming a permutation sequence:

Step 1. The value i_1 is randomly selected in the range of the interleaver size $A\in 1..L$ according to the random distribution law with probability $p(i_1)=1/L$.

Step 2. Further values of the permutation i_n $(n=2..L)$ in the range A are $1..L$ are chosen randomly so that the current value $i\in A$, $i_n\neq i_1, i_2,..., i_{n-1}$ does not coincide with any as previously recorded. The generator generates the value of i_n according to a random law with the probability $p(i_n)=1/(L-i+1)$.

At the same time, a check is performed for the uniqueness of the element.

Step 3. When $i=L$, the last value for permutations in the interleaver $\pi(L)$ will be generated.

Estimating the maximum delay in generating the output sequence for a random interleaver has certain difficulties since the implementation of the random number generator and its possible distribution is random. But on average, when implementing a sequence of permutations by static recording in memory, the time of permuting an interleaver of length L is determined by the time t_d of writing an element into the original sequence according to its position multiplied by the size of the interleaver. Some elements of the sequence after a random permutation may end up in the same positions as before the interleaving. That is, the minimum possible distance between the elements can be equal to 1, which, of course, has a significant negative effect on the correcting ability of the turbo code.

To eliminate this shortcoming, in [6] it was proposed to introduce a condition for checking the value of the distance between elements in the original sequence with a given value of S. This type of interleaver was called pseudo-random S-type. With an S-type pseudo-random interleaver, two sequentially input elements (i, i+1) will be separated by a distance of at least S. At the same time, S was initially chosen to be smaller or equal to $\left\lfloor \sqrt{L/2} \right\rfloor$, because at large values, the complexity of finding the permutation sequence became significant.

One of the characteristics of a permutation that evaluates its effectiveness is the minimum element permutation distance, which is the minimum distance between the input element i and the output element after the interleaver $\pi(i)$. In the process of analyzing the code sequences after the S-type interleaver in work [8] it was noted that information sequences of low weight $w\epsilon 1, 2, 3$ generate code words also with low weight w_{tc}, which reduce the correcting ability of the code. Therefore, modifications of the S-type interleaver were considered in other works. For example, in [9], a modification of the condition for the formation of a permutation element for an interleaver with code matching was proposed using the method of excluding codewords with low weight, which made it possible to increase the value of the minimum distance with the same parameter S. That, in general, makes it possible to increase the efficiency of the turbo code itself, in which this interleaver is used.

Further search for more random sequences for turbo codes drew attention to the chaotic distribution law, as in [10]. In the work, the random distribution law is based on the Lozi equation. But there are many mathematical works that indicate that, under certain circumstances, the Duffing equation better forms a random sequence [11]. To date, very few works have studied interleavers with the Duffing equation as part of turbo codes. However, in these works, the distance spectrum of the code is not analyzed, and in the algorithms for the formation of the interleaver, the distribution of the distance spectrum does not affect the formation of the elements in any way. All this allows us to state that it is advisable to conduct a study of the modification of the chaotic interleaver with the change of the equation to the Duffing equation and taking into account the distance spectrum of the code with the condition of increasing the distance between the code words with low weight.

3. The aim and objectives of the study

The purpose of this study is to develop a modification of the chaotic interleaver with a change to the Duffing equation in the composition of turbo codes and taking into account the distance spectrum of the code with the condition of increasing the distance between code words with low weight. This will make it possible to better approach the random formation of elements than in a chaotic interleaver with the Lozi equation. That, in the practical part, will increase the immunity and efficiency of the Turbo code.

To achieve the goal, the following tasks were set:

– to choose the Duffing equation, which gives a random sequence better than the Lozi equation in a standard chaotic interleaver;

– to take into account the distance spectrum of the code and form a condition that will increase the distance between code words with a small weight after the modified chaotic interleaver.

4. The study materials and methods

The object of our study is the modification of a chaotic interleaver with one of the Duffing equations as part of Turbo codes, taking into account the distance spectrum of the code.

The research hypothesis assumes that the development of a modification of the chaotic interleaver as part of Turbo codes with a change to the Duffing equation and taking into account the distance spectrum of the code could give a more random sequence of elements than without modification.

Accepted assumptions and simplifications – only Duffing equations are considered with the limitation of the interleaver length to 8192 bits in the Turbo codes.

Research methods: methods of coding theory (for building, analyzing, comparing, determining the effectiveness of codes), statistical analysis using methods of probability theory, mathematical statistics, mathematical modeling methods (for modeling processes in information communication systems).

Mathematical modeling was carried out on the basis of the developed software for simulating information communication channels, encoders, Turbo code decoders and various decoding algorithms and was verified in the MATLAB software environment. The simulation results were chosen as the worst performance for 5 identical attempts. This makes it possible to obtain the most realistic value of the characteristics.

5. Results of investigating the modification of the chaotic interleaver according to the Duffing equation as part of turbo codes

5. 1. Definition of the Duffing equation, which gives a random sequence better than the Lozi equation in a chaotic interleaver

The chaotic interleaver is based on inequalities that use the Lozi equation:

$$\begin{aligned} &x(i)=f\left(x(i-1)\right),\\ &x(i)=1-a\left|x(i-1)\right|+y(i-1),\\ &y(i)=b\cdot x(i-1), \end{aligned} \tag{1}$$

where i is the element before the interleaver.

In the general case, the coefficients are chosen with the following values: a=1.7, b=0.5.

The basic algorithm reported in [10] involves the following steps:

Step 1: the entire sequence $\{i\}_{i=1}^{L}\in Z$ is related to the correspondence of the numerical sequence $\{x_i\}_{i=1}^{L}\in R$ according to inequalities (1).

Step 2: permutation $\{s(i)\}_{i=1}^{L}\in Z$ is carried out in the sequence $\{x(i)\}_{in=1}^{L}$, where $s(i)$ is the position of the index $x(i)$ due to sorting by increasing value.

Step 3: The chaotic interleaver indices will be defined as:

$$\pi\left(\{s(i)\}_{n=1}^{L}\right)=\{i\}_{n=1}^{L}.$$

We proposed using one of the Duffing equations in the chaotic interleaver instead of the Lozi equations, which also gives the chaotic distribution dynamics. Analyzing works with a chaotic interleaver based on the Duffing equations [11] and considering many other Duffing equations, several Duffing equations as part of turbo codes were considered and their characteristics were analyzed. They made it possible to obtain different minimum distances between elements. The best distribution is close to random and the maximum value of the minimum distance between elements was achieved by the equation:

$$\begin{aligned} &x_i=f\left(x_{i-1}\right),\\ &x_i=a\cdot x_{i-1}^2/2+y_{i-1},\\ &y_i=b\cdot x_{i-1}^4/4. \end{aligned} \tag{2}$$

The value of the minimum distance between the elements according to this equation is given in Table 1 and compared to the Lozi equation.

We chose the coefficients with the following values: a=2.4, b=0.6.

In order to achieve the best value of the minimum distance between the elements, it is necessary to form the first condition for the fulfillment of the minimum distance below which the distance between the elements cannot be. We suggested that, as in S-type interleavers, the first condition of formation should be chosen according to:

$$S_{\min} = \sqrt{L/2},$$

$$D_{\min} \geq S_{\min} / 2. \quad (3)$$

The testing of the results was carried out by methods of generating elements according to the given algorithm and comparing them with other interleavers in the composition of turbo codes.

Table 1 gives the distance spectra of turbo codes with polynomials G=37/21, represented in octet form of recording, at the speed of the turbo code R_{tc}=1/2 for a standard S-type interleaver with the interleaver length L=1024 bits at the value of S=18. Also, Table 1 shows the distance spectra of the chaotic interleaver according to the Lozi equation with parameters a=1.7, b=0.5 and the modified chaotic interleaver according to the Duffing equation, the interleaver with a=2.4, b=0.6 and $S_{\min}$=18. In Table 1, the results of the modified chaotic interleaver according to the Duffing equation are given both with and without the condition of the maximum distance of code words with low weight, based on the consideration of the distance spectrum. The values of the minimum code distance of the turbo code ($D_{\min}$) are given, which are calculated according to work [12]. At the same time, the calculation of the minimum code distance $D_{\min}$ is carried out on the basis of all code words with weights of the information sequence

From the data in Table 1, it can be seen that the selected Duffing equation makes it possible to improve the value of the minimum code distance to 25 compared to the Lozi equation, where the minimum code distance is 24. The minimum code distance for the modified interleaver and the interleaver without modification with other turbo-code polynomials is given in Table 2.

5. 2. Taking into account the distance spectrum of the code and forming the condition for increasing the distance between code words with low weight

As shown in [9], to improve the characteristics of turbo codes, it is necessary to minimize the distance spectrum of code words with low weight. That is, it is necessary to form conditions that will make it possible to acquire the maximum values of the distance between code words with low weight. This will contribute to increasing the immunity of the code. The distance spectrum of the turbo code shows how many codewords $A_{w_{TK}}$ of weight w_{TK} are present in the output sequence after the decoder. The distance spectrum makes it possible to estimate the number and influence of code words of different weights on the correcting ability of the turbo code.

Taking into account all of the above, in the general case, the modified algorithm of the chaotic interleaver according to the Duffing equation and the minimization of the distance spectrum of codewords with low weight can be formulated in the following form:

Step 1. The length of the interleaver L is set.

Step 2. The array π(i) with dimension L is initialized, which will be filled with the ordinal numbers of the permutation x_i, iє1..L.

Step 3. The serial number of the permutation i is set.

Step 4. Form a chaotic sequence using the Duffing equation (2) and obtain two sequences x_i and y_i.

Step 5. Sort the sequence of x_i by increasing values.

Step 6. Change the serial numbers to a number from the sequence x_i.

Step 7. Check for the criterion of the minimum distance between the elements using equations (3).

Step 8. When fulfilling the criterion, we write this number from x_i into the array π(i).

Step 9. If all elements of the sequence L satisfy step 7, then all elements of the interleaver π(i) have been formed, if not, then you need to return to step 4 and repeat the operations.

Step 10. Check the conditions for the exclusion of information sequences of low-weight code words with a short distance:

a) $\left|\pi(i_3)-\pi(i_2)\right| \bmod \mu \neq 0,$ when input elements:

$$\left|(i_3)-(i_2)\right| \bmod \mu \neq 0 \text{ and}$$

$$k_3+k_2<\left(d_{\max}^2-6\right)\left(z_{\min}-2\right),$$

where μ is the minimum distance between units in the input se-

Table 1

Minimum code distance and distance spectra of the turbo code for different interleavers

Interleaver type	Polynomial G	Weight w	Number of code words A_w of weight w											$D_{\min}$
Pseudo-random S-type	37/21	–	24	25	27	27	28	29	30	31	32	33	34	24
		w=1	0	0	0	0	0	0	0	0	0	0	0	
		w=2	28	0	0	32	0	0	44	0	0	96	0	
		w=3	3	0	0	0	0	0	0	0	0	0	0	
		w=4	52	0	0	184	0	0	304	12	12	672	0	
Chaotic interleaver with the Lozi equation	37/21	–	24	25	26	27	28	29	30	31	32	33	34	24
		w=1	0	0	0	0	1	0	0		0	0	0	
		w=2	24	0	0	19	0	12	35	97	24	0	0	
		w=3	56	0	0	21	0	0	0	27	0	0	0	
		w=4	43	0	2	0	9	0	32	0	43	0	0	
Modified chaotic interleaver with no code word validation condition	37/21	–	25	26	27	28	29	30	31	32	33	34	35	25
		w=1	0	0	0	0	0	1	0	0	0	0	0	
		w=2	87	8	0	28	0	6	14	9	0	0	0	
		w=3	4	27	0	32	5	0	0	18	0	29	0	
		w=4	71	0	12	34	0	0	32	0	42	0	0	
Modified chaotic interleaver with the condition of maximum distance of code words with low weight	37/21	–	29	30	31	32	33	34	35	36	37	38	39	29
		w=1	0	0	9	0	0	32	0	15	0	21	0	
		w=2	0	12	0	78	0	0	1	0	12	7	2	
		w=3	36	35	21	4	46	0	0	52	0	0	47	
		w=4	109	4	0	0	25	8	62	0	22	0	0	

quence; k – the element after the interleaver $\pi(i)$, $z_{\min}$ – the minimum weight of the verification elements; $d^{w}_{\max}$ is the maximum weight of the code word formed by the input sequence of weight w;

b) $\left|\pi(i_3)-\pi(i_1)\right| \bmod \mu \neq 0,$ when input elements:

$\left|\left|(i_3)-(i_1)\right|\right| \bmod \mu \neq 0$ and

$k_1+k_2+k_3+k_4<\left(d^{4}_{\max}-12\right)\left(z_{\min}-2\right);$

c) $s>L_s$, where the value of L_s is chosen equal to the weight w of the input sequence, which generates a single error with the weight $<d^{w}_{\max}$.

Step 11. If the condition of point 10 is not met, return to point 4.

Step 12. The value of x_i is written into the array $\pi(i)$. Increase the serial number of the permutation by one ($i=i+1$). Check for the condition of not exceeding the dimensions of the array $i\leq L$. If the condition is met, proceed to the generation of a new permutation element in point 6.

Step 13. The array of permutations $\pi(i)$ is formed.

In Table 1, the results show that the new condition for the maximum distance of low-weight codewords proposed in the algorithm significantly improved the minimum distance from 25 to 29 for low-weight codewords w=1..4.

In order to evaluate how the modification affects other turbo codes with different polynomials and parameters, simulations and calculations were carried out similarly. Turbo codes are selected with different polynomials G, code limit k, code speed R_{tc}=1/2, according to different permutation lengths L=256 ...8192 bits. The results are given in Table 2.

Table 2

Characteristics of turbo codes using a chaotic interleaver and a modified chaotic interleaver with the condition of the maximum distance of code words with a small weight, length L=256, ..., 8192 bits and information sequences with weight $w_i\varepsilon$1..4

k	G	$D_{\min}$ chaotic interleaver with Lozi equation L=256/1024/8192 bits	$D_{\min}$ modified chaotic interleaver with Duffing equation and the condition of maximum distance of code words with low weight L=256/1024/8192 bits
3	7/5	12/19/21	15/22/24
3	5/7	13/18/22	16/23/25
4	17/13	15/23/29	19/29/34
4	17/11	16/24/30	18/29/34
5	37/23	20/26/36	24/31/39
5	37/21	21/25/37	24/29/41
5	35/23	22/26/40	26/32/42
6	75/53	23/33/41	27/37/45
6	67/45	25/34/41	27/37/46
6	53/75	28/37/42	30/40/46
7	131/173	31/39/46	33/43/49

Data in Table 2 show the increase in the value of the minimum distance when applied as part of a chaotic interleaver turbo code with a new formation condition for all considered turbo codes compared to the values of a chaotic interleaver without modification. But with a small value of the code limit, the minimum distance increases by 33 %, while when the value of the code limit of turbo codes increases, the minimum distance increases only up to 10 %. That is, the best gain will be for turbo codes with a small value of the code limit.

To evaluate the interference resistance of the turbo code, the dependences of the bit error probability value after decoding the turbo code with a modified chaotic interleaver according to the Duffing equation and without modification were considered.

Evaluation of the energy efficiency of the code (EEC) of the turbo code with a modified chaotic interleaver is carried out by Monte Carlo simulation [4]. Modeling the FM-2 communication channel with Gaussian error distribution and calculating the probability of a bit error (P_b) of decoding on the received side when the value of the signal-to-noise ratio (E_b/N_o) changes. With FM-2 signal, one bit of information is transmitted, the signal energy is equal to the bit energy ($E=E_b$) and the signal-to-noise ratio can be considered as the ratio of bit energy to noise energy (E_b/N_o). Figure 1 shows the dependences of the bit error probability for the turbo code with the polynomial G=37/21 for the interleaver length L=1024 and L=4096 bits and different types of interleavers. Turbo code speed R=0.66 and the Log-MAP iterative decoding algorithm with the number of iterations equal to 5. Diagram 1 shows the dependence of the bit error probability for the turbo code with the polynomial G=37/21, with a chaotic interleaver with the Lozi equation by the value of S=18 and the length of the interleaver L=1024 bits. Diagram 2 shows the dependence of the probability of a bit error for a turbo code with a polynomial G=37/21, with an S-type interleaver with a value of S=18 and an interleaver length of L=1024 bits. Diagram 3 shows the dependence of the bit error probability for the turbo code with the polynomial G=37/21, with a modified chaotic interleaver with the Duffing equation for S=18 and the interleaver length L=1024 bits. Diagram 4 shows the dependence of the probability of a bit error for the turbo code with the polynomial G=37/21, with a chaotic interleaver with the Lozi equation for S=18 and the interleaver length L=4096 bits. Diagram 5 shows the dependence of the probability of a bit error for a turbo code with a polynomial G=37/21, with an S-type interleaver with a value of S=18 and an interleaver length of L=4096 bits. Diagram 6 shows the dependence of the bit error probability for the turbo code with the polynomial G=37/21, with a modified chaotic interleaver with the Duffing equation for S=18 and the interleaver length L=4096 bits.

From the given dependences shown in Fig. 1, one can see:

– application in the modification of the chaotic interleaver with a change to the Duffing equation and the condition of the maximum distance of code words with a small weight compared to the standard chaotic interleaver based on the Lozi equation for a given probability of a bit error makes it possible to increase the value of the signal-to-noise ratio by 0.3, ..., 0.5 dB for length L=1024 bits;

– if the length of the interleaver is already L=4096 bits, the gain in the modification of the chaotic interleaver is already smaller and ranges from 0.05, ..., 0.25 dB;

– it can also be seen that the modified chaotic interleaver according to the Duffing equation as part of the turbo code makes it possible to acquire better EVC characteristics compared to the standard S-type interleaver and the chaotic interleaver.

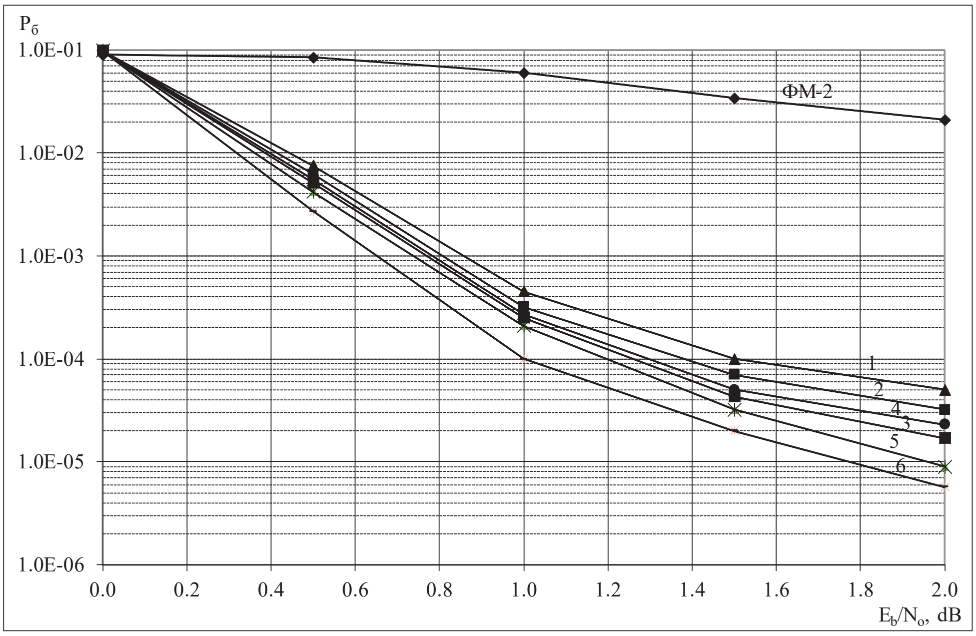

Fig. 1. Dependence of the bit error probability at the output of the turbo decoder on the signal-to-noise ratios and the type of interleaver

6. Discussion of results of investigating the modification of the chaotic interleaver according to the Duffing equation as part of turbo codes

Our research results are explained by the values in Tables 1, 2, showing the increase in the minimum distance of the modified chaotic interleaver with the Duffing equation and taking into account the distance spectrum of the code due to the one we proposed compared to Duffing with the interleaver without modification. Also, when applying distance spectrum analysis of the code and due to the condition of choosing a larger distance between elements, permutations according to low-weight codewords, the minimum distance increased from 10 % to 33 % compared to chaotic interleavers without modification. This increase in the minimum distance made it possible to increase the energy efficiency of turbo codes from 0.05 to 0.5 dB compared to chaotic interleavers without modification, which is shown in Fig. 1. The practical application of the modified chaotic interleaver with the Duffing equation and taking into account the distance spectrum of the code as part of the turbo code is possible in information communication channels of mobile, wired, and satellite communication to improve immunity.

The peculiarity of the proposed modification is not only in the selection of a more successful Duffing equation, which gives a more random distribution than the Lozi equation, for example, as in [11], as well as in the application of the analysis of the distance spectrum of the code and the condition of choosing a greater distance between elements by permutations by code words with a small by weight.

A limitation of this study is that only the group of Duffing equations with coefficients a=2.4, b=0.6 was considered; other equations can give a more random sequence.

The disadvantages of this study are:

– only one decoding algorithm of LogMAP turbo codes was used;

– interleaver lengths up to 8192 bits were considered.

Advancing this research is the search for equations with a law closer to random than the selected Duffing equation. In addition, the development of the research is the analysis of the interleaver for large values of the sequence length.

7. Conclusions

1. The Duffing equation was applied in the modified chaotic interleaver, which made it possible to increase the value of the minimum distance between elements compared to the interleaver according to the Lozi equation by up to 10 % as part of the turbo code.

2. Determining the distance spectrum of the code and taking into account the condition of increasing the distance between code words with a small weight in the modification of the chaotic interleaver with a change to the Duffing equation made it possible to increase the minimum distance to 33 % for small values of the code limit. When the code limit of turbo codes increases, the minimum distance increases by no less than 10 %. The application of a modified chaotic interleaver with a change to the Duffing equation and taking into account the distance spectrum of the code for a given probability of a bit error made it possible to increase the value of the signal-to-noise ratio by 0.05, ..., 0.5 dB compared to turbo codes with an interleaver without modification. This characterizes the improvement of the energy efficiency of turbo codes when applying the modification chaotic interleaver according to the Duffing equation and taking into account the distance spectrum of the code.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

References

1. Berrou, C., Glavieux, A., Thitimajshima, P. (1993). Near Shannon limit error-correcting coding and decoding: Turbo-codes. Proceedings of ICC '93 - IEEE International Conference on Communications. doi: https://doi.org/10.1109/icc.1993.397441
2. Mousavi, H., Amiri, I. S., Mostafavi, M. A., Choon, C. Y. (2019). LTE physical layer: Performance analysis and evaluation. Applied Computing and Informatics, 15 (1), 34–44. doi: https://doi.org/10.1016/j.aci.2017.09.008
3. Alebady, W. Y., Hamad, A. A. (2023). Concatenated turbo polar-convolutional codes based on soft cancellation algorithm. Physical Communication, 58, 102010. doi: https://doi.org/10.1016/j.phycom.2023.102010
4. Heegard, C., Wicker, S. B. (1999). Turbo Coding. Springer, 206. doi: https://doi.org/10.1007/978-1-4757-2999-3
5. Andrews, K., Heegard, C., Kozen, D. (1997). A Theory of Interleavers. Available at: https://www.researchgate.net/publication/2264878_A_Theory_of_Interleavers
6. Jinhong Yuan, Vucetic, B., Wen Feng. (1999). Combined turbo codes and interleaver design. IEEE Transactions on Communications, 47 (4), 484–487. doi: https://doi.org/10.1109/26.764913
7. Dyrda, V., Dyrda, O. (2002). Shchodo pobudovy efektyvnykh heneratoriv psevdovypadkovykh chysel. Naukovi pratsi ONAZ im. O.S. Popova, 1, 71–75.
8. Dolinar, S., Divsalar, D. (1995). Weight Distributions for Turbo Codes Using Random and Nonrandom Permutations. TDA Progress Report 42-12, 56–65. Available at: https://www.researchgate.net/publication/243773610_Weight_Distributions_for_Turbo_Codes_Using_Random_and_Nonrandom_Permutations
9. Topalov, V., Zaharchenko, N., Kononovich, V. (2008). Modifikatsiya peremezhitelya s kodovym sootvetstviem. Eastern-European Journal of Enterprise Technologies, 35, 26–30.
10. Sahnoune, A., Berkani, D. (2021). On the performance of chaotic interleaver for turbo codes. SN Applied Sciences, 3 (1). doi: https://doi.org/10.1007/s42452-021-04147-w
11. Urrea, C., Kern, J., López-Escobar, R. (2022). Design of Chaotic Interleaver Based on Duffing Map for Turbo Code. Symmetry, 14 (12), 2529. doi: https://doi.org/10.3390/sym14122529
12. Bazzi, L., Mahdian, M., Spielman, D. A. (2009). The Minimum Distance of Turbo-Like Codes. IEEE Transactions on Information Theory, 55 (1), 6–15. doi: https://doi.org/10.1109/tit.2008.2008114