

The object of this study is a tractor with a front plow without a support wheel. One way to avoid the use of ballast is to use front-mounted plows that operate under the "push" mode. As a rule, such plows are equipped with at least one supporting wheel. The presence of the latter complicates the structure of the plow and, of course, affects the degree of vertical load on the steered wheels of the mobile vehicle.

With the help of the constructed mathematical model and the corresponding amplitude and phase frequency characteristics, the dynamics of vertical oscillations of the front axle of a tractor with a front mounted plow without a support wheel were investigated. Vertical fluctuations of the total force acting on the tractor from the side of the plow were considered as a disturbing influence. According to the simulation results, an increase in the vertical load of the front axle of the tractor by 600 kg causes a desired decrease in the value of the amplitude and an increase in the phase of external disturbances of the dynamic system. The higher the frequency of disturbance oscillations, the more acceptable these characteristics become. It was established that in order to improve the response of the studied dynamic system to disturbances, it is necessary to reduce the stiffness coefficient of the tires of the front wheels. In practice, this is achieved by adjusting the air pressure in the tires. The amplitude-frequency characteristics of the system almost do not change when the damping coefficient of the tires of the front wheels of the tractor is increased in the range from 1 to 3 kN·s/m, while the phase-frequency characteristics improve. This is especially noticeable at the frequencies of oscillations of the disturbing influence in the range of 0–10 s<sup>-1</sup>.

The results could be used as a basis for evaluating the efficiency of tractors with a front plow without a support wheel in tillage operations. Such efficiency can be achieved under the condition of practical implementation of the recommendations proposed in this paper regarding the selection of design parameters of tires for the front wheels of the tractor

**Keywords:** plow unit, front attachment mechanism, "push-pull", front plow, tire stiffness coefficient, tire damping coefficient

# DETERMINING VERTICAL OSCILLATIONS OF FRONT-PLOW TRACTOR WITHOUT SUPPORT WHEEL

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## 1. Introduction

Soil cultivation is one of the most time-consuming and energy-intensive technological processes and is the most important component in the system of agrotechnical measures for the production of plant products [1]. The choice of a set of tillage tools has a significant impact on the indicators of the economic efficiency of the production of agricultural products. About 27 % of labor and 30 % of energy costs from the total costs of growing and harvesting crops fall on soil cultivation, of which about 48 % is plowing [2]. As a rule, the choice of a set of machines for plowing depends on the soil-climatic zone, physical and mechanical properties of the soil, requirements for the quality of processing. When choosing a plow unit, these factors should be taken into account in order to ensure favorable conditions

for the course of physical-chemical and biological processes in the soil [3].

During operation of a conventional plow unit, vertical loads are distributed along the tractor axles [4]. The essence of this phenomenon is that under the influence of the force of traction resistance of the plow, which in the longitudinal-vertical plane is inclined to the horizon, the rear axle of the tractor is loaded, and the front axle, on the contrary, is unloaded [5]. To avoid this, ballast loads are usually installed on this axis. It should be noted that adding ballast to the tractor reduces its skidding but increases fuel consumption and soil compaction. Therefore, it is necessary to eliminate the unloading of the front axle of the tractor without using its ballast.

Thus, the question of researching the dynamics of tractor axle loads, constructing mathematical models and corre-

sponding software based on them remains relevant. This could improve the efficiency of the plowing process and reduce fuel consumption and pressure on the soil.

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## 2. Literature review and problem statement

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The soil tillage machine described in work [6] can be considered a prototype of a modern plow. Modern plow designs are characterized by several fundamental differences. The method of aggregating the plow with the tractor is one of the most important differences. In particular, this is the use of a hinge system, which can be three-point or two-point. The three-point hitch system became the most common. Its main design parameters are scientifically based [4] and even standardized [7, 8]. At the same time, the research results reported in [4] cannot be applied in the case of using frontal plows in a unit configured according to the “push-pull” scheme. The requirements for attachment mechanisms set forth in standards [7, 8] do not take into account the statics or dynamics of the movement of plowing units.

Nevertheless, practice shows that the use of rear-mounted plows leads to an undesirable redistribution of vertical reactions on the tractor axles. In particular, they increase on the rear wheels and decrease on the front ones [9]. In order to find out the distribution of reactions on tractor axles with a three-point hitch system, the authors of [10–12] conducted a number of studies.

The authors of work [10] developed an adjustable dynamometer of 50 kN with a three-point attachment for measuring the force on the tractor and the attached implement. The dynamometer was installed on two frames fixed between the tractor drawbars and the tillage tool. The amount of effort was measured by a strain gauge installed between the frames. Measurements were performed in a wide range of tool sizes. The dynamometer can be used when testing all mounted tillage tools of categories II and III (plows, cultivators, harrows), with the exception of mounted tools with a drive from the power take-off shaft. Several field tests were conducted to measure the force required to pull the plow under clay-loam soil conditions. Field tests have shown that the plow can effectively work as intended without any mechanical problems. However, a significant limitation to the wide use of the results of this study is the type of soil and the conditions of the study.

The authors of study [11] recommend using a front three-point attachment when aggregating tillage implements. This increases the load on the front axle, which leads to a decrease in traction force. Also, when using a front linkage, one should use a tractor with four-wheel drive or with a mechanical drive on the front wheels. The results of the study are applied to the scheme of fastening the cultivator in front and the harrow in the back, which made it possible to save 21.9 % of time and 18.9 % of fuel. However, with a three-point plow attachment scheme under the same experimental conditions, such savings in time and fuel are unlikely to be achieved. The results are relevant only for the experimental conditions described in study [11].

Work [12] reports an algorithm for calculating the total force and moment acting on the tractor with a three-point attachment of the plow unit. Also, the algorithm makes it possible to calculate the forces acting at the point of contact of the wheels with the ground. The model is three-dimensional, describes the kinematics of the system with a three-

point attachment, including vertical and lateral movements. Forces are determined on the basis of static balance. The results of laboratory and field studies proved the high accuracy of determining the magnitude and component forces of the tractor plow load according to the developed algorithm. The presented calculation algorithm can be used as a basis for constructing mathematical models that will describe the three-point attachment of a tractor with a plow unit.

The authors of works [13, 14] believe that the redistribution of vertical reactions on the tractor axles can be changed by adjusting the rear hitch of the tractor, but it cannot be completely eliminated.

The studies reported in [13] are similar to those in [10]. In the cited work, the geometric parameters of the three-point hitch of the tractor were determined by constructing the trajectories of the movement of the upper and lower points of the hitch based on the kinematic analysis of the hitch mechanism. With different positions of the pivot point of the upper link and corresponding adjustment of the length of the lifting links, the three-point hitch system of the tractor met all the requirements of the standards for hitching systems of the I and II categories. Fixing the top link to the highest point reduced the change in orientation of the soil tillage implement during lifting and provided better weight transfer from the implement to the rear axle of the tractor. The presented kinematic analysis makes it possible to choose the best method of attachment of the attachment system in order to effectively use the power of the tractor during various tillage operations.

The results from [14] became the basis for the development of a computer program in Visual Basic 6.0. The program helps determine the virtual point of adhesion of the tractor to the soil depending on the depth of cultivation and to optimize the parameters of the three-point linkage. That makes it possible to align the virtual point of coupling with the line of traction force. The program can be useful for modeling both the three-point linkage and tillage implement parameters depending on the tillage depth requirements.

In work [15], the authors proved that since the front wheels of the tractor are usually steerable, the reduction of their vertical load negatively affects the controllability of the horizontal movement of the plow unit. This, in turn, leads to deterioration of the operational characteristics of the plowing process.

To solve this problem, the installation of ballast on the front axle of the tractor is considered in [16]. In contrast to the predecessors in [10–16], the justification of the level of mechanical ballasting of wheeled tractors was carried out taking into account the permissible maximum pressure of the tractor wheels on the soil and the maximum load capacity of the wheel tires. The procedure is based on taking into account the ecophilic properties of the tire. The work calculates the maximum permissible level of pressure of the tractor wheels on the soil, taking into account the maximum bearing capacity of the tires. The magnitude of the vertically applied load on each of the tractor axles with the simultaneous action of both the horizontal and vertical components of the traction resistance of the used tillage tool is also determined. The practical effectiveness of this approach consists in determining the possibility and level of ballasting of a specific wheeled tractor working as part of a plowing unit.

However, in [17] it is noted that adding ballast to a tractor reduces wheel slip and increases fuel consumption and soil compaction. It would be more appropriate to eliminate

the unloading of the front axle of the tractor without the use of a ballasting mechanism.

One possible solution to this problem is the use of plows with a front hitch, which work under the “push” mode. Work [5] considers the working conditions of a frontal plow without a support wheel. In such a design, the plowing depth is fixed by an adjustable limiter. It is attached to the tractor frame at one end, and to one of the lower links of the tractor’s front linkage at the other end. The total vertical additional load on the front axle of the tractor is taken as an estimate of the performance of the front plow. It was established that the influence of the plowing depth and the specific resistance of the front plow on the total vertical load on the front axle of the tractor is less significant than the influence of the weight of the plow and its working width. The angle of inclination of its lower rods should be within  $0-5^\circ$ . The angle of inclination of the central thrust of this system can vary within  $25-30^\circ$ .

The best results are demonstrated by the use of a front two-body plow simultaneously with a three-body plow with a rear hitch [18]. This technique makes it possible to increase the dynamic load on the front axle of the tractor by 16–18 %; reduce the processing time of one hectare of land by 34 %; reduce specific fuel consumption by 26.5 %.

As in the case of rear hinged devices, the procedure for selecting design parameters of front hinged systems is standardized [8]. However, this procedure does not take into account the force interaction between the front plow and tractor at either static or dynamic levels.

According to [19], incorrect inclination of the lower rods of the front attachment system of a mobile vehicle can lead to the opposite effect – unloading the front steered wheels of the tractor with the front plow. Moreover, the support wheel of the front plow plays a significant role in this process. In addition to the fact that it affects the amount of vertical load on the front axle of the tractor, it is also the reason for the complexity and increase in the cost of the design of the front plow.

Taking into account the above, it is advisable to investigate the possibility of removing the support wheel from the structure of this tillage unit. This could improve the performance of the system and prevent the front axle of the tractor from being unloaded when working with this type of plow.

Although the state of static loading of the front axle of the tractor by a front plow without a support wheel is described in detail in works [5, 16], the dynamics of this process were almost not investigated.

Work [20] partially presents the analysis of the dynamics of the plowing process with a unit with a front plow without a support wheel. The work is aimed at the development of a specialized computer program for calculating the forces acting on the central and lower traction of the tractor’s front attachment system. However, the design proposed by the authors does not contain a mechanism that would limit the depth of entry of the front plow without a support wheel. Because of this, the forces and reactions acting on the plow do not reflect its influence on the vertical load of the front axle of the tractor. To study the dynamics of the impact of the front plow on the used tractor, this simplification of the diagram of active forces has limited application for further in-depth scientific analysis. The force diagram reported in work [20] does not take into account the angle of inclination of the lower rods of the tractor’s front linkage. As shown by studies [21, 22], the contribution of this design parameter to the distribution of active forces and reactions acting on the front plow is significant and requires in-depth study.

From our review of the literature [6–22], it follows that most research results cannot be applied in the case of using frontal plows in a unit configured according to the “push-pull” scheme since they do not take into account the statics and dynamics of the movement of plowing units. And there are also a few scientific works that consider the operation of a plow unit with a front plow without a support wheel [5, 18–22]. However, in those works, it was not established under which conditions, under which stiffness and damping coefficients of the tires of the front drivers of the tractor, the additional vertical load of the front axle of the power vehicle is ensured. Therefore, it is advisable to conduct a study on the vertical vibrations of a tractor with a front plow without a support wheel.

### 3. The aim and objectives of the study

The purpose of our study is to determine the dynamics of vertical oscillations of a tractor with a front plow without a support wheel. This will make it possible to ensure a reduction in skidding of tractor propulsors and specific fuel consumption, improve controllability and stability of movement, and therefore the productivity of plowing machine-tractor units according to the “push-pull” scheme.

To achieve the goal, the following tasks were set:

- to determine the transmission function that reproduces the nature of the vertical oscillations of the front axle of the tractor under the influence of external disturbance;
- to investigate the amplitude-frequency characteristics (AFC) of the influence of a front plow without a support wheel on the dynamics of the vertical load on the front axle of the tractor;
- to investigate the phase-frequency (PFC) characteristics of the impact of a frontal plow without a support wheel on the dynamics of the vertical load on the front axle of the tractor.

### 4. The study materials and methods

For our calculations, the KhTZ-160 tractor (Ukraine) with a two-body front plow without a support wheel was adopted (Fig. 1).



Fig. 1. KhTZ-160 tractor with two-body front plow without support wheel

The KhTZ-160 tractor is equipped with 16.9R38 tires: radii  $r_o=0.77$  m,  $r_c=0.21$  m. The air pressure in these tires varied in the range of 0.10–0.16 MPa.

According to the developed working hypothesis, the additional vertical load of the front axle of the power vehicle under the influence of the frontal plow without a support wheel can be ensured if the values of the stiffness coefficients and damping of the tires of the tractor's front drivers are correctly selected.

During plowing, the front axle of the tractor functions as a dynamic system that reacts to external disturbances. This reaction leads to vertical oscillations of the front axle of the tractor ( $z$ ). In addition, it is assumed that during movement as part of a plowing unit with a front plow, the front wheels of the tractor maintain constant point contact with the longitudinal profile of the cultivated field. In other words, there are no vertical vibrations of the front axle of the agricultural vehicle, which could lead to separation of the wheels from the supporting surface. Long-term practice of operating plowing units has shown that this type of working movement prevails under regular field conditions.

We assume that the front wheels of the tractor are rigidly attached to the beam. The only elastic element of the beam is the tires of each wheel, which have both elastic and dissipative properties. The elasticity of the tire is reflected by the relationship between the vertical displacement of the wheel axis ( $z$ ) and the vertical force acting on it. For practical purposes, most scientists and researchers accept that this dependence corresponds to a linear law with sufficient accuracy [23].

With this approach, tire elasticity can be represented by a constant stiffness coefficient ( $C$ ) with a unit of N/m. The dissipative properties of the tire are described by the coefficient of resistance to deformation ( $K$ ), which is considered proportional to the vertical speed of the wheel axis ( $z$ ) and has a measurement unit of N s/m. Taking this into account, the front axle of the tractor can be considered a dynamic system, the stiffness coefficient of which is equal to twice the value of  $C$  ( $2C$ ), and the coefficient of resistance to deformation is twice the value of  $K$  ( $2K$ ).

The scheme of connecting the front plow to the tractor was built on the basis of study [5] and is shown in Fig. 2.

The plow support wheel is replaced by a link of adjustable length, denoted as  $KF$ , attached to the tractor at point  $F$ . The reaction force  $P_d$  acting on this link produces a vertical component  $N_o$ . Similarly, the vertical components  $N_v$  and  $N_n$  form reactions acting on the central link ( $P_v$ ) and lower links ( $P_n$ ) of the tractor's front attachment system.

The scheme of forces acting on the front axle of the tractor is shown in Fig. 3.

Since weight acts on the front axle of the tractor (Fig. 3):

$$M = M_b + (N_o + N_v + N_n) / g, \tag{1}$$

where  $M_b$  is the mass of the front axle of the tractor, kg;  
 $g$  is the acceleration of free fall, 9.81 m/s<sup>2</sup>.

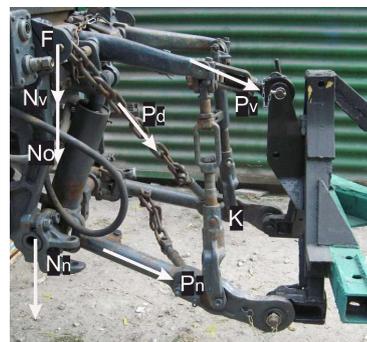


Fig. 2. The scheme of connecting the front plow to the tractor

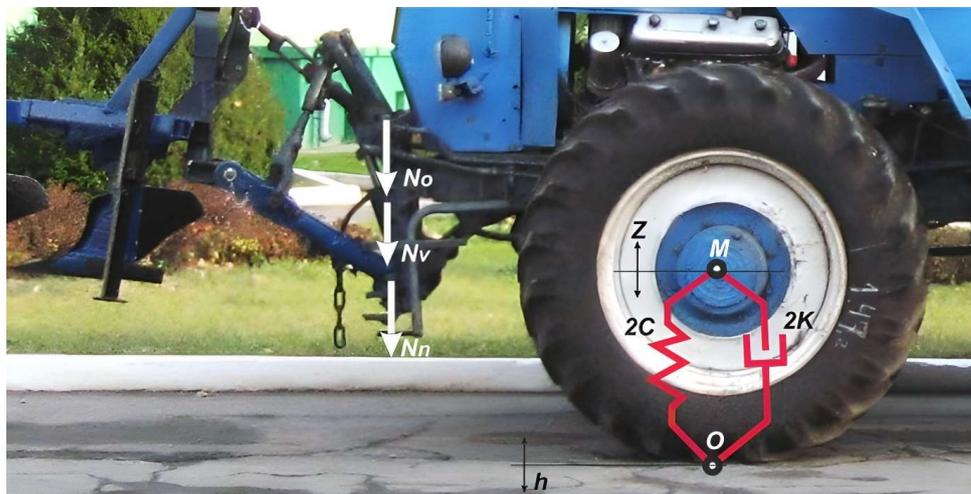


Fig. 3. Diagram of the action of forces on the front axle of the tractor

One of the input factors of the disturbance that causes vertical movements of the dynamic system ( $z$ ) is the vertical oscillations of the longitudinal profile of the field ( $h$ ). With one degree of freedom (generalized coordinate) of the front axle of the tractor, it is enough to write down only one differential equation in the form of a Lagrange equation of the second kind:

$$\frac{d}{dt} \cdot \frac{\partial E_k}{\partial \dot{z}} - \frac{\partial E_k}{\partial z} + \frac{\partial E_p}{\partial z} + \frac{\partial D_f}{\partial \dot{z}} = Q, \tag{2}$$

where  $E_k, E_p$  – kinetic and potential energies of the dynamic system, N·m;

$D_f$  – the energy dissipation function of the dynamic system, N·m;

$t$  – time, s;

$z$  – vertical movements of the dynamic system, m;

$\dot{z}$  – speed of vertical movements of the dynamic system, m/s;

$Q$  – total force, N.

The kinetic energy of the system is determined from the following equation:

$$E_k = \frac{M \cdot \dot{z}^2}{2}. \tag{3}$$

The potential energy of this dynamic system is a function of the tire deflection of the tractor's front wheels. In an analytical expression, these deflections can be represented as the difference between the amplitudes of the vertical oscillations of the front axle of the tractor ( $z$ ) and the longitudinal profile of the field ( $h$ ):  $z-h$ . Taking this into account, the

equation for calculating the potential energy of vibrations of the front axle of the tractor together with the front plow will take the following form:

$$E_p = \frac{2 \cdot C \cdot (z-h)^2}{2} = C \cdot (z-h)^2, \quad (4)$$

where  $C$  is the tire stiffness coefficient, kN/m;  
 $h$  – height of the longitudinal profile of the field, m.

Similarly, the dissipative function of the considered dynamic system is determined:

$$D_f = \frac{2 \cdot K \cdot (\dot{z}-\dot{h})^2}{2} = K \cdot (\dot{z}-\dot{h})^2, \quad (5)$$

where  $K$  is the tire deformation resistance coefficient, kN;

As for the generalized force of the dynamic system, it is the sum of the three aforementioned forces:

$$Q = N_s = N_v + N_n + N_o. \quad (6)$$

Taking the corresponding partial derivatives and substituting them together with equations (3) to (6) into the original equation (2), we obtain after transformations:

$$M \cdot \ddot{z} + 2K \cdot \dot{z} + 2C \cdot z - 2K \cdot \dot{h} - 2C \cdot h = N_s,$$

or:

$$M \cdot \ddot{z} + 2K \cdot \dot{z} + 2C \cdot z = 2K \cdot \dot{h} - 2C \cdot h + N_s. \quad (7)$$

It should be noted that the solution of the differential equation (7) is not trivial. The main difficulty of this process is related to the stochastic nature of variations of external disturbances in the form of parameters  $h$  and  $N_s$  of oscillations. Representing them in the resulting equation (7) is rather problematic. And even if it can be done, the solution of equation (7) will be significantly complicated.

A practical solution to this problem is to transform the differential form of equation (7) into a simple algebraic structure. This is achieved, as is known, by means of the Laplace transform mechanism [24]. To this end, the appropriate operator  $p$  is used in the form:

$$p = \frac{d}{dt}. \quad (8)$$

So, it is quite obvious that:

$$p^2 = \frac{d^2}{dt^2}. \quad (9)$$

By applying transformations (8) and (9) to equation (7), we obtain:

$$\begin{aligned} z(p)(M \cdot p^2 + 2K \cdot p + 2C) = \\ = h(p)(2K \cdot p + 2C) + 1 \cdot N_s(p), \end{aligned}$$

or

$$K_1 \cdot z(p) = K_2 \cdot h(p) + 1 \cdot N_s(p), \quad (10)$$

where:

$$K_1 = M \cdot p^2 + 2K \cdot p + 2C;$$

$$K_1 = Mp^2 + 2Kp + 2C; K_2 = 2(Kp + C),$$

$$K_2 = 2 \cdot (K \cdot p + C).$$

The resulting equation (10) is a mathematical model of oscillations of the front axle of a tractor with a front mounted plow without a support wheel under the influence of external disturbances, which are:

– ordinate of vertical fluctuations of the longitudinal profile of the field, denoted by ( $h$ );

– the total force ( $N_s$ ) acting from the front mounted plow on the front axle of the tractor, which is determined by expression (6).

To check the adequacy of the mathematical model under field conditions, the longitudinal field profile and vertical oscillations of the front axle of the KhtZ-160 tractor (Ukraine) were measured. A front two-hull plow was installed on the front attachment mechanism of this tractor (Fig. 1).

A profilograph was used to measure the field profile (Fig. 4). One end of its rotary lever is equipped with a wheel with a diameter of 8 cm, which, when moving the profilograph along the support beam, came into contact with the unevenness of the path.



Fig. 4. Device for measuring field profile fluctuations

The second end of the lever, fixed on the body of the profilograph, was in contact with the rotor of the variable resistance SP-3A (Ukraine) with a rated value of 470 Ohms and a linear characteristic. Angular oscillations of the profilograph lever correspondingly changed the value of the alternating resistance, and therefore the value of the voltage drop across it. The total path of measurement of the longitudinal profile of the path was at least 50 m.

Measurements of vertical oscillations of the front axle of the tractor were carried out using a three-axis gyroscope MPU-6050 GY-521 (made in China) installed on it. Electrical signals from the profilograph and gyroscope were sent to the Arduino UNO (made in Italy) and recorded in digitized form with an interval of 0.1 s on a micro-SD.

The resulting data were processed in the Microsoft Excel environment (USA) to obtain such statistical characteristics as mean square deviations (i.e., standards) and normalized spectral densities. To calculate the standards of fluctuations of the field profile and the front axle of the tractor, the obtained data arrays were treated using the Analysis Toolpak program (Descriptive Statistics-Microsoft Excel, Version 2208). The calculation of normalized spectral densities of fluctuations of the specified parameters was carried out with the help of a special program developed by us in the Mathcad 15.0 environment.

Using the obtained statistical characteristics, the experimental frequency response was calculated:

$$AFC_{exp} = \frac{\sigma_b}{\sigma_p} \cdot \sqrt{\frac{S_b}{S_p}},$$

where  $\sigma_b, \sigma_p$  are mean square deviations (cm) of field profile and tractor front axle fluctuations, respectively;

$S_b, S_p$  – normalized spectral densities (s) of field profile fluctuations and tractor front axle.

The theoretical frequency response was calculated using model (10). At the same time, fluctuations of the unevenness of the field profile  $h(p)$  were taken as an input parameter. The initial parameter was the vertical oscillations of the front axle of the tractor  $z(p)$ .

### 5. Results of investigating the dynamics of vertical oscillations of a tractor with a front plow without a support wheel

#### 5.1. Determining the transfer function from disturbance relative to the front axle of the tractor

The input parameter of this dynamic system is the amplitude of vertical oscillations of the front axle of the tractor, which is denoted by  $z$ . In essence, equation (10) is an analytical mechanism for transforming the input disturbances  $h(p)$  and  $N_s(p)$  of the dynamic system with respect to the initial value  $z(p)$ . It is known from the theory of automatic control for such systems [25] that a certain operator can express its transforming properties. The primary operators for many dynamic systems are transfer functions and amplitude and phase frequency characteristics, which are formed on their basis.

The transfer function  $W_z(p)$  from the disturbance  $N_s(p)$  is considered in relation to the vibrations of the front axle of the tractor,  $z(p)$ :

$$W_z(p) = \frac{1}{M \cdot p^2 + 2K \cdot p + 2C}. \tag{11}$$

Given that  $p=i\omega$  (where  $i=\sqrt{-1}$ , and  $\omega$  is the frequency of fluctuations of the input disturbance), then from function (11) we can obtain equations for calculating AFC  $[A(\omega)]$  and PFC  $[F(\omega)]$ :

$$A(\omega) = \frac{1}{\sqrt{(M \cdot \omega^2 + 2C)^2 + 4K^2 \cdot \omega^2}}; \tag{12}$$

$$F(\omega) = -\arctan\left(\frac{2K \cdot \omega}{M \cdot \omega^2 + 2C}\right). \tag{13}$$

During the operation of the plowing unit, the tractor driver observes the furrow left by the previous pass and sets the control action of the tractor control mechanism. Although our study does not consider the controlling influence, the dynamic system under investigation is a tracking system. Specific requirements for ideal AFC and PFC are established for such systems [26]. In the case of a tracking system that effectively cancels the input disturbance (as in this case), the ideal (or desired) AFC should be zero. This means that the dynamic system should not react to the disturbance, treating it as an unwanted signal. As a phase delay of the response of a dynamic system to an external disturbance, PFC should tend to infinity. In other words, the later the dynamic system reacts to an external disturbance, the better.

In summary, we can say that for the dynamic tracking system under consideration, the ideal (or desired) AFC and PFC should meet the following conditions:

$$AFC = 0; \tag{14}$$

$$PFC \rightarrow \infty. \tag{15}$$

Conditions (14), (15) make it possible to formulate an algorithm for mathematical modeling of the process of functioning of the dynamic system under consideration. The essence of this algorithm is as follows: the desired values of the parameters included in equations (12), (13) lead to the approximation of the actual AFC to zero, and the actual PFC to infinity.

The tire stiffness factor  $C$  included in equations (12), (13) was determined from the equation obtained by the Hedekel formula [27]:

$$C = 2\pi p_t \cdot \sqrt{r_c \cdot r_o}, \tag{16}$$

where  $p_t$  – air pressure in the tire, Pa;

$r_o$  – radius of the unloaded tire, m;

$r_c$  – radius of the cross section of the tire, m.

Based on the technical characteristics of the tires of the KhTZ-160 tractor and the condition that the air pressure in the tires varied in the range of 0.10–0.16 MPa, the value of the stiffness coefficient  $C$  varies within 250–400 kN/m. As for the coefficient  $K$ , according to the results of research, its value varied within 1–3 kN·s/m.

Taking into account the operating weight of the KhTZ-160 tractor of 8150 kg, the load on the front axle is 5400 kg. According to studies [5], the force  $N_s$  varies between 3000–9000 N, which corresponds to a mass of 305–917 kg. Taking this into account and using dependence (1), the value of the parameter  $M$  in the process of mathematical modeling varied within 5700–6300 kg.

The test of the constructed model (10) for compliance with the experiment involved comparing the theoretical and experimental amplitude frequency characteristics when working out by the dynamic system of the input disturbance in the form of oscillations of the longitudinal profile of the field (Fig. 5).

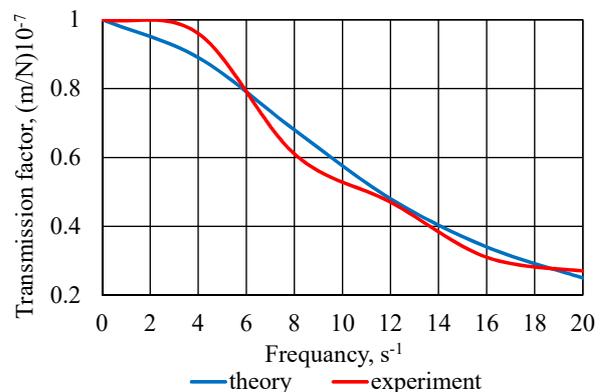


Fig. 5. Theoretical and experimental amplitude-frequency characteristics

As a result of such a comparison, the measurement error was determined, and it was established that the maximum difference between calculated (curve 1, Fig. 5) and natu-

ral (curve 2) data does not exceed 10 %. Such convergence of modeling data and field measurements indicates the possibility of using the constructed mathematical model (10). And this, in turn, gives reasons to consider the results of mathematical modeling represented below acceptable for further determination of AFC and PFC of vertical oscillations.

**5. 2. Studying the amplitude-frequency characteristic of the influence of the front plow on the dynamics of the vertical load of the front axle of the tractor**

Analysis of the calculated frequency response shows that with an increase in the frequency of oscillations of the total force  $N_s$ , the coefficient of its amplification by the dynamic system decreases rapidly (Fig. 6).

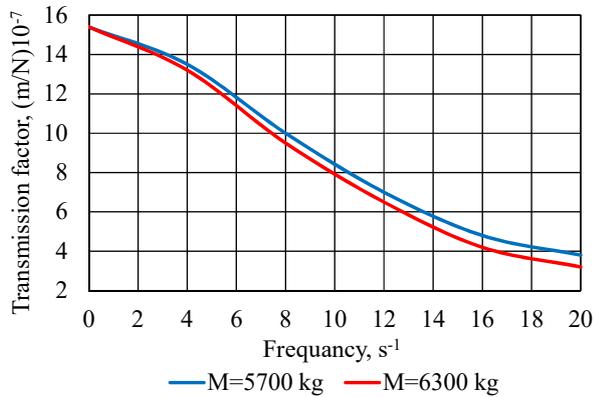


Fig. 6. Amplitude-frequency characteristic of working out by a dynamic system of the disturbance from vertical fluctuations of the total force

When calculating these frequency characteristics, the stiffness coefficient of the wheels of the front axle of the tractor was constant and was 325 kN/m. The value of the deformation resistance coefficient of these tires was equal to 2 kN/m.

In general, it should be emphasized that the AFC of the considered dynamic system shown in Fig. 6 is quite close to ideal. The maximum value ( $15.4 \cdot 10^{-7}$  m/N) shows that with the stationary action ( $\omega=0$  s<sup>-1</sup>) of the force  $N_s$  at the level of kN, the amplitude of oscillations of the front axle of the tractor with the front plow in the vertical direction ( $z$ ) is 1.54 mm. At the maximum value of the specified force at the level of 9 kN, this fluctuation does not exceed 14 mm. With the most probable frequency of force oscillations  $N_s$  at the level of  $\omega=1$  Hz (i.e., 6.28 1/s) and its maximum value of 9 kN, the oscillations of the parameter  $z$  for both values of  $M$ , as follows from the analysis of Fig. 3, will be within 9–10 mm.

The nature of the dependence of AFC of the dynamic system of the considered input disturbance on the value of the stiffness coefficient of the tires of the wheels of the front axle of the tractor is shown in Fig. 7.

The average value of the vertical load of this bridge (parameter  $M$ ) in this case is 6000 kg. The value of the coefficient  $K$  is unchanged and equal to 2 kN·s/m.

According to the results of mathematical modeling, it was established that, in contrast to the stiffness coefficient of the tire  $C$ , the change in the coefficient of resistance to its deformation  $K$  in the range from 1 to 3 kN·s/m practically does not have a noticeable effect on the AFC of the disturbance created by the dynamic system (Fig. 8). Qualitatively and most importantly, quantitatively, the change of these frequency characteristics in the entire frequency range of force oscillations

$N_s$  from  $\omega=0$  to  $\omega=20$  1/s is practically the same. Graphically, this is manifested in the coincidence of curves 1 and 2.

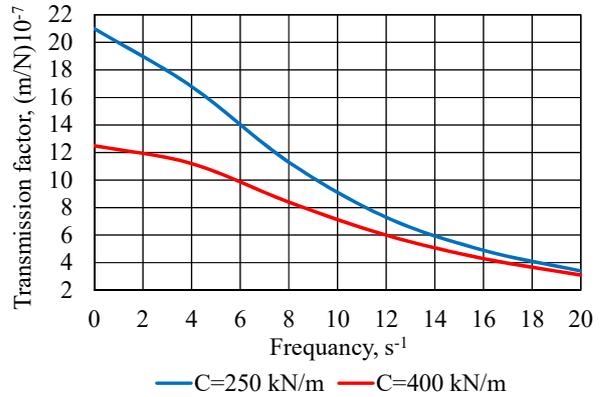


Fig. 7. Amplitude-frequency characteristic of working out by a dynamic system of the disturbance of from fluctuations in the tire stiffness coefficient

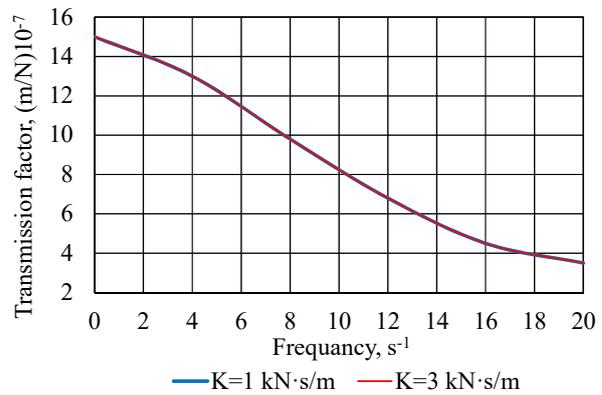


Fig. 8. Amplitude-frequency characteristic of working out by a dynamic system of the disturbance from the coefficient of resistance to the deformation of the tires of the front wheels of the tractor

The average value of the vertical load of this bridge (parameter  $M$ ) in this case was also 6000 kg, the value of the coefficient  $C$  remained unchanged at 325 kN/m.

**5. 3. Studying the phase-frequency characteristic of the influence of the front plow on the dynamics of the vertical load of the front axle of the tractor**

As is known, the phase-frequency characteristics of a dynamic system by its very nature are a phase delay (in degrees and time) of the reactions of the dynamic system to an external disturbance. In this study, it is the vertical fluctuations of the total force  $N_s$ . Fig. 9 shows the dependence of PFC for different values of the design parameter  $M$ , which is at two levels: 5700 and 6300 kg.

Shown in Fig. 9, PFC were obtained with the following values of stiffness coefficients and resistance to deformation of the wheels of the front axle of the tractor:  $C=325$  kN/m and  $K=2$  kN·s/m.

The nature of the change in the reaction delay of this dynamic system to the fluctuations of the vertical load of the front axle of the tractor for different values of the stiffness coefficient of the wheel tires is shown in Fig. 10.

The calculations of these frequency characteristics were performed based on the average value of the vertical load

of the front axle of the tractor  $M=6000$  kg and the coefficient of resistance to the deformation of the tires of its front wheels  $K=2$  kN·s/m.

Fig. 11 reproduces the process of the influence of changes in the coefficient of resistance to the deformation of the tires of the front wheels of the tractor on the dynamics of the delay in its response (i.e., PFC) to the input disturbance in the form of vertical fluctuations of the total force  $N_s$ .

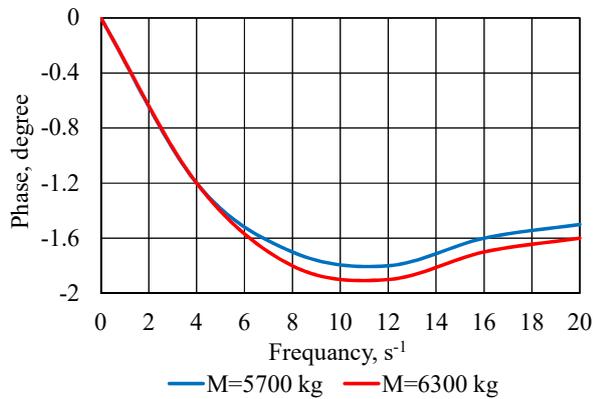


Fig. 9. Phase-frequency characteristic of working out by a dynamic system of the disturbance from vertical fluctuations of the total force

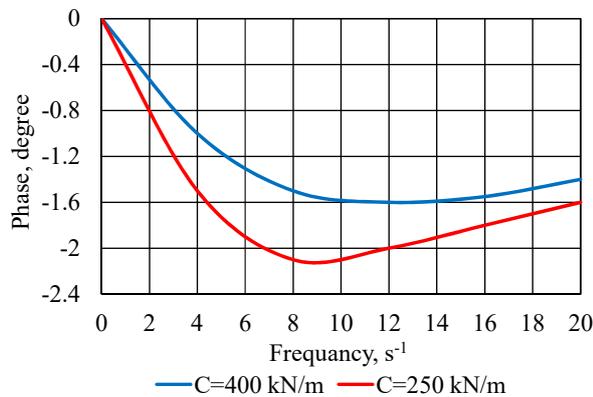


Fig. 10. Phase-frequency characteristic of working out by a dynamic system of the disturbance from fluctuations of the stiffness coefficient of the tires of the front wheels of the tractor

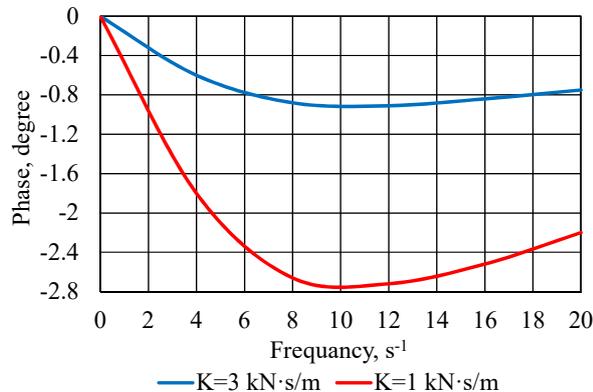


Fig. 11. Phase-frequency characteristic of working out by a dynamic system of the disturbance from the coefficient of resistance to the deformation of the tires of the front wheels of the tractor

As in the previous versions, the calculations of these PFC were carried out with the following parameter values:  $M=6000$  kg;  $C=325$  kN/m.

### 6. Discussion of results of investigating the dynamics of vertical oscillations of a tractor with a front plow without a support wheel

The peculiarity of our research results is the wide use of frequency methods for evaluating the functioning of the considered dynamic system. Despite their wide distribution [25, 26, 28, 29], the use of AFC and PFC for analyzing arable aggregates according to the “push-pull” scheme has not been found. It is the lack of frequency analysis methods for units with a front plow without a front wheel that emphasizes the scientific and practical value of the reported research results.

The limiting factor of the model is the relationship between the input and output coordinates of the considered dynamic system. As a rule, it is nonlinear, which characterizes the dynamic system as non-stationary. In this case, non-stationarity can be based on both mathematical expectation and variance. Options for their combined (total) manifestation are quite possible. Due to the complexity of such systems, the methods of mathematical assessment of the nature of their functioning are at an insufficient level of development.

At this level of research, its stationary linear version was used. Such an idealization of the system is often effective for agricultural machinery, the dynamics of which have not yet been sufficiently studied. In this case, the linearization of the dynamic system makes it possible to physically comprehend the obtained results and accumulate practical design experience.

Analysis of the calculated AFC shows that with an increase in the frequency of oscillations of the total force  $N_s$ , the coefficient of its amplification by the dynamic system decreases rapidly (Fig. 6). The intensity of this process is practically the same for the values of the  $M$  parameter both at the level of 5700 kg and 6300 kg. The decrease in the considered AFC is explained by the inertial properties of the dynamic system, which is the front axle of the KhtZ-160 tractor together with the front plow. Most likely, as the frequency of oscillations of the input disturbance (force  $N_s$ ) increases, it increasingly differs from the frequency of the system’s own oscillations. The greater the value of the  $M$  parameter, the lower this frequency. In view of this, it is quite clear why an increase in the mass of  $M$  from 5700 to 6300 kg leads to a decrease in the AFC value, albeit a small one (by a maximum of 8.8 %, curve 2, Fig. 6). The results are correlated with the research data in [30]. In this work, it is proven that at a constant speed or depth of tillage, the largest vertical vibration acceleration occurs on the front axle. An increase in the vertical load of the front axle of the tractor leads to a decrease in AFC values. The authors also carried out a multi-purpose optimization of the stiffness and damping parameters of the front axle suspension. However, the conditions of the experiment and the purpose of study [30] differ from those presented in our paper, so the results can only be partially compared.

From the analysis of the influence of the stiffness coefficient of the wheels of the front axle of the KhtZ-160 tractor on the nature of change in the obtained AFC, it can be seen that an increase in the disturbance frequency to 20 1/s leads to a desired decrease in the AFC for all accepted values of the coefficient  $C$  (curves 1 and 2, Fig. 7). At the same time, increasing the

tire stiffness factor from 250 to 400 kN/m is quite effective and therefore more acceptable. This is especially evident at frequencies of oscillations of the disturbance force (i.e., force  $N_s$ ) close to zero. So, for example, at  $\omega=2$  1/s, the AFC value decreases from  $19 \cdot 10^{-7}$  m/kN (curve 1, Fig. 7) to  $12 \cdot 10^{-7}$  m/kN (curve 2). That is, the desired reduction of the gain of the input disturbance by the dynamic system is 36.8%.

The reason for our result is as follows. As follows from equation (16), the value of the  $C$  coefficient depends most on the air pressure in the tires of the wheels ( $\rho_t$ ). The smaller the value of this parameter, the smaller the value of the coefficient  $C$ . At the same time, the smaller the value of  $\rho_t$ , the greater the elastic properties of the tire, and, therefore, the greater the ability of the wheel to vibrate in the vertical plane. With the same frequency and amplitude of the disturbance, the wheel for which the values of the parameters  $\rho_t$  and  $C$  are smaller will oscillate more. At a frequency of fluctuations of the input signal of about 20 1/s, the dynamic system, due to its inertial properties, becomes practically invariant with respect to the stiffness coefficient of the tires of the front axle of the tractor.

Analysis of the corresponding phase-frequency characteristics shows that when the frequency of these oscillations increases to approximately  $\omega=4$  1/s, the estimated PFC for both values of the system parameter  $M$  (5700 and 6300 kg) do not have a significant difference (curves 1 and 2, Fig. 9).

In this case, the phase shift of the dynamic system when it works out the action of the disturbance reaches  $1.2^\circ$  or 0.02 rad. In the time dimension at  $\omega=4$  1/s it is only  $0.02/4=0.005$  s. In practice, this means that in the frequency range of oscillations of the disturbance force  $N_s$ , from 0 to 4 1/s, the dynamic system reacts almost instantly to its (force) changes. In principle, this fact is undesirable.

After such a peculiar point of bifurcation, the character of PFC changes, albeit slightly (Fig. 9). Namely, an increase in the value of the system parameter  $M$  from 5700 to 6300 kg causes a desirable increase in the response delay of the dynamic system to the disturbance. In principle, such a result is quite logical since a more massive (by 600 kg) dynamic system reacts to disturbances later due to the manifestation of its inertial properties.

Starting from the frequency  $\omega \geq 8$  1/s, the difference between PFC becomes constant and equals approximately  $0.1^\circ$  or  $2 \cdot 10^{-3}$  rad. However, this difference at different values of  $\omega$  characterizes the degree of lateness of the response of the dynamic system to disturbances in different ways. For example, at  $\omega=12$  1/s, the value of the difference in phase delays is  $1.7 \cdot 10^{-3}$  s, and at a frequency of  $\omega=20$  1/s, this difference is equal to  $10^{-4}$  s. This is 41% less, which is an undesirable trend. In a practical sense, it can be stated that an increase in the parameter of the dynamic system  $M$  from 5700 to 6300 kg has almost no effect on the delay in its response to disturbances.

A quantitatively different situation occurs when the stiffness factor  $C$  of the tires of the front wheels of the tractor changes (Fig. 10). In general, a decrease in the value of this coefficient from 400 to 250 kN/m leads to a desired increase in the phase shift when working out the dynamic system of the action of the disturbance. The logic of this result follows from the following. The smaller the value of the  $C$  coefficient, the less rigid the tire is and the later, thanks to its greater deformation capacity, it reacts to disturbances.

The biggest difference (curves 1 and 2, Fig. 10) between the obtained PFC occurs at a frequency  $\omega$  of approximately 9 1/s. At this frequency, the phase shift (by absolute value)

for the PFC of system with a coefficient of  $C=400$  kN/m is  $1.55^\circ$  or 0.027 rad. The reaction delay of the dynamic system to the disturbance in this case is  $3.0 \cdot 10^{-3}$  s. For PFC with  $C=250$  kN/m, the phase shift is larger and is equal to  $2.10^\circ$  or 0.037 rad. In the time dimension, this is  $4.1 \cdot 10^{-3}$  s. As a result, at a frequency of  $\omega=9$  1/s, the difference between the compared PFC (Fig. 10) is  $0.55^\circ$  or 0.01 rad. In practice, this corresponds to the difference between the delay of the reaction of the dynamic system to the impact of the disturbance at the level of  $1.1 \cdot 10^{-3}$  s.

At the same time, at the frequency  $\omega=20$  1/s, the difference between the compared PFC is only  $0.2^\circ$  (0.003 rad), and the difference in phase shifts is  $0.15 \cdot 10^{-3}$  s. Compared to the previous version, it is 86% less, which is an undesirable trend. With an increase in the disturbance frequency after 9 1/s, the effectiveness of reducing the stiffness coefficient of the tires of the front wheels of the KhTZ-160 tractor gradually decreases from 400 to 250 kN/m. In other words, the reduction of the value of the stiffness coefficient  $C$  remains effective due to the fluctuations of the disturbance in the frequency range  $\omega=0 \dots 9$  1/s.

Joint analysis of Fig. 6–10 shows that when the action of the disturbance is worked out, an increase in the value of the  $C$  coefficient, on the one hand, improves the amplitude-frequency characteristics, and on the other hand, leads to a deterioration of the phase-frequency characteristics. Decreasing the value of this parameter leads to the opposite result: the AFC of the dynamic system deteriorates, and the PFC improves. It is known from the basics of the theory of automatic control [28, 29] that the magnitude of the phase shift of this response plays a decisive role with a small amplitude response of the dynamic system to the action of the disturbance. And it is, as you know, reflected by the relevant PFC. The conclusion is logical: in order to improve the nature of processing by the dynamic system of the external disturbance, which is considered in the form of vertical fluctuations of the load on the front axle of the tractor, the value of the stiffness coefficient of the tires of the front wheels should be reduced. As follows from the analysis of equation (16), in practice this can be achieved by a corresponding decrease in the air pressure in these tires.

As the analysis of our PFC reveals (Fig. 10), the character of their change in qualitative terms is almost completely similar to the ones considered above (Fig. 9, 10). In turn, it differs significantly in quantitative terms (Fig. 11).

The resulting maximum difference between the compared frequency characteristics occurs at the frequency of oscillations of the disturbance action  $\omega=10$  1/s. For a system with a coefficient of  $K=3$  kN·s/m, PFC reaches  $-0.9^\circ$  or 0.016 rad. For the same system, but with  $K=1$  kN·s/m, the phase shift (i.e., PFC) increases (in absolute terms) to  $-2.6^\circ$  or 0.046 rad. Thus, the difference between the compared PFC at the frequency  $\omega=10$  1/s is approximately  $1.7^\circ$  or 0.03 rad. Such a phase shift corresponds to the difference between the response delay of the dynamic system to a disturbance with a value of  $3.0 \cdot 10^{-3}$  s.

As for the simulation results presented above (Fig. 9–11), a similar trend is observed with an increase in the frequency of disturbance oscillations above 10 1/s. Namely, the effectiveness of reducing the coefficient of resistance to the deformation of the tires of the front wheels gradually decreases. It is advisable to reduce the value of the coefficient  $K$  in cases where the frequency of oscillations of the external disturbance (force  $N_s$ ) does not exceed 10 1/s. It follows from

this that the availability of information about the frequency spectrum of oscillations of the specified force is a determining factor and requires an additional cycle of special research. Our results are partially confirmed by research data in [20].

The current study was carried out taking into account the elastic and dissipative properties of only the front tires of the tractor wheels. To some extent, this can be considered a shortcoming of this work since power tools with sprung bridges appear at the modern stage. This fact requires supplementing the dynamic system with additional elasticity elements in the form of springs and energy dissipation elements in the form of hydraulic shock absorbers. In the differential equations of the vertical oscillations of the front axle of the tractor, this will be reflected by the appearance of the corresponding stiffness and damping coefficients.

At the same time, even with the application of such restrictions, the problem of ensuring a sufficient level of vertical additional loading of the front wheels of the tractor when it is aggregated with a front plow without a support wheel is solved in this work. This was done through the appropriate selection of rational values of the stiffness coefficients and damping of the tires of the front drivers of the power vehicle.

Further studies of the considered problem involve the study of the influence of fluctuations of the longitudinal profile of the field [ $h(p)$ ] on the dynamics of the vertical load of the front axle of the tractor. To that end, the amplitude and phase frequency characteristics calculated on the basis of the transfer function, which reflects the relationship of the output parameter  $z(p)$  of the mathematical model (10) with the external disturbance  $h(p)$ , should be analyzed. Based on the analysis of the obtained data, it becomes possible to formulate such initial requirements for the amplitude and frequency of fluctuations of the longitudinal profile of the field, the provision of which will contribute to the improvement of the agrotechnical culture of soil cultivation.

The research results discussed above can already at this stage be used in the development of new arable machine-tractor units according to the "push-pull" scheme. Their characteristic feature is the use of frontal plows of a simplified design. The condition for ensuring the desired efficiency of the aggregation of such plowing tools is to adjust the tires of the front axle of the tractor according to the requirements for the values of their stiffness and damping coefficients. The practical implementation of the received recommendations is potentially capable of reducing the skidding of tractor propulsors and specific fuel consumption, improving the controllability and stability of movement, and therefore the productivity of the work of such plowed MTA.

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## 7. Conclusions

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1. A transmission function was determined that reproduces the nature of the vertical movements of the front axle of the tractor under the influence of force fluctuations created by a front plow without a support wheel. By its na-

ture, this function represents a classic oscillating link, the elastic properties of which are represented by the stiffness coefficient, and the dissipative properties are represented by the damping coefficient of the wheels of the front axle of the tractor. The inertial properties of the transfer function are represented by the mass that falls on the front bridge of the power tool, taking into account the force of the frontal plow.

2. Our analysis of the resulting amplitude-frequency characteristics revealed that an increase in the vertical load on the front axle of the tractor by 600 kg leads to a desired decrease in its vertical oscillations. And this decrease is greater, the higher the frequency of the disturbing influence. The estimated AFC practically does not change with an increase in the damping coefficient of the tires of the front wheels of the tractor in the range from 1 to 3 kN·s/m. At the same time, the value of the stiffness coefficient of the tires of the tractor's front drivers should be as small as possible. In practice, this can be achieved by adjusting the air pressure in the tires.

3. Based on the analysis of the derived phase-frequency characteristics of working out the input disturbance by the dynamic system, it was established that additional loading of the front axle of the tractor with a mass of up to 600 kg causes an increase in the delay of its reaction to external influences. This is especially noticeable when the frequency of the input oscillations of the disturbance is from 0 to 10 1/s with an increase in the damping coefficient of the tires of the front wheels of the tractor from 1 to 3 kN·s.

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## Conflicts of interest

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The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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## Data availability

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The data will be provided upon reasonable request.

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## Use of artificial intelligence

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The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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