The work examines an analytical solution for calculating the fluxmetric demagnetizing factor of cylindrical magnets at large values of magnetic susceptibility and an arbitrary value of elongation. The application of the analytical solution for calculating the demagnetizing factor significantly simplifies the modeling and calculation of magnetic characteristics of cylindrical technical objects. A simplified analytical model of the scalar potential of the magnetic field of a cylinder with infinite magnetic favorability, inductively magnetized in a uniform magnetic field, was constructed using an approximate representation of the distribution of fictitious magnetic charges on its surface. The method of spherical harmonic analysis for the magnetic field was used, which made it possible to obtain an analytical representation of the demagnetization field in the central cross section of the cylinder. Limitation of the harmonic series of this representation by seven first harmonics is proposed, and an additional amplitude factor is applied to correct the contribution of the first harmonic to the demagnetization field. This made it possible to compensate for the distortion of the magnetic field near the ends of the cylinder and bring the simplified analytical model closer to the target mathematical model with a uniform demagnetization magnetic field. The reliability of the results of calculating the fluxmetric demagnetizing factor according to the derived formula was evaluated by comparing them with the known results obtained using the numerical method of calculation and according to empirical formulas. It is shown that the proposed approach makes it possible to obtain reliable results of calculating the fluxmetric demagnetizing factor with a deviation of up to 5 % at infinite favorability in the range of cylinder elongation values from 0.01 to 500

Keywords: cylinder fluxmetric demagnetizing factor, inductive magnetization, spherical harmonics of the magnetic field

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FINDING AN ANALYTICAL SOLUTION FOR THE CYLINDER'S FLUXMETRIC DEMAGNETIZING FACTOR USING SPHERICAL HARMONICS

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1. Introduction

An important characteristic used in the study and modeling of magnetism of regularly shaped bodies is the demagnetizing factor. The demagnetization magnetic field created by magnetization inside the body is conveniently related to the magnitude of magnetization using a proportionality constant between them – the demagnetizing factor (DF). However, such a simple representation of DF is mathematically rigorous only for bodies whose boundary surface is a closed surface of the second order. This is due to the fact that only for such magnetized bodies the created internal demagnetization field is uniform. However, the prevalence of the cylindrical shape in technology predetermines the practical significance of the application of DF in modeling and designing cylindrical magnetoactive parts of various devices. In radio engineering, the DF of a cylinder is used in the calculation of antennas, inductors, chokes, etc. In electrical engineering, the calculation of magneto-active parts of automation systems, complete devices and power electromagnets is based on models using the cylinder DF. In metrological studies of the characteristics of magnetic materials, samples of a cylindrical shape and calculation models based on the DF of a cylinder are usually used. Depending on the features of the magnetic model of the technical object, the concepts of magnetometric or cylinder fluxmetric demagnetizing factors are used. The magnetometric demagnetizing factor is based on the use of the volume average value of magnetization, that is, the specific magnetic moment of a cylindrical sample. The fluxmetric coefficient is entered as the ratio of the average integral value of the demagnetization field to the average integral value of magnetization in the central cross section of the cylinder. According to the method of experimentally determining such a ratio, the DF was termed fluxmetric. However, an analytically rigorous idea of the distribution of magnetization in the middle of an inductively magnetized cylinder is still missing, which limits the possibilities of substantiating and obtaining formulas for calculating the DF of a cylinder. A successful example of the application of additional conditions in the construction of a model for analytical calculation of DF is the use of the condition of homogeneity of the distribution of residual magnetization exclusively in the middle of a cylindrical permanent magnet. This corresponds to the case of calculating the DF of an inductively magnetized cylinder with a value of magnetic permeability close to zero. The situation with determining the spatial distribution of magnetization in the middle of an inductively magnetized cylinder is significantly complicated at large values of magnetic favorability, which is inherent in most magnetic materials. In this case, the inhomogeneity of the magnetization distribution is affected by the shape of the magnet and the inhomogeneity of the demagnetization field.

Therefore, to meet the practical need for the calculation of DF, approximate formulas are usually used, which were obtained by mathematical processing of the experimentally determined magnetic characteristics of cylindrical magnets. In addition, recently, computer numerical methods for calculating the magnetic field have been widely used, which significantly increased the accuracy of determining the DF, for a given geometry and magnetic susceptibility of the cylinder.

However, the search for an analytical solution even limited by additional conditions for the inductive magnetization and subsequent calculation of the DF of the cylinder remains an urgent task.

2. Literature review and problem statement

Works by the classics of electromagnetism contain the first attempts to build a complete analytical model of a magnetized cylinder, by analogy with the successful application of potential theory to obtain an idea of a magnetized ellipsoid. However, the approximate model of an ultrathin cylinder with a linear distribution of magnetization obtained by them can be considered conditionally successful.

In the absence of a rigorous analytical solution to meet the practical needs of applying the cylinder fluxmetric demagnetizing factor, its asymptotic representations were proposed. Most of such formulas, such as in [1], were developed on the basis of the correction of the known analytical representation for the demagnetizing factor of a spheroid, and their reliability was proven experimentally. However, the formula from [1] has restrictions on its use and is used only for elongated cylinders for which the elongation is greater than ten. A peculiar addition to the range of use of such a formula is the representation reported in [2], in which a formula is proposed for calculating the fluxmetric demagnetizing factor of short cylinders with elongation from two to ten.

In order to avoid limitations on the area of definition of the analytical representation of the demagnetizing factor, various analytical methods of modeling the inductive magnetization of the cylinder were used. Thus, in [3], a method was proposed in which the cylindrical core and the air gap are described by equivalent magnetic circuits. The correctness of the method is confirmed by a good agreement between the results of the calculation and the experiment. However, the use of this method is not based on obtaining a direct formula for calculating the fluxmetric demagnetizing factor. The application of another analytical method based on the solution of the Poisson-Neumann equation in the form of cylindrical harmonics is considered in [4]. The results contain integral representations, the practical use of which is impossible without the use of special methods of their mathematical calculation. The development of this method on the basis of a simplified mathematical representation for the external magnetic field [5], in which the cylinder is located, also does not lead to the possibility of representing the magnetization of the cylinder with formulas convenient for analytical calculation.

Current papers related to the research and modeling of an inductively magnetized cylinder are based on the optimization of numerical methods, based on the results of which it is possible to calculate the fluxmetric demagnetizing factor, for example, as in [6]. For this purpose, the work solves a more advanced problem of calculating the magnetic characteristics for both a ferromagnetic and a diamagnetic cylinder. The accuracy of the results obtained in this way for the case of large values of cylinder elongation at infinite favorability is partially considered in [7]. In [8], a numerical calculation of the fluxmetric demagnetizing factor and its comparison with other available data was carried out based on the model of fictitious surface magnetic charges. But in these works, without obtaining an approximation formula, data are given only for some specific values of the ratio of the length to the diameter of the cylinder at different values of relative favorability. The use of a specialized software package for numerical calculations of various physical processes COMSOL Multiphysics when determining the volume average value of the cylinder demagnetizing factor is considered in [9]. The obtained results for ten variants of the geometry of the cylinder agree well with the experimentally obtained data by other authors and justify the possibility of applying the magnetization curve of the material in numerical calculations of open magnetic systems. Different approaches to modeling the magnetization of a cylinder in the form of various sources based on magnetic charges and currents and their comparison when applying the finite element method are considered in [10]. In this work, the comparison of the obtained results is also carried out from the point of view of simplifying the requirements for numerical calculation tools. The most complete data in terms of the breadth of presented results of numerical calculation of fluxmetric and magnetometric cylinder demagnetizing factors are reported in [11]. The paper presents demagnetizing factors calculated by two-dimensional and one-dimensional models for a wide range of specific values of elongation and magnetic susceptibility in graphical and tabular form. Paying the main attention to the peculiarities of the application of numerical calculation methods, obtaining an analytical representation for the cylinder demagnetizing factor was not considered in these works.

Although the use of numerical calculation methods is the most reliable way to obtain the value of the cylinder demagnetizing factor with the required accuracy, they all have a common drawback. The application of the numerical method of calculation does not allow obtaining a strict analytical representation for the demagnetizing factor of the cylinder, even with additional conditions for simplifying the magnetization model. Although an attempt to obtain an approximate asymptotic formula for the demagnetizing factor for a diamagnetic cylinder, using the results of a numerical calculation, is considered in [12]. However, the obtained result cannot be applied to ferromagnetic cylinders with a large value of magnetic favorability.

Our review of the current state of calculation of the fluxmetric demagnetizing factor of a cylinder gives grounds for asserting that the existing analytical representations have limitations on the application in the form of a finite segment of possible elongation values. Therefore, it is appropriate to build and study a model of an inductively magnetized cylinder, the mathematical analysis of which allows obtaining a formula for direct calculation of fluxmetric demagnetizing factor for arbitrary values of elongation.

3. The aim and objectives of the study

The purpose of our work is to obtain an analytical representation for the fluxmetric demagnetizing factor of a cylinder based on spherical harmonic analysis of its magnetic field.

This will make it possible to apply the demagnetizing factor of a single formula in engineering calculations at arbitrary values of cylinder elongation for the case of large values of permeability.

To achieve the goal, the following tasks are to be solved:

– to build a simplified analytical model of a cylinder inductively magnetized in a uniform external magnetic field, using an approximate representation of the distribution of fictitious magnetic charges on its surface;

– to perform a spherical harmonic analysis of the demagnetization magnetic field in the central cross-section of the cylinder, limiting the number of applied harmonics to the seventh power;

– to perform an assessment of the reliability of the results of calculating the fluxmetric demagnetizing factor according to the obtained formula by comparing them with the known results obtained using the numerical method of calculation and according to empirical formulas.

4. The study materials and methods

In the work, the idea of an inductively magnetized solid cylinder within the framework of the concept of the basic equation of magnetostatics [13] is subject to theoretical research. Denoting through $\vec{M}$ the magnetization vector, for each observation point determined by the radius vector $\vec{r}$, inside the magnetized body, the following equality holds:

$$\vec{M}(\vec{r}) = \chi\vec{H}(\vec{r}), \tag{1}$$

where χ is the magnetic susceptibility of the material, $\vec{H}$ is the total (true) magnetic field at the point of observation. An additional condition that significantly simplifies the analytical representation is the constant value of the magnetic favorability χ, which is taken as a constant that does not depend on the value of the magnetic field modulus or its direction. The true value of the magnetic field in the middle of a magnetized body in (1) means the sum of two components: $\vec{H}_0$ – the magnetic field of external sources, and $\vec{H}_d$ – the magnetic field of demagnetization created by the body itself [11]:

$$\vec{H} = \vec{H}_0 + \vec{H}_d. \tag{2}$$

Under such conditions, the model of an inductively magnetized body with $\chi=\infty$ is equivalent to a model based on fictitious magnetic charges located on its surface [14]. Denoting by σ the surface density of fictitious charges, the transition to this model occurs according to the formula:

$$M_n = \sigma, \tag{3}$$

where M_n is the normal projection of magnetization on the surface of the body.

Then, by analogy with the model of the electric field in the middle of the conductor, the total magnetic field in (2) is zero [14].

The determination of the fluxmetric demagnetizing factor N_f of the cylinder under such preliminary conditions takes the form [6]:

$$N_f = -\frac{\frac{1}{S}\int_S \left(\vec{H}_d\right)_z \mathrm{d}S}{\frac{1}{S}\int_S \left(\vec{M}\right)_z \mathrm{d}S}, \tag{4}$$

where S is the surface of the central cross-section of the cylinder, the axis of which coincides with the applicate axis. The minus sign in (4) emphasizes the physics of creating a demagnetization field, which in the central section is directed opposite to magnetization.

The demagnetization field is potential and is represented by the scalar potential U, which has a functional relationship with surface charges according to the provisions of the potential theory [15]:

$$\vec{H}_d(\vec{r}) = -grad(U) = -grad\left(\frac{1}{4\pi}\oiint_{S_c} \frac{\sigma(\vec{r}')}{\left|\vec{r}-\vec{r}'\right|}\mathrm{d}S_c\right), \tag{5}$$

where S_c is the complete surface of the cylinder, which is integrated over all points of location of surface charges $\vec{r}'$.

Representation in (5) of the inverse distance $1/\left|\vec{r}-\vec{r}'\right|$ through spherical harmonics according to [16]:

$$\frac{1}{\left|\vec{r}-\vec{r}'\right|} = \sum_{n=1}^{\infty}\sum_{m=0}^{n} \frac{r^n}{(r')^{n+1}} \frac{(n-m)!}{(n+m)!}\left(2-\delta_m^0\right)P_n^m\left(\cos\theta_P\right)\times$$
$$\times P_n^m\left(\cos\theta_Q\right)\cos\left(m\left[\varphi_P-\varphi_Q\right]\right),\ r<r' \tag{6}$$

allows conducting a spherical harmonic analysis of the demagnetization magnetic field represented in the spherical coordinate system r, θ, φ.

5. Results of research on the fluxmetric demagnetizing factor of an inductively magnetized cylinder

5. 1. Simplified analytical model of a cylinder inductively magnetized in a uniform magnetic field

An analogy with the well-known model of a uniformly magnetized sphere [14] was used to construct a simplified analytical model of magnetization of a cylinder. According to it, in the case of an axially symmetrical arrangement on the surface of the sphere (Fig. 1) of fictitious magnetic charges with a density σ_s, which is proportional to $\cos(\theta_s)$, a uniform demagnetization field is created.

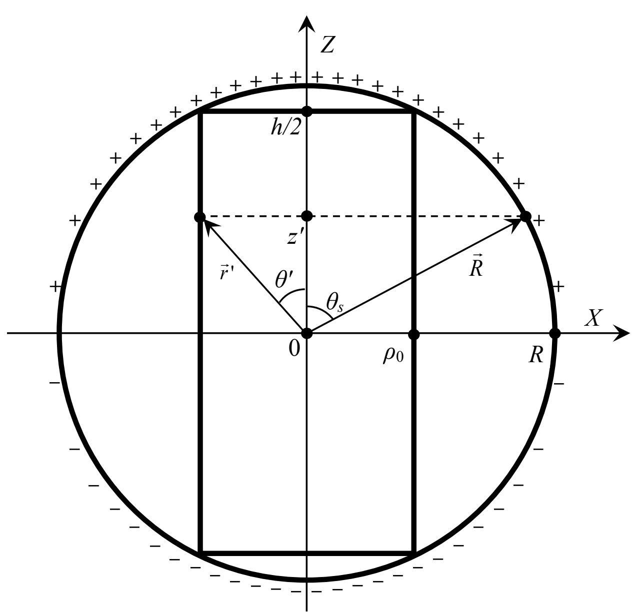

Fig. 1. Longitudinal section of a cylinder located inside a sphere charged by $\cos(\theta_s)$

Moving from spherical to cylindrical coordinates according to the geometry in Fig. 1, we get that the density of fictitious charges on the sphere is proportional to the applicate $\sigma_s \sim z'$. Then, for a cylinder with height h and radius ρ_0, we can assume a similar dependence $\sigma_c(z')$ of fictitious charges on the side surface:

$$\sigma_c(z') = \frac{\sigma}{\rho_0} z', \tag{7}$$

where σ/ρ_0 is a constant factor [A/m^2] chosen in a convenient form for further analysis.

It automatically follows from (7) that with negative values of the applicate, the surface charges on the cylinder have a negative sign, and the maximum and minimum values of the density correspond to the circles that are common to the ends. Since with such a geometry of the problem, the value of applicates at the ends does not change, the surface charge density at the ends is determined by two constants:

$$\sigma_c\left(\pm\frac{h}{2}\right) = \pm\frac{\sigma}{\rho_0}\frac{h}{2}, \tag{8}$$

where the plus sign corresponds to the upper end of the cylinder in Fig. 1.

Using a simplified model of the surface density of fictitious magnetic charges given by (7) and (8), the scalar potential of the demagnetization magnetic field is represented according to (5) as the sum of three terms:

$$U = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\rho_0} \frac{\sigma}{\rho_0}\frac{h}{2}\frac{\rho' d\rho'}{|\vec{r}-\vec{r}'|}\mathrm{d}\varphi' + \frac{1}{4\pi}\int_0^{2\pi}\int_{-h/2}^{h/2} \frac{\sigma}{\rho_0}\frac{z' dz'}{|\vec{r}-\vec{r}'|}\rho_0 \mathrm{d}\varphi' +$$
$$+\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\rho_0} \frac{-\sigma}{\rho_0}\frac{h}{2}\frac{\rho' d\rho'}{|\vec{r}-\vec{r}'|}\mathrm{d}\varphi', \tag{9}$$

where $\vec{r}'$, ρ', φ', z' are the radius vector and the cylindrical coordinates of fictitious magnetic charges on the surface of the cylinder.

The terms in (9) are contributions to the potential, respectively, from the upper end, from the side surface, and from the lower end.

After substituting (6) into (9) and integrating over the cyclic coordinate φ', which in this case is common to the cylindrical and spherical coordinate systems, we have the representation for the scalar potential:

$$U = \frac{\sigma}{2}\frac{h}{2\rho_0}\int_0^{\rho_0}\frac{1}{r'}\sum_{n=1}^{\infty}\left(\frac{r}{r'}\right)^n P_n(\cos\theta)P_n(\cos\theta')\rho'\mathrm{d}\rho' +$$
$$+\frac{\sigma}{2}\int_{-h/2}^{h/2}\frac{1}{r'}\sum_{n=1}^{\infty}\left(\frac{r}{r'}\right)^n P_n(\cos\theta)P_n(\cos\theta')z'\mathrm{d}z' -$$
$$-\frac{\sigma}{2}\frac{h}{2\rho_0}\int_0^{\rho_0}\frac{1}{r'}\sum_{n=1}^{\infty}\left(\frac{r}{r'}\right)^n P_n(\cos\theta)P_n(\cos\theta')\rho'\mathrm{d}\rho', \tag{10}$$

where $P_n(\cos\theta)$ are Legendre polynomials of power n.

It should be noted that for each of the terms on the right-hand side of (10), the relation of cylindrical and spherical coordinates of the location of surface charges differs and is determined according to the geometry of the problem. For the component of the potential from the charged upper end (first term), the relation of cylindrical and spherical coordinates of the location of charges is determined by the formulas:

$$\begin{cases} r' = \dfrac{h/2}{\cos\theta'}, \\ \rho' = \dfrac{h}{2}\mathrm{tg}\theta', \\ d\rho' = \dfrac{h/2}{(\cos\theta')^2}d\theta'. \end{cases} \tag{11}$$

The transition for the side surface (the second term) is described by the relation:

$$\begin{cases} r' = \dfrac{\rho_0}{\sin\theta'}, \\ z' = \rho_0\mathrm{ctg}\theta', \\ d\rho' = -\dfrac{\rho_0}{(\sin\theta')^2}d\theta'. \end{cases} \tag{12}$$

The relationship of cylindrical and spherical coordinates of the location of charges for the lower end (for the third term) is as follows:

$$\begin{cases} r' = -\dfrac{h/2}{\cos\theta'}, \\ \rho' = -\dfrac{h}{2}\mathrm{tg}\theta', \\ d\rho' = -\dfrac{h/2}{(\cos\theta')^2}d\theta'. \end{cases} \tag{13}$$

For the convenience of writing formulas, the cylinder elongation factor γ is usually used, which is defined as the ratio of its height to the diameter:

$$\gamma = \frac{h}{2\rho_0}. \tag{14}$$

To move from integration along the cylindrical coordinate in the three terms (10) to integrals along the spherical coordinate θ', the limits of integration are replaced according to the intervals:

$$\begin{cases} [0;\, \mathrm{arcctg}(\gamma)], \\ [-\mathrm{arcctg}(\gamma);\, \mathrm{arcctg}(\gamma)], \\ [\pi;\, -\mathrm{arcctg}(\gamma)]. \end{cases} \tag{15}$$

The application of the transition formulas in (10) allows us to represent the scalar potential as the sum of three integrals along the angular spherical coordinate:

$$U = \frac{\sigma}{2}\frac{h}{2\rho_0}\times$$
$$\times\int_0^{\mathrm{arcctg}(\gamma)}\sum_{n=1}^{\infty}\frac{h}{2}\left(\frac{r\cos\theta'}{h/2}\right)^n P_n(\cos\theta)P_n(\cos\theta')\frac{\mathrm{tg}\theta'}{\cos\theta'}\mathrm{d}\theta' -$$
$$-\frac{\sigma}{2}\int_{-\mathrm{arcctg}(\gamma)}^{\mathrm{arcctg}(\gamma)}\sum_{n=1}^{\infty}\rho_0\left(\frac{r\sin\theta'}{\rho_0}\right)^n P_n(\cos\theta)P_n(\cos\theta')\frac{\mathrm{ctg}\theta'}{\sin\theta'}\mathrm{d}\theta' +$$
$$+\frac{\sigma}{2}\frac{h}{2\rho_0}\int_{\pi}^{-\mathrm{arcctg}(\gamma)}\sum_{n=1}^{\infty}\frac{h}{2}\left(\frac{-r\cos\theta'}{h/2}\right)^n P_n(\cos\theta)P_n(\cos\theta')\frac{\mathrm{tg}\theta'}{\cos\theta'}\mathrm{d}\theta'. \tag{16}$$

Paying attention to the symmetry of the contributions of the upper and lower end, as well as using the symmetry of the limits of integration of the lateral surface charges (in the second term), we obtain representations without even harmonics in infinite sums. In addition, the transition from integration over θ′ to the new variable $x=\cos\theta'$ allows us to simplify (16), which takes the form convenient for analysis:

$$U=\frac{\sigma\rho_0}{2}\times$$
$$\times\sum_{n=1}^{\infty}{}'\left(\frac{r}{\rho_0}\right)^n P_n(\cos\theta)\left\{\begin{array}{l}\gamma^{2-n}\left[\begin{array}{l}\int\limits_{-1}^{1}x^{n-1}P_n(x)\mathrm{d}x-\\-\int\limits_{-\gamma/\sqrt{1+\gamma^2}}^{\gamma/\sqrt{1+\gamma^2}}x^{n-1}P_n(x)\mathrm{d}x\end{array}\right]+\\+\int\limits_{-\gamma/\sqrt{1+\gamma^2}}^{\gamma/\sqrt{1+\gamma^2}}x\left(1-x^2\right)^{\frac{n-3}{2}}P_n(x)\mathrm{d}x\end{array}\right\},\quad(17)$$

where $\sum_{n=1}^{\infty}{}'$ is the sum in which the terms are added only for odd values of n.

The difference of the integrals in braces of the right part of (17) determines the contribution from the charges located on the ends, and the last integral is the contribution of the charged side surface to the scalar potential of the demagnetization field.

5. 2. Spherical harmonic analysis of the magnetic field of demagnetization in the central cross section of the cylinder

Analysis of the image in braces (17) shows a decrease in each subsequent term as n increases. Accordingly, the infinite series of spherical harmonics in (17) for $r<\rho_0$ is sufficiently rapidly decreasing. Limiting the infinite series to the seventh harmonic, we obtain for analysis four harmonics with $n=1, 3, 5, 7$. We denote by $I_n(\gamma)$ the function that is in braces (17) and is determined depending on n. Then the functional coefficient $I_n(\gamma)$ of the parameter γ at the first harmonic ($n=1$) has the form:

$$I_1(\gamma)=\gamma\left[\int\limits_{-1}^{1}\mathrm{d}x-\int\limits_{-\gamma/\sqrt{1+\gamma^2}}^{\gamma/\sqrt{1+\gamma^2}}\mathrm{d}x\right]+\int\limits_{-\gamma/\sqrt{1+\gamma^2}}^{\gamma/\sqrt{1+\gamma^2}}\frac{x^2}{\left(1-x^2\right)}\mathrm{d}x=$$
$$=2\left(\gamma-\frac{\gamma^2}{\sqrt{1+\gamma^2}}-\frac{\gamma}{\sqrt{1+\gamma^2}}+\frac{1}{2}\ln\frac{\sqrt{1+\gamma^2}+\gamma}{\sqrt{1+\gamma^2}-\gamma}\right).\quad(18)$$

The first pair of terms in the right part of (18) is the contribution from the ends, and the last pair of terms is the contribution from the side surface.

All $I_n(\gamma)$ with even n have a zero value, so the following is the functional coefficient at the third harmonic, which is defined as:

$$I_3(\gamma)=-\frac{\gamma^2(\gamma-1)}{\left(\sqrt{1+\gamma^2}\right)^5}.\quad(19)$$

Entering $k=\gamma\big/\sqrt{1+\gamma^2}$, to shorten the notation, for the coefficient at the fifth harmonic we have:

$$I_5(\gamma)=-\frac{7}{4}k^9+\frac{19}{4}k^7-\frac{17}{4}k^5+$$
$$+\frac{5}{4}k^3-\frac{7}{4\gamma^3}k^9+\frac{5}{2\gamma^3}k^7-\frac{3}{4\gamma^3}k^9.\quad(20)$$

The functional coefficient at the seventh harmonic is as follows:

$$I_7(\gamma)=\frac{1}{8}\left(\begin{array}{l}\left(1-\gamma^{-5}\right)\left[33k^{13}-63k^{11}+35k^9-5k^7\right]-\\-78k^{11}+\frac{605}{3}k^9-189k^7+77k^5-\frac{35}{3}k^3\end{array}\right).\quad(21)$$

The general solution (17) for the scalar potential of the demagnetization field in the case of a series of spherical harmonics limited to $n=7$ has the representation:

$$U=\frac{\sigma\rho_0}{2}\sum_{n=1}^{7}{}'I_n(\gamma)\left(\frac{r}{\rho_0}\right)^n P_n(\cos\theta),\quad(22)$$

where r, θ are the spherical coordinates of the field observation point.

By analogy with the model of a conductor in a uniform external electric field, an inductively magnetized cylinder with an infinite value of magnetic susceptibility does not contain a magnetic field in the middle. This fact according to (2) for a full field means that the demagnetization magnetic field $\vec{H}_d$ must be uniform, equal, and opposite to the external magnetic field $\vec{H}_0$. However, the found solution for the scalar potential (17) contains nonzero higher harmonics with $n>1$, i.e. the obtained demagnetization field is not homogeneous. This means that the rigorous analytical solution obtained is not for the demagnetization field of an inductively magnetized cylinder but for another, close to this problem, in which the surface magnetization is given by (7) and (8).

For a better approximation of solution (22) to the problem of an inductively magnetized cylinder, the contribution of the first harmonic to the demagnetization field is corrected by applying an additional amplitude coefficient. The representation for such an amplitude coefficient is justified as follows. To level the total contribution of higher harmonics, the coefficient $I_1(\gamma)$ is reduced by the sum of all $I_n(\gamma)$ with $n>1$. At the same time, it is convenient to search for the form of such a difference when $\gamma=1$. In addition, the correction of the magnitude of the uniform field (the very first harmonic) occurs by a similar reduction by the sum of $I_n(1)$ for all $n>1$. Then the correction factor takes the value:

$$I_1(1)-2\sum_{n=3}^{\infty}{}'I_n(1)=0.93432-2\cdot 0.0372\approx 0.86.\quad(23)$$

To reduce the influence of the effect of distant ends at large values of γ, the correction factor calculated by (23) is applied only to the part $I_1(\gamma)$, created by charges on the side surface. Next, instead of $I_1(\gamma)$, determined by (18), a new representation of this coefficient is used with the application of a correction in the form:

$$I_1^k(\gamma)=2\left(\gamma-\frac{\gamma^2}{\sqrt{1+\gamma^2}}+0.86\left[\begin{array}{l}\frac{-\gamma}{\sqrt{1+\gamma^2}}+\\+\frac{1}{2}\ln\frac{\sqrt{1+\gamma^2}+\gamma}{\sqrt{1+\gamma^2}-\gamma}\end{array}\right]\right).\quad(24)$$

Using the geometry of the problem to determine the fluxmetric demagnetizing factor N_f according to (4), we have the following representation in spherical coordinates:

$$N_f=-\int\limits_0^{\rho_0}\left(\vec{H}_d\right)_z r\mathrm{d}r\Bigg/\int\limits_0^{\rho_0}M_z r\mathrm{d}r,\quad(25)$$

where the values of the magnetization and field projections are taken at the value of the spherical angular coordinate of the observation point $\theta=\pi/2$.

To determine the projection of the demagnetization field onto the axis of the applicates in (25) in the central cross-section ($\theta=\pi/2$) of the cylinder, its connection with spherical projections was used:

$$\left(\vec{H}_d\right)_z=\left(\vec{H}_d\right)_r\cos\theta-\left(\vec{H}_d\right)_\theta\sin\theta=-\left(\vec{H}_d\right)_\theta. \qquad (26)$$

Given (22) and (26), for the integral in the numerator (25) we have:

$$\int_0^{\rho_0}\left(\vec{H}_d\right)_z r\mathrm{d}r=-\int_0^{\rho_0}\left(\vec{H}_d\right)_\theta r\mathrm{d}r=-\int_0^{\rho_0}\left(\frac{1}{r}\frac{\partial}{\partial\theta}U_\theta\right)r\mathrm{d}r=$$
$$=-\frac{\sigma\rho_0}{2}\left(\sum_{n=1}^{7}{}' I_n(\gamma)\int_0^{\rho_0}\left(\frac{r}{\rho_0}\right)^n\mathrm{d}r\frac{\partial}{\partial\theta}P_n(\cos\theta)\right). \qquad (27)$$

Using the known properties of Legendre polynomials at $\theta=\pi/2$ [14], after differentiation and integration for (27), we obtain a simplified form:

$$\int_0^{\rho_0}\left(\vec{H}_d\right)_z r\mathrm{d}r=\frac{\sigma\rho_0^2}{2}\sum_{n=1}^{7}{}'(-1)^{\frac{n+1}{2}}\frac{n!!}{(n+1)!!}I_n(\gamma)=$$
$$=\frac{\sigma\rho_0^2}{2}\left(-\frac{1}{2}I_1^k(\gamma)+\frac{3}{8}I_3(\gamma)-\frac{5}{16}I_5(\gamma)+\frac{35}{128}I_7(\gamma)\right). \qquad (28)$$

To calculate the denominator in (25), the representation of magnetization based on its scalar potential U_m is used, in complete analogy to [4]. According to the concept of the spatial distribution of magnetization based on cylindrical harmonics, uniformly charged ends create a homogeneous component directed along the axis of symmetry. The linear distribution of surface charges on the side surface is also a well-known analytical model [17], so the magnetization potential can be represented as:

$$U_m=\sigma\gamma z+\frac{2\sigma}{\pi\rho_0}\int_0^{\infty}\frac{I_0(\lambda\rho)}{\lambda^3 I_1(\lambda\rho_0)}\sin\left(\lambda\frac{h}{2}\right)\sin(\lambda z)\mathrm{d}\lambda, \qquad (29)$$

where ρ, z are the cylindrical coordinates of the magnetization observation point inside the cylinder; $I_0(\lambda\rho)$ is a modified Bessel function.

The denominator in (25) after substitution (29) and integration taking into account the fact that in the central cross section $z=0$, and the spherical coordinate r coincides with the cylindrical coordinate ρ, takes the form:

$$\int_0^{\rho_0}M_z\rho\mathrm{d}\rho=\int_0^{\rho_0}\frac{\partial}{\partial z}(U_m)\rho\mathrm{d}\rho=\frac{\sigma\rho_0^2}{2}\gamma(1+\gamma). \qquad (30)$$

After substituting in (25) the numerator of (28) and the denominator of (30), we obtain the desired relationship for the fluxmetric demagnetizing factor:

$$N_f=\frac{\frac{1}{2}\left(I_1^k(\gamma)-\frac{3}{4}I_3(\gamma)+\frac{5}{8}I_5(\gamma)-\frac{35}{64}I_7(\gamma)\right)}{\gamma(1+\gamma)}, \qquad (31)$$

where $I_n(\gamma)$ should be understood as formulas (24), (19) to (21), respectively, for $n=1$, 3, 5, and 7.

5. 3. Evaluating the reliability of results of calculating the fluxmetric demagnetizing factor of the cylinder according to the derived formula

To assess the correctness of our formula for calculating the fluxmetric demagnetizing factor of the cylinder at infinite favorability, a direct comparison was made with the known results of calculating N_f by the numerical method [11]. The choice of the results obtained by the numerical method as the basis for comparison is explained by its best accuracy compared to other calculation and experimental methods. Table 1 gives the applied results of calculating N_f by the numerical method and the corresponding results of calculating $N_f(\gamma)$ according to (31), as well as the relative deviation Δ of the latter compared to the basic ones. The calculation of the value of the relative deviation in percent was carried out according to the formula:

$$\Delta=\frac{N_f-N_f(\gamma)}{N_f}100\,\%, \qquad (32)$$

where N_f and $N_f(\gamma)$ are the results obtained by the numerical method and according to formula (31) with the same elongations of the cylinder γ, respectively.

Table 1

Results of calculating the fluxmetric demagnetizing factor, obtained by the numerical method and according to formula (31), and their comparison

γ	N_f, numerical method	$N_f(\gamma)$, according to formula (31)	Δ, %
0.01	0.9544	0.972620698	1.9
0.02	0.9191	0.946088906	2.9
0.05	0.8349	0.87156652	4.4
0.1	0.7321	0.76392848	4.3
0.2	0.5929	0.604587172	2.0
0.3	0.499	0.50192746	0.6
0.5	0.3773	0.382353638	1.3
0.7	0.3009	0.305900874	1.7
1	0.22752	0.22879019	0.6
1.5	0.15786	0.157594671	0.2
2	0.11805	0.118582713	0.5
3	0.07526	0.076927483	2.2
5	0.04001	0.041775566	4.4
10	0.01545	0.016142258	4.5
20	0.005391	0.005571152	3.3
50	0.0012111	0.00121994	0.7
100	0.00037203	0.0003665	1.5
200	0.00011046	0.000106845	3.3
500	0.000021359	0.0000202861	5.0

For an indirect comparison of the correctness of the proposed formula (31), the well-known formulas for calculating the fluxmetric demagnetizing factor of a cylinder at infinite favorability, applied at certain limited intervals γ, were used. For the cylinder elongation interval $\gamma\in[2;\ 10]$ we use the formula proposed in [2]:

$$N_f(\gamma)=\frac{2.94}{4\pi}\left(\gamma^{-0.87}-0.0783\right). \qquad (33)$$

The formula from [1], which is used for the interval $\gamma \in [10; 250]$ has been successfully proven experimentally:

$$N_f(\gamma) = \frac{1}{\gamma^2}\left(\ln(1.2\gamma) - 1\right). \tag{34}$$

A good coincidence of the results of calculating $N_f(\gamma)$ according to the found representation (31), as well as according to (33), (34), and the results of the numerical calculation is illustrated in Fig. 2.

To quantify the accuracy of determining the fluxmetric coefficient of the cylinder using various approximate asymptotic analytical representations according to (32), the corresponding relative deviations Δ were calculated in comparison with the result obtained by the numerical method. Data on the calculated relative deviations are given in Table 2 and illustrated in Fig. 3.

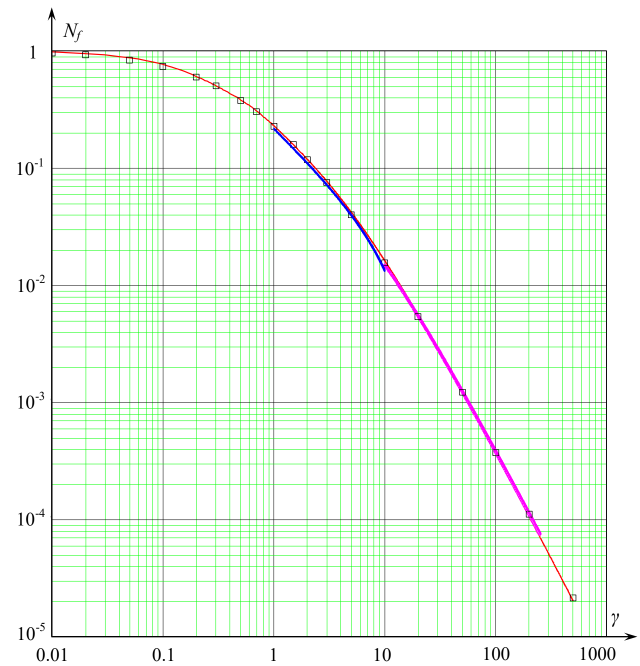

Fig. 2. Results of calculating $N_f(\gamma)$ according to Okoshi — blue line, according to Rozenblat — purple line, the proposed formula — red line; points obtained by the numerical method — black squares

Table 2

Results of calculating the relative deviation Δ when determining $N_f(\gamma)$ using approximate asymptotic formulas from the data obtained by the numerical method

γ	Okoshi		Rozenblat		This work	
	$N_f(\gamma)$	Δ, %	$N_f(\gamma)$	Δ, %	$N_f(\gamma)$	Δ, %
1	0.21564	5.2	–	–	0.2287902	0.6
1.5	0.14609	7.5	–	–	0.1575947	0.2
2	0.10969	7.1	–	–	0.1185827	0.5
3	0.07164	4.8	–	–	0.0769275	2.2
5	0.03936	1.6	–	–	0.0417756	4.4
10	0.01324	14.3	0.0148491	3.9	0.0161423	4.5
20	–	–	0.0054451	1.0	0.0055712	3.3
50	–	–	0.0012377	2.2	0.0012199	0.7
100	–	–	0.0003787	1.8	0.0003665	1.5
200	–	–	0.0001120	1.4	0.0001068	3.3
500	–	–	0.0000216	1.1	0.0000203	5.0

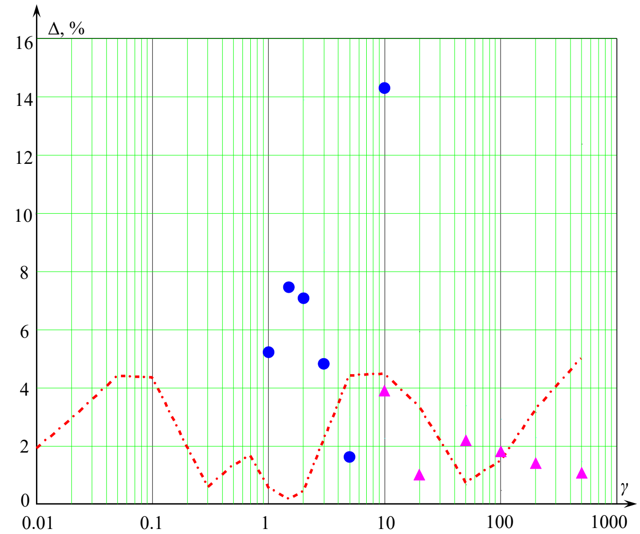

Fig. 3. Relative deviation Δ in the calculation of $N_f(\gamma)$ compared to the data obtained by the numerical method, when using the formulas proposed by: Okoshi — blue circles, Rozenblat — purple triangles, the authors of this work — red dashed line

Since Table 2 gives the results of calculating $N_f(\gamma)$ according to various asymptotic formulas, which have different intervals of application based on the parameter γ, the part of the cells that are not included in the corresponding interval were not filled.

6. Discussion of results from the analysis of determining the fluxmetric demagnetizing factor of a cylinder with infinite favorability

The main result of our work is the derived analytical representation (31) for the fluxmetric demagnetizing factor $N_f(\gamma)$, which can be applied in the entire range of possible values of cylinder elongation γ, in contrast to formulas from [1, 2]. The result obtained during the study of the approximate model of the magnetized cylinder is a confirmation of the idea that the dependence of the demagnetizing factor $N_f(\gamma)$ is functionally determined primarily by the elongation γ (the effect of the body shape) than by the real distribution of magnetism. That is why a sufficiently rough and approximate model of the magnetization of a cylinder, but with the same elongation γ (shape), can be corrected and used to calculate $N_f(\gamma)$ with a small relative deviation.

The presence of fluctuations in the given graphic image (Fig. 3) of the relative deviation Δ from the exact $N_f(\gamma)$ value is associated with the limited accuracy of the applied model of the inductively magnetized cylinder. This is the result of the application of initial assumptions about the distribution of magnetization when building the model in contrast to the model based on cylindrical harmonics in [4, 5]. Therefore, a further increase in the number of spherical harmonics from an infinite series used in the calculation of the demagnetization field actually does not change the value of the relative deviation Δ. This is explained by the rapid decrease in the contributions of the higher order harmonics in (17).

Based on an indirect comparison of the accuracy of the proposed representation for $N_f(\gamma)$ using the analysis of relative deviations Δ (Fig. 3) for the results obtained according to known formulas from [1, 2], the following can be asserted. The value of the relative deviation Δ of $N_f(\gamma)$ calculation according to (31) is limited to 5 % for the entire range of possible values of cylinder elongation γ. This actually coincides with the limits for relative deviations from the exact value when using known approximate formulas. But the advantage of the proposed representation is that there is no restriction on its use beyond a narrow range of values of the elongation of the cylinder γ. The disadvantages of using (31) in comparison with the formulas obtained in [1, 2] include a large number of terms with different degrees of elongation γ, which somewhat complicates the practical use of the formula. However, the current level of software and hardware for mathematical calculations almost completely eliminates this inconvenience.

From the point of view of a possible improvement of the accuracy of the model for calculating $N_f(\gamma)$, the application of a more complex but more accurate dependence of the distribution of the inductive magnetization on the surface of the cylinder looks promising. This is due to the fact that the applied simpler cylinder magnetization model actually uses only the first term of the power series expansion of the surface magnetization function. It can be expected that the use of a more advanced analytical model of the inductive magnetization of the cylinder could increase the accuracy of the calculation of the fluxmetric demagnetizing factor. Therefore, the search for a rigorous analytical representation of the distribution of inductive magnetization of a cylinder remains relevant. One of the possible ways to build such a model is to use another type of spatial harmonics, which are better adapted to describe the elongated shape of the cylinder.

From a practical point of view, our formula makes it possible to expand the possibilities of modeling, designing, and calculating the parameters of various devices, the magnetically active part of which is a magnetized cylindrical core. In particular, the reliability of the metrological assessment of the characteristics of cylindrical iron structures during their production can be increased.

7. Conclusions

1. A simplified model of magnetization has been built based on fictitious magnetic charges located on the surface of the cylinder, with the additional condition of linear dependence between magnetization and magnetic field at infinite favorability. The model is based on the assumption that the surface distribution of fictitious magnetic charges on the surface of a cylinder is analogous to the surface distribution on the surface of a sphere, which creates a uniform demagnetization field. This has made it possible to obtain for a simplified model analytical ideas about the ratio of the average integral values of magnetization and the demagnetization field in the average cross-section of a cylindrical magnet.

2. As a result of our spherical harmonic analysis of the magnetic field of demagnetization in a simplified model of a magnetized cylinder, representations were obtained for the direct calculation of contributions of harmonics having power $n \leq 7$. It is proposed to use a correction factor equal to 0.86 of the amplitude to change the contribution of the first harmonic to the demagnetization field. This has made it possible to approximate the simplified model to the real distribution of the inductive magnetization of the cylinder for the case of an infinite value of magnetic favorability, as well as to obtain a formula for the fluxmetric demagnetizing factor based on the spherical harmonics of the magnetic field of the cylinder.

3. As a result of our research into the accuracy of the proposed formula for direct analytical calculation of the cylinder's fluxmetric demagnetizing factor, the following was established. The value of the relative deviation of the result of the calculation of the fluxmetric coefficient using the proposed formula, compared to the numerical calculation, does not exceed 5 %. This provides the possibility of its application in engineering calculations for the range of determining the elongation of an inductively magnetized cylinder from 0.01 to 500. An additional condition for the correctness of the practical application of the proposed formula is the fulfillment of the inequality for the magnetic favorability of the magnet, which must be greater than one hundred elongations of the cylinder.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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