In this paper, the object of the study is determining air traffic scheduling management by optimizing mathematical models. The problem in this study is that the COVID-19 pandemic has significantly disrupted air traffic, resulting in changes to regulations, travel restrictions and a decrease in passenger demand. One of the problems that must be resolved is how to organize and adapt flight schedules to current conditions by focusing on mathematical models that are used to optimize or increase the efficiency of managing flight schedules or air traffic. Mathematical models can help find ways to optimize the use of available resources, such as airport capacity, flight routes and flight frequency. The results obtained in this research are a mathematical model that specifically takes into account the variables involved in setting air traffic schedules during the COVID-19 pandemic so that capacity limits at airports and airspace are always normal. An optimization model was developed from previous research, namely the model that takes into account ground and air delays as well as the use of alternative paths and avoids deviations from the initial more accurate flight plan, which overall indicates that the maximum time and maximum distance values for each item have been optimized to achieve better values. This research has the novelty of producing a mathematical model using variables, objective functions, capacity limits, flight structure limits and variable domains, which then produces an algorithm with data input processes, determining optimization models, determining variables, determining objective functions, determining problems. The results of this model can be recommended to airlines in scheduling flights during the pandemic

Keywords: flight scheduling management, mathematical models, algorithms, optimization, COVID-19 pandemic

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MATHEMATICAL MODEL FOR OPTIMIZATION IN AIR TRAFFIC SCHEDULING MANAGEMENT DURING THE COVID-19 PANDEMIC

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1. Introduction

The increasing spread of the COVID-19 virus has caused the aviation sector to be most significantly impacted by travel restrictions, requiring process improvements in flight scheduling management [1, 2]. One way to increase the efficiency of flight schedule management processing is to use mathematical model optimization [3, 4]. Mathematical model optimization aims to be a solution to meet conflicting needs [5]. The use of mathematical models is not only for efficiency in scheduling but will be a solution for security, operational efficiency and recovery on the economic side [6]. In addition, mathematical models will play a role in identifying patterns and trends in many scenarios, which will help aviation in providing and making informed decisions [7]. Several previous studies have conducted research related to flight scheduling. The work [8] carried out air traffic management by extracting information using XAI. Another research [9] conducted flight traffic management by looking at workload from the context of shift work features so that it can influence alertness. The research [10] carried out air traffic flight scheduling using a deep learning approach at Turkish airports. The research [11] conducted selecting an algorithm to determine routes for scheduling flight traffic, which produces a model that can maximize scheduling during the pandemic.

In the context of Air Traffic Flow Management (ATFM), which is a system that has a concept in traffic management, which is regulated to carry out the task of optimizing the flow of aircraft traffic with the aim of ensuring the safety and smoothness of flight traffic is guaranteed, the use of optimization in mathematical models has an important role as the main solution to obtain solutions for security, operational efficiency and economic recovery [12]. Mathematical model optimization will be used to analyze and make predictions on air traffic in determining more optimal flight paths. In optimizing the mathematical model, variables such as weather, flight schedules, airport capacity and geographical restrictions will be taken into consideration. The use of techniques and formulas from mathematics such as optimization, simulation and algorithms will produce models that can help find the best solutions to maximize efficiency and meet safety regulations in relevant health protocols [13, 14]. During the pandemic, the use of mathematical models in ATFM will adjust flight schedules to dynamic changes in health protocols. Mathematical models can be used to design flight schedules by decision makers. The integration of mathematical model optimization and Air Traffic Flow Management will provide a better basis for managing adaptive, efficient and safe flight schedules, especially in pandemic situations. Therefore, research on the optimization of mathematical models in scheduling traffic is relevant for use in the aviation industry, which can help decision-making in air traffic management.

2. Literature review and problem statement

The research [15] used stochastic modeling techniques related to the aviation industry. The stochastic model will be applied to non-stationary optimization problems. In solving this problem, an additional layer of complexity will be added to the stochastic model, this model is able to determine decision-making. This model will be developed using big data so that it can develop operational models for air transportation. The use of this model is to address skin problems in a deterministic setting.

The research [16] used a mixed integer linear programming formulation proposed to investigate the trade-off between various performance indicators of interest, in solving the problem of managing real-time aircraft takeoff and landing operations. This paper will discuss safety considerations. Testing was carried out at 2 airports in Europe on landing and takeoff tests. The optimal value is obtained using an integer linear model. In this paper, computational analysis is the selection of solutions that are able to find values with various indicators. However, the use of a mixed integer linear programming formulation has the weakness of calculating complex computational solutions for the many parameters used so complexity often occurs.

The research [17] determined flight routes and schedules using a Simulated Annealing (SA) based heuristic algorithm combined with mixed integer mathematics for routing and scheduling in the aviation industry, which will consider the proposed variable data. The complexity of this problem is determining the next route when passengers change planes to the next destination and there is a problem of uncertainty so that analysis and use of mixed integer mathematics can overcome this problem. However, this research has weaknesses in terms of complicated calculations, especially when discrete form variables are processed into the model, which makes optimal solutions difficult to achieve.

The research [18] developed a new mixed integer mathematical model that will represent energy consumption in machines directly and indirectly. This research has compared with traditional models using the GEP algorithm or gene expression programming, which will produce rules in the delivery function. Integer mathematical models are used to determine the best direction of evolution by optimizing values from historical data. This research carries out analysis by applying multiple attributes to a mathematical model for optimal delivery. However, this research has the weakness of being very sensitive to changes in parameters or constraints, making it difficult to use the model in situations where parameters fluctuate.

The approach [19] uses a linear formula to optimize the scheduling of incoming and outgoing trucks in the distribution of goods, which has the advantage of getting efficient time and the proposed model is effective. However, when using linear formulas, there are shortcomings or weaknesses, such as limitations in non-linear relationships, which affect the accuracy of the model, especially in the context of scheduling and then linearity problems, which complicate the relationship between variables.

The work [20] proposes a mathematical model to be a solution in solving scheduling problems, which utilizes linearization so as to obtain efficient time. This model is also combined with heuristics to produce good efficiency optimization. However, the use of linearized models has disadvantages such as loss of accuracy due to linear approximation of non-linear variables, which can reduce the accuracy of the model, especially in situations where non-linearity has a significant impact.

The work [21] applies a mixed integer linear programming (MILP) formulation, which will be presented and used to make time efficient with sequential parameters. An optimization algorithm will be used to view scheduling in terms of predetermined time and location. However, this research has a weakness, namely it is very sensitive to changes in parameters or constraints, making it difficult to use the model in situations of fluctuating parameters. Based on this research, the use of mathematical models is very necessary to obtain optimal values in scheduling.

The work [22] applies a mathematical model to approach flight schedule recovery with limited resources, the mathematical model used is an adaptation model of the Resource-Constrained Project Scheduling Problem (RCPSP), which collects features from time, management, allocation and routing variables. This model will be tested on busy flights. The results obtained are cost-effective so they can be used on limited resources and are useful for flight scheduling planning. However, the use of the Resource-Constrained Project Scheduling Problem (RCPSP) model has weaknesses, such as the high complexity of the RCPSP problem itself, which is a challenge in developing an adaptation model that is efficient and can produce optimal solutions.

In [23], there is an optimal and branched transportation problem. In optimal branching, there is a distribution with large and small capacities so that they require optimal transportation routes and that mathematically they will be used μ and v on, and a payoff function h the planner wants to minimize $M_{ə}(T) - ⌊_x \, hd(əT)$ to transportation T from μ to v with $\tilde{\mu} \leq \mu$ and $\tilde{v} \leq v$.

In [24], the use of hyperbolic mathematical formulations with Maxwell's equations involves dielectric permittivity, where each variable has a different eigenvalue. With Maxwell's equations, we will be able to overcome the problem of system complexity, which can change according to the input used. However, this research has weaknesses such as the complexity of the hyperbolic model, which can be a challenge, especially when calculations or numerical solutions are required.

The research [25] applies the second-order Painlevé differential equation, which is a solution in a mathematical model that is solved using the I-variation iteration algorithm. In this formulation, we will use additional parameters that will optimize the solution of the scheduling problem from the numerical side. This research concludes that to obtain optimal model values, several mathematical formulations such as the Painlevé differential equation must be used.

3. The aim and objectives of the study

The aim of the study is to develop a mathematical model for scheduling air traffic using optimization techniques. This will possibly be a solution for security, operational efficiency and economic recovery.

To achieve the aim, the following objectives were set:

– to determine the objective function;

– to determine capacity limits;

– to determine flight structure constraints;

– to determine the variable domain.

4. Materials and methods

In the context of Air Traffic Flow Management (ATFM), the use of optimization in mathematical models has an important role as the main solution for obtaining security solutions, operational efficiency and economic recovery. The main hypothesis in this research is air traffic scheduling by utilizing mathematical models. This is based on the fact that the impact of the COVID-19 virus on the aviation industry is quite worrying due to the existence of health protocols that require the implementation of travel restrictions. The problem that will be solved in this research is to optimize the mathematical model and increase efficiency in managing air traffic scheduling. The approach that will be taken in applying optimization to the mathematical model will use variables such as weather, flight schedules, airport capacity and geographical restrictions to be considered. The use of techniques and formulas from mathematics such as optimization, simulation and algorithms will produce models that can help find the best solutions to maximize efficiency and meet safety regulations in related health protocols. In this research, there are limitations such as dependence on certain variables, flexibility in dealing with sudden changes. In this research, there are research subjects such as mathematical models that will be used to develop and optimize traffic scheduling management as well as management processes with setting flight times and routes and traffic scheduling management. In this research, there are research objects such as optimization, which aims to optimize the scheduling process using a mathematical approach, air traffic, which involves aspects such as the number of aircraft, capacity and flight routes. This research will use hardware, namely a Core i3 laptop and software such as Microsoft Word, Anaconda, Jupyter. The limitations in this research when using models outside the context of a pandemic are the need to adjust variables and limitations in terms of the ability to make predictions to overcome gaps and uncertainty regarding changes in air traffic regulations. Apart from that, this research has assumptions as follows:

– the assumption of this research is that the mathematical model used can be applied to air traffic scheduling management. This describes that the model variables and parameters describe the dynamics of the pandemic situation;

– the assumption that the data to be used in implementing the mathematical model is available, such as actual traffic data;

– the assumption that the application of mathematical models can make more optimal air traffic scheduling.

The hypothesis in this research is that the application of an optimized mathematical model can increase efficiency in the context of flight scheduling management and this research will increase resource use, reduce delays, and will provide a more optimal scheduling solution than conventional methods. The object of this research is the air traffic scheduling management system, which includes the entire procedure in terms of policies for determining flight schedules and carrying out air traffic management, while the subject of this research involves the process of applying mathematical models in terms of optimizing traffic scheduling, which includes the parameters used as well as model performance. This research will begin with a simulation architecture by setting the points, schedules and flight routes as shown in Fig. 1.

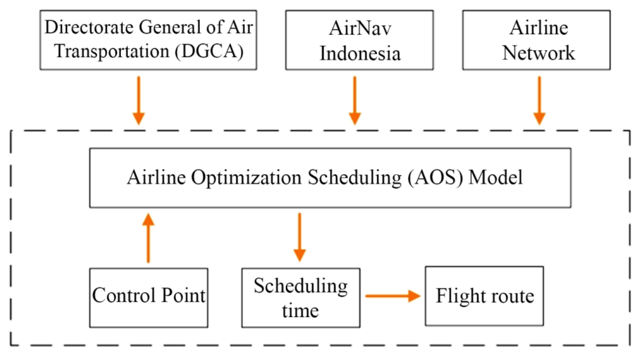

Fig. 1. The proposed architecture

Fig. 1 shows the Airline Optimization Scheduling (AOS) model, which aims to be the main architecture of the model. The flight plan determines the route and time that is always scheduled for each flight. These are airports and waypoints (air traffic control points at the border or in each sector) that aircraft must pass through, as well as the time schedule for passing each of these points. In applying the model in determining air traffic scheduling, it will specifically be illustrated through a mathematical model where there will be four air sectors. (I, II, III and IV), three airports (k_1, k_2 and k_3) and eight waypoints (W_1, ..., W_8). Apart from that, for each airport k_1 there are 2 boundary nodes, k_i^{in} and k_i^{out}, each shows the entrance and exit of the airport k_i. The following is an illustrative image of the model in Fig. 2.

Fig. 2 shows a model that will implement three flights, $f=1$ departing from k_1 and heading to k_3, $f=2$ from k_1 toward k_2, and $f=3$ from k_2 toward k_3.

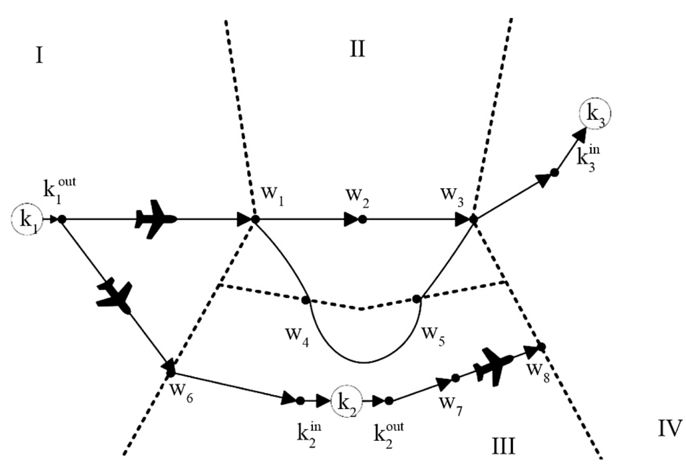

Fig. 2. Air traffic model illustration

Note that first, two alternative routes have been considered. So, $G_1=(N_1, A_1)$ is defined as follows:

$$N_1=\left\{k_1,k_1^{out},w_1,w_2,w_3,w_4,w_5,k_3^{in},k_3\right\} A_1 = \\ =\left\{\begin{matrix}\left(k_1,k_1^{out}\right),\left(k_1^{out},w_1\right),\left(w_1,w_2\right),\left(w_1,w_1\right),\left(w_1,w_3\right),\\ \left(w_3,k_3^{in}\right),\left(w_4,w_5\right),\left(w_5,w_3\right),\left(k_3^{in},k_3\right)\end{matrix}\right\}. \quad (1)$$

Each flight has a certain time period to take off from the departure airport and a scheduled number of time periods to pass through each arc in the chart. Using this information, it is possible to obtain the scheduled arrival times at each node in the route (including airports). As stated previously, if when using the original/scheduled flight plan for all flights in the network, capacity at the airport or air sector would be compromised, then the decision to change some flight plans should be considered. For each flight f, the following decisions can be made by setting a ground delay, changing the aircraft's speed as it crosses one or more arcs on its route, and selecting an alternative route. To consider the change in speed, expressed by $\ell_{f,m,n}$ number of scheduled time periods for which the flight f spends time traversing the arc (m, n). Then the delay and speed increase can be formulated by defining $\ell_{f,m,n}$ and $\ell_{f,m,n}$ upper and lower bounds, respectively, on the number of flight time periods f can spend traversing the arc (m, n) for $(m, n)\in A_f$ $(\ell_{f,m,n}\leq\ell_{f,m,n}\leq\ell_{f,m,n})$.

Flight plan modifications can incorporate ground delays, speed changes, and rerouting, so that flights can be affected by all of these actions simultaneously. Every decision has an associated cost and the goal of this problem is to obtain a feasible flight plan for all flights with minimum costs while considering airport and sector capacity constraints. It should be noted that because continuous flights are operated by the same aircraft, all decisions regarding continuous flights can affect subsequent flights. So, if a flight is delayed, the next flight will also be delayed. Therefore, the variables responsible for the willingness to fly in the case of sustainable aviation must be considered.

The mathematical formulation developed in this study is a continuation of research conducted by [25–27]. Before introducing the model, previous models are briefly introduced to show the evolution of the problem and the contributions they have made. The notation used in the model built is in Table 1 below.

Table 1

Notation Set and Notation Parameter

Set	Information
T	$\{1, ..., T\}$ A set of time periods
K	A set from the airport
J	A set of air sectors
F	A set of flights
Q	$\{(f', f)\}$ A set of continuous flights such as $f\in F$ continued with the flight $f\in F$
N_f	A set of nodes that defines all routes available for flights $f\in F$
A_f	$\{(m,n)\|m,n\in N_f\}$ A set of arcs that defines all available routes, in other words, traffic sequencing, for a flight $f\in F$. The subset $A_f^*\subseteq A_f$ includes arcs that define scheduled routes for flights f
N^j	A set of nodes that belong to a sector $j\in J$
A_f^j	$\left\{(m,n)\in A_f \mid m,n\in N_f\cap N^j\right\}$ A set of bows for flight $f\in F$ included in this sector $j\in J$
$\Gamma_f^-(n)$	$\{m\|(m,n)\in A_f\}$ A set of nodes (m, n) is the incoming arc of the node $n\in N$ for flights $f\in F$
τ_f^n	A set of feasible time periods for flight f to arrive at the node, $f\in F$, $n\in N_f$. For example T_f^n last element in T_f^n

After presenting the notation set in Table 1 and providing information, the mathematical model will describe the notation parameters contained in Table 2 below.

Table 2

Notation Parameter

Parameter	Information
$\tau_f^d\in\tau$	Scheduled departure time for a flight
$\tau_f^a\in\tau$	Arrival time at the scheduled destination for the flight $f\in F$ if following its scheduled route
$k_f^d\in K$	Departure airport for the flight $f\in F$
k^{out}	Airport node limits for any flight departure, $k\in K$
$k_f^a\in k$	Arrival airport for the flight $f\in F$
K^{in}	Airport node limits for any flight arrival, $k\in K$
$\ell_{f,m,n}$	Scheduled travel time (i.e. number of time periods) for the flight $f\in F$ to traverse an arc $(m,n)\in A_f$

Note: $\ell_{f,k_f^d,k_f^{d^{out}}}=0$ and $\ell_{f,k_f^{a,in},k_f^a}=0$; $\tau_f^a=\tau_f^d+\sum(m,n)\in A_f^*\ell_{f,m,n}$

$\overline{\ell}_{f,m,n}$ and $\underline{\ell}_{f,m,n}$ – the maximum and minimum travel time (i.e. number of time periods) allowed for a flight $f\in F$ to traverse an arc $(m,n)\in A_f$. $\underline{\ell}_{f,m,n}\leq\overline{\ell}_{f,m,n}$ for each inner arch A_f. Specifically, $\underline{\ell}_{f,k_f^d k_f^{d^{out}}}=\ell_{f,k_f^d k_f^{d^{out}}}=\overline{\ell}_{f,k_f^d k_f^{d^{out}}}\overline{\ell}=0$ and $\underline{\ell}_{f,k_f^{a^{in}},k_f^a}=\ell_{f,k_f^{a^{in}},k_f^a}=\overline{\ell}_{f,k_f^{a^{in}},k_f^a}=0.$

$\tau_{f',f}$ – turnaround time, in other words, the time required to prepare for the flight f after arrival f', $(f', f)\in Q$. It is assumed that $\tau_{f'}^a+\tau_{f',f}\leq\tau_f^d$.

$C_k^{t,dep}$ – departure capacity from the airport $k\in K$ at time.

$C_k^{t,arr}$ – arrival capacity at the airport $k\in K$ at time $t\in\tau$.

C_k^t – capacity along with departures and arrivals from the airport $k\in K$ at time $t\in\tau$.

Nodes: $C_k^t<C_k^{t,dep}+C_k^{t,arr}$.

C_j^t – capacity of the sector $j\in J$ at time $\in\tau$.

5. Results of the development of mathematical model optimization as a solution for air traffic scheduling

5. 1. Determining the objective function

$v_{f,n}^t=1$, if flight f arrives at the node n with time t, and 0 if not, $\forall f\in F$, $n\in N_f$, $t\in\tau_f^n$. Note that the variable (due to the use of the word «with» in the definition) is a step variable, in other words, indicating if the flight f has arrived at the node n at time t or earlier. So if $v_{f,n}^t=1$, then $v_{f,n}^{t+1}=.....=v_{f,n}^{T_f^n}=1$. An example of ATFM as an illustration when discussing different models is presented in the research, an example is described based on the situation depicted in Fig. 1 for aviation $f=1$. Consider that the scheduled departure time for the flight is $\tau_1^d=1$, and it was possible to delay its departure by at most two periods. In other words, flights should not depart later than $t=3$. Consider also that the scheduled times and maximum travel times for each arc of the route are those shown in Table 3.

Table 3

Travel time for $f=(1)$ in the example

Parameter	(k_2, I)	(I, II)	(II, IV)	(IV, k_3)
$\ell_{1,m,n}$	0	1	3	2
$\ell_{f,m,n}$	0	2	5	2

Note that with $\tau_1^{k_1}$ have a possible departure time of the flight, in other words $\tau_1^{k_1}=\{1,2,3\}$, remaining from the set τ_1^n can be computed easily using the travel time information in Table 1. Specifically, given the node n and $\tau_1^n,\tau_1^{n+1}=\{\underline{T_1^n}+\ell_{1,n,n+1,\ldots,}T_1^n+\overline{\ell}_{1,n,n+1}\}$, where T_1^n is the first time period in which the node n can be achieved with $\underline{T_1^n}=\min\{t\in\tau_1^n\}$. Note that there is a formula $T_f^n=\max\{t\in\tau_f^n\}$.

Fig. 3 shows a graph depicting the flight route of the example, along with the sectors the flight passes through, travel time, and set T_1^n. Vertical dotted lines depict sector boundaries.

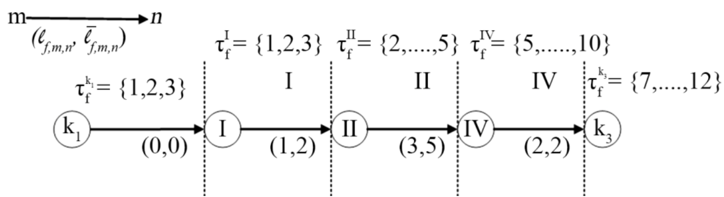

Fig. 3. Air traffic flow management problem with the node for $f=1$ in the example

To conclude the example for this formulation, consider that a new flight plan for $f=1$ includes the following modifications: one period of departure delay and two periods of in-air delay when crossing sectors II (arc (II, IV)). In this case, the decision variable will take the following values:

$v_{1,k_1}^1=0,$
$v_{1,k_1}^2=1,$
$v_{1,k_1}^3=1,$

$v_{1,II}^2=0,$
$v_{1,II}^3=1,$
$v_{1,II}^4=1,$
$v_{1,II}^5=1,$

$v_{1,II}^2=0,$
$v_{1,II}^3=1,$
$v_{1,II}^4=1,$
$v_{1,II}^5=1,$

$v_{1,IV}^5=0,$
$v_{1,IV}^6=0,$
$v_{1,IV}^7=0,$
$v_{1,IV}^8=1,$
$v_{1,IV}^9=1,$
$v_{1,IV}^{10}=1,$

$v_{1,k3}^7=0,$
$v_{1,K3}^8=0,$
$v_{1,k3}^9=0,$
$v_{1,k3}^{10}=1,$
$v_{1,k3}^{11}=1,$
$v_{1,k3}^{12}=1.$

Deviations from the original flight plan are expressed in an objective function taking into account ground and air delays. The total ground delay is assigned to the flight f. Computed as the difference between the departure time in plan and in reality:

$$g_f=\sum_{t\in\tau_f^k:k=k_f^d}\left(t-\tau_f^d\right)\left(v_{f,k}^t-v_{f,k}^{t-1}\right)=$$
$$=\sum_{t\in\tau_f^k:k=k_f^d}t\left(v_{f,k}^t-v_{f,k}^{t-1}\right)-\tau_f^d.$$

Pay attention to the amount, all the requirements $(v_{f,k}^t-v_{f,k}^{t-1})$ will be equal to 0, except for one, which will be equal to 1. Specifically, if the flight departs at time t', $(v_{f,k}^t-v_{f,k}^{t-1})=1$ and $g_f=t'-\tau_f^d$. Similarly, air delay is the difference between the planned and actual arrival time minus the delay due to being detained on the ground:

$$a_f=\sum_{t\in\tau_f^k:k=k_f^a}\left(t-\tau_f^a-g_f\right)\left(v_{f,k}^t-v_{f,k}^{t-1}\right)=$$
$$=\sum_{t\in\tau_f^k:k=k_f^a}t\left(v_{f,k}^t-v_{f,k}^{t-1}\right)-\tau_f^a-g_f.$$

These two quantities are each multiplied by a cost coefficient c_f^g and c_f^a, leading to the following objective function to minimize:

$$\min\sum_{f\in F}\left(c_f^g g_f+c_f^a af\right). \quad (2)$$

Note that because ground delays are safer than air delays, they should take precedence, in other words $c_f^g<c_f^a$.

5. 2. Determining capacity constraints

This research will determine capacity limits with the parameters used to carry out flight traffic management. The mathematical formula used is as follows, found in (3)–(6):

$$\sum_{f\in F:\left\{k_f^d=k,t\in\tau_f^k\right\}}\left(v_{f,k}^t-v_{f,k}^{t-1}\right)\le C_k^{t,dep}\forall k\in K,t\in\tau s, \quad (3)$$

$$\sum_{f\in F:\left\{k_f^a=k,t\in\tau_f^k\right\}}\left(v_{f,k}^t-v_{f,k}^{t-1}\right)\le C_k^{t,arr}\forall k\in K,t\in\tau, \quad (4)$$

$$\sum_{f\in F:\left\{k_f^a=k,t\in\tau_f^k\right\}}\left(v_{f,k}^t-v_{f,k}^{t-1}\right)+$$
$$+\sum_{f\in F:\left\{k_f^d=k,t\in\tau_f^k\right\}}\left(v_{f,k}^t-v_{f,k}^{t-1}\right)\le C_k^t\forall k\in K,t\in\tau, \quad (5)$$

$$\sum_{f\in F}\left(v_{f,j}^t-v_{f,j+1}^t\right)+c_j^t\forall j\in j,t\in\tau. \quad (6)$$

After explaining the mathematical formulation for capacity constraints, this formulation is used to optimize the mathematical model in managing flight traffic.

5. 3. Determining flight structure constraints

This research will determine the boundaries of the aviation structure with the parameters used to carry out aviation traffic management. The mathematical formula used is as follows, found in (7)–(10):

$$v_{f,n+1}^{t+\ell_{f,n,n+1}}-v_{f,n}^t\le 0\forall f\in F,n\in N_f\setminus\left\{k_f^a\right\},t\in\tau_f^n, \quad (7)$$

$$v_{f,k_f^d}^t-v_{f',k_{f'}^a}^{t-\tau_{f',f}}\le 0\forall\left(f',f\right)\in Q,t\in\tau_f^{k_f^d}, \quad (8)$$

$$v_{f,n}^t-v_{f,n}^{t-1}\ge 0\forall f\in F,n\in N_{f,t}\in\tau_f^n, \quad (9)$$

$$v_{f,k_f^d}^{T_f^{k_f^d}}=1\forall f\in F. \quad (10)$$

After explaining the mathematical formulation of flight structure capacity limits, which includes the number of routes, number of aircraft fleet and number of passengers, it will be used to optimize the mathematical model in carrying out air traffic management.

5. 4. Variable domain

This research will use mathematical formulations on domain variables with the aim of being able to carry out air traffic management as in the following (11):

$$v_{f,n}^{t} \in \{0.1\} \forall f \in F, n \in N_{f,t} \in \tau_f^n. \quad (11)$$

In the model, constraints (2)–(5) guarantee that capacity in the airport and air sector is respected at all times. Constraint (6) establishes connectivity between nodes on the route and ensures that each flight f uses the least amount of scheduled time ($\ell_{f,n,n+1}$) from one node to another. Constraint (7) is for continuous flight connections (f', f): flights f must not leave before the flight f' has landed and is taking its time $\tau_{f',f}$ at the airport preparing the plane. Constraint (8) is for time connectivity: it is stated that if a flight has not arrived on time t on a node in the route, then it also has not arrived before. Finally, constraint (9) ensures that all flights take off. After obtaining the modeling in subsections 5.1–5.4, an algorithm will be produced, which can be seen in Fig. 4, 5.

After Fig. 4, the design of the air traffic planning algorithm during the pandemic will develop at the intersection conditions, which can be seen in Fig. 5

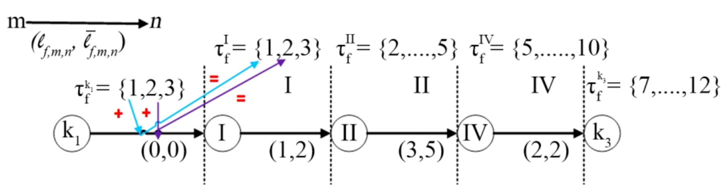

Fig. 4. Air traffic planning algorithm design during the pandemic

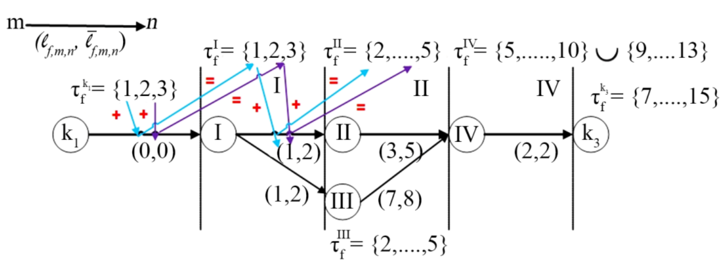

Fig. 5. Air traffic planning algorithm design with intersection conditions during the pandemic

From the two images above, the following algorithm can be designed:

# **Specify input data**
plane=[$p1$, $p2$, ..., pn] # aircraft list
flight=[$f1$, $f2$, ..., fn] # flight list
airport=[$a1$, $a2$, ..., an] # list of airports
# **Define the optimization model**
model=model()
# **Define variables**
$x[i, j]$=1 if aircraft i is used for flight j, 0 otherwise
$y[i, j]$=1 if airport i is used for flight j, 0 otherwise
$t[i, j]$=the time required for flight j using aircraft i
$d[i, j]$=the distance traveled by aircraft i during flight j
Set the objective function
minimize sum($c[i]$*$x[i, j]$ for all i, j)+sum(k*$t[i, j]$ for all i, j)+sum(h*$d[i, j]$ for all i, j)

Set constraints
for all i,j:
$x[i, j]$<=1
$y[i, j]$<=1
For all *j*:
sum($x[i, j]$ for all i) == 1 # each flight must use one aircraft sum($y[i, j]$ for all i)==1 # each flight must use one airport
$t[i, j]$<=flight[j].max_time # flight time must not exceed the maximum permitted time
$d[i, j]$<=flight[j].max_distance # the distance traveled must not exceed the maximum distance permitted
For all *i*:
sum($x[i, j]$ for all j)<=planes[i].capacity # plane capacity must not be exceeded
sum($y[i, j]$ for all j)<=airports[i].capacity # Airport capacity must not be exceeded
For all *i*,*j*,*k*:
If j!=k: # flights j and k are not the same flight
$t[i, j]$+Flights[j].turnaround_time+$t[i, k]$<= flights [k].max_time # total time between flights j and k must not exceed the maximum time allowed
$d[i, j]$+flights [j].turnaround_distance+
$d[i, k]$<=flights [k].max_distance # the total distance between flights j and k must not exceed the maximum distance permitted
 # Solve optimization problems
model.solve()
#**Print optimal solution**
For *j* in flight:
print("flights", j.id)
For *i* on the plane:
If $x[i, j]$.solution_value==1:
print(" - Plane:", Plane [i].id)
For *i* at the airport:
if $y[i, j]$.solution_value==1:
print(" - airport:", airport[i].id).

The algorithm first defines the input data and creates an optimization model object. It then defines decision variables representing whether an aircraft is assigned to the flight, whether the airport is used by the flight, the time and distance required for the flight using the aircraft, and the cost of using the aircraft.

The objective function is set to minimize the total schedule cost, which includes the cost of using each aircraft, the time required for each flight, and the distance traveled by each aircraft. The algorithm then sets a number of constraints to ensure that each flight is assigned exactly one plane and one airport, the flight time and distance do not exceed maximum limits, the capacity of the aircraft and airport is not exceeded, and the time and distance between consecutive flights do not exceed maximum limits. Finally, the optimization problem is solved using the solve () method of the optimization model object, and the optimal solution is printed, which includes the aircraft and airport assigned to each flight.

6. Discussion of the results of optimizing mathematical models in scheduling air traffic

Air traffic scheduling in this context will apply mathematical formulas that produce a model that can be used to carry out air traffic management to ensure efficiency and guarantee the safety of flight operations. The results of this research are in the form of a mathematical model that can be a solution in air traffic management. The resulting model is a mathematical formula to determine the function that can minimize the costs contained in equation (2). Then produce a formulation to determine capacity limits in equations (3)–(6), produce a formula to determine capacity limits, which produces mathematical formulations in equations (7)–(9) and equation (10) and produce a formulation to determine the variable domain contained in equation (11). In this model, equations (2)–(5) guarantee that capacity at the airport and aviation sector is always met. Equation (7) determines the connectivity between nodes on the route. Equation (7) is for a continuous flight connection. Equation (8) is for time connectivity. Finally, equation (9) ensures that all flights take off. The results of optimizing the mathematical model will involve several factors, including:

a) flight route efficiency.

By using a mathematical model, you will get optimal routes for flights such as travel time, fuel, so that the mathematical model can optimize by considering weather conditions, air currents, air traffic capacity and airline capacity;

b) flight schedule management.

Applying optimization to mathematical models can help in preparing schedules by making more optimal use of resources such as the use of runways and aircraft parking lots;

c) handling delays and operational disruptions.

By optimizing the mathematical model, you can find alternative solutions if handling delays and operational disruptions occur.

From points (a–c), the method proposed in this research is produced to continue the development of the research carried out by [23–25]. The features from the results of using the mathematical model used are then optimized, which produces the algorithm depicted in Fig. 4, 5, which can define input data and create optimization model objects. It then defines decision variables representing whether the aircraft is assigned to the flight, whether the airport is used by the flight, the time and distance required for the flight using the aircraft, and the cost of using the aircraft. The objective function is set to minimize the total schedule cost, which includes the cost of using each aircraft, the time required for each flight, and the distance traveled by each aircraft. The algorithm then sets a number of constraints to ensure that each flight is assigned exactly one plane and one airport, the flight time and distance do not exceed maximum limits, the capacity of the aircraft and airport is not exceeded, and the time and distance between consecutive flights do not exceed maximum limits. Finally, the optimization problem is solved using the solve() method of the optimization model object, and the optimal solution is printed, which includes the aircraft and airport assigned to each flight.

Comparisons with known results in literature sources will include comparisons with the application of mathematical models produced in the research including:

– in the literature study, only 1 parameter is used, namely the number of flights, while the application of the mathematical model will determine the objective function, which will overcome the problem of flight route efficiency such as travel time, fuel and weather conditions;

– in the literature study, there are models that use stochastic variables, while the application of mathematical models will determine capacity limits that will handle scheduling management such as preparing schedules by utilizing resources more optimally such as using runways and aircraft parking lots and determining flight structure limits that will overcome operational delays and disruptions.

There are shortcomings in the optimization of mathematical models in air traffic management on the scheduling side, such as the application limits on the results obtained with the models that are built are very sensitive to input data, changes to the input data can affect the mathematical model that has been optimized. Then the weakness of the results obtained is the complexity in applying the mathematical model. So, to overcome this problem, it is necessary to analyze the input data that is most relevant to the mathematical model that will be applied both in terms of objective function variables, capacity limitations, flight structure limitations and domain variables.

Research related to optimizing mathematical models can be developed by increasing the depth of complexity of mathematical models such as integration with machine learning-based technology, which can increase knowledge and improve model capabilities regarding air traffic situations and then incorporate human factors into mathematical models such as decision-making and air traffic control. In its development, difficulties will be faced mathematically, such as changes that often occur in input data in the form of passenger data, airport data, number of routes and weather factors.

7. Conclusions

1. The mathematical model contains variables. The objective function was determined, which is one of the results of air traffic scheduling by considering flights that use the word «with» as the main variable accompanied by arrival and time parameters. In this model, departures are always considered with the rule that the departure can be postponed for up to two periods. The simulation has been carried out using the graph, which shows the flight sectors that are traversed, which have characteristics in flight traffic scheduling such as travel time and location that have been determined. There are modifications to the model with one period of departure delay and two periods for the maximum number of delays. So that air delay is the difference between the planned arrival time and the actual time so that the model with objective function variables can provide cost and safety coefficients because delays on the ground are safer than delays in the air, so these delays must take priority.

2. The mathematical model produced in this research is in the form of variables in determining capacity limits, which are used as parameters for carrying out air traffic management. For this variable, 4 formulas are used to solve the problem: air traffic runways and air navigation systems, airport operational time, weather problems, passenger capacity, security and airport operations. All of these equations can be used to optimize mathematical models in air traffic management.

3. This research produces a mathematical model with variable flight structure constraints in air traffic management. This variable will produce 4 formulas, which are used to solve the problem: number of routes, number of aircraft fleets, number of passengers, conditions outside the input parameters so that traffic management can be effective and efficient using mathematical models.

4. The mathematical model for conducting air traffic management will produce domain variables for carrying out management. This variable will ensure that capacity at the airport and aviation sector is always met by always implementing connectivity between nodes on the route and each flight using the least scheduled time. Connectivity will refer to the arrival time of the aircraft, which will then ensure that all flights take off with security maintained from an air traffic management perspective. So this research will obtain an algorithm that uses mathematical models in scheduling air traffic by overcoming complexity problems, which are results that can be superior compared to known results.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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