UDC 514.18
DOI: 10.15587/1729-4061.2024.301711

# DETERMINING THE SHAPE OF A FLEXIBLE THREAD IN THE FIELD OF HORIZONTAL AND VERTICAL FORCES

**Tetiana Volina**
*Corresponding author*
PhD, Associate Professor*
E-mail: t.n.zaharova@ukr.net
**Serhii Pylypaka**
Doctor of Technical Sciences, Professor, Head of Department*
**Viktor Nesvidomin**
Doctor of Technical Sciences, Professor*
**Mykhailo Kalenyk**
PhD, Professor, Dean
Department of Mathematics, Physics and Methods of their Education***
**Dmytro Spirintsev**
PhD, Associate Professor, Head of Department
Department of Mathematics and Physics
Bogdan Khmelnitsky Melitopol State Pedagogical University
Naukovoho mistechka str., 59, Zaporizhzhia, Ukraine, 69097
**Serhii Dieniezhnikov**
PhD, Associate Professor**
**Iryna Hryshchenko**
PhD, Associate Professor*
**Alla Rebrii**
Senior Lecturer
Department of Engineering Systems Design
Sumy National Agrarian University
Herasyma Kondrativa str., 160, Sumy, Ukraine, 40021
**Tetiana Herashchenko**
PhD, Associate Professor**
**Viktoriia Soloshchenko**
PhD, Associate Professor
Department of Theory and Practice of Romano-Germanic Languages***
*Department of Descriptive Geometry, Computer Graphics and Design
National University of Life and Environmental Sciences of Ukraine
Heroiv Oborony str., 15, Kyiv, Ukraine, 03041
**Department of Management of Education and Pedagogy of High School***
***Sumy State Pedagogical University named after A. S. Makarenko
Romenska str., 87, Sumy, Ukraine, 40002

***In theoretical mechanics, the equilibrium of a flexible, inextensible thread is considered, to which the tension force of its ends and the distributed force of weight along the length of its arc are applied. An unsolved problem is finding the shape of the thread under the action of other distributed forces. This study has considered the equilibrium of a completely flexible thread, to which, in addition to this force, a transverse distributed force is applied. A sail serves as an example. Wind of equal intensity in the plane of the orthogonal section of the sail can be considered a distributed force. The sail can be cut into narrow strips with the same shape of the curves of the cross-section, which are equal to the cross-section of the sail as a whole. The theory of flexible thread is applied in the current study. The task is reduced to finding the curve of the cross-section of the sail.***

***The object of research is the formation of a cylindrical surface from a flexible thread under the action of distributed forces applied to it.***

***An important characteristic of the shape of a flexible thread is its curvature. Its dependence on the length of the arc was found and it was established that the found curve is a chain line (catenary). This is the feature of the current research and its distinguishing characteristics. The significance of the results stems from the derived analytical dependences, according to which the change in the ratio between the distributed forces acting on the flexible thread deforms it, but it retains the shape of the catenary. At the same time, the angle of deviation of its axis of symmetry from the vertical changes. In the absence of a horizontal distributed force and the presence of only a distributed force of weight, the axis of symmetry of the chain line is directed vertically – at an angle of 90° to the horizontal. If they are equal, this angle is 45°. Scope of application includes structures with stretched supporting wires, conveyor belts, flexible suspended ceilings, the shape of which can be calculated by using our results***

***Keywords: chain line, curvature, axis of symmetry, angle of inclination, inextensible thread***

*Received date 02.02.2024*
*Accepted date 03.04.2024*
*Published date 30.04.2024*

***How to Cite:*** *Volina, T., Pylypaka, S., Nesvidomin, V., Kalenyk, M., Spirintsev, D., Dieniezhnikov, S., Hryshchenko, I., Rebrii, A., Herashchenko, T., Soloshchenko, V. (2024). Determining the shape of a flexible thread in the field of horizontal and vertical forces. Eastern-European Journal of Enterprise Technologies, 2 (7 (128)), 24–30. https://doi.org/10.15587/1729-4061.2024.301711*

## 1. Introduction

The shape of a flexible tape subjected to vertical and horizontal distributed forces is an interesting subject of research in various fields of science and technology. For example, this form is important for analyzing the stressed-strained state of conveyor belts, conveyor systems, electric cables, bridge structures, etc. To carry out such research, it is necessary to have a mathematical model of the shape of a completely flexible tape, which depends on the intensity and direction of the applied forces. A physical model of the tape with such a load is a sail, for which the vertical distributed force is the force of gravity, and the horizontal force is the force of the wind. Our study assumes that both forces are constant, that is, there is a constant intensity of the action of these forces on the tape along its entire length, which is also constant.

Scientific research into this area is important as it makes it possible to significantly expand existing approaches in solving the problem of forming surfaces from flexible material under the action of distributed forces. It is needed in practice because it enables calculation of the shape of the surface, as well as the flexible thread, depending on the ratio of the weight of the material and the applied forces. The relevance of research relates to the fact that the solution to a known problem is extended to more general cases that can occur in practice.

## 2. Literature review and problem statement

The task to determine the shape of a flexible tape subjected to distributed forces has many applications in various fields of science and technology. Scientists use different methods and approaches to solving this problem. One of the first to deal with this issue was Euler who in 1744 obtained a solution for the form of a flexible tape, which is suspended by two ends and deforms under the influence of its own weight. It is also necessary to mention the catenary studies by Galileo, Bernoulli, Leibniz, and Huygens. Ultimately, the solution was built on the basis of a second-order differential equation that describes the curvature of the tape axis.

However, in the initial statement, the problem was solved on the basis of a completely natural request: determining the deformation of a completely flexible inextensible thread under the action of the force of its own weight, that is, a distributed vertical force uniformly distributed along its length. Analytical studies are aimed at the practical needs for determining the shape of a flexible thread under the additional action of other distributed forces. That predetermined the purpose of the work.

An alternative method is the variational method, which is based on the principles of continuum mechanics and the minimum potential energy of the system. This method makes it possible to obtain a solution in the form of a series for some functions that satisfy the boundary conditions. This approach was proposed by Kossel in 1933 and proved to be effective in solving the problem of finding the shape of a flexible tape with distributed forces.

Most works by modern authors relate to the practical use of flexible threads and tapes. For example, in [1], the basics of statics, kinematics, and dynamics of threads in relation to the textile industry are considered. Study [2] considers the stressed-deformed state of a flexible belt of a tubular conveyor. However, in those studies, there is no mathematical model of the shape of the tape, which could be used in a wide range of research.

In [3], a comprehensive review of axially moving flexible structures in problems involving distributed contact of the structure with a solid body is given. The outlined features apply only to axially moving structures based on models with simplified support conditions.

The issue of the dynamics of moving contact lines, which are much more complicated than the geometric flow, and their application in interphase science and technological fields is addressed in [4]. The authors emphasize the role of the dynamics of moving contact lines in numerous advanced technologies and stress the need to focus the attention of scientists on a better understanding of the physics of motion.

The theoretical model of a moving contact line built by removing a flat solid body from a liquid was devised in [5]. The representation of laminar flow is based on a combination of stream functions defined in the molecular domain dominated by molecular forces and in the macroscopic domain dominated by viscous forces. The mathematical model realistically reflects the details of the viscous flow near the contact line and accurately corresponds to the experimental data, but it can only be partially applied to the case of a suspended system.

Oscillations of low-viscosity liquid droplets spreading over textured surfaces arise from pinning of the three-phase contact line, which in turn is associated with hysteresis in the wetting behavior of real surfaces.

In [6, 7], the study of dynamic loads of cargo on a flexible suspension is described, using the example of the simultaneous movement of the turning and lifting mechanisms of a jib crane. However, the mathematical models obtained by scientists do not take into account the action of horizontally distributed forces.

The movement of the tape along the inner surface of a stationary cylinder is considered in [8]. Paper [9] reports the construction of mathematical models of the stability of the functioning of a mechanical system under the condition of its fastening. The resulting models cannot be used for the case of a suspended state of the system.

The dynamic model of the three-dimensional angle of the contact line is proposed in [10]. It quite completely predicts the evolution of the contact angle on surfaces, including those that perform oscillatory movements. However, the limitation of the approach proposed by the authors is the gas-liquid state of the media.

All of the above allows us to state that it is appropriate to conduct a study on the construction of a mathematical model of the cross-section of an absolutely flexible inextensible tape, which it acquires under the action of vertical and horizontal distributed forces.

## 3. The aim and objectives of the study

The purpose of our study is to determine the shape of the cross section of a completely flexible non-stretchable tape, which it acquires under the action of vertical and horizontal distributed forces. This will make it possible to purposefully search for the shape of cylindrical surfaces made of flexible material that deform under the action of distributed forces, and thus optimize the volume they occupy.

To achieve the goal, the following tasks were set:

– to compile the equilibrium equation of the tape element on the coordinate axis and find the parametric equations of the curve of its cross section;

– to describe the properties of the found curve depending on the ratio of vertical and horizontal distributed forces.

## 4. The study materials and methods

The object of research is the shape formation process of a cylindrical surface from a flexible tape under the action of distributed forces applied to it.

It is known from the course of theoretical mechanics that a flexible thread under the action of a vertical distributed force of weight acquires the form of a chain line, the equation of which includes a constant value. It was hypothesized that when a horizontal distributed force was applied to the thread, its shape would change but it would remain a chain line with a different constant value. To confirm it, the apparatus of analytical and differential geometry was used. Mathematical transformations and integration of the resulting dependences were carried out using the symbolic mathematics software package "Mathematica" (USA). Construction of the curves of the cross-section of the tape based on the equations built was performed using the software suite "MATLAB" (USA). Some graphic illustrations to explain the material of the paper were drawn in the "AutoCAD" environment (USA). The study assumed that the tape or thread is inextensible and adopted simplification that they are perfectly flexible.

## 5. Research results on determining the shape of the cross-section of an absolutely flexible non-stretchable tape

### 5. 1. Construction of differential and parametric equations of equilibrium of the tape on the coordinate axis

Let a completely flexible thread, which is part of a similar tape, is fixed at point $A$ (Fig. 1). Two constant distributed forces act on it: the weight of a unit length of tape $m$ vertically downwards and a lateral force $q$, which can be the force of the wind. At each point of the thread, there is a tension force $T$: at point $A$ of the suspension, it has a value of $T_0$. As the point of the curve goes down, the tension force changes and at point $B$ it takes a value of $T_1$.

The equation of thread equilibrium takes the form:

$$\frac{d}{ds}\left(T\frac{dx}{ds}\right)+F_x=0, \qquad (1)$$

$$\frac{d}{ds}\left(T\frac{dy}{ds}\right)+F_y=0,$$

where $T$ – thread tension at the current point.

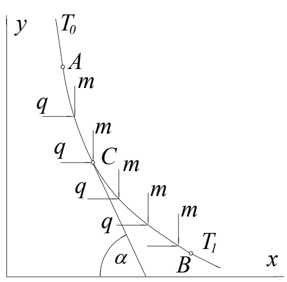


Fig. 1. Graphic illustration of the shape of a flexible thread, which it acquires under the action of vertical $m$ and lateral $q$ distributed forces

The distributed forces $m$ and $q$ in the projections onto the coordinate axis will be written:

$$F_x=q; \qquad (2)$$

$$F_y=-m;$$

After substituting (2) into (1):

$$\frac{d}{ds}\left(T\frac{dx}{ds}\right)=-q; \qquad (3)$$

$$\frac{d}{ds}\left(T\frac{dy}{ds}\right)=m.$$

Taking into account that $\frac{dx}{ds}=\cos\alpha,$ $\frac{dy}{ds}=\sin\alpha,$ equations (3) take the form:

$$\frac{d}{ds}(T\cos\alpha)=-q; \qquad (4)$$

$$\frac{d}{ds}(T\sin\alpha)=m.$$

In equations (4), the product in round brackets must be differentiated for the variable $s$ – the length of the thread:

$$\frac{dT}{ds}\cos\alpha-T\frac{d\alpha}{ds}\sin\alpha=-q; \qquad (5)$$

$$\frac{dT}{ds}\sin\alpha+T\frac{d\alpha}{ds}\cos\alpha=m.$$

The angle α (Fig. 1) depends on the length of the tape arc $s$. It is known from differential geometry that $\frac{d\alpha}{ds}=k,$ where $k$ is the curvature of the curve at the current point. Based on this:

$$\frac{dT}{ds}=\frac{dT}{d\alpha}\cdot\frac{d\alpha}{ds}=\frac{dT}{d\alpha}k. \qquad (6)$$

After substituting (6) into (5), two equations can be obtained as a function of one variable α:

$$\frac{dT}{d\alpha}k\cos\alpha-Tk\sin\alpha=-q; \qquad (7)$$

$$\frac{dT}{d\alpha}k\sin\alpha+Tk\cos\alpha=m.$$

Equations (7) include two unknown functions: $T=T(\alpha)$ and $k=k(\alpha)$. The solution to the system of equations (7) with respect to $k$ and $\frac{dT}{d\alpha}$ takes the following form:

$$k=\frac{1}{T}(m\cos\alpha+q\sin\alpha), \qquad (8)$$

$$\frac{dT}{d\alpha}=T\frac{m\sin\alpha-q\cos\alpha}{m\cos\alpha+q\sin\alpha}. \qquad (9)$$

In differential equation (9), the variables can be separated. After its integration:

$$T=\frac{T_0}{m\cos\alpha+q\sin\alpha}, \qquad (10)$$

where $T_0$ is the thread tension at the starting point (Fig. 1).

Substituting (10) into (8) gives the result:

$$k=\frac{1}{T_0}(m\cos\alpha+q\sin\alpha)^2. \qquad (11)$$

Equation (11) determines the dependence of the curvature $k$ of the curve on the angle of inclination α of the tangent to it at the current point. In fact, the curve is given, but to build it, you need to go to the coordinate record. The transition from dependence (11) to the parametric equations of the curve is known from differential geometry:

$$x=\int\frac{\cos\alpha}{k}\mathrm{d}\alpha;\quad(12)$$

$$y=\int\frac{\sin\alpha}{k}\mathrm{d}\alpha.$$

Substituting (11) in (12) yields expressions that can be integrated:

$$x=\frac{2T_0m}{\left(m^2+q^2\right)^{3/2}}\mathrm{Arctanh}\frac{m\tan\alpha/2-q}{\sqrt{m^2+q^2}}-$$

$$-\frac{T_0q}{m^2+q^2}\left(m\cos\alpha+q\sin\alpha\right);\quad(13)$$

$$y=\frac{2T_0q}{\left(m^2+q^2\right)^{3/2}}\mathrm{Arctanh}\frac{m\tan\alpha/2-q}{\sqrt{m^2+q^2}}+$$

$$+\frac{T_0m}{m^2+q^2}\left(m\cos\alpha+q\sin\alpha\right).$$

According to equations (13), Fig. 2, *a* shows the constructed curves of the cross-section of the flexible tape at $T_0$=1; $m$=0,5 and $q$=1. Curve 1 is constructed when the angle α changes within 0...139°, and curve 2 – within 30°...127°. In Fig. 2, *b*, curve 2 is enlarged and combined with curve 1 so that their ends coincide. This gives an idea of the shape of the cross-section of the sail at different lengths of the cross-section curve between the attachment points.

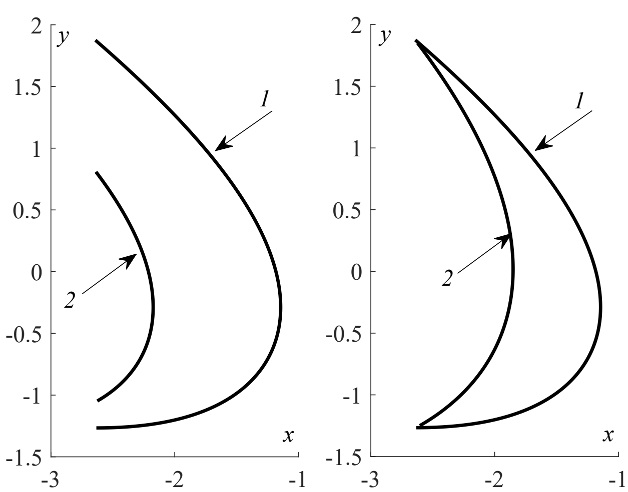


Fig. 2. The shape of the cross-section of a flexible tape on the example of a sail loaded with distributed forces: vertical $m$=0.5 and horizontal $q$=1 at different limits of angle change α: *a* – curves are drawn at a common scale; *b* – curve 2 is enlarged and combined with curve 1 so that the ends of the curves coincide

In equations (13), the tension force $T_0$ can be taken out of parentheses, that is, it does not affect the shape of the curve but plays the role of a scaling factor.

**5. 2. Studying the properties of the found curve depending on the ratio of vertical and horizontal distributed forces**

If necessary, you can find the length $s$ of the curve between the attachment points. Based on the fact that $\frac{d\alpha}{ds}=k$, taking into account expression $k$ (11):

$$s=\int\frac{\mathrm{d}\alpha}{k}=\int\frac{T_0\mathrm{d}\alpha}{\left(m\cos\alpha+q\sin\alpha\right)^2}=$$

$$=\frac{T_0\sin\alpha}{m\left(m\cos\alpha+q\sin\alpha\right)}.\quad(14)$$

When substituting in (14) the values of the angle α at the points of attachment of the curve, the length of the arc $s$ between them can be found. Excluding the angle α in expressions (11) and (14), it is possible to obtain the dependence of the curvature $k$ of the curve on the length $s$ of its arc, that is, the natural equation of the curve:

$$k=\frac{T_0m^2}{m^4s^2+\left(T_0-mqs\right)^2}.\quad(15)$$

When $q$=0, that is, when there is only a distributed force of weight $m$, we get a known natural equation of the chain line:

$$k=\frac{a}{s^2+a^2},\quad(16)$$

where the expression $a=\frac{T_0}{m^2}$. acts as a constant $a$.

Algebraic transformation of equation (15) can lead to the form:

$$k=\frac{a}{\left(s-s_0\right)^2+a^2},\quad(17)$$

where $a=\frac{T_0}{m^2+q^2}$, $s_0=\frac{T_0q}{m^3+mq^2}$. Thus, equations (16) and (17) describe the chain line at different values of the constant $a$. The $s_0$ value does not affect the shape of the curve but only the reference point of the arc on it. At $q$=0, they become identical.

The chain line has an axis of symmetry that is vertical at $q$=0. Analyzing Fig. 2, which is drawn when $q\neq0$, it can be assumed that the axis of symmetry is deviated from the vertical by a certain angle. It is impossible to find the expression of this angle from equation (17) since the natural equation does not depend on the rotation of the curve in the plane. If the chain line is turned by an angle $\alpha_0$, then its symmetry axis is turned by this angle relative to the $Oy$ axis and is also tangent at point $A$ in relation to the $Ox$ axis (Fig. 3). Point $A$ is the point with the greatest curvature. Parametric equations (13) describe the rotated chain line as a function of the angle α, which is the angle between the tangent to the curve at the current point and the $Ox$ axis. It is required to find such a value of the angle α at which the curvature of the chain line is maximum. The expression of the curvature as a function of the angle α is given in (11). The task boils down to determining the extremum of this expression. Equating the derivative of expression (11) to zero and solving with respect to α, we can obtain:

$$\alpha_0 = \arccos\frac{m}{\sqrt{m^2+q^2}}. \quad (18)$$

At $q$=0, $\alpha_0$=0, that is, the axis of symmetry is vertical, which coincides with the known result. At $m$=0, $\alpha_0$=90°, i.e., the axis of symmetry is horizontal, which corresponds to the horizontal distributed force with a weightless sail.

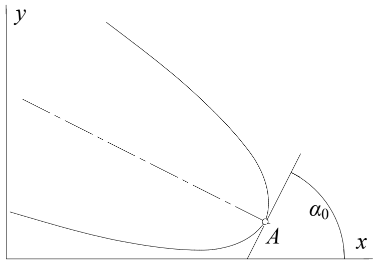


Fig. 3. Graphic illustration for determining the angle $\alpha_0$ of the inclination of the chain line

It should be noted that parametric equations (13), in addition to being bulky, due to their specificity do not give the desired result at $m$=0 (the second equation becomes identical to zero). However, for any ratio of the distributed forces $m$ and $q$, it is very simple to construct the corresponding curve according to the known parametric equations of the chain line as a function of the arc length $s$:

$$x = a\,\mathrm{arcsinh}\frac{s}{a}; \quad (19)$$

$$y = \sqrt{a^2+s^2},$$

which correspond to the natural equation (16). According to equation (17), the constant $a$ is found:

$$a = \frac{T_0}{m^2+q^2}.$$

This curve must be turned at angle $\alpha_o$ (18). It should be noted that the angle $\alpha_o$ (18) depends only on the ratio of the distributed forces $m$ and $q$. If the forces $m$ and $q$ are known, then the position of the axis of symmetry of the chain line is known. But the shape of the chain line depends on the constant $a = \frac{T_0}{m^2+q^2}$, that is, its shape is affected by the tension force $T_0$, which, as mentioned above, plays the role of the scale factor. Taking this into account, it is possible to set a condition for finding the value of $T_0$, provided that the curve passes through the given two points (fixing points). At the same time, it is not necessary to determine the length of the arc between the attachment points since it is the variable $s$ itself.

One can set a simpler problem: to find the tension force $T_0$ when the ratio of the distributed forces $m$ and $q$ changes, provided that the catenary remains congruent, that is, the constant $a$ does not change. Let the new values of the forces $m$, $q$, and $T_0$ be denoted with the index 1. By equating the constant $a$ with the previous and new values and solving the resulting equation with respect to $T_{01}$, we can obtain:

$$T_{01} = T_0\frac{m_1^2+q_1^2}{m^2+q^2}. \quad (20)$$

An important conclusion follows from expressions (18), (20): when changing the values of the distributed forces $m$ and $q$ to new $m_1$ and $q_1$, the shape of the catenary taken by the flexible tape will not change but the tension at the attachment point will change according to (20), as well as angle α according to (18) at new values of distributed forces $m_1$ and $q_1$. In the case when the distributed forces grow proportionally, that is, they increase by $n$ times, then neither the shape of the curve nor the angle of inclination of its axis of symmetry changes, but only the tension force increases by $n^2$ times, that is, $T_{01}=T_0 n^2$.

In Fig. 4, congruent curves are constructed according to equations (19) with subsequent rotation by the angle $\alpha_o$. The initial data are tension force $T_0$=10 and distributed forces $m$=3 and $q$=5. According to formula (18), the rotation angle $\alpha_o$=1.03 rad or $\alpha_o$=59°. The curve based on these data is plotted in Fig. 4, *a*. Let's take distributed forces with a new ratio: $m_1$=4 and $q_1$=3. From expression (20) we find: $T_{01}$=7.4, and from expression (18): $\alpha_{o1}$=0.64 rad or $\alpha_o$=37°. The corresponding curve is plotted in Fig. 4, *b*.

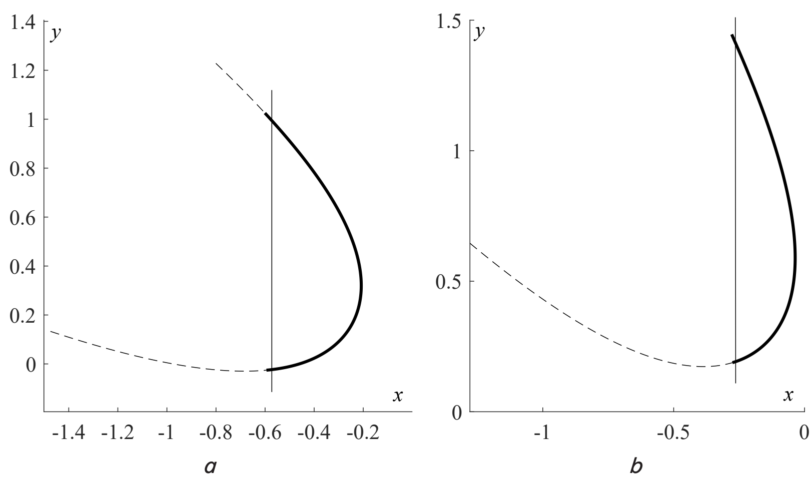


Fig. 4. Congruent curves constructed with different ratios of distributed forces $m$ and $q$: $a$ – $T_0$=10; $m$=3; $q$=5; $\alpha_o$=59°; $b$ – $T_{01}$=7,4; $m_1$=4; $q_1$=3; $\alpha_{o1}$=37°

The boundaries of the flexible tape between the attachment points on a vertical line are marked with a thick line in the figures. The limits of the variable $s$ are selected in such a way that the length of the thickened lines in both figures is the same and equal to 1.4 linear units.

## 6. Discussion of the procedure for finding the shape of a flexible tape, which it acquires under the action of distributed forces

In contrast to [2], in which a mathematical model of a flexible tape under the action of a vertical applied force was built, we constructed differential equations of equilibrium of a flexible tape under the action of vertical and horizontal distributed forces. They make it possible to derive parametric equations (13) that describe its shape in the vertical plane. It follows from these equations that the initial tension force $T_0$ at one of the attachment points is a scaling factor and does not affect the shape of the curve. It is obvious that the tension force $T_0$ depends on the length of the flexible tape between the attachment points. In the absence of a horizontal distributed force, a known result was obtained, in which the flexible tape takes the form of a catenary under the action of only a vertical distributed force. In the presence of a horizontal distributed force, the curves obtained by equation (13) were similar to the arc of a turned catenary (Fig. 2). It was hypothesized that the obtained curve is also a catenary, in which the axis of symmetry is turned by a certain angle from the vertical position. By transformations using known dependences of the differential geometry of the curves, the natural equation of the curve was built, which confirmed the correctness of the hypothesis. In addition, the value of the constant was found, which affects the shape of the catenary, as well as the angle of deviation of its axis of symmetry from the vertical position. Its value depends only on the ratio of vertical and horizontal distributed forces. With the same magnitude of these forces, the deflection angle is 45°, and with a weightless tape – 90°, that is, under the action of only the horizontal distributed force, the axis of the catenary is located horizontally. In contrast to [6, 7], the transition to the natural equation made it possible to study the shape of the chain line with a different ratio of distributed forces. According to formula (20), it was established that with a proportional increase of both forces by $n$ times, the shape of the curve does not change, but the tension force at one of the attachment points increases by $n^2$ times.

Similarly to [8], the limitations of our approach are that the tape is completely flexible and inextensible, that is, it does not resist bending and does not lengthen as a result of tensile forces.

The disadvantage is the impossibility of finding an expression for determining the coordinates of the attachment points on a vertical line. To that end, numerical methods should be used. For example, in Fig. 4, attachment points are located on a vertical line at $x=-0.6$ (Fig. 4, *a*) or at $x=-0.3$ (Fig. 4, *b*). Therefore, the equation $x=x(\alpha)$ or $x=x(s)$ must be equated to this value and find the value of the variable $\alpha$ or $s$, of which there must be two, since the curve at the given value of $x$ has two points. Such an equation cannot be solved analytically, so numerical methods must be used. By substituting the found values of $\alpha$ or $s$ into the equation $y=y(\alpha)$ or $y=y(s)$, the numerical values of the coordinates at both points are found. The advantages of this research compared to existing ones are that when finding the shape of the flexible tape, the natural equation of the curve was used, which identifies the curve in a fixed coordinate system regardless of its rotation in this system. This approach has made it possible to solve part of the problem, which eliminated the use of the variation method and to obtain results based on the natural essence of the problem. The essence of using our results is that when changing the ratio of distributed forces, it is not necessary to look for the shape of a flexible tape every time but to find it according to a simplified scheme in the form of a catenary and the angle of its deviation from the vertical direction. This presents opportunity to solve specific research and practical problems. The development of the current research may consist in finding the shape of a flexible tape under the action of distributed forces of variable magnitude.

## 7. Conclusions

1. The equation of equilibrium of a flexible inextensible tape under the action of constant vertical and horizontal distributed forces in projections onto the coordinate axis has been constructed. As a result of transformations and the application of known formulas of the differential geometry of curves, two differential dependences were established: the curvature and the tension force of the flexible tape depending on the variable $\alpha$ – the angle of inclination of the tangent to the curve with the $x$ axis. As a result of further transformations, a transition was made to one equation – the dependence of curvature of the curve on the angle $\alpha$. Such a dependence, according to the known formulas of the differential geometry of curves, allows one to proceed to the parametric equations of the curve by integrating the expressions, which in this case make it possible to carry out the integration and derive the equation in the final form. According to the equations built, the curves of the cross-section of the flexible tape for the specified values of the vertical and horizontal distributed forces were constructed.

2. A partial case of forming the curve of the cross-section of a flexible tape is the absence of a horizontal distributed force, i.e., the case when only the vertical distributed force of weight acts on it. For this case, known result was obtained, according to which the curve is a catenary with a vertical axis of symmetry. For the general case, in the presence of two distributed forces, the transition to the natural equation is made and it is shown that the curve would also be a catenary, the axis of symmetry of which forms a certain angle with the vertical direction. The expression of the constant was obtained, which is included in the natural equation of the catenary, due to the distributed forces. An expression for determining the angle of inclination of the axis of symmetry of the catenary was also obtained. It has been mathematically confirmed that with a proportional increase of both distributed forces by $n$ times, the shape of the catenary and its angle of inclination do not change, but only the tension force increases by $n^2$ times.

## Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

## Funding

The study was conducted without financial support.

## Data availability

All data are available in the main text of the manuscript.

## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References


1. Khamrayeva, S., Kadirova, D., Rakhimkhodjayev, S. (2021). Study on the mechanics of textile thread in woven. E3S Web of Conferences, 304, 03035. https://doi.org/10.1051/e3sconf/202130403035
2. Kyriia, R. V., Larionov, H. I., Larionov, M. H. (2019). Mathematical model of the stress-strain state of belt with the load of tubular belt conveyer. System Technologies, 3 (122), 42–54. https://doi.org/10.34185/1562-9945-3-122-2019-06
3. Scheidl, J., Vetyukov, Y. (2023). Review and perspectives in applied mechanics of axially moving flexible structures. Acta Mechanica, 234 (4), 1331–1364. https://doi.org/10.1007/s00707-023-03514-5
4. Mohammad Karim, A. (2022). A review of physics of moving contact line dynamics models and its applications in interfacial science. Journal of Applied Physics, 132 (8). https://doi.org/10.1063/5.0102028
5. Diaz, M. E., Cerro, R. L. (2021). A general solution of dewetting flow with a moving contact line. Physics of Fluids, 33 (10). https://doi.org/10.1063/5.0065168
6. Loveikin, V., Romasevych, Y., Kadykalo, I. (2023). Dynamic analysis of the joint movement of the hoisting and slewing mechanisms of a boom crane. Machinery & Energetics, 14 (4), 75–85. https://doi.org/10.31548/machinery/4.2023.75
7. Romasevych, Y., Hubar, Y. (2023). Propeller thrust tower crane slewing mechanism model identification. Machinery & Energetics, 14 (3), 72–78. https://doi.org/10.31548/machinery/3.2023.72
8. Pylypaka, S., Zaharova, T., Zalevska, O., Kozlov, D., Podliniaieva, O. (2020). Determination of the Effort for Flexible Strip Pushing on the Surface of a Horizontal Cylinder. Advanced Manufacturing Processes, 582–590. https://doi.org/10.1007/978-3-030-40724-7_59
9. Kozachenko, O., Syedykh, K., Shkrehal, O. (2021). Mathematical modeling of stability of mechanical system of discator. Machinery & Energetics, 12 (4). https://doi.org/10.31548/machenergy2021.04.061
10. Dwivedi, R. K., Jain, V., Muralidhar, K. (2022). Dynamic contact angle model for resolving low-viscosity droplet oscillations during spreading over a surface with varying wettability. Physical Review Fluids, 7 (3). https://doi.org/10.1103/physrevfluids.7.034002