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DETERMINING THE PATTERNS OF ASYMMETRIC INTERACTION OF PLASTIC MEDIUM WITH COUNTER-DIRECTIONAL METAL FLOW

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The plane problem of rolling theory is analytically solved using the method argument of functions of a complex variable. The solution to the plane problem has been strengthened from the point of view of the asymmetry of the process, which made it possible to consider the applied problem as the interaction of differently directed zones in the deformation zone. The interaction of lagging and advancing zones is represented as a combination of multidirectional processes in a single deformation zone. With a change in kinematic, power characteristics in local zones, the process parameters change in the entire deformation zone. Stressed states of intermediate loading schemes between stable and unstable rolling are considered. A feature of the interaction of zones with the opposite flow of metal is the analogy with the action of back tension on the deformation zone in literally all parameters - this is the presence of tensile stresses in the lagging zone, a decrease in local specific pressures, a shift in maximum normal stresses towards the exit from the rolls, a change in the length of the advance zone, reduction in rolling force.

The studies confirm and repeat the generally accepted provisions of the theory of rolling but reveal the effects of changes in the stress state under different loading models.

The results of the work make it possible to determine the modes of rolling processes visually and computationally under conditions of strong and weak interaction of zones with an oppositely directed metal flow.

The effects of plastic deformation with a decrease in the total effort in the processes that are within the reach of the limiting focus of crimping under conditions of increasing kinematic load when the gripping angles vary between 0.077...0.168 are given

Keywords: loading asymmetry, counter-directional metal flow, plane problem, effects of plastic deformation

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1. Introduction

The effectiveness of new rolling technologies is largely determined by the ability to solve issues related to increasing mill productivity, reducing energy consumption, and improving the quality of rolled products. In most cases, this is a controversial and poorly solved problem. The complexity of this task increases many times if it is necessary to implement it simultaneously across the entire range of changing production conditions and parameters.

The reason for this complexity is largely the lack of knowledge of the features, effects, and processes that determine the interaction of the lagging and advance zones in a single deformation zone. The interaction of the zones manifests itself in the fact that changes in kinematic and force parameters in a local zone lead to changes in the process parameters of the entire deformation zone. The effect of asymmetric interaction of multidirectional zones in a single deformation zone occurs. The total effect of interacting zones can be different and adjustable.

Issues of process sustainability are a controversial phenomenon. One of the options may be processes with negative or zero advance, interaction parameters determined by the limiting focus of deformation.

Hereafter, the limiting zone of deformation will be considered, which corresponds to the process of loss of stability or the process of strip slipping in rolls.

In relation to this task, interaction effects are difficult to track, for various reasons, to show their attractive sides and the possibilities of their use. Research into development of new approaches to solving problems in the theory of rolling, taking into account their clarity, is relevant. Under these conditions, the most appropriate is a theoretical analysis based on modern generalized schemes for solving problems in continuum mechanics, including the theory of plasticity, with minimal simplifications and assumptions.

2. Literature review and problem statement

When rolling, the determining factor is the gripping ability of the rolls, on which the further course of the entire process depends.

Theoretical studies [1, 2] of rolling processes under a stable mode, taking into account internal and external forces, confirm the possibility of the rolling process with a negative advance [3].

At the contact, powerful shear deformations occur in the form of relative sliding of the metal along the surface of the rolls. This significantly changes the kinematics of the flow of the plastic medium. Features of the process appear that affect the power parameters, including features of multidirectional metal flow. In [4], the process of localization of plastic flow in near-contact layers of metal is considered. Based on the mathematical model, a new numerical approach is proposed that makes it possible to simulate fully localized plastic flow. However, the use of numerical methods in the calculation of machines and structures, taking into account their interaction with the elastoplastic medium, is largely determined by the complexity or even the impossibility of analytical calculations due to the complexity of structural schemes, heterogeneity of material properties, and unevenness of soil layers. Solving the problems considered in [5] is possible only within the framework of numerical methods, the most common of which is the finite element method. Some ways to overcome such difficulties are proposed in [6] using a procedure for constructing computational models of joint deformation and the mutual influence of rigid structures and the plastic external environment.

The requirements for controlled rolling technologies complicate the production process. Rolling accuracy decreases and metal consumption increases. In this regard, technologies are needed that reduce the rolling force. There is a need to devise new theoretical approaches.

The evolution of the theory is moving towards the development of generalized methods for solving problems of continuum mechanics, identifying the features of the stress state under different physical, boundary, and extreme loading conditions of the metal. Paper [7] describes well-known models of the behavior of a solid deformable body, represented in a systematic form: elastic deformable body, viscoelastic deformable body, plastic and fluid models.

With the rapid development of technology, the boundary conditions of many applied problems become more complex, which necessitates the expansion and complexity of process models, finding not the solutions themselves but the conditions for their existence and generalizations. In this regard, an important circumstance is the response of the literature to the problem of generalizations in the theory of differential equations. Generalized solution methods include variational principles of mechanics, tested in many theoretical studies, which are used in different areas of continuum mechanics [8]. A rather long recalculation complicates the analysis of solutions, which becomes more complicated upon further consideration. In addition, the finite element method may not always be sufficient to ensure the reliability of the obtained result due to simplifications in the theory of solving the variational problem itself. The finite element method is largely based on variational principles of mechanics. In [9, 10] it is shown that varying the displacement functional corresponds to solving only differential equilibrium equations and the corresponding boundary conditions. Varying the functional across stresses meets other criteria of the problem. The reliability of the result in accordance with the data from [11] occurs when a closed variational problem is solved. From the point of view of the approach of a closed variational problem, the variation of the functional should be carried out both in terms of displacements and stresses, which is not often implemented in practice due to computational difficulties.

The mathematical theory of plasticity is based on certain ideas, covering its range of applied problems, and allows one to generalize these solutions with approaches that are inaccessible to other schemes [12]. The issue is the nonlinearity of problems in plasticity theory, including boundary conditions, and the complexity of their analytical solution and generalization.

In [13], based on the modified theory of pair stresses, a mathematical model of Euler-Bernoulli beams was developed, and the deformation theory of plasticity was applied. For metal beams, taking into account geometric and physical nonlinearity, the phenomenon of changing boundary conditions, i.e., structural nonlinearity, was discovered. This is close to the processes considered in our study.

In [14], metal loadings during plastic deformation are examined. But loads can be considered somewhat more broadly, for example, asymmetrical and symmetrical, which narrows the range of issues to be resolved.

When stating problems in mechanics, systems of equations are considered that can be classified as equations of mathematical physics. First of all, these are hyperbolic, elliptic, and parabolic partial differential equations. Methods for solving them include separation of variables, characteristics, Riemann, Fourier, d'Alembert methods, integral transformations.

The closed problem of plasticity theory was proposed in [15]. The method of argument of functions of a complex variable is used, which is generalized to various problems of continuum mechanics. The method linearizes the solution using various substitutions that can close the problem using two functions designated by the argument. But the authors did not fully develop the theory of asymmetric loading, which limits the result and does not allow taking into account the peculiarities of the interaction of zones with the opposite flow of metal.

Many solutions to the theory of plasticity and elasticity use the method of stress functions [16], which shows a reliable result (it can be effectively used, with certain simplifications, as a test of the theory of argument of functions of a complex variable).

Paper [17] shows inconsistencies between the stressed and deformed state of a plastic medium due to mismatch of fields. Solutions are shown that make it possible to eliminate the contradictions between theoretical and experimental data due to the identified effects of twisting of conjugate slip planes. Such approaches are productive because they allow one to imagine the process of plastic deformation in a new way, but they still require improvement. Processes with effects with multidirectional flow of metal during rolling are proposed in a new way, allowing one to obtain not only theoretical, but also practical results.

Work [18] reports the analysis of differences in the nature of plastic fracture of plates under plane tension in the far zone and plane strain bending. A critical understanding of this problem is provided based on the results of a detailed numerical study of fracture tests using the Gurson material model. This is especially true for large-scale industrial applications.

In [19], transition conditions for additional variables are used (analogy of function arguments). This makes it possible to relate different types of differential equations. The fact that this idea is used is important. However, it is not extended to other generalized approaches (for example, finding conditions for the existence of solutions). Work [20] shows cyclic loading in the case of simple shear, which finds a corresponding response of internal stresses. A basic trigonometric function is introduced into consideration. Its capabilities are shown when boundary conditions change. But there is no correspondence between it and other basic functions.

Article [21] provides an example of using the argument of a function as a coordinate one, which significantly expands the possibilities of the solution. In the future, they can be used as closing functions of the problem being solved. However, the proposed approaches only consider two extreme states: these results do not take into account the three-dimensional nature of the structures.

In [22], loading at the base of a certain discontinuity was studied. The heterogeneity of the stress state was characterized by trigonometric and exponential expressions. A simplified analytical method is proposed that approximates the actual elastic-plastic behavior of the material. The combination of functions allows one to expand the scope of their application through simplification of boundary conditions. All this allows us to assert that it is advisable to conduct a study to consider the influence of various factors on the force and deformation parameters of the rolling process.

Work [23] shows a solution to one of the main problems of engineering design: predicting the behavior of a material during fracture using a fundamental substitution. In article [24], when solving a similar problem, it is proposed to take into account the design features of the part.

However, the purpose of the proposed substitution in the last two works is different. The use in [24] of one argument of the exponential dependence function does not allow its use in solving the problem.

The heterogeneity of the stress-strain state negatively affects the strength characteristics and durability of products [24, 25]. But as a result of the action of inhomogeneities, combinations of features are possible, leading to the effects of shape changes and loading of the tool.

Work [26] shows the R-function method, which can be a justification for the function argument method. However, it is further shown that its use is involved in other schemes, for example in the variational principles of mechanics. Differential relations for transition from one variable to another are used, but they are not applicable for generalizations using arguments of functions.

Work [27] demonstrates how the boundary conditions of the problem are mathematically justified based on the collocation method. Their determination is a problem in continuum mechanics. In addition, the nonlinearity of the boundary conditions does not allow one to find options for transformations in the solutions themselves.

In dynamic problems of the theory of elasticity, a wave equation of the hyperbolic type is often used. However, the function argument method proposed in [15] was used to solve another hyperbolic type equation in a dynamic problem. Generalized approaches to solving these equations were obtained using the Cauchy-Riemann differential relations, which made it possible to specify d'Alembert's result [28].

In the Cartesian coordinate system, a solution to the problem of the theory of elasticity was proposed, also using the method of arguments of functions of a complex variable [29]. Another type of differential equations was worked out. An applied problem of the theory of elasticity under loading of a semi-infinite space is shown. The simplest Cauchy-Riemann relations led to complete coincidence of the final result obtained using the stress function method proposed by other authors.

A similar approach was applied to solving the problem of elasticity theory in polar coordinates [30]. The system of staged equations is represented by other differential equations, where the defining equation, as in [29], is the Laplace equation. The argument method of functions of a complex variable was used. There are greater opportunities to satisfy more complex boundary conditions. Works [26, 27, 29, 30] are presented as fundamental for solving plane problems of the theory of elasticity using the method of argument functions. As a continuation of studies [28–30], the theory was refined and the applied problem of the action of a transverse force at the end of a console with a rigid embedment was solved [31]. Some difficulties arose in satisfying the stress boundary conditions. However, when the solution was complicated by introducing a hyperbolic cosine, the sine for the second argument of the function, the boundary conditions were satisfied. An interesting final result was obtained, a gradual reduction in the load at the end of the beam, affecting the overall stress state, which is confirmed by the Saint-Venant principle.

Our review of the literature [2, 5, 17] reveals that new challenges arise related to the latest experimental and theoretical research in the mechanics of deformed solids and mechanics in general. These statements suggest that there is a need to more fully use modern achievements of theory and science in solving complex technological problems.

A complex technological problem arises associated with the development of rolling schemes, in which it is possible to simultaneously increase the reduction and reduce the strength characteristics of the process. In this case, the general unsolved task is the discovery of the effects of plastic deformation, which makes it possible to implement a control effect on the deformation zone.

3. The aim and objectives of the study

The purpose of this work is to determine the patterns of influence of the interaction of areas of a single deformation center with the counter-directional flow of metal, the effects of the gripping ability of the rolls, on the force parameters of the process, under conditions of changing deformation loading. This will make it possible to use an effective approach to visually study the stress-strain state of the metal, taking into account intermediate schemes, multidirectional force, and deformation loading modes.

To achieve the goal, the following tasks were set:

– to build a physical and mathematical model of a plane problem of rolling theory under conditions of changing asymmetric interaction of zones with counter-directional flow of metal;

– to investigate and identify the features of plastic deformation with two-zone and single-zone deformation, changing interaction of areas with counter-directional flow of metal, the effects of the influence of the gripping ability of rolls on process parameters;

– to analyze the stressed state of the metal in the area of reach of the limiting source of deformation with increasing deformation loading.

4. The study materials and methods

The object of our study is the stressed state of the metal under conditions of a two-zone and single-zone deformation zone. The changing interactions of areas with counter-directional flow of metal, including in the area of reach of the limiting deformation zone with increasing deformation loading, are analyzed.

The research hypothesis is to confirm, within the framework of solving the problem, the asymmetry of the interaction of zones with counter-directional flow of metal in a single deformation zone.

The assumptions are presented as follows. The asymmetry of interaction between zones of a single deformation zone and the counter-directional flow of metal is introduced into consideration. The conditions of a two-zone and single-zone deformation zone in the region of reach of the limiting deformation zone with increasing deformation loading are considered.

Loading and asymmetry of interaction are determined by technological production factors: the deformation zone shape factor, contact friction, and other parameters. The shape factor l/h_{av} must be within 1...15. This is especially true for large values, because when rolling a thin strip, the stressed state of the metal is more "sensitive" to external influences and the manifestation of the effects of plastic deformation. The friction coefficient is assumed to be in the range of 0.05...0.5, which allows one to adjust the load over a wide range. The computational model of the process must respond to physical processes with counter-directional flow of metal, under conditions of a single-zone and two-zone deformation zone. To do this, it is necessary to obtain a new solution to the problem that takes into account the increasing asymmetry of loading. It is necessary to show a physical model of the process that explains the appearance of the effects of interaction of zones with counter-directional flow of metal, to build a mathematical model that responds to a single-zone deformation zone and the availability of process stability to force and deformation loading parameters.

5. Results of determining the patterns of asymmetric interaction of the plastic medium with the counter-directional flow of metal

5. 1. Construction of a mathematical model of a plane problem of rolling theory under conditions of asymmetric interaction of zones

The determining influence of the lagging zone through tensile stresses on the force parameters of the process with a change in compression is shown in [32, 33]. The stressed state of the metal under conditions of strip slipping is characterized. In this case, the diagram of normal contact stresses has a dip along the entire length of the deformation zone, and the stress state coefficient is less than unity in this section. A pressing issue is confirmation of the reliability of the results obtained. To enhance the reliability of the result, there is a need to use fundamental approaches in solving problems of continuum mechanics and, in particular, the theory of plasticity. One of the fundamental approaches in mechanics is the direction associated with finding the result in a closed form. In this case, the solution in stresses must be confirmed by solutions to the problem in strain rates or strains, a solution to the heat conduction equation, and boundary conditions. If the problem can be closed with generalizing functions and a mathematical model of a changing plastic medium can be obtained, then, according to the authors of [9–11], the reliability of the proposed solution is ensured. In accordance with this, the closed problem of plasticity theory is considered.

In [15], a closed plane problem of plasticity theory was stated and solved. The statement of the closed problem is a system of equations of the plane theory of plasticity, including the heat equation:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0;\ \frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0;$$

$$\left(\sigma_x-\sigma_y\right)^2+4\tau_{xy}{}^2=4\cdot k^2;\ \frac{\sigma_x-\sigma_y}{2\tau_{xy}}=\frac{\xi_x-\xi_y}{\dot{\gamma}_{xy}}=F_1;$$

$$\xi_x+\xi_y=0;\ \frac{\partial^2\xi_x}{\partial y^2}+\frac{\partial^2\xi_y}{\partial x^2}=\frac{\partial^2\dot{\gamma}_{xy}}{\partial y\partial x};$$

$$\ldots\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial x^2}=0,$$

where σ_x is normal stress; τ_{xy} – tangential stress; k – resistance to plastic shear deformation (variable value); ξ_x, ξ_y, γ_{xy} – linear and shear strain rates; t is the temperature of the metal at a given point.

The boundary conditions are specified in stresses and hence in strain rates:

$$\tau_n=-\left[\frac{\sigma_x-\sigma_y}{2}\cdot\sin\left(2\phi\right)-\tau_{xy}\cos\left(2\phi\right)\right],$$

$$\dot{\gamma}_n=-2\left[\frac{\xi_x-\xi_y}{2}\cdot\sin\left(2\phi\right)-\dot{\gamma}_{xy}\cos\left(2\phi\right)\right],$$

where γ_n is the shear strain rate characterizing the boundary condition; τ_n – tangential stress characterizing the boundary condition.

On the basis of a closed solution, an applied problem of the theory of plasticity is proposed, which makes it possible to demonstrate the capabilities of the method of argument of functions of a complex variable. More complex physical processes of rolling production associated with the counter-directional flow of metal under conditions of complex asymmetric loading are considered. Considering that the stressed state of a plastic medium is analyzed, the solution is represented in stresses, which simplifies the statement of the problem:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0;\ \frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0;$$

$$\left(\sigma_x-\sigma_y\right)^2+4\tau_{xy}^2=4k^2, \tag{1}$$

boundary conditions:

$$\tau_n=-\left[\frac{\sigma_x-\sigma_y}{2}\sin\left(2\phi\right)-\tau_{xy}\cos\left(2\phi\right)\right]. \tag{2}$$

From equations (1) it is clear that the number of unknowns is equal to the number of equations. The expression for determining the intensity of tangential stresses of a plane problem is written for further transformations in the form:

$$T_i=\frac{1}{2}\sqrt{\left(\sigma_x-\sigma_y\right)^2+4\tau_{xy}^2}. \tag{3}$$

Using (3), we can write down the difference in normal stresses and, taking into account (1), obtain a generalized equilibrium equation [29]:

$$\sigma_x-\sigma_y=2\sqrt{T_i^2-\tau_{xy}^2}, \tag{4}$$

$$\frac{\partial^2\tau_{xy}}{\partial x^2}-\frac{\partial^2\tau_{xy}}{\partial y^2}=\pm 2\frac{\partial^2}{\partial x\partial y}T_i\sqrt{1-\left(\frac{\tau_{xy}}{T_i}\right)^2}. \tag{5}$$

To go to the plasticity expression (1), it is necessary to use the Huber-Mises condition in the form:

$$T_i=k.$$

In this case, there is nonlinearity, both in the boundary conditions (2), (4), and in the differential equation (5), which is fundamental for determining the tangential stresses. Dependences (4), (5) can be linearized, i.e., simplified, using trigonometric substitution:

$$\frac{\tau_{xy}}{T_i}=\sin \mathrm{A}\Phi, \tag{6}$$

where $\mathrm{A}\Phi$ is an unknown function, considered as the first argument of the solution function. Taking into account (6), we have simplifications of the boundary conditions and linearization of equation (5):

$$\tau_n=-T_i\sin\left(\mathrm{A}\Phi-2\phi\right), \tag{7}$$

$$\frac{\partial^2\left(T_i\sin \mathrm{A}\Phi\right)}{\partial x^2}-\frac{\partial^2\left(T_i\sin \mathrm{A}\Phi\right)}{\partial y^2}=\pm 2\frac{\partial^2}{\partial x\partial y}T_i\cos \mathrm{A}\Phi. \tag{8}$$

In accordance with [31, 34], for linear partial differential equations we use a fundamental substitution of the form:

$$T_i=C_\sigma\exp\theta, \tag{9}$$

where θ is an unknown function, considered as the second argument of the problem function. A distinctive feature of fundamental substitution (9) from [34] is that this function is not specified but determined. Expression (9) has worked well for solving symmetric problems [15], however, with different boundary conditions at the ends, it is not possible to link the solution into a single mathematical model under asymmetric loading. In this case, it is advisable to use in the solution not the constant value C_σ in the product (9) but to switch to a variable, i.e.:

$$T_i=H_\sigma\exp\theta, \tag{10}$$

where H_σ is an unknown coordinate function determined in the process of obtaining the final result.

For the desired functions of equation (8), the tangential stress, in combination with (6) and (10), can be represented:

$$\tau_{xy}=H_\sigma\exp\theta\sin \mathrm{A}\Phi. \tag{11}$$

With the introduction of two argument functions, the statement of the problem changes somewhat; its statement can be as follows: what conditions must the argument function of expression (11) meet in order to close the solution to the problem, turning equation (8) into an identity.

As further analysis shows, the optimal solution approach is to use a complex variable function [35]. Let us represent the tangential stress (11) in the form:

$$\tau_{xy}=H_\sigma\frac{\exp\left(\theta+i\mathrm{A}\Phi\right)-\exp\left(\theta-i\mathrm{A}\Phi\right)}{2i}. \tag{12}$$

The right-hand part of expression (5):

$$T_i\sqrt{1-\left(\frac{\tau_{xy}}{T_i}\right)^2}=H_\sigma\frac{\exp\left(\theta+i\mathrm{A}\Phi\right)+\exp\left(\theta-i\mathrm{A}\Phi\right)}{2}. \tag{13}$$

The sequence of determining the tangential stress function of equation (8) is considered in detail. Taking into account (12), the second derivative with respect to the x coordinate of equation (8) is determined:

$$\frac{\partial^2\tau_{xy}}{\partial x^2}=\frac{1}{2i}\left\{\begin{array}{l}\exp\left(\theta+i\mathrm{A}\Phi\right)\left[\begin{array}{l}\left(H_\sigma\right)_{xx}+2\left(H_\sigma\right)_x\times\\ \times\left(\theta_x+i\mathrm{A}\Phi_x\right)+\\ +H_\sigma\left(\theta_{xx}+i\mathrm{A}\Phi_{xx}\right)+\\ +H_\sigma\left(\theta_x+i\mathrm{A}\Phi_x\right)^2\end{array}\right]-\\ -\exp\left(\theta-i\mathrm{A}\Phi\right)\left[\begin{array}{l}\left(H_\sigma\right)_{xx}+2\left(H_\sigma\right)_x\times\\ \times\left(\theta_x-i\mathrm{A}\Phi_x\right)+\\ +H_\sigma\left(\theta_{xx}-i\mathrm{A}\Phi_{xx}\right)+\\ +H_\sigma\left(\theta_x-i\mathrm{A}\Phi_x\right)^2\end{array}\right]\end{array}\right\}, \tag{14}$$

where θ_{xx}, $(H_\sigma)_x$, $\mathrm{A}\Phi_x$ are partial derivatives with respect to the x coordinate. The mixed derivative of the right-hand part of the generalized equilibrium equation when substituting (13) will be written:

$$\frac{\partial^2}{\partial x\partial y}\left(T_i\sqrt{1-\left(\frac{\tau_{xy}}{T_i}\right)^2}\right)=$$

$$=\frac{1}{2}\left\{\begin{array}{l}\exp\left(\theta+i\mathrm{A}\Phi\right)\left[\begin{array}{l}\left(H_\sigma\right)_{xy}+\left(H_\sigma\right)_y\left(\theta_x+i\mathrm{A}\Phi_x\right)+\\ +\left(H_\sigma\right)_x\left(\theta_y+i\mathrm{A}\Phi_y\right)+\\ +H_\sigma\left(\theta_{xx}+i\mathrm{A}\Phi_{xx}\right)+\\ +H_\sigma\left(\theta_x+i\mathrm{A}\Phi_x\right)\left(\theta_y+i\mathrm{A}\Phi_y\right)\end{array}\right]+\\ +\exp\left(\theta-i\mathrm{A}\Phi\right)\left[\begin{array}{l}\left(H_\sigma\right)_{xy}+\\ +\left(H_\sigma\right)_y\left(\theta_x-i\mathrm{A}\Phi_x\right)+\\ +\left(H_\sigma\right)_x\left(\theta_y-i\mathrm{A}\Phi_y\right)+\\ +H_\sigma\left(\theta_{xx}-i\mathrm{A}\Phi_{xx}\right)+\\ +H_\sigma\left(\theta_x-i\mathrm{A}\Phi_x\right)\times\\ \times\left(\theta_y-i\mathrm{A}\Phi_y\right)\end{array}\right]\end{array}\right\}. \tag{15}$$

Using expressions (14), (15) in equation (8), taking into account the second derivative of the y coordinate, we have:

$$\frac{1}{2i}\exp(\theta+i\mathrm{A}\Phi)\times\left\{\begin{array}{l}\left[(H_\sigma)_{xx}-(H_\sigma)_{yy}-2i(H_\sigma)_{xy}\right]+\\+2(H_\sigma)_x\left[\begin{array}{l}(\theta_x+\mathrm{A}\Phi_y)-\\-i(\theta_y-\mathrm{A}\Phi_x)\end{array}\right]-\\-2(H_\sigma)_y\left[\begin{array}{l}i(\theta_x+\mathrm{A}\Phi_y)-\\-(\theta_y-\mathrm{A}\Phi_x)\end{array}\right]+\\+H_\sigma\left[\begin{array}{l}(\theta_{xx}-\theta_{yy}+2\mathrm{A}\Phi_{xy})+\\+i(\mathrm{A}\Phi_{xx}-\mathrm{A}\Phi_{yy}-2\theta_{xy})\end{array}\right]+\\+H_\sigma\left[(\theta_x+\mathrm{A}\Phi_y)-i(\theta_y-\mathrm{A}\Phi_x)\right]^2\end{array}\right\}-\frac{1}{2i}\times$$

$$\times\exp(\theta-i\mathrm{A}\Phi)\left\{\begin{array}{l}\left[\begin{array}{l}(H_\sigma)_{xx}-(H_\sigma)_{yy}+\\+2i(H_\sigma)_{xy}\end{array}\right]+\\+2(H_\sigma)_x\left[\begin{array}{l}(\theta_x+\mathrm{A}\Phi_y)+\\+i(\theta_y-\mathrm{A}\Phi_x)\end{array}\right]-\\-2(H_\sigma)_y\left[\begin{array}{l}(\theta_y-\mathrm{A}\Phi_x)-\\-i(\theta_x+\mathrm{A}\Phi_y)\end{array}\right]+\\+H_\sigma\left[\begin{array}{l}(\theta_{xx}-\theta_{yy}+2\mathrm{A}\Phi_{xy})-\\-i\left(\begin{array}{l}\mathrm{A}\Phi_{xx}-\\-\mathrm{A}\Phi_{yy}-2\theta_{xy}\end{array}\right)\end{array}\right]+\\+H_\sigma\left[\begin{array}{l}(\theta_x+\mathrm{A}\Phi_y)+\\+i(\theta_y-\mathrm{A}\Phi_x)\end{array}\right]^2\end{array}\right\}=0. \quad (16)$$

In the operators of both exponentials of equation (16), the same brackets (θ_x+$\mathrm{A}\Phi_y$) and (θ_y–$\mathrm{A}\Phi_x$) appeared in the process of transformations. Moreover, these brackets create nonlinearity, and it is quite reasonable to take them equal to zero. As a result, we arrive at the Cauchy-Riemann relations of the form:

$$\theta_x=-\mathrm{A}\Phi_y,$$

$$\theta_y=\mathrm{A}\Phi_x. \quad (17)$$

Moving on to the second derivatives in ratios (17), we get:

$$\theta_{xx}+\theta_{yy}=0,$$

$$\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy}=0. \quad (18)$$

These are Laplace's equations, the solutions of which are harmonic functions. Using relations (17), (18), the problem of identifying the arguments of the functions θ and AΦ is eliminated. It should be added that differential relations (17), (18) are generalized invariant characteristics (conditions) [26], which make it possible to close the problem for equation (8). After substitution (17) and simplifications, differential equation (16) takes the form:

$$\frac{1}{2i}\exp(\theta+i\mathrm{A}\Phi)\left\{\begin{array}{l}\left[(H_\sigma)_{xx}-(H_\sigma)_{yy}-2i(H_\sigma)_{xy}\right]+\\+H_\sigma\left[\begin{array}{l}(\theta_{xx}-\theta_{yy}+2\mathrm{A}\Phi_{xy})+\\+i\left(\begin{array}{l}\mathrm{A}\Phi_{xx}-\mathrm{A}\Phi_{yy}-\\-2\theta_{xy}\end{array}\right)\end{array}\right]\end{array}\right\}-$$

$$-\frac{1}{2i}\exp(\theta-i\mathrm{A}\Phi)\left\{\begin{array}{l}\left[\begin{array}{l}(H_\sigma)_{xx}-(H_\sigma)_{yy}+\\+2i(H_\sigma)_{xy}\end{array}\right]+\\+H_\sigma\left[\begin{array}{l}(\theta_{xx}-\theta_{yy}+2\mathrm{A}\Phi_{xy})-\\-i\left(\begin{array}{l}\mathrm{A}\Phi_{xx}-\\-\mathrm{A}\Phi_{yy}-2\theta_{xy}\end{array}\right)\end{array}\right]\end{array}\right\}=0.$$

One can show that the operators in square brackets are zero, that is:

$$\theta_{xx}-\theta_{yy}+2\mathrm{A}\Phi_{xy}=0,\quad \mathrm{A}\Phi_{xx}-\mathrm{A}\Phi_{yy}-2\theta_{xy}=0.$$

Hence:

$$\frac{1}{2i}\exp(\theta+i\mathrm{A}\Phi)\left\{\left[(H_\sigma)_{xx}-(H_\sigma)_{yy}-2i(H_\sigma)_{xy}\right]-\right.$$

$$\left.-\frac{1}{2i}\exp(\theta-i\mathrm{A}\Phi)\left\{\left[(H_\sigma)_{xx}-(H_\sigma)_{yy}+2i(H_\sigma)_{xy}\right]=0.\right.\right. \quad (19)$$

Under the condition H_σ=C_σ=const, equation (19) turns into an identity because partial derivatives of constant quantities are equal to zero. One of the solutions to the system of differential equations (19) for the function H_σ may be the expression:

$$H_\sigma=\frac{C_0\left(\frac{l}{2}-x\right)+C_1\left(\frac{l}{2}+x\right)}{l}, \quad (20)$$

where C_0, C_1 are constants that determine unequal stresses at the entrance and exit from the deformation zone; l is the length of the deformation zone.

It should be emphasized that the function introduced into the consideration has a special purpose associated with describing the asymmetry of the plastic interaction process. As a result, an analytical solution of the inhomogeneous, nonlinear second-order partial differential equation (8) was obtained:

$$\tau_{xy}=H_\sigma\exp\theta\sin\mathrm{A}\Phi, \quad (21)$$

provided:

$$\theta_x=-\mathrm{A}\Phi_y,\quad \theta_y=\mathrm{A}\Phi_x,$$

$$\theta_{xx}+\theta_{yy}=0,\quad \mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy}=0.$$

The attractiveness of expression (21) is that not the solutions themselves to differential equations (8), (16) are obtained but the conditions for their existence.

Further, it should be noted that the introduction of the function H_σ into consideration turned out to be a successful mathematical technique since in the differential equation (16) the unknown derivatives of this variable with respect to coordinates were combined with the Cauchy-Riemann relations. This made it possible to exclude them from further consideration and simplify the task.

From the equilibrium equations (1), with a known functional dependence (21), it is possible to determine the normal components of the stress tensor. From the equilibrium equations [32, 36] taking into account the deviatoric component:

$$\frac{\partial(\sigma_x-\sigma_0)}{\partial x}=-\begin{bmatrix}(H_\sigma)_y \exp\theta\sin A\Phi+\\ +H_\sigma\theta_y \exp\theta\sin A\Phi+\\ +H_\sigma A\Phi_y \exp\theta\cos A\Phi\end{bmatrix};$$

$$\frac{\partial(\sigma_y-\sigma_0)}{\partial y}=-\begin{bmatrix}(H_\sigma)_x \exp\theta\sin A\Phi+\\ +H_\sigma\theta_x \exp\theta\sin A\Phi+\\ +H_\sigma A\Phi_x \exp\theta\cos A\Phi\end{bmatrix}.$$

After separating the variables and integrating them, let's write:

$$\sigma_x=H_\sigma\exp\theta\cdot\cos A\Phi-T_1+\sigma_0'+f(y),$$

$$\sigma_y=-H_\sigma\exp\theta\cos A\Phi-T_2+\sigma_0'+f(x),$$

where:

$$T_1=\int\left[(H_\sigma)_y\exp\theta\cos A\Phi+(H_\sigma)_x\exp\theta\sin A\Phi\right]dx,$$

$$T_2=\int\left[(H_\sigma)_x\exp\theta\cos A\Phi-(H_\sigma)_y\exp\theta\cos A\Phi\right]dy.$$

It can be shown that they can be taken as equals:

$$T_1=T_2=T.$$

In this case, the normal stresses are as follows:

$$\sigma_x=H_\sigma\exp\theta\cos A\Phi-T+\sigma_0'+f(y),$$

$$\sigma_y=-H_\sigma\exp\theta\cos A\Phi-T+\sigma_0'+f(x).$$

Taking into account the plasticity condition, another dependence can be obtained:

$$\sigma_x=-H_\sigma\exp\theta\cos A\Phi+f(y),$$

$$\sigma_y=-3H_\sigma\exp\theta\cos A\Phi+f(x).$$

This is realized due to the average stress σ_0', equal to:

$$\sigma_0'=-2H_\sigma\exp\theta\cos A\Phi+T. \quad (22)$$

To substantiate expression (22), it should be emphasized that in the closed system of equations the condition of continuity of deformations is used, which in the spherical form of the stress state is determined by the following equation [28, 29]:

$$\Delta^2 n\sigma_0'=\frac{\partial^2 n\sigma_0'}{\partial x^2}+\frac{\partial^2 n\sigma_0'}{\partial y^2}=0. \quad (23)$$

This is Laplace's equation, denoted in the solution by equations (18). As the analysis shows, the solution to equation (23) can be dependence (22) provided that the argument functions of the Cauchy-Riemann relations are satisfied, i.e.:

$$\sigma_0'=-2H_\sigma\exp\theta\cos A\Phi+T,$$

at:

$$\theta_x=-A\Phi_y,\ \ \theta_y=A\Phi_x,$$

$$\theta_{xx}+\theta_{yy}=0, A\Phi_{xx}+A\Phi_{yy}=0.$$

As a result, using the method of argument of functions of a complex variable, an analytical solution to the plane problem of the theory of plasticity under stress was obtained, taking into account the variable value of H_σ:

$$\sigma_x=H_\sigma\exp\theta\cos A\Phi+\sigma_0+f(y),$$

$$\sigma_y=-H_\sigma\exp\theta\cos A\Phi+\sigma_0+f(x),$$

$$\tau_{xy}=H_\sigma\exp\theta\sin A\Phi,$$

$$\sigma_0=\mp nH_\sigma\exp\theta\cos A\Phi,$$

at:

$$\theta_x=-A\Phi_y,\ \ \theta_y=A\Phi_x,$$

$$\theta_{xx}+\theta_{yy}=0, A\Phi_{xx}+A\Phi_{yy}=0. \quad (24)$$

The value of boundary conditions lies in the fact that they are different at the input and exit from the source of deformation (asymmetry). In the substitution we have:

$$C_0=\frac{k_0\xi_0}{\exp\theta_0\cos A\Phi_0},\ C_1=\frac{k_1\xi_1}{\exp\theta_1\cos A\Phi_1},$$

$$\psi_0=\tan A\Phi_0,\ \ \psi_1=\tan A\Phi_1,$$

where $A\Phi_0$ and $A\Phi_1$, θ_0 and θ_1, k_0 and k_1, ξ_0 and ξ_1 – the value of the functions AΦ and θ, shear resistance; coefficients that take into account the influence of support and tension during rolling at the entrance and exit from the deformation zone.

For further analysis, it is necessary to determine the argument of the function AΦ and θ. Having solved the Laplace equations, we reconcile them with the Cauchy-Riemann conditions. We have the first argument function for the trigonometric relationship:

$$A\Phi'=AA_6'\left(\frac{l}{2}+x\right)y-AA_6''\left(\frac{l}{2}-x\right)y-2\phi=$$
$$=AA_6'\left[(x-X_0)+\left(\frac{l}{2}+X_0\right)\right]y+$$
$$+AA_6''\left[(x-X_0)-\left(\frac{l}{2}-X_0\right)\right]y-2\phi, \quad (25)$$

where $\varphi=(l-x)/R$ – angle of inclination of the contact area; X_0 – position of the neutral section relative to the origin; AA_6', AA_6'' – constants that determine the values of trigonometric functions along the edges of the deformation zone. For example, given boundary conditions; $x=l/2$, $y=h_1/2$,

$\varphi=0$, $A\Phi'=A\Phi_1-\alpha$, at $x=-l/2$, $y=h_0/2$, $\varphi=\alpha$, $A\Phi'=-A\Phi_0$ the integration constants are determined:

$$AA_6' = 2\frac{A\Phi_1-\alpha}{lh_1},\quad AA_6'' = 2\frac{A\Phi_0+2\alpha}{lh_0}. \qquad (26)$$

As the analysis shows, expressions (26) to some extent characterize the interaction of the lagging and advance zones. Taking into account the Cauchy-Riemann relations and Laplace equations (17), (18), the function θ is found. We have:

$$\theta = -\frac{1}{2}\left(AA_6'+AA_6''\right)\left[\left(x+X_0\right)^2-y^2\right]-$$
$$-\left(AA_6' l_{lag}-AA_6'' l_{adv}\right)\left(x-X_0\right). \qquad (27)$$

Taking into account (25) and the boundary conditions, a neutral angle shall be determined showing the position of the neutral cross-section at the contact at the source of deformation:

$$\gamma = \frac{\alpha}{2}\frac{A\Phi_1-\alpha}{\left(A\Phi_0+2\alpha\right)\left(1-\frac{1}{2}\varepsilon\right)}. \qquad (28)$$

The analysis shows that it is possible to accept:

$$A\Phi_1 = A\Phi_0 = f\left(a-bf\right), \qquad (29)$$

where α is the grip angle; ε – relative compression; f – friction coefficient; *a, b* are constant coefficients; as a first approximation, $a=b=1$ can be taken. Using expression (27), taking into account the boundary conditions, allows us to find the constants θ_0 and θ_1:

$$\theta_0 = -\frac{1}{2}\left(AA_6'+AA_6''\right)\left(l_{lag}^2-\frac{h_0^2}{4}\right)+$$
$$+\left(AA_6' l_{lag}-AA_6'' l_{adv}\right)l_{lag},$$

$$\theta_1 = -\frac{1}{2}\left(AA_6'+AA_6''\right)\left(l_{adv}^2-\frac{h_1^2}{4}\right)+$$
$$+\left(AA_6' l_{lag}-AA_6'' l_{adv}\right)l_{adv}.$$

By substituting the value for the expression C_i, taking into account (25) to (29), we obtained working formulas for calculating the stressed state of the strip during rolling, including for a single-zone deformation zone (with limited gripping ability of the rolls):

$$\sigma_x = -\frac{\frac{k_0}{\cos A\Phi_0}\left(\frac{l}{2}-x\right)\exp\left(\theta-\theta_0\right)+\frac{k_1}{\cos A\Phi_1}\left(\frac{l}{2}+x\right)\exp\left(\theta-\theta_1\right)}{l}\cos A\Phi+k_0,$$

$$\sigma_y = -3\frac{\frac{k_0}{\cos A\Phi_0}\left(\frac{l}{2}-x\right)\exp\left(\theta-\theta_0\right)+\frac{k_1}{\cos A\Phi_1}\left(\frac{l}{2}+x\right)\exp\left(\theta-\theta_1\right)}{l}\cos A\Phi+k_0, \qquad (30)$$

$$\tau_{xy} = \frac{\frac{k_0}{\cos A\Phi_0}\left(\frac{l}{2}-x\right)\exp\left(\theta-\theta_0\right)+\frac{k_1}{\cos A\Phi_1}\left(\frac{l}{2}+x\right)\exp\left(\theta-\theta_1\right)}{l}\sin A\Phi.$$

Mathematical analysis (30) reveals that, taking into account the boundary conditions, two damped mutually inverse functions penetrating each other appear at the edges of the deformation zone, forming overlap zones. These overlap zones can influence the distribution of generalized functions along the length of the deformation zone. From the point of view of the physical model, the indicated mathematical overlap zones and different boundary conditions along the edges of the deformation zone analytically characterize the interaction of multidirectional metal flows in a single deformation zone. Using formulas (30), the values of relative contact stresses were calculated.

5. 2. Studying the features of plastic deformation under conditions of a two-zone and single-zone deformation zone

Fig. 1, 2 show diagrams of contact normal and tangential stresses for processes with different values of the friction coefficient f and the shape factor l/h_{av}. Relative stresses are designated as $\sigma_y/2k$ and τ_{xy}/k, relative length of the deformation zone x/l. The external influence is represented by parameters through the friction coefficient and the deformation zone shape factor. Fig. 1 shows the stress distribution along the length of the deformation zone depending on the friction coefficient.

Tangential stresses represent a two-zone and single-zone source with a minimum friction coefficient of 0.05. In the lag zone, the normal stress diagrams mainly have a concave shape, which is characterized by a weakening of the load in the area; in the advance zone – convex, which is characterized by increased load. This leads to increased asymmetry of contact stresses between the lagging and advance zones within a single deformation region. The tangential stresses in the lagging zone are indicated by maximum values and advance zones of varying lengths.

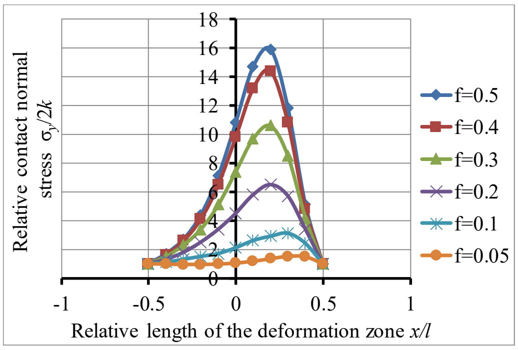
Fig. 2 shows the stress distribution along the length of the deformation zone depending on the shape factor l/h. As it increases, several parameters of the diagrams change. The magnitude of the stresses increases, and the shape of the stress diagrams changes significantly.

The maximum values of normal stresses shift towards the entrance to the deformation zone, and the tangential stresses in the lag zone also shift in the same direction. The length of the advance zone is the same for all loading modes. In general, such qualitative indicators of the distribution of contact stresses are known and supported by experimental studies [37, 38]. The peculiarity of our data is that the theoretical problem was solved for a single deformation zone, which increases the reliability of the result (30).

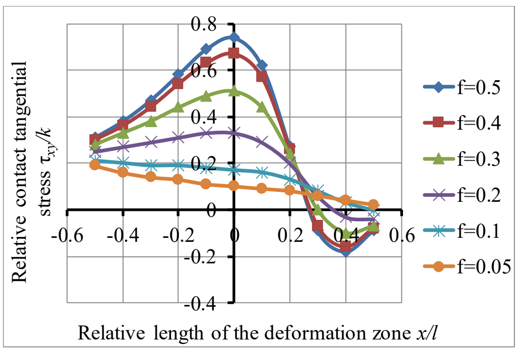
The presented distribution of contact stresses is confirmed by experimental data [37], Fig. 3.

This asymmetry of stress distribution is based on a certain physical model of the process, characterized by the interaction of zones of multidirectional metal flow. In the lag zone, Fig. 4, there are buoyant contact forces of normal pressure p_x and retractive friction forces τ_x, which are directed in opposite directions, causing the appearance of longitudinal tensile stresses in the plastic medium.

In the advancing zone, these forces are co-directed and located opposite the rolling progress, Fig. 4.

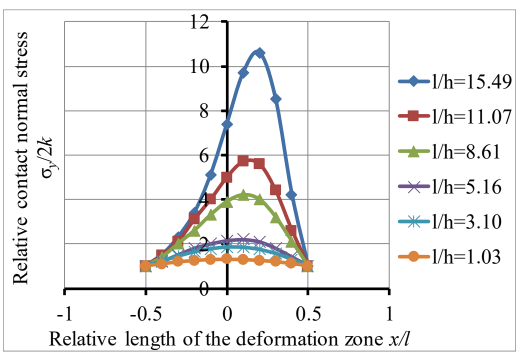


a



b

Fig. 1. Distribution of relative contact stresses along the length of the deformation zone depending on contact friction, at l/h=15.49; α=0.077; f=0.05...0.5: *a* – distribution of normal stresses; *b* – distribution of tangential stresses



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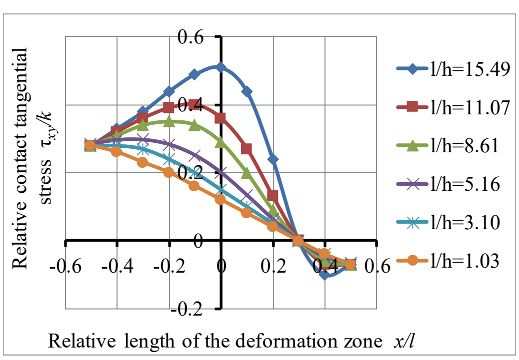
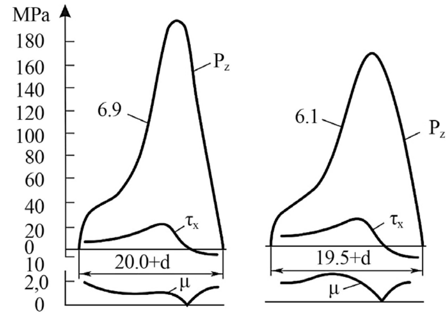
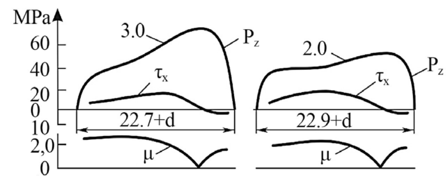


b

Fig. 2. Distribution of contact stresses along the length of the deformation zone depending on the shape factor at l/h=1.03...15.49; α=0.077; f=0.3: *a* – distribution of normal stresses; *b* – distribution of tangential stresses



a



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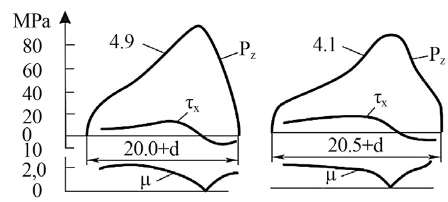

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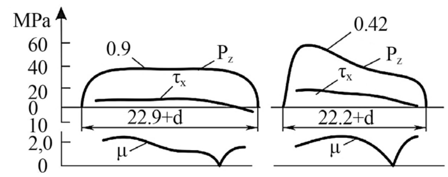


d

Fig. 3. Experimental data on the distribution of contact stresses as a function of the shape factor l_d/h_{av}: *a* – l_d/h_{av}=6.9; 6.1; *b* – l_d/h_{av}=4.9; 4.1; *c* – l_d/h_{av}=3.0; 2.0; *d* – l_d/h_{av}=0.9; 042; l_d – deformation zone length: h_{av} – average height [37]

In the first case, there is a decrease in the stressed state of the strip; in the second, there is an increase in the influence of supporting contact forces in the advance zone. With increasing compression (decreasing the gripping ability of the rolls), the effect of the buoyant force increases, which increases the tensile component. The interaction of the zones increases; the influence of the supporting forces of contact friction in this part of the deformation zone is weakened [32]. At the same time, the deflection of the normal stress diagrams increases (decreased contact stresses).

According to Fig. 1, Table 1 gives the results of measurements of the stressed state of the strip depending on the external influence associated with the friction coefficient.

Table 1

Results of the strip stress state study at l/h=15.49; α=0.077; f=0.05...0.5

Coefficient of friction f	0.5	0.4	0.3	0.2	0.1	0.05
Maximum stress state coefficient n_σ	15.9	14.4	10.6	6.5	3.1	1.5
Average stress state coefficient n_σ	6.9	6.3	5.0	3.4	1.9	1.1
Parameter f/α	6.49	5.19	3.90	2.60	1.30	0.65

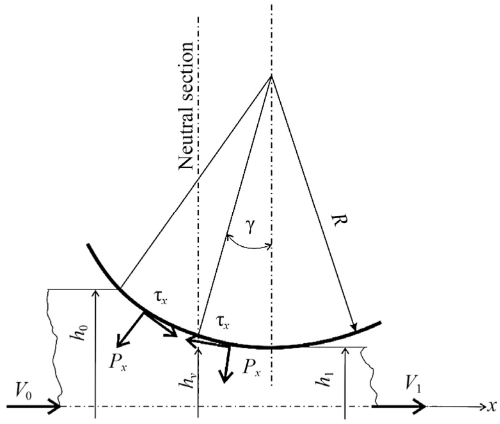


Fig. 4. Strength characteristics of the deformation zone during rolling

When analyzing the data in Table 1 and Fig. 1 some features of the process appear. With a friction coefficient of 0.05, the stability coefficient of the rolling process is less than one and equal to 0.65. With this parameter of the stability coefficient, there should be no metal gripping by the rolls. This conclusion is confirmed by the tangential stress diagram in Fig. 1. A single-zone deformation zone is shown (the diagram does not cross the neutral line). At the same time, Table 1 and Fig. 1 show that the local and general indicators of the stress state are greater than one, and accordingly equal to n_σ=1.1;1.5. This indicates a stable rolling process. There is some contradiction that needs to be justified. This phenomenon can be explained by the fact that the rolling process is realized with a negative advance, or an advance equal to zero [1–3].

Fig. 2, with a friction coefficient f=0.3 shows diagrams of the distribution of contact stresses with a change in the thickness of the strip, i.e., the shape factor. The friction coefficient is significantly greater than the grip angle, which eliminates the appearance of a single-zone deformation zone. As the shape factor decreases, the contact stresses decrease, which corresponds to studies [37] and Fig. 3.

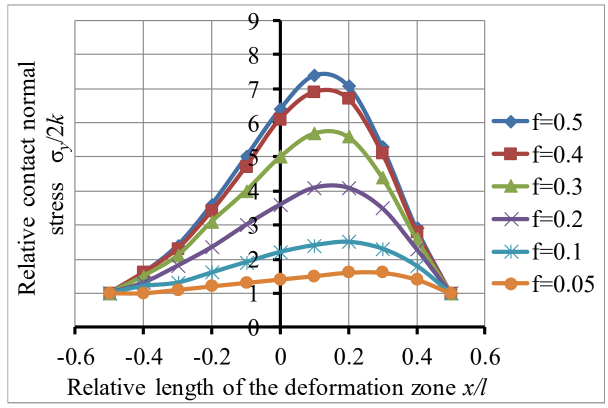
Table 2 gives the results of a study of the stressed state of the strip depending on the deformation zone shape factor l/h_{av}. Continuing the analysis, we are convinced that there are general trends in changes in the stressed state of the metal, Fig. 2, Table 2.

Table 2

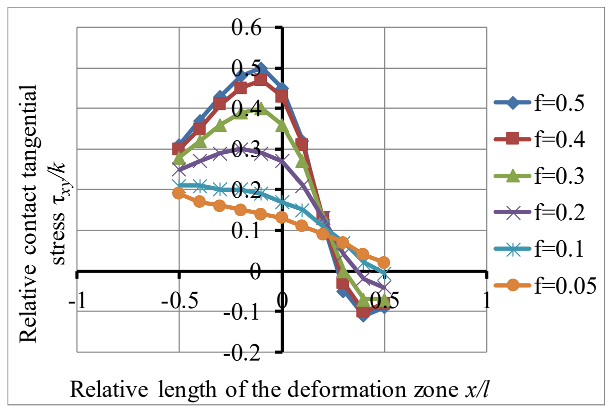
Results of studying the stressed state of the strip depending on the deformation zone shape factor at l/h=15.49...1.033; α=0.077; f=0.3

Shape factor l/h_{av}	15.49	11.07	8.61	5.16	3.10	1.03
Maximum stress coefficient n_σ	10.6	5.7	4.2	2.2	1.9	1.3
Average stress factor n_σ	5.0	3.3	2.6	1.7	1.5	1.2
Parameter f/α	3.90	3.90	3.90	3.90	3.90	3.90

A similar dependence of the influence of the shape factor takes place in Fig. 3. Comparative analysis enhances the reliability of the resulting mathematical model (30). The influence of one more indicator is considered: relative compression or grip angle, which is largely decisive when gripping metal with rolls. For a correct assessment, processes with the same shape factors and sets of friction coefficients, but with different grip angles, are considered. Fig. 5 shows the distribution of contact stresses depending on the friction coefficient f, without slipping of the rolls, with the same value of the shape factor equal to 11.07, with a minimum grip angle equal to 0.077.



a



b

Fig. 5. Distribution of contact stresses along the length of the deformation zone at l/h=11.07, α=0.077 depending on the friction coefficient f: *a* – distribution of normal stresses; *b* – distribution of tangential stresses

Table 3 gives the results of a study of the stressed state of the strip depending on the friction coefficient for a given deformation zone shape factor l/h_{av}.

The general patterns that occur during rolling with the shape factor l/h=15.49 are present in processes with the shape parameter l/h=11.07. As the friction coefficient increases, the stress state coefficients increase. A stable process is realized, including a single-zone deformation zone.

Table 3

Results of studying the stressed state of the strip depending on the friction coefficient at l/h=11.07; α=0.077; f=0.5...0.05

Coefficient of friction f	0.5	0.4	0.3	0.2	0.1	0.05
Maximum stress state coefficient n_σ	7.4	6.9	5.7	4.1	2.5	1.6
Average stress state coefficient n_σ	4.0	3.8	3.3	2.6	1.8	1.3
Parameter f/α	6.49	5.19	3.90	2.60	1.30	0.65

Comparing the data in Tables 1, 3, one should pay attention to the differences. In Table 3, at the same friction coefficients, the stress state coefficients are lower, which is a reliable fact.

The process is considered when the friction coefficient is less than the grip angle. In this case, the capture index is f/α=0.65. It shows that at a friction coefficient of f=0.05, a violation of the stability of the rolling process should be expected. And this is confirmed by the diagram of the distribution of contact tangential stresses, in which a single-zone deformation zone is indicated, Fig. 5. The diagram of the distribution of normal contact stresses shows that with such a friction coefficient there are no characteristic signs of strip slipping, no signs of concavity of the diagram along the length of the deformation zone, including the lag zone. This indicates that there is no partial slippage of the strip. This contradiction can be explained by the fact that the rolling process is possible with a negative advance, or an advance equal to zero, [1–3]. It follows that the physical and mathematical models (30) take into account this process factor and allow them to respond adequately to it.

Analysis of Table 3 provides confirmatory data for the adoption of a physical model of the process, which will allow one to evaluate the stressed state of the strip during the slipping process. A comparison was made of the normal stress diagram (shape factors 15.49 and 11.07, Fig. 1, 5, for a friction coefficient of 0.05) and the data given in Tables 1, 3. There is a discrepancy between the general tendency of the influence of these factors on the stress state in the deformation zone. Indeed, with a larger value of the shape factor l/h=15.49, the stress state (n_σ=1.5;1.1) should be greater than for the shape factor l/h=11.07 (n_σ=1.6;1.3). However, Tables 1, 3 show inverse dependences under conditions of a single-zone deformation zone. This can be explained from the standpoint of the accepted physical model of the process. Near the limiting source of deformation, the influence of buoyancy forces increases, and with a decrease in the cross-sectional area of the strip (shape factor l/h=15.49), the effect of tensile stresses in the lagging zone increases significantly. This causes a decrease in contact stresses in this zone and a greater concavity of the normal stress diagram, Fig. 1. This leads to another conclusion: the concavity of the normal stress diagrams is a characteristic of the influence of tensile stresses in the lagging zone, which is reflected in the physical and mathematical models of the process.

In continuation of the analysis of the stressed state of the metal under the same loading parameters, a process with a large capture angle is considered. Fig. 6 shows the distribution of normal and tangential stresses that respond to the loss of stability of the rolling process, with comparable friction coefficients and grip angles. In this case, the compression was increased from α=0.077 to α=0.129, with the deformation zone shape l/h=11.04 in the range of friction coefficients f=0.05...0.5.

Therefore, the mathematical model (30) must react not only to the coefficient of friction, the factor of shape, but also to the angle of capture, which increases the possibilities of evaluating the process and identifying features.

Table 4 gives the results of a study of the stressed state of the strip depending on the friction coefficient for a given deformation zone shape factor l/h_{av}. With an increase in the friction coefficient, the stress state coefficients increase, as was the case in the data in Tables 1, 3. However, comparing the results of Tables 1, 3 and Fig. 1, 5, it can be seen that in the latter case the stress state coefficients are greater. This indicates the response of the model to the grip angle, which has increased. In the case of the capture angle α=0.129, already two tangential stress curves (f=0.05; 0.1) do not intersect the zero line, i.e., already two processes are realized under the conditions of a single-zone deformation zone.

A process with a complete loss of stability with a minimum coefficient of friction has emerged.

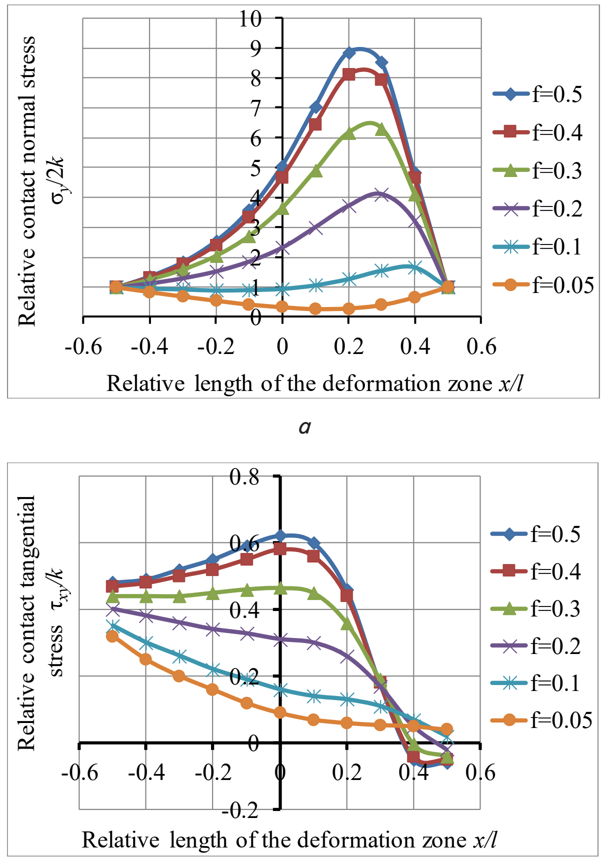


Fig. 6. Distribution of contact stresses along the length of the deformation zone depending on contact friction, at l/h=11.04, α=0.129, f=0.05...0.5: *a* – distribution of normal stresses; *b* – distribution of tangential stresses

Characteristics of the normal stress diagram during loss of process stability: minimum friction coefficient f=0.05, grip angle α=0.129; parameter characterizing the stability of the rolling process f/α=0.39<1; stress factor n_σ<1.

Table 4

Results of studying the stressed state of the strip depending on the friction coefficient at: l/h=11.04, α=0.129, f=0.05...0.5

Coefficient of friction f	0.5	0.4	0.3	0.2	0.1	0.05
Maximum stress factor n_σ	8.9	8.1	6.3	4.1	1.7	0.7
Average stress factor n_σ	4.2	3.9	3.2	2.2	1.1	0.6
Parameter f/α	3.88	3.10	2.33	1.55	0.78	0.39

The distribution of contact normal stresses has a clearly concave shape along the entire length of the deformation zone for n_σ less than unity. Tangential stresses define a single-zone deformation zone with a margin. With such diagrams, rolling with a negative advance is impossible. Using this form of the diagram of normal and tangential stresses, the process of loss of stability, i.e., metal slipping in the rolls, will be determined in the future, Fig. 6. It is noteworthy that the mathematical model (30) characterizes the distribution of tangential and normal stresses during the process of strip slipping in rolls. From a comparison of Fig. 5, 6 it is clear that at the same process parameters, with increasing compression, the maximum contact stresses increased. The deflection of the diagrams in the lag zones increases; the peak of normal stresses shifts towards the entrance to the deformation zone. However, complete slipping, according to [32], occurs when the normal stress diagram is realized with a complete concavity of the diagram along the length of the deformation zone. In the second single-zone deformation zone, stable rolling is realized, in accordance with works [1–3]. There is no concavity of normal stresses along the entire length.

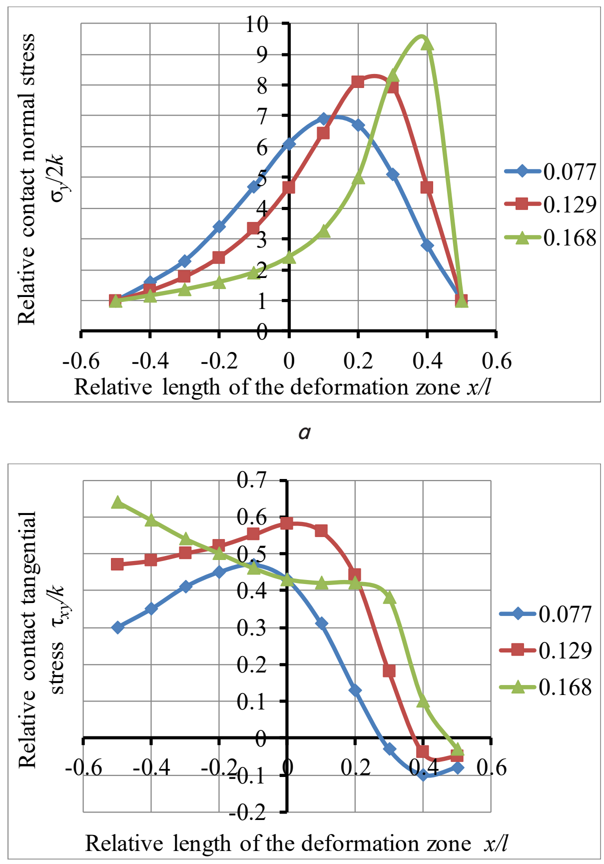
To identify interaction effects, it is possible to compare the data in Tables 3, 4 in progress, with different small friction coefficients. Comparing the data with a friction coefficient of 0.2 and different grip angles, we have: with less compression f=0.2; α=0.077; n_σ=4.1;2.2; at a larger angle f=0.2; α=0.129; n_σ=4.1;2.2. If the local stress state coefficients are equal, a smaller angle corresponds to a larger average stress state coefficient, and a larger angle corresponds to a smaller one. Thus, the coefficients characterizing the interaction of zones with the opposite flow of metal decreased by 15.4 %, which is a rather interesting fact. Under conditions of changed interaction, the average stress state does not increase in accordance with compression but decreases. This provision will be subsequently strengthened.

When moving to the friction coefficient f=0.1, a single-zone deformation zone appears, which is confirmed by the parameter f/α=0.78, less than unity. Comparing the data in Tables 3, 4, we analyze the following values of the stress state coefficients: with grip angle α=0.077; n_σ=2.5;1.8, with grip angle α=0.129; n_σ=1.7;1.1. With an increase in compression by 40.3 %, the stress state coefficients decreased by 32.0 % and 38.9 %, respectively. Despite the decrease in stress state coefficients, the process at a given friction coefficient proceeds without slipping of the rolls. This is confirmed by the fact that the stress state coefficients in the advance zone are greater than one, and there is no dip in the normal stress diagram along the entire length of the deformation zone. Thus, according to the gripping parameters, we have a single-zone deformation zone, and according to the stress state coefficients, a stable rolling process, which confirms the previously drawn conclusions of [1–3].

Further, comparing the data in Tables 3, 4, with a friction coefficient f=0.05, we have the process parameters: with less compression f=0.05; α=0.077; n_σ=1.6;1.3, with greater compression f=0.05; α=0.129; n_σ=0.7;0.6. The capture parameter f/α=0.39 shows that there is a single-zone deformation zone, Fig. 6 (tangential stresses). The coefficients of maximum and average support dropped to values n_σ=0.7;0.6, i.e., to values of both coefficients less than one. According to all the given indicators, roll slippage occurs, which is shown by the calculated parameters and the type of concave diagram of contact normal stresses, Fig. 5. There is a fact of restructuring of the stressed state of the metal under conditions of strip slipping. The change in the stressed state of the strip under these conditions, i.e., loss of stability, is clearly demonstrated. This proves the adequacy of model (30) when the external influence changes under extreme conditions.

The very fact of the influence of the grip angle on the parameters of the stress state in the complex of influence of various factors was shown, Fig. 1, 5, 6. A peculiarity was determined that at this stage of the study a certain pattern of decreasing force load with increasing deformation load is visible. This fact should be studied in more detail, leaving all process parameters constant, except for the capture angle, to obtain the result in terms of clarity of the process, confirmed capabilities of the mathematical model (30). It is useful to consider several processes with the same shape factor and friction coefficient, but at different grip angles.

Fig. 7 shows diagrams of the distribution of contact stresses depending on the grip angle for the same values of the friction coefficient and shape factor. According to the diagrams of the distribution of tangential stresses, there are two-zone centers of deformation (we are not talking about extreme rolling processes). Features of the obtained diagrams, Fig. 7: there are processes that combine opposite changes in local force loading parameters and those common to the entire deformation zone.



a

b

Fig. 7. Distribution of contact stresses along the length of the deformation zone depending on the grip angle: at l/h=11.04, α=0.077;0.129;0.168, f=0.4: *a* – distribution of normal stresses; *b* – distribution of tangential stresses

The obtained result is given in Table 5. With the friction coefficient f=0.4, all three processes are far from prohibitive. Stability coefficients are in the range of 2.38...5.19. This characterizes them as sustainable processes. Tangential stresses also respond to compression at the entrance to the deformation zone. Despite the stability of the rolling process under all deformation modes, the tangential stresses for a grip angle of 0.168 have a minimum length and magnitude of the tangential stress in the advance zone [29, 32].

Table 5

Results of studying the stressed state of the strip depending on the grip angle at l/h=11.04, f=0.4

Grip angle α	0.77	0.129	0.168
Maximum stress state coefficient n_σ	6.9	8.1	9.4
Average stress state coefficient n_σ	3.8	3.9	3.3
Parameter f/α	5.19	3.10	2.38

Under this mode, in the lagging zone, tensile stresses reach maximum values, which affects the deflection of the normal stress diagram. An increase in compression has a dual effect on the deformation site. The maximum normal stresses increase and at the same time the stresses in the lag zone decrease. Due to the increasing deflection of the diagrams in the lag zone, with increasing compression, the average normal stresses decrease along the entire length of the deformation zone. From Table 5 it can be seen that at grip angles, respectively α=0.077;0.129;0.168, the maximum stress coefficient is equal to n_σ=6.9;8.1;9.4. This is natural when, with increasing compression, the stressed state of the deformation zone in a given zone increases. The maximum support coefficients accordingly increase by 13.8...14.8 %. However, the average stress coefficients change differently. They do not grow but decrease. This is affected by the influence of the interaction of zones with the opposite flow of metal within the deformation zone. Indeed, at the beginning, the stress state did not actually change when moving to a grip angle of 0.128, and then its decrease reached a significant value. For average stress state coefficients, changes take place n_σ=3.8;3.9;3.3. With an increase in compression by 54 %, the decrease in the average stress coefficient was 15.4 %. There is only one explanation for this effect: the presence of increasing tensile stresses in the lagging zone.

5. 3. Analysis of the stressed state of the metal under conditions of interaction of zones with counter-directional flow of metal

The interaction of zones with counter-directional metal flow should be interpreted as the effect of rear external tension on the deformation zone [37].

The effect of posterior external tension on the deformation site is characterized by:

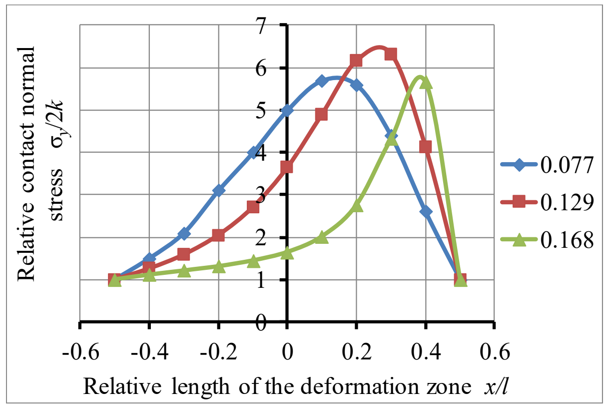
– reduction of specific pressures in the lag zone;

– displacement of the maximum normal stress towards the exit of the metal from the rolls;

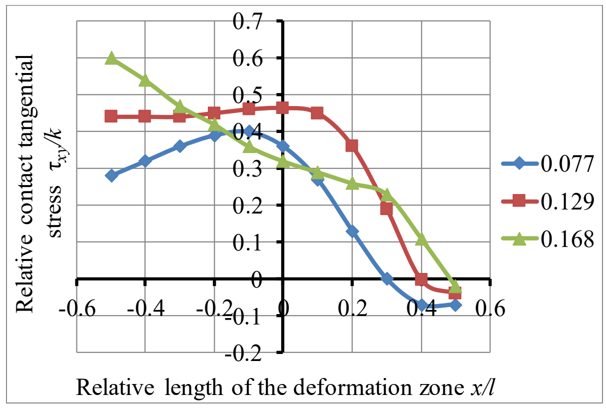
– reducing the advance zone.

The same parameters characterize the effect of interaction of zones with counter-directional flow of metal in the deformation zone. It follows from this that the appearance of tensile stresses due to the counter-directional flow of metal is enhanced by a high index of the deformation zone shape factor. This noticeably increases tensile stresses due to the small rolling height. It is of interest to evaluate the rolling process, which in its parameters approaches the limiting process. This is possible by reducing the friction coefficient.

Another illustrative example of the operation of model (30) is offered. Fig. 8 shows the distribution of contact stresses depending on compression, with shape factor l/h=11.04, friction coefficient f=0.3. The process is similar to the previous one, only with a lower friction coefficient.



a



b

Fig. 8. Distribution of contact stresses along the length of the deformation zone depending on the grip angle, at: l/h=11.04; α=0.077;0.129;0.168; f=0.3: *a* – distribution of normal stresses; *b* – distribution of tangential stresses

Table 6 gives the values of the stress state coefficients and parameters f/α, which decreased, at the same grip angles α=0.077; 0.129; 0.168. Reducing the parameters f/α brings the process under study closer to the limiting process, to metal slipping in rolls, which can change the conditions of interaction of zones in the deformation zone. In this regard, qualitative changes should occur in the contact stress diagrams, Fig. 8. There is a restructuring of processes from stable formation modes to modes under which there is a partial or complete loss of stability of the process, with different diagrams of contact stresses.

Table 6

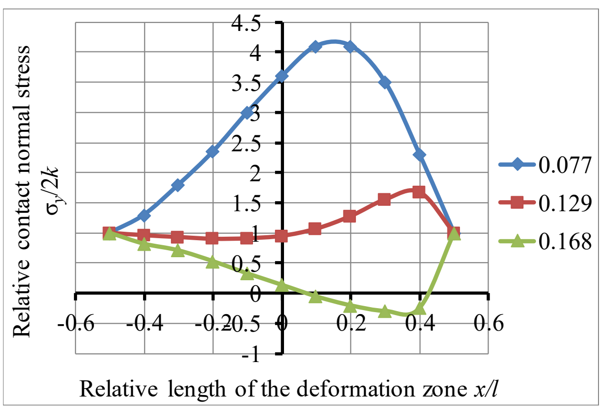
Results of studying the stressed state of the strip depending on the grip angle at l/h=11.04; f=0.3

Grip angle α	0.077	0.129	0.168
Maximum stress state coefficient n_σ	5.7	6.3	5.7
Average stress state coefficient n_σ	3.3	3.2	2.1
Parameter f/α	3.90	2.33	1.79

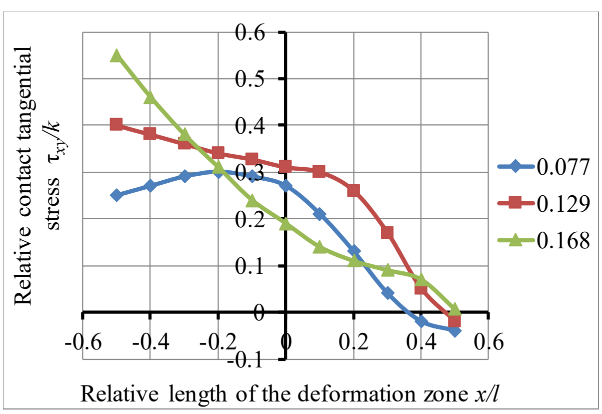
The result from Table 6 is considered in comparison with the data in Table 5, Fig. 7. There is a decrease in the stress state with a decrease in the friction coefficient for all studied parameters for both processes, which is understandable. However, with a friction coefficient of 0.3, there is a decrease in the average stress coefficient with increasing grip angle, Table 6. Significant deflections of the normal stress diagrams are observed, especially for maximum compression, which is confirmed by a decrease in the average stress state coefficient within the range of 2.1...3.3. A two-zone deformation zone is realized. This process according to Table 6 and diagrams in Fig. 7, 8 seems stable.

It should be noted that against the background of changes, there is a redistribution of all force parameters presented in Fig. 8. Feature: reduction of maximum stress state coefficients, which is the determining factor before the process loses stability. In Fig. 8, the tangential stresses at the beginning of the deformation zone are in accordance with the compression, as for the diagrams, Fig. 7. Prerequisites for approaching the limiting process appear.

The next stage of research: the process is considered at the same compression, deformation zone shape coefficient, but with a lower friction coefficient equal to 0.2, comparable to the maximum grip angle. Fig. 9 shows diagrams of contact normal and tangential stresses at the corresponding process parameters. A feature of these processes is the completion of restructuring changes in strength characteristics in the deformation zone, which were already identified in previous studies.



a



b

Fig. 9. Distribution of contact stresses along the length of the deformation zone depending on the grip angle, at: l/h=11.04; α=0.077;0.129;0.168; f=0.2: *a* – distribution of normal stresses; *b* – distribution of tangential stresses

Table 7 gives the results of studying the stressed state of the strip depending on the grip angle, with l/h=11.04 and friction coefficient f=0.2.

Table 7

Results of studying the stressed state of the strip depending on the grip angle at l/h=11.04; f=0.2

Grip angle α	0.077	0.129	0.168
Maximum stress state coefficient n_σ	4.1	1.7	
Average stress state coefficient n_σ	2.6	1.2	
Parameter f/α	2.60	1.55	1.19

From the data in Table 7 it can be seen that the minimum grip angle of 0.077 radians corresponds to the maximum stress coefficient equal to n_σ=4.1. For the same grip angle, the maximum is the average stress state coefficient equal to n_σ=2.6. An intermediate angle of 0.129 radians, which is 40.3 % greater than the minimum, corresponds to a maximum stress coefficient equal to n_σ=1.7, which is 58 % lower than the previous one. The average stress factor, equal to n_σ=1.2, is also 54 % less than the previous one. At the maximum value of the grip angle, which was 54 % greater than the minimum value, the pressure dropped below the critical level and the rolling process lost stability. It is clearly shown that under conditions that are as close as possible to the limiting rolling process, there is not a gradual decrease in pressure on the rolls, but a sharp drop in load, accompanied by a complete loss of stability. To assess the interaction process at the source of multidirectional metal flow zones in development, it is advisable to evaluate and compare the data of the last three processes.

For clarity, Table 8 compares the stress state indicators of tables with a shape factor of 11.04 at maximum and average values of the stress state coefficients, friction coefficients f=0.4; 0.3; 0.2. Together we consider restructuring processes characterized by the peculiarities of the interaction of zones in a single deformation zone.

Table 8

Comparative analysis of the results of studying the stressed state of the strip depending on the grip angle and friction coefficient, at l/h=11.04

Parameter name	Grip angle								
	0.077			0.129			0.168		
Friction coefficient	0.4	0.3	0.2	0.4	0.3	0.2	0.4	0.3	0.2
Maximum stress factor	6.9	5.7	4.1	8.1	6.3	1.7	9.4	5.7	less than 1
Average stress factor	3.8	3.3	2.6	3.9	3.2	1.2	3.3	2.1	less than 1

From Table 8 it can be seen that each grip angle corresponds to data on three friction coefficients, for the maximum and average stress state coefficients. Therefore, it becomes possible to compare processes not only by friction coefficients but also by grip angles. All three grip angles are characterized by a general tendency for the stress state coefficients to change depending on the friction coefficient. However, no clear trends are observed when changing capture angles. If, at a friction coefficient of 0.4, the maximum values of the stress state change in accordance with the grip angles, then for the average coefficients of the stress state this correspondence is violated. It can be argued that there is an accessibility zone determined by the inverse influence of the grip angle on the force parameters of rolling. With an increase in the grip angle, the parameters of the stress state do not increase but decrease. When the friction coefficient decreases to 0.3, these trends intensify. The maximum stress coefficients remain practically unchanged with increasing grip angle. And the average decreases by 36 %. At a friction coefficient of f=0.2, metal slipping in the rolls or a complete loss of stability of the process is observed. This is accompanied by a dip in the normal stress diagram along the entire length of the deformation zone. The coefficient determining the stability of the process f/α is within unity, which is quite acceptable.

The accessibility zone can be characterized as the proximity of a process with a limiting source of deformation, through the friction coefficient, to the process under consideration. It should be emphasized that the identified effects of plastic deformation presented above take place only in the accessibility zone. Otherwise, studies show that such effects are not observed.

The constructed physical model is based on the different nature of the stress state in the lagging and advance zones. Contact stresses weaken in the lagging zone and increase in the advance zone, which affects the asymmetry of stress distribution along the deformation zone. This determines the nature of the interaction and the rolling force.

In relation to the accessibility zone, the asymmetric interaction of sections with counter-directional flow of metal by mutually inverse damping functions penetrating each other, forming an overlap zone, is mathematically characterized. This made it possible to use a fairly effective approach to visually study the stress-strain state of the metal, taking into account intermediate schemes and multidirectional force and deformation loading modes.

In the work, the reliability of the result is confirmed in several areas. These include experimental data [37]; fundamental approaches related to the statement and solution of a closed problem in the theory of plasticity; development and testing of the method argument of functions of a complex variable.

Experimental data [37] confirm the qualitative calculated characteristics of the stress distribution along the length of the deformation zone, which respond equally to the effects of plastic deformation. Indeed, comparing the calculation results in Fig. 2 and experiment, Fig. 3, it is clear that the influence of the shape factor is unambiguous, both in theory and in experiment; the effects of the influence of the counter-directional flow of metal in the lagging and advance zones appear, equally influencing the shape of the normal stress diagrams. The maximum values of contact stresses with increasing thickness shift to the middle of the deformation zone, and with a shape factor equal to unity, the diagram acquires a symmetrical shape, as during upsetting, which is confirmed by numerous experimental data. Such coinciding analogies indicate confirmation of the physical model of the process and the response of mathematics to complex force loading patterns of mutually penetrating zones of lag and advance.

Fundamental approaches related to the statement and solution of a closed problem in the theory of plasticity enhance the reliability of the results obtained. This is due to the fact that the solutions in stress are confirmed by the solutions of the deformation problem and are consistent with each other through physical coupling equations. Ultimately, generalized defining functions appear that make it possible to close the problem by constructing a mathematical model of a plastic variable medium. This approach is classic and should not be neglected.

A generalized method for solving problems in continuum mechanics was repeatedly confirmed by publications in highly rated journals in various fields. Including the theory of elasticity [29–31], plasticity [15], and the theory of dynamic processes [28]. There are publications in such journals on geomechanics [39]. The method presents generalizations related not to finding solutions to differential equations but to identifying the conditions for their existence, which include differential invariant Cauchy-Riemann relations and Laplace equations. This makes it possible, while maintaining the qualitative indicators of the mathematical model, to change the quantitative result due to the boundary conditions, which is typical for solving semi-inverse problems of continuum mechanics.

6. Discussion of results of studying the stressed state of metal under conditions of interaction of zones with counter-directional flow of metal

Using the data of this study, it is possible to make a number of fundamental generalizations when solving problems in continuum mechanics, and in particular in the theory of plasticity. Using the example of solving an applied problem in the theory of rolling, the capabilities of the method of argument of functions of a complex variable in the theory of plasticity are shown. The peculiarity of the method is that it is not the solutions to the problem themselves that are found but the conditions for their existence, which allows them to be applied to different areas of continuum mechanics.

Using the same approaches to obtaining results in the theory of elasticity [26–28], dynamic processes [28], plasticity theory [15], and geomechanics [39], analytical solutions were obtained that reflect the processes of various applied problems.

Generalizations in applied problems of continuum mechanics are invariant relations imposed on the function argument in the form of Cauchy-Riemann differential relations and Laplace equations. The approaches of the argument method of functions of a complex variable in the theory of plasticity and, in particular, the theory of rolling, made it possible to solve the problem and construct a mathematical model of asymmetric rolling. In a single deformation zone, the interaction of zones with counter-directional flow of metal was taken into account. Such objects are (20) to (22), (24), (30), Fig. 1–9, Tables 1–7.

A special feature of the proposed approach is the consideration of a single source of deformation, which made it possible to determine the real physical and corresponding mathematical model of the process; take into account the interaction of zones with counter-directional flow of the medium in the deformation zone. The physical model is clearly presented in Fig. 4. In the lag zone, the force impact from the roll on the metal is fundamentally different from the force impact from the roll in the advance zone. The pushing and pulling forces in the lagging zone are directed in opposite directions, which, according to the literature data by various authors [32], causes the appearance of tensile stresses in this zone. In the advance zone, a different loading pattern is observed; these reactive forces are codirected in the direction opposite to the movement of the strip in the rolls, which enhances the influence of the supporting forces of contact friction on contact stresses. Thus, in the deformed metal of the lagging zone, supporting stresses from contact friction and tensile stresses from multidirectional forces from the roll act, which reduces the effect of supporting forces of contact friction on normal stresses. This physical model of the process was adopted as the determining one in the construction of the mathematical model.

This made it possible to identify the effects of plastic deformation associated with the combination of loading and unloading processes, kinematic and force parameters of rolling. The influence of local impact on the processes of the entire deformation zone is shown. It is shown that within the zone of accessibility of the limiting source of deformation, the effects of a decrease in the force load with an increase in the deformation effect are observed. This phenomenon occurs both for a two-zone deformation zone and for a single-zone deformation zone, which characterizes the rolling process under conditions of negative or zero advance. It should be emphasized that such effects are not observed beyond the accessibility of the limiting deformation zone. Repeated testing of the method in different models of continuum mechanics, comparison of the simulation results of this study with experimental and theoretical data by other authors confirm the reliability of the result obtained.

The limitations of this method are determined in the process of using it in relation to various applied problems of continuum mechanics. It becomes obvious that the method is limited to the use of a fundamental substitution that characterizes one of the basic functions.

The disadvantages of this study include the lack of data on the heterogeneity of the stress-strain state of the plastic medium throughout the entire source, not only on the contact surface.

The development of this research may consist in developing a process model that takes into account the influence of the heterogeneity of the distribution of the stress-strain state of the metal throughout the entire volume of a single deformation zone.

7. Conclusions

1. Physical and mathematical models of the plane problem of rolling theory have been constructed under conditions of changing asymmetry of the interaction of zones with multidirectional metal flow, under conditions of a two-zone and one-zone single deformation zone. A feature of the physical model is the asymmetric interaction of zones in a single deformation zone under conditions of complex counter-directional loading of the metal. A special feature of the mathematical model is that it takes into account the interaction of deformation zones that inversely penetrate each other through overlap zones. As a result, a method for solving problems of continuum mechanics in relation to the theory of plasticity, the method of argument of functions of a complex variable, was further advanced.

2. On the basis of the resulting mathematical model, the features of the stressed state of the plastic medium were studied. A zone of accessibility of the limiting source of deformation has been identified when the grip angles change within the range of 0.077...0.168, i.e., the zone of influence of the grip angle on the effects of reducing the force load with an increase in the deformation effect.

3. An analysis of the stressed state of metal was carried out under conditions of increasing interaction of zones with counter-directional flow of metal. It is shown that the interaction of the zones is equivalent to the action of posterior tension in the deformation zone. The influence of local impact on the processes of the entire deformation zone is shown with a shape factor in the range of 1.0...15.00.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The manuscript has associated data in the data warehouse.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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