

The object of this study is photovoltaic modules with different options for the angle of their installation to the horizon at different geographical latitudes. The scientific problem to solve was determining the dependence of the average annual efficiency of solar photovoltaic modules on the mounting angle of photovoltaic modules and the value of geographical latitude. It has been proven that the efficiency of installation of solar photovoltaic modules can be increased by reducing the angle of their inclination to the horizon depending on the value of the geographical latitude at which they are installed. The average annual efficiency of photovoltaic modules with different mounting angles to the horizon at different geographical latitudes was determined as the annual weighted average value of the cosine of the angle of incidence of solar rays on the plane of the photovoltaic module. The maximum of the average annual efficiency of photovoltaic modules corresponds to a smaller value of the angle of their installation to the horizon than the value of the geographical latitude. So, with a latitude value of 10°, 20°, 30°, 40°, 50°, and 60°, the mounting angle of photovoltaic modules to the horizon will be 9.5°, 18.8°, 28°, 37°, 45.8°, and 54°, respectively. A dependence was derived that allows determining the mounting angle of photovoltaic modules to the horizon dependent on the value of the geographical latitude at which they are installed. A mathematical expression was also constructed that makes it possible to determine the average annual efficiency of photovoltaic modules depending on the angle of their installation to the horizon for different values of geographic latitude.

The results could be used in calculating the average annual efficiency of photovoltaic modules based on the adjusted values of the angle of their installation to the horizon at different geographical latitudes

Keywords: photovoltaic module, angle of incidence of solar rays, efficiency of module installation, geographical latitude, efficiency of photovoltaic modules

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DETERMINING THE INFLUENCE OF MOUNTING ANGLE ON THE AVERAGE ANNUAL EFFICIENCY OF FIXED SOLAR PHOTOVOLTAIC MODULES

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1. Introduction

Due to the consistently high demand for energy, environmental problems related to global warming, and the rapid

development of alternative energy technologies, photovoltaic energy has become an effective method for producing “clean energy” from the sun’s energy [1]. The current worldwide use of photovoltaic energy increases by more than 50 % per year

and this indicator continues to grow [2]. Therefore, one of the scientific tasks of the photovoltaic industry is to increase the average annual efficiency of electricity production by a photovoltaic module, which can be implemented by two main methods [3, 4]. The first is to improve the efficiency of converting solar energy into electrical energy by developing or directly improving the design of the photovoltaic module [5]. The second one involves determining the optimal technique of installing the photovoltaic module. In this case, the value of the optimal angle of inclination of the photovoltaic module to the horizon should be determined depending on the geographical latitude at which it is installed [6].

The scientific literature includes several studies on improving the efficiency of a photovoltaic module by installing it with an optimal angle of inclination to the horizon depending on the geographical latitude at which the module is installed [5–8]. However, the reported results have a local application since the studies were carried out under different conditions at different geographical latitudes, so it is difficult, and sometimes even impossible, to use them under other conditions.

Therefore, solving the scientific problem related to the systematization and generalization of known results and establishing the dependence of the average annual efficiency of the photovoltaic module on the angle of its inclination and geographical latitude will have a high practical value. This will make it possible to reduce the number and complexity of experimental studies, significantly reduce time and investment in project work. It will also be possible to carry out a quantitative and qualitative assessment of the process of electricity generation by a photovoltaic module with high accuracy for engineering work. The above will help improve the average annual efficiency of photovoltaic modules.

2. Literature review and problem statement

Fundamental research on the evaluation of efficiency of the installation of photovoltaic modules is reported in [9]. The work, in addition to examples of the current structures of photovoltaic modules, and examples of their operation, presents the results of many years of research into the dependence of the energy conversion coefficient on the orientation of the photo panel and the angle of inclination of the module frame to the horizon. A mathematical expression for determining the average annual efficiency of a photovoltaic module is given, which, although it yields adequate calculation results in many applications, especially in a simplified form, is, in fact, the result of mathematical transformations. Based on the modern realities of the development of photovoltaic technologies, that study should be supplemented. In particular, in order to increase the accuracy of the calculations, additional design parameters (besides the panel area) should be introduced into the presented mathematical expression, as well as parameters that take into account the angle of incidence of solar radiation on the photovoltaic panel. In addition, the mathematical expression does not take into account how the value of the average annual efficiency of the module changes when the climate zone changes, the geographical latitude at which it is installed, etc.

This problem was partially solved in [10]. The authors presented a mathematical expression that is the starting point for calculating the average annual efficiency of photovoltaic modules in many scientific studies. According to the expression, the average annual efficiency of photovoltaic modules is

defined as the ratio of the power of the module to the product of the values of three parameters: the surface area of the module panel, the angle of incidence of the sun's rays on the photo panel, and the intensity of solar radiation. It is clear from the expression that it is possible to increase the average annual efficiency of photovoltaic modules, along with improving their design, by ensuring the optimal method of their installation. The disadvantage of the study is that the design parameters of the module, as in the previous work [9], are expressed only through the area of the photovoltaic panel. The influence of the type of semiconductor materials, the number and type of photovoltaic cells, the design of the protective shell, the design of the connection (cable connectors, mounting frames) is taken into account only indirectly. However, the authors derived the dependence of the angle of incidence of solar radiation on the angle of inclination of the photovoltaic module and the geographical latitude at which the panel of the module is installed on the earth's surface. That significantly increased the accuracy of calculations of the average annual efficiency of photovoltaic modules.

The authors of [11, 12] performed a number of studies on the optimization of the design of photovoltaic modules depending on the mounting angle of these modules in the case when the mounting angle of the modules is set based on landscape requirements (roofs of buildings, complex reliefs of mounting surfaces).

For example, in [11], the dependence of the efficiency of converting solar energy into electrical energy during the operation of four different designs of panels of photovoltaic modules was investigated. The panels studied were bonded frameless structures with taped cellular matrices sandwiched between facing/insulating layers. The specified structures were installed on the back panel made of fiberglass or carbon fiber. The suitability of the materials for use was checked using scanning electron microscopy, thermal stability analysis, and the nanoindentation method. Depending on the design of the panel and its texture, the efficiency of converting solar energy into electrical energy ranged from 19.98 to 20.71 %. Higher efficiency values corresponded to larger angles of incidence of solar rays. That work is still incomplete since the operating parameters and characteristics of only slightly modified panel structures (with reduced cell power for economic reasons) were determined. This makes it impossible to generalize the results of the study, and it is unclear whether they could be scaled without loss of efficiency. However, the results of the cited work have a high practical value and can be used for further research on the relationship between the design parameters of the module panel and the angle of inclination of the module frame to the horizon in order to increase the average annual efficiency of the module.

Work [12] complements the previous study. In particular, the authors, based on the mathematical expressions given in [11], built a mathematical model that allows simulating the electrical behavior of the photovoltaic module with a sufficient degree of reliability and predicting its average annual efficiency. A limitation to the application of the study is that only a small number of input parameters can be entered into the model, which makes its universal application impossible. As of today, the obstacle to the completion of the research is the refusal of the manufacturers of photovoltaic modules to provide the authors with the full technical characteristics of their products (in particular, the value of resistance in series and parallel connections of photocells). Completion of the works will make it possible to simulate with a high

degree of reliability the electrical processes that occur in the photovoltaic panel of the module, which, in the future, will contribute to the increase of the indicator of the average annual efficiency of the module.

In turn, the authors of paper [13], using the achievements of their predecessors, conducted large-scale studies aimed at establishing the dependence of the degree of average effective radiation absorption by the panel surface on the design and installation parameters of the module. The methodology for calculating the share of incident solar radiation, which is used in the productive mode of operation of a solar photovoltaic panel, is given. Using a number of variables (day of the year, latitude value, roof/frame slope, panel orientation, percentage of diffuse radiation, radiation discreteness), optimal values of glass thickness, extinction coefficient, and degree of solar reflection by the coating were calculated. However, the complexity of the given functions and the significant number of variables to be entered make it impossible to predict with confidence what the value of the average effective absorption of the distiller would be under a certain series of experimental conditions. As part of the study, it was established that the discreteness of insolation has a negligible effect on the average annual efficiency of the photovoltaic module, and the greater the proportion of diffuse radiation, the lower the degree of absorption of solar radiation by the panel coating. The given equations make it possible to determine the degree of average effective absorption of radiation by the surface of the solar panel for any certain combinations of initial variables and, indirectly, the average annual efficiency of the photovoltaic module. In addition, the authors of the work note that it would be advisable to supplement the research with data on the shadow effect from surrounding architectural structures, which could increase the accuracy of calculations.

This problem was solved in study [14]. The authors, in accordance with various urban development plans, built and tested a mathematical model of electricity generation by photovoltaic systems. First, the authors devised an annual dynamic shadow model of the building. The model was then used to calculate hourly annual electricity production, optimize the design of photovoltaic panels, analyze shading coefficient values, and average annual efficiency of electricity production by photovoltaic panels for different building plans. The results show that the highest value of the annual shading coefficient is 60.16 %, while the highest values of the efficiency of electricity generation by photovoltaic panels were observed. It is advisable to use the obtained results in the early design of buildings with further research into the indicator of the average annual efficiency of photovoltaic systems.

The authors of paper [15] note that although a lot of work has been done in the direction of increasing the average annual efficiency of photovoltaic modules, modern installations often do not achieve maximum efficiency due to incorrect mounting angles. Also, in high-latitude cold regions, the actual efficiency of photovoltaic generation is often overestimated due to weather conditions not being taken into account. The authors have built a new model that allows determining the value of the local optimal mounting angle of photovoltaic modules, which ensures the maximization of electricity production by photovoltaic modules, based on data on climatic and weather conditions. The results emphasize the important role of weather conditions in determining the efficiency of photovoltaic modules, as they are the reason for the drop of the module efficiency index to 14.7 % of rated values. Installing a photovoltaic module with an optimal

angle of inclination to the horizon increases the average annual efficiency of the module by approximately 4.8–12.3 % compared to conventional mounting angles. The proposed methodology for determining the optimal mounting angle of the module takes into account the impact of weather conditions on the efficiency of electricity generation, potentially serving as a bridge between climate change adaptation and future efforts in photovoltaic energy production.

Work [16] reports the results of 57 studies related to the assessment of the impact of increased roof albedo values on the efficiency of energy production by one- and two-sided photovoltaic modules. The studies were carried out under the given constant characteristics of the module and for certain conditions of the selected climatic zones. It is noted that an increase in the albedo of the roof by 0.1 increases the average annual efficiency of energy production by one-sided photovoltaic modules by 0.7 %, and by two-sided – by 4.55 %, additionally contributing to the reduction of the negative impact of the thermal effect in cities. Corresponding parametric relations for various module designs are proposed. The influence of geographical and installation parameters on the average annual efficiency of energy production by modules was studied. It has been experimentally confirmed that the correctly chosen method of installing photovoltaic modules increases their average annual efficiency by 4–12 %. However, the authors note that not all of the research objectives were solved. In particular, the authors could not present an algorithm for the optimal placement of photovoltaic modules on flat roofs of an irregular shape since the availability of space on such roofs is a significant limiting factor.

The solution to the specified scientific problem is highlighted in [17], which presents an algorithm for optimal placement of rows of photovoltaic modules installed on flat roofs of an irregular shape. The algorithm takes into account the shape of the roof, the presence of building structures (chimneys, machine rooms of elevators, ventilation shafts), the phenomenon of self-shading, and geometric parameters of modules, the configuration of the fastening system, the dimensions of technological passages, data on minimizing the effect of shading. With the help of the algorithm, the optimal layout of the photovoltaic modules on the roof, their angle of inclination to the horizon, the mounting scheme, etc. were determined. The results were compared with the recommendations for the installation of photovoltaic modules according to the IDAE technical report, the Lorenzo equation, and the Jacobson equation. The optimal installation of the module on the roof according to the algorithm increases the effective area of the photo panel by 35.52 % (in relation to the area calculated by the Jacobson equation) and by 32.29 % (in relation to the area recommended by IDAE). At the same time, it provides an increase in its average annual efficiency by 27.83 % (compared to the data obtained according to the Jacobson equation) and by 24.84 % (compared to the data according to the IDAE report). However, the authors of the work note that a more significant increase in the efficiency of the modules, in particular due to the determination of the optimal method of their installation, currently requires the collection, analysis, comparison, parameterization and, finally, systematization of the available information. Nevertheless, the results have a high value, and it is advisable to use them in the calculations of various parameters of photovoltaic systems.

Having analyzed scientific works [9–17], it can be noted that the efficiency of the photovoltaic module largely depends on its tilt angle. In some works, this angle is represented in mathematical expressions, in others it is expressed through

indirect parameters. The value of the tilt angle varies depending on the type of module, climate zone, intensity of solar radiation, weather conditions and geographical latitude where the module is installed. Along with that, there are a number of studies in which the authors substantiated that the highest efficiency of the photovoltaic module is achieved when the angle of inclination of the module is equal to the geographical latitude at which it is installed.

In particular, in work [18], the authors note that in the intertropical region, the maximum annual production of solar energy was achieved by installing photovoltaic modules at an angle equal to the local latitude. At the same time, the panels of the modules were oriented towards the equator. However, the authors also note that their statement for regions with a hot climate (Central and West Africa) is tentative. The reason is that, due to the high intensity of solar radiation, the deviation from the optimal orientation and tilt up to 20° led to a decrease in the level of exposure of the panel by less than 5%. That work is purely practical and does not contain mathematical justifications or generalizations for use by a wide range of consumers. Also, the results reported in the paper are suitable, mainly, for hot climatic zones.

In [19], the optimal values of the inclination angles of photovoltaic modules were determined for six cities of the western province of Sichuan (China), located at an altitude of 821 m above sea level. The province is characterized by a cold climate and incoming solar energy from the south-west direction. The angles of inclination of panels of photovoltaic modules for these cities varied in the range from 20° to 35° . The fundamental reason for the difference in the values of the optimal angle of inclination between the six cities is the ratio of direct, diffuse and reflected radiation from the ground throughout the year, seasons, and months. In the conclusions, the authors recommend introducing the value of the azimuthal angle in the range from -20° to $+20^\circ$ and the angle of inclination of the module within the range of latitude $\pm 25^\circ$ to the current standards. Much to our regret, the results of the study, although of practical importance, can only be applied to conditions similar to those under which this study was conducted.

Contrary to the results of studies [18, 19], the authors of works [20–22] note that the maximum average annual efficiency of photovoltaic modules is achieved when the mounting angle of the module is slightly smaller than the latitude at which it is installed.

So, for example, according to the results of study [20], the maximum average annual efficiency of photovoltaic modules occurred when the angle of inclination of the photovoltaic panels of the module was 20° . Research was conducted in the city of Guangzhou (China), which is located at 23° latitude. As we can see, the mounting angle of the module is 3° less than the latitude.

In work [21], four photovoltaic modules with tilt angles of 0° , 15° , 35° , 55° , installed in the city of Kita-Kyushu (latitude 33.89°), located in the north of the island of Kyushu, in Fukuoka prefecture, were investigated. Diffuse solar radiation, radiation intensity, reflection coefficient from the ground, solar declination and latitude were selected as variable factors of the study. Data were processed by parametric analysis. According to the results of experiments, the optimal angle of inclination of the module panel was from 25° to 32° , which is also a slightly smaller value than the value of geographical latitude.

In study [22], using the Bernard-Menguy-Schwartz (BMS) model, the optimal values of the seasonal and annual tilt angles of photovoltaic module panels were determined.

The research is based on the proven fact that the maximum average annual efficiency of photovoltaic modules is achieved at the optimal angle of inclination. This study was conducted in Albania (hot climate zone). Photoelectric modules were installed in Vlora (latitude 40.4661°), Kuksi (latitude 42.0807°), and Tirana (latitude 41.3275°). According to the research results, the optimal tilt angles for these cities were: Vlora – 37° , Kuksi – 39° , and Tirana – 38° . In addition to the results of observations, the authors also presented a mathematical model for determining the optimal angle of inclination of panels. It is worth noting that the annual optimal angle of inclination of the panels of photovoltaic modules, calculated according to the given mathematical model, is also somewhat smaller than the geographical latitude. The shortcoming of the model of the study is that it does not take into account the influence on the magnitude of the angle of inclination of the error caused by the lunar period.

There are a number of works in which the authors, through mathematical modeling, tried to calculate the optimal angle of inclination of the photovoltaic module panel for a certain region, taking into account various local influencing factors. According to [23], the local factors of influence on the dependence of the angle of inclination of the panel on the geographical latitude are chosen: the sign of latitude, longitude, height of the area, diffuse fraction or albedo of radiation, humidity, temperature, etc. Many of the above calculations are based on the use of the Perez model [24, 25]. However, even in these works, few observational data are presented to make generalizations about the value of the optimal tilt angle of the panel as a function of latitude, or as a function of latitude and other local influencing factors [26, 27].

There are several models in the literature that can calculate the annual optimal tilt angle of the module panel as a function of latitude, but these are integrated into databases that do not contain sufficient observational data for each latitude. With such calculations of the optimal tilt angle of the module panel, the error can reach more than 10%. This suggests that for the same latitude, the optimal inclination angle may depend on various local influencing factors [28]. The authors of studies [27, 29] also share this point of view.

After reviewing the literature [9–29], it can be concluded that there are enough works on substantiating the value of the optimal angle of inclination as a function of geographical latitude, and the influence of this angle on the average annual efficiency of the photovoltaic module. Some works are of a purely practical nature and the reported results could be applied only for conditions similar to those provided in studies. Other works contain mathematical models that, if the local influence factors are entered incorrectly, give an error of $<10\%$, which makes their use impossible.

Therefore, a study aimed at establishing the dependence of the average annual efficiency of solar photovoltaic modules on their angle of inclination to the horizon depending on the value of the geographical latitude at which they are installed is necessary. The results should be represented in the form of generalized mathematical expressions that make it possible to take into account the influence of the correction angle on the average annual efficiency of the photovoltaic module.

3. The aim and objectives of the study

The purpose of our study is to establish the dependence of the average annual efficiency of solar photovoltaic mod-

ules on the mounting angle of photovoltaic modules and the value of geographical latitude. This will increase the efficiency of electricity generation by photovoltaic modules.

To achieve the goal, the following tasks were set:

- to determine the average annual efficiency of fixed photovoltaic modules depending on the angle of their installation at different values of geographical latitude;
- to derive generalized expressions for determining the average annual efficiency of installation of stationary photovoltaic modules depending on the amount of correction of the angle of their installation at different values of geographical latitude.

4. The study materials and methods

Photoelectric modules are the object of our research.

The subject of the study is the dependence of the correction of the mounting angle of photovoltaic modules on the value of geographical latitude.

The main hypothesis of the research assumes that by adjusting the mounting angle of photovoltaic modules for different values of geographical latitude it is possible to increase the efficiency of installing solar photovoltaic modules.

The main assumptions and simplifications adopted in the work: the shape of the Earth was assumed to be a sphere; cloudiness, since it does not affect the angle of incidence of sunlight on the photovoltaic module, but only affects the intensity of solar radiation, was ignored in the calculations; it was assumed that the Earth is in a parallel flow of solar rays.

Researching the average annual efficiency of photovoltaic modules is an extremely large-scale and long-term process. If we take into account the planning of the experiment and ensuring its reliability, it takes either a lot of time to carry out the necessary number of repetitions, or a significant number of parallel studied modules. The results of the study are influenced by a significant number of factors – starting from climatic changes and ending with a decrease in the efficiency of photovoltaic modules over time.

To implement a large-scale study, a significant number of research modules are required, which are installed in territories with a precise geographical location. Laboratory models that determine the efficiency of solar PV modules at different latitudes also require significant capital investment and time. At the same time, the results of such experiments will differ significantly from the data obtained under real conditions due to the influence of a significant number of external factors.

The above significantly increases the value of simulation models, which allow conducting research in numerical form instead of high-cost field experiments. Such models, one of which is described in [30], make it possible to optimize the studied systems in the digital space. After analyzing the data obtained by simulation modeling, field experiments can be performed in the ranges determined as optimal by the results of simulation modeling.

When conducting research, the angular length of daylight was determined by the following expression [30]:

– if:

$$\delta < 0 \rightarrow a = 2\arctg\sqrt{\frac{\text{ctg}^2\varphi}{\text{tg}^2\delta}};$$

– if:

$$\delta \geq 0 \rightarrow a = 2\pi - 2\arctg\sqrt{\frac{\text{ctg}^2\varphi}{\text{tg}^2\delta}} - 1, \tag{1}$$

where a is the angular length of daylight; δ is the declination angle (angular position of the Sun at noon relative to the plane of the equator); φ is the geographical latitude of the installation point of the panel on the earth's surface.

The angle of incidence of the sun's rays relative to the z -axis, which is perpendicular to the surface of the solar panel, according to the geometry of the three-dimensional space, was determined by the expression:

$$\cos\theta_z = \sqrt{1 - \cos^2\theta_H - \cos^2\theta_V} = \sqrt{\sin^2\theta_H - \cos^2\theta_V}, \tag{2}$$

where θ_H is the angle of incidence of the sun's rays relative to the y -axis, which is placed in the plane of the solar panel parallel to the equatorial plane; θ_V is the angle of incidence of the sun's rays relative to the x -axis, which is placed in the plane of the solar panel in the meridional plane; θ_Z is the angle of incidence of the sun's rays relative to the z -axis, which is perpendicular to the plane of the solar panel.

The average annual efficiency of installing photovoltaic modules was determined as the annual weighted average value of the cosine of the angle of incidence of solar rays $\cos\theta_Z^m$ on the plane of the solar panel according to the following expression [30]:

$$\cos\theta_Z^m = \frac{\sum_{i=1}^{365} a_i \cos\theta_{Zi}^d}{\sum_{i=1}^{365} a_i}, \tag{3}$$

where a_i is the angular length of the i -th day, degrees; $\cos\theta_{Zi}^d$ – daily efficiency of the installation of photovoltaic modules, which was determined as the weighted average daily value of the cosine of the angle of incidence of solar rays on the plane of the solar panel:

$$\cos\theta_{Zi}^d = \frac{\sum_{j=0}^{a_j} a_j \cos\theta_j}{\sum_{j=0}^{a_j} a_j}, \tag{4}$$

where a_j is the current value of the angular length of the j -th day from sunrise to sunset, degrees; $\cos\theta_j$ is the corresponding to the angular length of the j -th day, the value of the cosine of the angle of incidence of the sun's rays.

The average annual efficiency of installation of fixed photovoltaic modules depending on the amount of correction of the angle of their installation at different values of geographic latitude and the generalization of the results was carried out on the basis of modeling in the Microsoft Excel environment (USA).

First, for each value of geographic latitude, the average annual efficiency of stationary photovoltaic modules was determined depending on the mounting angle of the panel at the corresponding latitude. After that, for each value of geographical latitude, the mounting angle of stationary photovoltaic modules was determined, at which the maximum value of the average annual efficiency was reached. Next, the difference between the value of the geographical latitude and the mounting angle of fixed photovoltaic modules, at which the maximum value of the average annual efficiency was achieved, was calculated. The resulting values of the difference between the value of geographical latitude and the mounting angle of fixed photovoltaic modules were compared with experimental data obtained by different authors

at different geographical latitudes. Subsequently, the calculated data obtained with the help of a mathematical model were approximated by the corresponding equations.

5. Results of determining the influence of mounting angle on the efficiency of stationary solar photovoltaic modules

5.1. Average annual efficiency of photovoltaic modules depending on the angle of their installation at different values of geographical latitude

According to the results of the mathematical model presented in [30], the value of the average annual efficiency of the installation of photovoltaic modules was determined for the values of geographical latitude from 0° (equator) to 60° with a step of 10°. Also, for the mounting angles of photovoltaic modules for latitude values from 0° (equator) to 60° with a step of ±10°.

The result of the study of the simulation model is the family of curves shown in Fig. 1. According to Fig. 1, for each value of geographical latitude when changing the mounting angle of the module panels, first the average annual efficiency of the installation of photovoltaic modules increases, reaching the maximum value, and then decreases. Thus, each curve shown in Fig. 1 has a maximum that corresponds to the value of the theoretical optimal mounting angle of the solar photovoltaic module at each geographic latitude. Moreover, only at the equator does the mounting angle of photovoltaic modules coincide with the value of geographic latitude (0°). At higher latitudes, the mounting angle of photovoltaic modules decreases, and the amount of the mounting angle decrease increases with increasing latitude.

Table 1 gives digital data obtained as a result of experimental studies on determining the angle of inclination of the panel of the photovoltaic module depending on the geographical latitude at which the module is installed.

Summarizing the results of analyzing the family of curves shown in Fig. 1, there is a graphical dependence demonstrated in Fig. 2. Since all the results shown in Fig. 2 are the maxima of the average annual efficiency, their values were not displayed on the plot to simplify perception. The

main purpose of constructing this plot is to reconcile the results of simulation modeling with the results of research by other scientists from around the world, given in Table 1. The sample (Table 1) contains the results of studies of the average annual efficiency of the tilt of the modules at different geographic latitudes and is aimed at establishing the adequacy of the model where a full-scale field experiment is impossible due to the scale and duration of the work. Fig. 2 shows two graphical dependences. The first one is built according to the values of the angles of inclination of the photovoltaic module, at which the maximum of the average annual efficiency is reached at each geographical latitude. The second is a curve of approximation of a sample of the results of experimental studies regarding the optimal mounting angles of photovoltaic modules (Table 1). Research conditions and results of statistical processing of the data given in Table 1 are reported in [20–22, 27, 31–34].

To check the adequacy of the simulation results, a method of comparison with the results of approximation of the data in Table 1 was used. The approximation was carried out using a polynomial of the second power, the equation of which takes the following form:

$$\beta = -0.0007 \cdot \varphi^2 + 1.135 \cdot \varphi - 1.6858, \tag{5}$$

where β is the value of the mounting angle of photovoltaic modules at a given geographic latitude, degrees; φ is the geographical latitude of the installation point of the panel on the earth's surface, degrees.

The reliability of the approximation is $R^2 = 0.8708$.

The values of the mounting angles of the modules, for which the average annual efficiency is maximum, obtained from the simulation model, are described by a polynomial approximation of the second power:

$$\beta = -0.0023 \cdot \varphi^2 + 1.024 \cdot \varphi + 0.1143. \tag{6}$$

The reliability of the approximation is $R^2 = 0.9992$.

The approximation results are correlated with the simulation results, the correlation coefficient between the model data and the data of the statistical sample given in Table 1 is $r = 0.9859$.

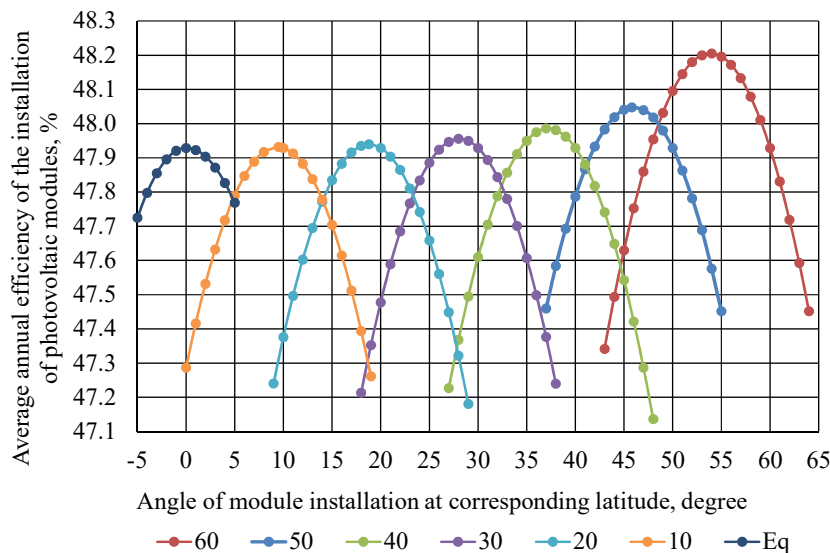


Fig. 1. Average annual efficiency of the installation of photovoltaic modules depending on the angle of their inclination at different values of geographical latitude

Table 1

Data from experimental studies of the angle of inclination of the photovoltaic module and the geographical latitude at which the module is installed

Country	City	Geographical latitude	Optimal angle of module installation	Source
1	2	3	4	5
China	Guangzhou	23°N	20°	[20]
Japan	Kita-Kyushu	33°53'N	32°	[21]
Albania	Vlora	41°20'N	38°	[22]
	Kuksi	42°05'N	39°	
	the tyrant	40°28'N	37°	
Algeria	Algeria	36°43'N	31°	[27]
Austria	Graz	47°N	33°	
Belize	Belize	17°32'N	16°	
Bosnia and Herzegovina	Banja Luka	44°47'N	33°	
Bulgaria	Fertile	42°08'N	30°	
Canada	Calgary	51°07'N	45°	
	Vancouver	49°11'N	34°	
	Montreal	45°30'N	37°	
China	Beijing	39°56'N	37°	
	Shanghai	31°10'N	23°	
	Kunming	25°01'N	25°	
Cuba	Santi Spiritus	21°56'N	21°	
Cyprus	Larnaca	34°53'N	30°	
Czech Republic	Ostrava	49°43'N	33°	
France	Lyon	45°44'N	30°	
	Bordeaux	44°50'N	33°	
Germany	Cologne	50°52'N	32°	
	Munich	48°08'N	33°	
Greece	Athens	37°54'N	29°	
Hungary	Debrecen	47°29'N	30°	
Iran	Tehran	35°25'N	31°	
	Travel	31°53'N	26°	
Ireland	Kilkenny	52°40'N	36°	
Israel	Beer Sheva	31°15'N	29°	
Italy	Catania	37°28'N	27°	
Japan	Osaka	34°47'N	30°	
Korea	Gwangju	35°08'N	29°	
North Korea	Pyongyang	39°02'N	36°	
Libya	Tripoli	32°40'N	27°	
Malaysia	Kuala Lumpur	3°07'N	1°	
Mexico	Mexico	19°26'N	17°	
Mongolia	Ulaanbaatar	47°56'N	43°	
Montenegro	Podgorica	42°22'N	36°	
Morocco	Casablanca	33°22'N	28°	
Norway	Oslo	59°54'N	40°	
Palestine	Jerusalem	31°52'N	28°	
Philippines	Manila	14°31'N	9°	
Poland	Bielsko-Biala	49°40'N	31°	
Portugal	Lisbon	38°44'N	35°	
Romania	Bucharest	44°30'N	32°	
Russia	St. Petersburg	59°58'N	40°	
Russia	Omsk	59°56'N	42°	
Saudi Arabia	Riyadh	24°42'N	24°	
Senegal	Dakar	14°44'N	14°	
Serbia	Belgrade	44°49'N	34°	
Singapore	Singapore	1°22'N	0°	
Slovakia	Kosice	48°42'N	33°	

Continuation of Table 1

1	2	3	4	5
Spain	Castellón de la Plana	39°57'N	36°	[27]
	Ceuta	35°53'N	31°	
Sweden	Stockholm	59°39'N	41°	
Switzerland	Geneva	46°15'N	32°	
Syria	Damascus	33°25'N	29°	
Taiwan	Taipei	25°04'N	17°	
Tunisia	Tunisia	36°50'N	28°	
Turkey	Ankara	40°07'N	29°	
Ukraine	Odesa	46°27'N	31°	
USA	Riley	35°52'N	32°	
	Bakersfield	35°26'N	29°	
	Austin	30°17'N	28°	
Uzbekistan	Tashkent	41°16'N	32°	
Venezuela	Caracas	10°36'N	10°	
Vietnam	Hanoi	21°12'N	16°	
Morocco	Tangier	35°45'N	32°	[31]
	Tetouan	35°34'N	32°	
	Nador	35°10'N	31°	
	Oujda	34°41'N	31°	
	Kenitra	34°15'N	30°	
	Fez	34°02'N	30°	
	Discount	34°01'N	30°	
	Meknes	33°53'N	30°	
	Mohammedia	33°41'N	30°	
	Casablanca	33°32'N	30°	
	Ifran	33°31'N	30°	
	El Jadid	33°14'N	29°	
	Settate	33°00'N	29°	
	Beni Mellal	32°20'N	28°	
	Safi	32°17'N	28°	
	Errachidia	31°55'N	28°	
	Marrakesh	31°37'N	27°	
Ouarzazate	30°55'N	27°		
Agadir	30°25'N	26°		
Helmim	28°59'N	25°		
Iran	Tabas	33°22'N	32°	[32]
Canada	Ottawa	45°N	36°–38°	[33]
	Toronto	44°N	32°–35°	
Iceland	Reykjavik	64°07'N	48°	[34]
Canada	Sherbrooke	45°24'N	36°	

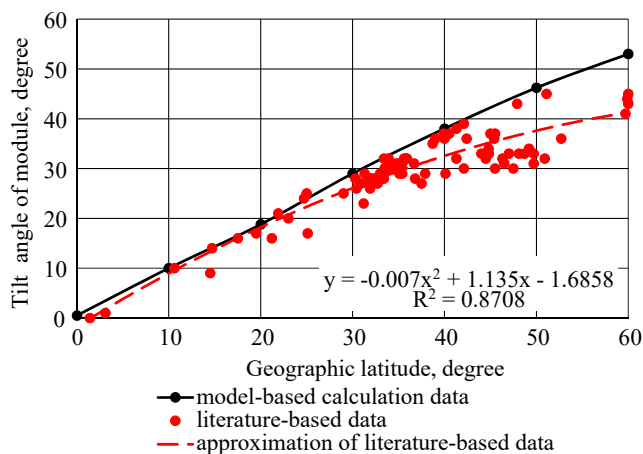


Fig. 2. Deviation of the mounting angle of photovoltaic modules from the latitude angle at which their maximum average annual efficiency is achieved at different values of geographic latitude

Despite the high level of agreement, there are differences regarding the optimal mounting angles of solar photovoltaic modules at latitudes higher than 30°. According to the results of field experiments, the mounting angle of the modules is smaller than the value of the angle obtained as a result of simulation. This is explained by the significant difference between the number of sunny days in summer and winter. As latitude increases, the number of sunny days in winter decreases significantly, which leads to a decrease in the efficiency of solar photovoltaic modules, and therefore to negative values of the mounting angles of the modules. Thus, the highest efficiency of energy production by modules is observed in the summer period, where the length of daylight also differs significantly in latitudes higher than 30°. If the last factor is taken into account by the mathematical model [30], then weather conditions, such as high cloud cover in certain regions, were not taken into account. This does not make the model wrong, but it is clear from the study that for higher latitudes it can be refined in further studies.

Approaching both approximation curves to each other in Fig. 2 shows the essentiality of the model. In the future, the introduction of additional data into the equation of the model (for example, annual statistics of the number of sunny days) will make it possible to reach the maximum values of the average annual efficiency of the modules in any part of the world with high accuracy.

5. 2. Generalized expressions for determining the efficiency of fixed photovoltaic modules taking into account the angle of their installation

Based on the simulation model [30], the values of the average annual efficiency of solar modules were calculated and the graphical dependence between the average annual efficiency of photovoltaic modules and their mounting angle for different values of geographical latitude was plotted, which is shown in Fig. 3.

The calculation data used to construct this plot are given in Table 2.

Based on the approximation of the calculated data shown in Fig. 3, a mathematical expression is obtained that makes it possible to determine the average annual efficiency of installing photovoltaic modules depending on the angle of their installation for different values of geographic latitude. This expression takes the following form:

$$\cos\theta_z^{an} = 47.929 - 0.0145 \cdot \varphi + 0.0039 \cdot \beta - 0.0053 \cdot \varphi^2 + 0.0124 \cdot \varphi\beta - 0.0068 \cdot \beta^2, \tag{7}$$

where $\cos\theta_z^{an}$ is the average annual efficiency of installation of photovoltaic modules, %.

Our results regarding the efficiency of the installation of photovoltaic modules make it possible to establish a relationship between the average annual efficiency of the installation of photovoltaic modules and the angle of their installation at different values of geographical latitude.

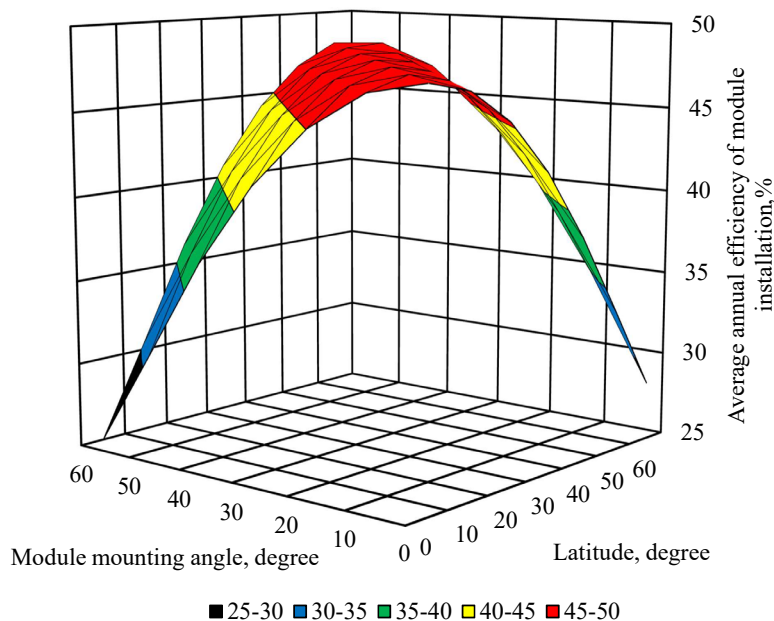


Fig. 3. Average annual efficiency of installation of photovoltaic modules depending on the angle of their installation for different values of geographical latitude

Table 2

Estimated data of average annual efficiency of photovoltaic modules

Mounting angle of the photovoltaic module, degree	Latitude, degree						
	60	50	40	30	20	10	0
60	47,929	46,614	44,239	40,709	36,068	30,43	23,953
50	48,096	47,929	46,795	44,492	41,004	36,398	30,798
40	46,802	47,787	47,929	46,923	44,694	41,261	36,707
30	44,086	46,194	47,61	47,929	47,026	44,87	41,501
20	40,031	43,197	45,838	47,478	47,929	47,115	45,034
10	34,759	38,887	42,677	45,585	47,376	47,929	47,199
0	28,431	33,396	38,219	42,307	45,383	47,287	47,929

6. Discussion of results of investigating influence of the mounting angle of photovoltaic modules on their average annual efficiency

To evaluate the efficiency of the installation of photovoltaic modules, the value of the annual average weighted by the cosine of the angle of incidence of solar rays on the plane of the solar panel was used [30].

Research results showed that the average annual efficiency of photovoltaic modules has a maximum for each value of geographical latitude. Moreover, only at the equator does the mounting angle of photovoltaic modules coincide with the value of geographic latitude (0°). In other cases, the maximum of the average annual efficiency of photovoltaic modules corresponds to a smaller value of the angle of their installation than the value of the geographical latitude. Thus, with a latitude of 10°, 20°, 30°, 40°, 50°, and 60°, the mounting angle of photovoltaic modules will be 9.5°, 18.8°, 28°, 37°, 45.8°, and 54°, respectively. It is obvious that at larger latitude values, the deviation of the mounting angle of photovoltaic modules from the latitude value increases.

The correlation coefficient of the set of experimental data of the mounting angle of photovoltaic modules from the value of geographic latitude and the corresponding set of calcu-

lated data according to (5) has a value of 0.83. Fig. 2 clearly shows that the largest deviation of the actually obtained values of the mounting angle of photovoltaic modules from the value of geographical latitude from the calculated data occurs at values of geographical latitude greater than 40°. This can be explained by the fact that significant cloudiness is observed at these latitudes in winter. At the same time, researchers are forced to choose the values of the mounting angles of photovoltaic modules, which more closely correspond to the values of the mounting angles of photovoltaic modules for the summer period. And these angles, in turn, at latitude values greater than 40°, are to a greater extent smaller than the values of geographic latitude.

The physical essence of the phenomenon of an increase in the average annual efficiency of photovoltaic modules when the angle of their installation is reduced compared to the value of the geographical latitude is explained as follows. When the mounting angle of photovoltaic modules decreases, the cosine of the angle of incidence of solar rays on the plane of the solar panel increases in summer and decreases in winter. On the contrary, when the mounting angle of photovoltaic modules increases, the cosine of the angle of incidence of solar rays on the plane of the solar panel decreases in summer and increases in winter. This determines the presence of maxima of the average annual efficiency of photovoltaic modules. The greater decrease in the mounting angle of photovoltaic modules to achieve the maxima of their average annual efficiency with an increase in the value of geographical latitude is explained as follows. As is known, the length of the day increases with the increase in the value of the geographical latitude. This means an increase in the average annual efficiency of photovoltaic modules due to a longer time of the sun's rays falling on the plane of the solar panel in the summer.

Our results are similar to the results reported in [19, 20]. In those works, the authors confirm in practice the dependence of the efficiency of the installation of photovoltaic modules on the reduction of the angle of their installation at different latitudes compared to the value of the latitude.

The chosen approach is determined by two components. The first is the determination of the values of the weighted average daily cosine of the angle of incidence of the sun's rays on the plane of the photovoltaic module. The second is to determine, on this basis, the weighted average annual value of the cosine of the angle of incidence of the sun's rays on the plane of the photovoltaic module. The results can be used to calculate the amount of reduction in the mounting angle of photovoltaic modules at different latitudes. The study explains the slight deviation of the optimal mounting angles of photovoltaic modules, installed experimentally, from the mounting angles determined by this method, and which have values corresponding to the values of the latitude of the location of the installation of photovoltaic modules. Our results, illustrated in Fig. 2, 3, and given in dependences (5), (7), have numerous experimental confirmations in the scientific literature [24–27].

Limitations of the study: the results allow establishing a relationship between the average annual efficiency of installing photovoltaic modules and the reduction of the angle of their installation at different geographic latitudes compared to the value of geographic latitude on the Earth's surface from the equator to the value of latitude 66°32'.

The main shortcoming of this study is that it does not take into account the unevenness of the flow of sunlight on

the Earth's surface in summer and winter, which mostly concerns geographical latitudes far from the equator.

The further development of this research should consist in determining the optimal values of the mounting angles of stationary photovoltaic modules separately for summer and winter time.

7. Conclusions

1. The average annual efficiency of photovoltaic modules has a maximum for each value of geographical latitude. The maximum average annual efficiency of photovoltaic modules corresponds to a smaller value of the angle of their installation than the value of the geographical latitude. Thus, with a latitude of 10°, 20°, 30°, 40°, 50°, and 60°, the mounting angle of photovoltaic modules will be 9.5°, 18.8°, 28°, 37°, 45.8°, and 54°, respectively. It is obvious that at larger latitude values, the deviation of the mounting angle of photovoltaic modules from the latitude value should increase.

2. A dependence was derived that allows determining the mounting angle of photovoltaic modules from the value of the geographical latitude at which they are installed. This dependence is an approximation by a second-power polynomial of the points corresponding to the maxima of the average annual efficiency of solar photovoltaic modules at different geographical latitudes. A mathematical expression was also built that makes it possible to determine the average annual efficiency of installing photovoltaic modules depending on the angle of their installation for different values of geographic latitude. This dependence describes the array of data obtained during the study of the simulation model by a two-factor equation of the second power. The graphical equivalent of the resulting dependence is the response surface, on which the regions of the maxima of the average annual efficiency of solar modules are clearly visible.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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