The subject of this study is a punch-elastic half-space system under compressive force. The paper solves the problem of determining contact stresses and displacements. The content of results is the constructed models and the assessment of their adequacy.

This work considers the problem of pressing a rigid plane double-connected punch on a homogeneous and isotropic elastic half-space. To obtain an analytical solution, a variant of the perturbation method based on the expansion of the potential of a simple layer distributed over a double-connected region by a small parameter was applied. The problem of pressing a flat punch in the form of a non-circular ring is reduced to a sequence of problems for a punch in the form of a circular ring. This allows us to use a known solution for a circular ring.

Finite element models were built using ANSYS. A group of models was constructed to take into account possible damage in the event that the punch-elastic half-space system is exposed to difficult natural conditions or an aggressive environment during a certain time of modeling. A database was formed for the purpose of further transferring it to CLIPS. Sets of rules and knowledge were compiled.

A generalizing algorithm was developed for the problems of constructing and analyzing mathematical and computer models of contact interaction between a rigid cylindrical punch with a flat double-connected base with an elastic half-space under the action of a compressive force. The problem of determining the geometric shape of the cross-section of an annular punch in the plan for the punch-elastic half-space system was solved for the case when the contact zone is not known in advance. The devised approach could be employed in engineering calculations for strength and durability

Keywords: spatial contact problem, analytical solution, finite-element method, ANSYS, CLIPS

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CONSTRUCTION OF MATHEMATICAL AND COMPUTER MODELS FOR CALCULATING CONTACT CHARACTERISTICS OF INTERACTION BETWEEN A RIGID PUNCH AND AN ELASTIC HALF-SPACE

Tetyana Zaytseva
PhD, Associate Professor, Head of Department*

Vladyslav Zhushman
Corresponding author
Postgraduate Student*
E-mail:zhushmanvlad@gmail.com

*Department of Computer Technologies
Oles Honchar Dnipro National University
Gagarina ave., 72, Dnipro, Ukraine, 49010

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1. Introduction

The mechanics of the contact interaction of bodies is an important part of the mechanics of a continuous environment, which is actively developing and is constantly at the center of modern research. This importance can be justified by the fact that all structures and mechanisms consist of contacting elements in one way or another, and the distribution of contact forces between these elements is not known in advance. Contact problems can be considered key within the mechanics of deformable solids due to the fact that contact is the main method of applying forces. In addition, stress concentration in the contact zone can lead to damage, and sometimes to destruction. Only a limited class of such problems can be solved analytically, therefore it is advisable to devise numerical and numerical-analytical methods for their solution, as well as to apply expert systems and modern information technologies. The finite element method (FEM) and boundary element method (BEM) are especially widely used in modern mechanics [1, 2]. Also proposed are approaches [3] that combine FEM for simulating interphase interactions at the macro level with BEM for solving the contact problem at the micro level. However, it should be noted that it is the analytical approach that shows a deep understanding of the behavior and characteristics of the analyzed system or phenomenon. Moreover, analytical solutions often reveal basic laws and asymptotic behavior that may remain hidden in purely numerical analysis [4]. So, for example, the approach to solving the problem of the interaction between a base and a sliding die is extended to the problem of collinear interphase cracks with different electrical conditions on their edges [5]. Research into the problems of contact interaction of crack edges taking into account friction is quite complex and extensive [6]. A modern trend is to study the contact mechanisms of solid bodies in tribosystems [7, 8]. Solving such problems plays a central role in the fabrication of bionic materials and the mitigation of friction-induced damage, which is of great interest in fields such as medical engineering.

The relevance of finding effective ways to solve problems of contact interaction is due to the high cost and duration of creating real prototypes, as well as the understanding of the empirical results. The use of complex structures made of metal, under the conditions of an aggressive environment, which is the reason for the change in their structural features, because of, for example, corrosion, requires constant control by scientists and workers. Degradation of the structure during its operation can be the cause of a potential accident. Ensuring the durability of such structures to reduce future costs is an extremely urgent issue. Therefore, the further improvement of methods for calculating the strength of machine parts, elements of engineering structures and buildings, the use of new materials, advanced technologies are important issues that require the constant attention of researchers and active development. The results of such studies have important practical significance because modern technological progress determines the appearance of new engineering structures and materials, which becomes the reason for the relevance of the study of contact problems. More intensive use of composite materials in production, as well as high-precision modeling of the behavior of structures at the design stage, requires improvement and expansion of existing methods of contact interaction analysis.

2. Literature review and problem statement

In work [9], an analytical solution to the problem was obtained for a ring punch in the form of a double series, in which the coefficients are calculated exactly from simple recurrence relations. Devising such an approach made it possible to implement the use of fairly simple mathematical transformations to find new analytical solutions to classical and non-classical problems of mechanics. But areas of a different shape were not considered. The advancement of the method proposed in [9] is the analytical method devised in [10] for areas close to ring ones. It is based on the use of the development of the potential of a simple layer and is applied to the problems of the propagation of cracks, which are close in shape to a circular ring. This allows us to assert the sufficient universality of the proposed method. But devising a method for a family of problems about pressing stamps on an elastic half-space with a cross-sectional shape close to a ring remained an unsolved issue. This is the approach used in [11]. However, as a drawback, it can be pointed out that the study of the stability of the solution obtained by such an analytical method was not considered there. This was done in work [12].

One of the unsolved issues of the previous studies mentioned above is the consideration of initial or residual stresses. Work [13] reports the results of research on the contact interaction of elastic bodies with initial stresses without taking into account frictional forces. The development of this method for the case of a rigid cylindrical ring punch and an elastic half-space was studied in [14, 15], within the limits of the linearized theory of elasticity with an arbitrary structure of the elastic potential, which simplified the statement of the problem itself.

Considering friction in the process of body contact is one of the most important issues. During the phenomenon of friction, various processes such as mechanical, electrical, thermal, chemical, and vibrational can occur simultaneously. That is, it is such a process that complicates mathematical modeling. For example, works [16, 17] propose a reduction to a quasi-static problem to speed up the analysis of contact characteristics. The new algorithm leads to significant savings in computational resources, providing satisfactory accuracy. However, this method is aimed only at solving a local problem. Most contact problems, both with and without friction, require a separate statement and solution methods.

Issues related to the optimization of contact stresses, which prevents local concentration of stress and promotes uniform distribution of the load, which in turn helps increase the efficiency and durability of structures, remained unresolved in the above review of the literature. For example, [18] developed a method for analytically solving a quasi-static contact problem taking into account frictional forces. The method is based on the proposed development of the potential of a simple layer. The problem of optimizing the punch shape and pressure distribution in the contact zone was solved. Unsolved issues are the solution of similar problems with a doubly connected contact zone. The reason for this may be objective difficulties associated with the extreme complexity of mathematical considerations for building an analytical solution method. Most contact problems, both with and without friction, require a separate statement and construction of solution methods.

The method of solving inverse problems was proposed by the authors of [19], in which the problem of shell theory in the variational statement was considered. This method was developed in [20], in which the authors solved the contact problem as an inverse one. The authors obtained a solution to the contact problem of pressing a punch into an elastic half-space, taking into account friction, in the presence of adhesion, separation, and sliding zones. The given approach is represented in the form of an inverse problem in which, as an additional condition, the Coulomb friction law was used in the friction regions. This solution method, together with the discretization procedure, makes it possible to determine the zones of micro-slip, adhesion, and delamination.

All this gives reason to assert that over the past decades, interest in finding new methods and ways of solving contact problems has only been growing. Obvious is the need to combine already existing classical approaches with modern information technologies and the latest methods and algorithms based on the use of systems, for example, expert systems. It is in the field of mechanics, where tasks often require complex calculations and the analysis of a large number of variables, that expert systems can help in the diagnosis and monitoring of mechanical systems, providing accurate and reliable solutions to maintain their performance. Comprehensive analysis can reveal optimal solutions that are usually not obvious with a narrower approach. The integration of various methods and techniques can lead to new results and improvements in the field of contact mechanics problems.

3. The aim and objectives of the study

The purpose of our research is to construct mathematical and computer models of the contact interaction of a rigid punch with an elastic half-space. The use of such models will make it possible to determine ways to solve practical problems, such as calculating the strength and wear resistance of power transmission line supports, foundations, hydraulic structures, etc.

To achieve the goal, the following tasks were set:

– to derive calculation formulas for the analytical solution of problems for flat two-link stamps;

– to build finite element models of the interaction of a flat absolutely rigid punch with an elastic half-space;

– to develop a generalized algorithm for solving the contact problem for the system punch – elastic half-space, to implement CLIPS functionality;

– to determine the contour of the damaged punch shape using the developed tools.

4. The study materials and methods

The object of our study is the stressed-strained state of the die – elastic half-space system, when the uniform and isotropic elastic half-space of an absolutely rigid flat die occupying a two-connected region is pressed. The main hypothesis assumes that devising a complex approach to the solution of this class of contact problems, namely, the construction of a generalizing algorithm, will increase the efficiency and quality of their solutions. Mathematical and computer modeling of the contact interaction of a cylindrical punch with a flat two-link base with an elastic half-space under the action of a compressive force will allow conducting research within the limits of the main hypothesis. The problem of researching pressing on a homogeneous and isotropic elastic half-space of an absolutely rigid flat punch in the form of a non-circular ring is considered. The developed generalization algorithm for creating and analyzing models will make it possible to effectively and sufficiently accurately model the behavior of the system under the specified conditions.

The following simplifications were adopted in the work: the punch is considered as an absolutely rigid body, and the elastic half-space is considered homogeneous and isotropic.

The analytical solution was obtained using a variant of the perturbation method, based on the small-parameter expansion of the potential of a simple layer distributed over a doubly connected region. In this way, the problem of pressing a flat punch in the form of a non-circular ring is reduced to a sequence of problems for a punch in the form of a circular ring [9–12, 21]. Special software for calculations and analysis of the results was developed in the publicly available C++ language (USA). The construction of the finite-element model takes place with the help of the ANSYS (USA) software package. The academic version of ANSYS STUDENT 2024 R1 was used, which provides free access within the framework of scientific research [22]. Mathematical modeling and finite element analysis were carried out within the Static Structural module. The use of hexahedra made it possible to regularize the finite-element mesh using the Face Meshing method. Face Sizing method was used to increase mesh detailing in the contact area. To access the functionality that is missing in the current program interface, a software application was developed in the APDL programming language [22]. As a result of postprocessing, the results were obtained in the form of arrays of stress and displacement values. The probe tool was used to acquire stress values from conditional sensors that were previously located on the elastic half-space along the contact contour of the punch.

In a general form, the problem of determining the contour of the cross section of the punch interacting with the elastic half-space is stated [20, 23, 24]. With the help of the free software system CLIPS, (C Language Integrated Production System) (USA) [25], an expert system is built to automate the search for the most suitable punch form. CLIPS is a software environment for developing expert systems capable of solving artificial intelligence problems using rules and inferential mechanisms.

The software for processing the results of the expert system and visualization is implemented in the C++ language. A knowledge base was arranged as one of the key components of the expert system. It included information on the geometric characteristics and properties of the materials of the stamps and half-space, information on the stressed-strained state of the punch-elastic half-space system, as well as a set of rules and facts. The purpose of the built expert system is to determine the shape of the cross-sections of stamps with a flat sole pressing on an elastic half-space.

The coordinates of points from the list of facts were obtained using a regular expression, which separates only the necessary information, regarding the coordinates of points of bodies with corresponding stress values, from the general text response of the expert system.

After receiving the coordinates of the points, the process of generating the shape of the cross section of the punch is carried out using the developed software in OpenGL (Open Graphics Library is a specification of the software interface for rendering 2D and 3D graphics).

It is worth noting that for the correctness of our results, the coordinates of the points were previously arranged in the sequence of movement along the contour of the cross section. A special calculateAngle function has been developed that accepts the coordinates of two points as parameters: the center of the circle and a specific point on the contour of the punch. It is designed to calculate the angle between the horizontal axis and the line that connects the center of the circle and the given point. The comparePoints function was developed and used to compare the coordinates of two points on the punch contour by their angles relative to the origin of the coordinate system. Thus, the size of the angle becomes the ordering criterion. The points were sorted by the order of increasing angles relative to the origin of the coordinate system for the Ox axis, which resulted in the positioning of the points in a clockwise direction around the center of the circle along the contour of the punch.

In the process of visualizing the shape of the two-link punch, the initial collection of points was divided into two collections for the outer and inner contours, respectively. Cubic spline interpolation was used to reproduce the outline of the punch. The cubic spline was chosen because of its smoothness and flexibility properties that make it well suited for many types of data and problems.

Assumptions regarding the adequacy of the models [26, 27] and the correctness of the results [28] in the study undergo the necessary stages of verification of a comparative nature [11], as well as others.

5. Research results regarding the development of a generalized algorithm for modeling the contact interaction of a punch with an elastic half-space

5. 1. Analytical method for solving the problem of pressing a two-link punch into an elastic half-space

The problem of pressing a rigid flat punch in the form of a non-circular ring on a homogeneous and isotropic elastic half-space $x_3 \leq 0$ is considered. The elastic half-space fills the entire part of the half-space contained on one side of the Ox_1x_2 plane. We introduce the coordinate system so that the elastic half-space coincides with the region $x_3 \leq 0$ (Fig. 1).

The problem considers the use of only isotropic and homogeneous materials operating within the limits of elasticity. For example: the punch material is steel: E=200000 MPa, μ=0.3, and for the elastic half-space – E=180 MPa, μ=0.2. On the Ox_1x_2 plane, a two-connected region Ω is considered, bounded by closed lines K_1 and K_2, which contains points located after deformation on the displaced surface of the punch base. The boundary conditions refer to the undeformed surface of the elastic body, that is, to the plane $x_3=0$. The base of the punch is considered absolutely smooth; therefore, it is assumed that the tangential stresses σ_{31}, σ_{31} are absent along the entire plane $x_3=0$:

$$x_3=0:\sigma_{31}=0,\sigma_{23}=0. \quad (1)$$

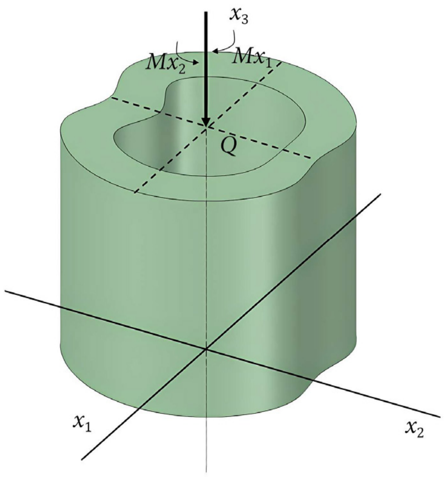


Fig. 1. General view of a flat punch in the form of a non-circular ring

Normal stresses are absent on the plane $x_3=0$ outside the region Ω of the contact of the punch with the elastic half-space. At the points of the region Ω, the elastic medium is subjected to a compressive load $p(x)$, therefore:

$$\sigma_{33}(x)=\begin{cases}0, & x\notin\Omega,\\ -p(x), & x\in\Omega.\end{cases} \quad (2)$$

Here, $x(x_1, x_2, 0)$ is a point on the Ox_1x_2 plane of three-dimensional space.

The function $p(x)$ characterizing the pressure distribution under the punch is not specified in advance. Under conditions (1), (2), the balance of the punch is described by the following equations:

$$P=\iint_{\Omega} p(x)\mathrm{d}x, \quad (3)$$

$$M_1=\iint_{\Omega} x_2 p(x)\mathrm{d}x,\ \ M_2=-\iint_{\Omega} x_1 p(x)\mathrm{d}x, \quad (4)$$

where P, M_1, M_2 are the main vector and main moments of forces applied to the punch.

Under the action of the load, the punch will move gradually and rotate around some y-axis of the Ox_1x_2 plane. Let us denote the gradual movement parallel to the vertical axis x_3 by δ, and by β_1, β_2 – the projections of the small rotation vector:

$$x_3=0,\ \ (x_1,x_2)\in\Omega:\ U_3=\delta-\beta_2 x_1+\beta_1 x_2. \quad (5)$$

In the case of a flat die, the condition for the vertical movement of the points of the region Ω is reduced to a two-dimensional integral equation of the first kind for the desired normal pressure distribution $p(x)$:

$$\delta-\beta_2 x_1+\beta_1 x_2=\frac{1-\nu}{2\pi G}\iint_{\Omega}\frac{p(x_1',x_2')\mathrm{d}x_1'\mathrm{d}x_2'}{\sqrt{(x_1-x_1')^2+(x_2-x_2')^2}}, \quad (6)$$

where G is the shear modulus. The values δ, β_1, β_2 are unknown in advance, and the equilibrium equations of the punch (3), (4) are used to determine them.

In addition, the punch must be pressed over the entire compression surface, so the desired pressure distribution satisfies the condition:

$$p(x_1,x_2)\geq 0,\ \ (x_1,x_2)\in\Omega. \quad (7)$$

It is assumed that the base, with which the punch is pressed against the half-space, has the shape of a smooth surface, the contact area occupies in the Ox_1x_2 plane a two-connected region Ω, bounded by closed lines K_1 and K_2, containing points located after deformation on the undisplaced surface z of the punch base.

It is further assumed that the unknown equations of lines K_1 and K_2 can be represented in the form of the following functions that depend on the small parameter ε. The parameter may take both geometric and mechanical, or any other, value, characterizing the shape and dimensions of the contact area:

$$K_1:\rho=a\cdot(1+f_1(\varepsilon,\theta)); \quad (8)$$

$$K_2:\rho=b\cdot(1+f_2(\varepsilon,\theta)), \quad (9)$$

where $a<b$, $\varepsilon<1$, the functions $f_1(\varepsilon,\theta)$, $f_2(\varepsilon,\theta)$ are continuous and single-valued functions such that they can be represented by series of powers of ε in the form:

$$f_1(\varepsilon,\theta)=\varepsilon f_{10}(\theta)+\varepsilon^2 f_{20}(\theta)+ +\varepsilon^3 f_{30}(\theta)+\ldots=\sum_{i=1}^{\infty}\varepsilon^i f_{i0}(\theta), \quad (10)$$

$$f_2(\varepsilon,\theta)=\varepsilon f_{01}(\theta)+\varepsilon^2 f_{02}(\theta)+ +\varepsilon^3 f_{03}(\theta)+\ldots=\sum_{k=1}^{\infty}\varepsilon^k f_{0k}(\theta), \quad (11)$$

coefficients of which $f_{ik}(\theta)$ are continuous functions on the interval $[0, 2\pi]$.

In the case when $f_1(\varepsilon,\theta)=f_2(\varepsilon,\theta)$, the boundary lines of the region are similar.

We considered the case when the equations of lines K_1 and K_2, limiting Ω, the area of contact of the punch with the half-space, can be represented in the form of the following functions:

$$\rho_1=a\cdot(1+f(\varepsilon,\theta)),\ \rho_2=b\cdot(1+f(\varepsilon,\theta)),\ (a<b,\varepsilon<1), \quad (12)$$

where ρ, θ are polar coordinates, $x_1=\rho\cos\theta$, $x_2=\rho\sin\theta$, $f(\varepsilon,\theta)$ is a continuous and single-valued function such that it can be represented by a power series of ε in the form:

$$f(\varepsilon,\theta)=\varepsilon f_1(\theta)+\varepsilon^2 f_2(\theta)+\ldots. \quad (13)$$

The punch is pressed into the half-space by a vertical force Q. Since the equations of the lines (12) bounding the region Ω depend on the small parameter ε, it is obvious that the desired distribution of normal pressures $p(\rho, \theta)$ also depends on ε. The solution to equation (6) is found in the form:

$$p(\rho,\theta)=\sum_{k=0}^{\infty}p_k(\rho,\theta)\varepsilon^k. \quad (14)$$

New variables (R, φ) are introduced, which are related to the old ones (ρ, θ) by the following dependences:

$$p = R\left(1 + f\left(\varepsilon, \theta\right)\right), \quad \theta = \varphi. \tag{15}$$

Here $f(\varepsilon, \theta)$ is determined by expression (13); at the same time, the region Ω bounded by lines (12) will turn into a circular ring D bounded by circles $R=a$, $R=b$.

In the new variables, the solution (14) is represented in the form of a power series ε:

$$p\left(\rho\left(R, \varphi, \varepsilon\right), \varphi\right) \equiv \sum_{k=0}^{\infty} P_k\left(R, \varphi\right) \varepsilon^k. \tag{16}$$

It follows from dependences (15), (13) that $\rho=R$ at $\varepsilon=0$. Taking this into account, we obtained the expressions for $P_k(R, \varphi)$ at $k=0, 1, 2$, necessary in the first approximations, which take the form:

$$P_0\left(R, \varphi\right) = p_0\left(R, \varphi\right), \tag{17}$$

$$P_1\left(R, \varphi\right) = p_1\left(R, \varphi\right) + p_0'\left(R, \varphi\right) R f_1\left(\varphi\right), \tag{18}$$

$$P_2\left(R, \varphi\right) = p_2\left(R, \varphi\right) + p_0'\left(R, \varphi\right) R f_2\left(\varphi\right) + \\ + p_1'\left(R, \varphi\right) R f_1\left(\varphi\right) + 0.5 p_0''\left(R, \varphi\right) R^2 f_1^2\left(\varphi\right). \tag{19}$$

The stroke means the derivative with respect to the variable R. The integral included in equation (6) is represented in the form of an expansion by powers of ε:

$$U = \iint_{\Omega} \frac{p\left(\rho, \theta\right)}{r} \mathrm{d}s = \sum_{k=0}^{\infty} U_k \varepsilon^k, \tag{20}$$

$$r^2 = \left|x - y\right|^2 = \rho^2 + \rho_0^2 - 2\rho\rho_0 \cos\left(\theta - \theta_0\right); \quad \left(\rho_0, \theta_0\right) \in \Omega. \tag{21}$$

Since the integrand function in the integral (20) turns to infinity at the point (ρ_0, θ_0), when finding the derivatives for the expansion of (20), it is necessary to first exclude this point from the region Ω. To this end, the point (ρ_0, θ_0) is cut out of Ω by a circle of radius α.

The derivatives included in expression (20) are derivatives of the parameter ε of the double integral, in which not only the integrand function depends on this parameter but also the boundary equations depend on ε. By differentiating with respect to the parameter ε and taking into account that the boundary equations of the region with the discarded point depend on the parameter, it is obtained by passing to the boundary at α, which goes to 0 and $\varepsilon=0$. By means of mathematical transformations, the following is obtained:

$$\delta = \sum_{k=0}^{\infty} \delta_k \varepsilon^k, \quad \beta_1 = \sum_{k=0}^{\infty} \beta_{1k} \varepsilon^k, \quad \beta_2 = \sum_{k=0}^{\infty} \beta_{2k} \varepsilon^k. \tag{22}$$

Then in equations (3), (4) the integrals included in these equations are represented in the form of series by powers of ε.

By substituting expansion (22) as well as (20) into equations (3), (4), (6), and equating expressions with the same powers of ε, the following systems of equations for determining $P_k(\rho, \theta)$, δ_k, β_{1k}, β_{2k} are obtained.

$$\frac{\delta_k 2\pi G}{1 - \upsilon} = \iint_D \frac{P_k\left(R, \theta\right)}{r} \mathrm{d}s + \Phi_k\left(P_0, P_1, .., P_{k-1}\right), \tag{23}$$

$$Q_k = \iint_D P_k\left(R, \theta\right) \mathrm{d}s + F_k\left(P_0, P_1, .., P_{k-1}\right), \tag{24}$$

$$\delta_k' = \delta_k + \beta_{1k} \rho \sin\theta - \beta_{2k} \rho \cos\theta, \tag{25}$$

$$M_{1k} = \iint_D P_k\left(\rho, \theta\right) \rho \sin\theta \ \mathrm{d}s + V_k\left(P_0, P_1, ..., P_{k-1}\right), \tag{26}$$

$$M_{2k} = \iint_D P_k\left(\rho, \theta\right) \rho \cos\theta \ \mathrm{d}s + W_k\left(P_0, P_1, ..., P_{k-1}\right),$$

$$\left(k = 0, 1, 2, \ldots\right). \tag{27}$$

The expression for the functions Φ_k, F_k, V_k, W_k at $k=0, 1, 2, \ldots$ is written out, which are specially introduced and used to simplify the notation when performing mathematical transformations:

$$\Phi_0 = 0, \quad \Phi_1 = L_1\left(P_0\right), \quad \Phi_2 = L_1\left(P_1\right) + L_2\left(P_0\right), \tag{28}$$

$$F_0 = 0, \quad F_1 = 2\iint_D P_0\left(\rho, \theta\right) f_1\left(\theta\right) \mathrm{d}s, \tag{29}$$

$$F_2 = \iint_D \begin{pmatrix} 2P_1\left(\rho, \theta\right) f_1\left(\theta\right) + 2P_0\left(\rho, \theta\right) f_2\left(\theta\right) + \\ + P_0\left(\rho, \theta\right) f_1^2\left(\theta\right) \end{pmatrix} \mathrm{d}s, \tag{30}$$

$$V_0 = 0, \quad V_1 = \iint_D P_0\left(\rho, \theta\right) 3 f_1\left(\theta\right) \rho \sin\theta \mathrm{d}s, \tag{31}$$

$$V_2 = \iint_D 3 \begin{pmatrix} P_1(\rho, \theta) f_1(\theta) + P_0\left(\rho, \theta\right) f_2\left(\theta\right) + \\ + P_0\left(\rho, \theta\right) f_1^2\left(\theta\right) \end{pmatrix} \rho \sin\theta \mathrm{d}s, \tag{32}$$

$$W_0 = 0, \quad W_1 = \iint_D P_0\left(\rho, \theta\right) 3 f_1\left(\theta\right) \rho \cos\theta \mathrm{d}s, \tag{33}$$

$$W_2 = \iint_D 3 \begin{pmatrix} P_1\left(\rho, \theta\right) f_1\left(\theta\right) + P_0\left(\rho, \theta\right) f_2\left(\theta\right) + \\ + P_0\left(\rho, \theta\right) f_1^2\left(\theta\right) \end{pmatrix} \rho \cos\theta \mathrm{d}s, \tag{34}$$

$$L_1\left(P_0\right) = \left(1 - \rho_0 \frac{\partial}{\partial \rho_0}\right) \iint_D \frac{P_0\left(\rho, \theta\right)}{r} f_1\left(\theta\right) \mathrm{d}s, \tag{35}$$

$$L_2\left(P_0\right) = \left(1 - \rho_0 \frac{\partial}{\partial \rho_0}\right) \iint_D \frac{P_0\left(\rho, \theta\right)}{r} f_2\left(\theta\right) \mathrm{d}s + \\ + \rho_0^2 \frac{\partial^2}{\partial \rho_0^2} \iint_D \frac{P_0(\rho, \theta)}{2r} f_1^2(\theta) \mathrm{d}s, \tag{36}$$

where D is a circular ring, $a \leq \rho \leq b$.

Thus, a sequence of problems (23) to (27) for the circular ring D was built for the region Ω. Calculation formulas were also derived for the analytical solution of problems for flat stamps of different shapes in the plan close to ring ones.

5. 2. Construction of finite-element models for the punch-elastic half-space system

The three-dimensional geometry of the punch body and the elastic half-space were designed (Fig. 2). The material for the elastic half-space was assumed to be isotropic and homogeneous.

A finite element mesh was constructed for these geometric objects. A hexahedron was chosen as the dominant finite element, which provided more controlled and accurate results in the modeling process. A total of 120 to 210 original discretized finite element models were built for each die cross-section configuration. The developed models made it possible to reproduce the processes of both axisymmetric loading and non-axisymmetric loading.

Structures and mechanisms in operation are under conditions where various types of damage occur due to various reasons. Such processes can take place both according to a certain law [29, 30] and randomly. Fig. 3 shows the results of a computer simulation of the interaction of a cylindrical flat in-plane punch in the shape of a circle with a homogeneous isotropic elastic half-space. Fig. 3, *a* demonstrates the distribution of normal stresses for a punch in the form of a circle without damage. Fig. 3, *b–d* successively shows the distribution of normal stresses for damage of the wear type. Wear damage occurred according to the following rule: for a central angle of 90°, a part of the cross section of the circle was removed gradually with a step of 0.04 from the value of the radius of the circle (using the spline function). Fig. 3, *b* demonstrates that the wear occurred on one side of the circle (central angle – 90°) to a depth of 0.12 times the radius. Fig. 3, *c* demonstrates that the wear occurred on two opposite sides of the circle (90°+90°) to a depth of 0.12 times the radius. Fig. 3, *d* demonstrates that the wear occurred on three sides of the circle (270°) to a depth of 0.12 times the radius. Fig. 3, *e* demonstrates that the wear occurred on four sides of the circle (360°) to a depth of 0.12 times the radius. Fig. 3, *f* shows the case of point damage of the crack type, which penetrates to a depth of 0.2 times the radius of the die.

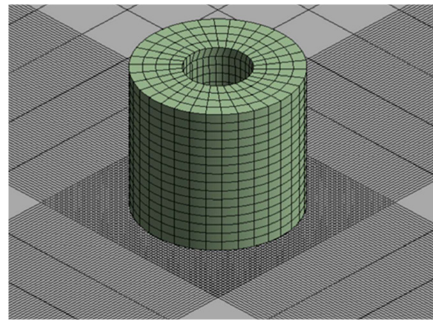
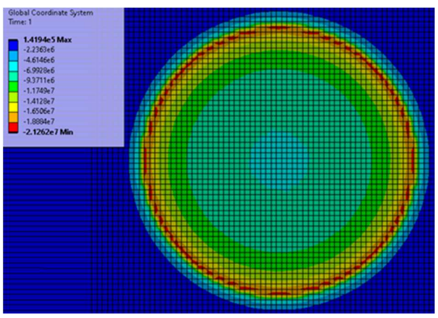
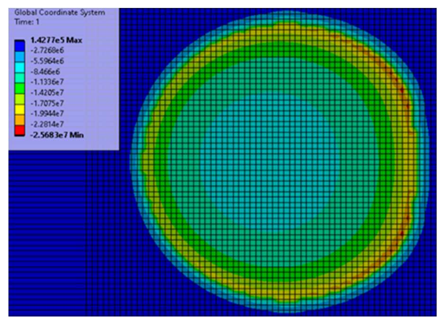


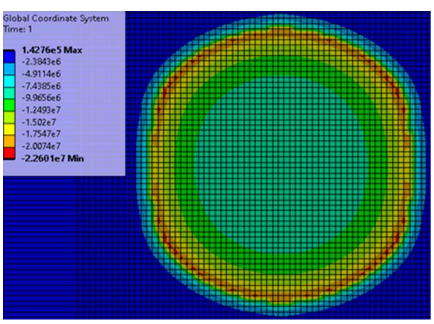
Fig. 2. Example of a finite-element model of a two-connected planar flat punch with an elastic half-space



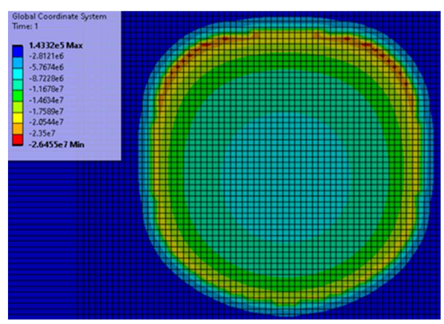
a



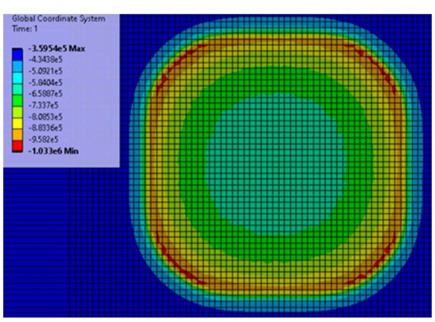
b



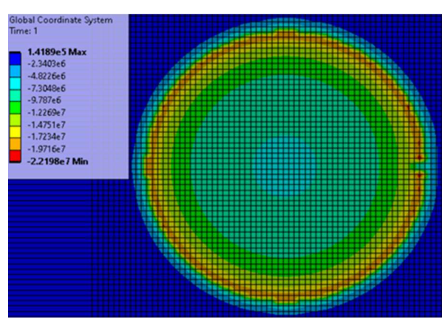
c



d



e



f

Fig. 3. Distribution of normal stresses in the half-space under the circular punch: *a* – circle without damage; *b* – a circle with damage to the edge of the punch within the central angle of 90°; *c* – a circle with damage to the edge of the punch within two vertical central angles (90°+90°); *d* – a circle with damage to the edge of the punch within the three central corners (90°+90°+90°); *e* – a circle with damage to the edge of the punch within the four central corners (90°+90°+90°+90°); *f* – a circle of damage of the crack type, which penetrates to a depth of 0.2 from the radius of the punch

Fig. 4 shows as examples the results of computer simulation of the interaction of a ring-shaped punch with a uniform isotropic elastic half-space. Fig. 4, *a* demonstrates the distribution of normal stresses for a ring-shaped die without damage. Fig. 4, *b*–e consistently shows the distribution of normal stresses for damage of the wear type. Wear damage occurred according to the following rule: for a central angle of 60°, a part of the cross-section was removed gradually with a step of 0.0625 from the value of the ring width (using the spline function). Fig. 4, *b* demonstrates that the wear occurred on one side of the outer contour of the ring to a depth of 0.125 from the width of the ring. Fig. 4, *c* shows that the wear occurred on the same side of the outer contour of the ring to a depth of 0.1875 from the width of the ring. Fig. 4, *d* – the wear occurred on the same side of the outer contour of the ring to a depth of 0.25 from the width of the ring. Fig. 4, *e* – the wear occurred on the same side of the outer contour of the ring to a depth of 0.3125 from the width of the ring. Fig. 4, *f* – the wear occurred on the same side of the ring's outer contour to a depth of 0.375 from the ring's width.

In this way, data arrays with normal and tangential stresses and displacements were formed. The software developed in the C++ programming language is used to transfer these data arrays. In the case of the manual option, the data is exported to an Excel file for further analysis and processing with the help of a js-script and the formation of files for the knowledge base. To simulate the shapes of stamps after corrosion deformation, the corrosion models of Gutman and Dolynsky [29, 30] were considered, which provide an opportunity to evaluate two different approaches to the mathematical description of metal corrosion processes.

5. 3. Building a generalizing algorithm

Since quite often in the case of solving practical problems, the geometric characteristics of stamps are not available for direct study, the evaluation of these parameters can be carried out in various ways [19, 20, 23–28]. In this study, a knowledge base was built, which included information on the geometric characteristics and properties of the materials of stamps and half-space, information on the stressed-strained state of the punch-elastic half-space system. A set of rules and facts was also formed. With the help of CLIPS, a system was built that can identify the contact zones of stamps with a flat sole that press on an elastic half-space.

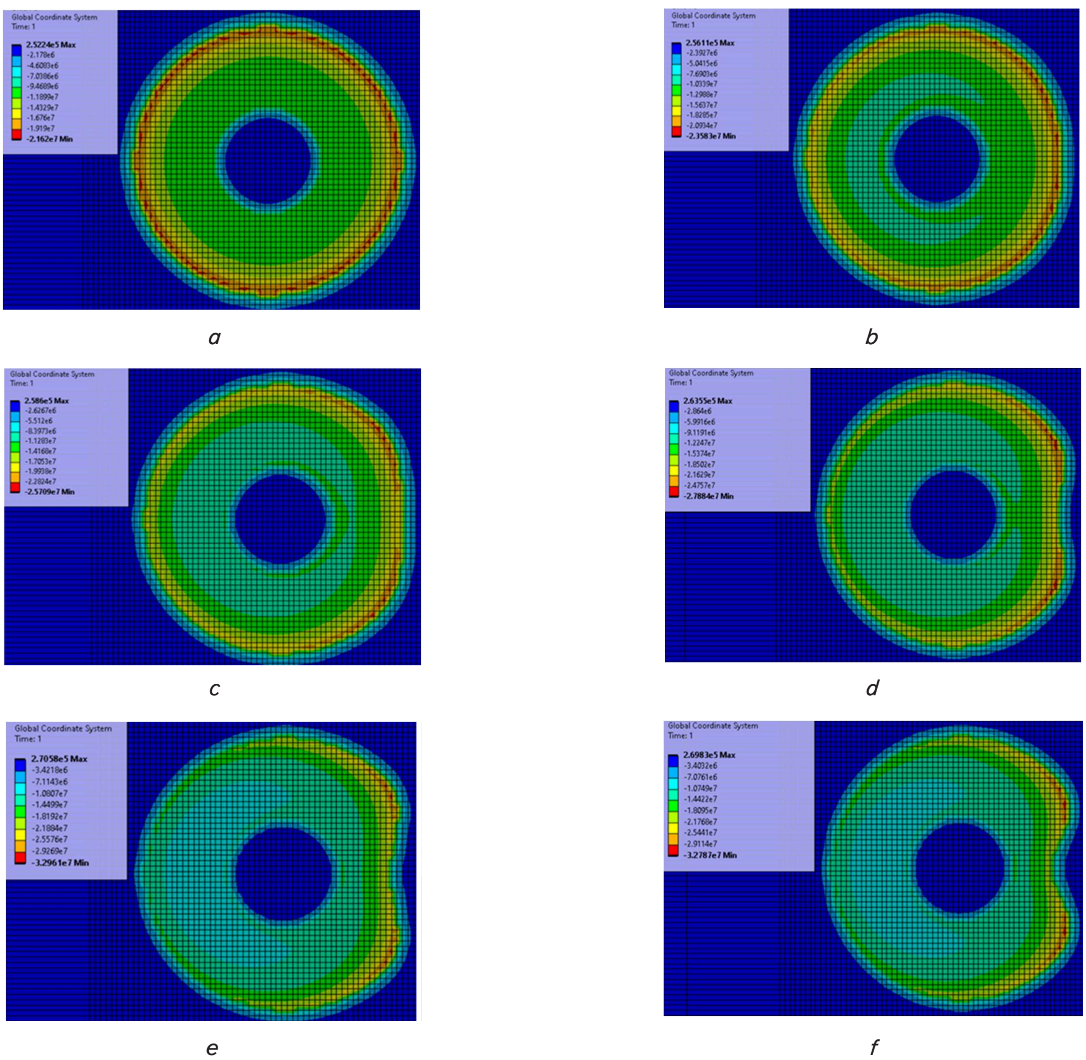


Fig. 4. Distribution of normal stresses in the half-space under the ring punch: *a* — circular ring without damage; *b* — a circular ring with damage at an angle of 60° with a depth of 0.125 times the width of the ring; *c* — a circular ring with damage at an angle of 60° with a depth of 0.1875 times the width of the ring; *d* — a circular ring with damage at an angle of 60° with a depth of 0.25 times the width of the ring; *e* — a circular ring with damage at an angle of 60° with a depth of 0.3125 times the width of the ring; *f* — a circular ring with damage at an angle of 60° with a depth of 0.375 times the width of the ring

A point fact template has been developed, which represents information about a point with specified x and y coordinates, as well as the value of normal and tangential stresses and displacement. Point coordinates, values of normal and tangential stresses, and displacements have data type FLOAT.

The known-stress fact template is used to store known stress values, displacements, and point coordinates. In this template: stress is the known value of normal stress, x, y are the coordinates of the point where the known value of normal stress was measured.

The contour template represents the contour of a figure with the specified coordinates of the points that form this contour. In this template: points – a string with the coordinates of the points that form the contour. In this case, this slot will be filled with a string of coordinates, for example, "($x1$ $y1$) ($x2$ $y2$) ($x3$ $y3$)". This fact template is used to accumulate results and output them as the results of the expert system.

The designed expert system follows a classify-contour rule, which performs shape contour classification based on comparison of point normal stresses with known normal stress values.

This rule contains a condition that collects data for each point with normal stress, a condition that retrieves the known normal stress values along with the coordinates, and a test function that compares the normal stress value of a point with a known value with some user-specified tolerance independently. By default, the error is 0.1. If the condition of the rule is met, an action is performed that creates a new fact of type contour, which contains the coordinates of a known point with the corresponding normal stress:

IF A THEN C,

where:

$$A=\left|\text{knownStress}-\text{stress}\right|\leq\varepsilon,$$

C – adding point coordinates to the resulting points fact.

An example of a program segment for the formation of rules and facts of an expert system in the specific intuitive language COOL, which is part of CLIPS [25], is given. In the example, a fragment of the knowledge base is provided, which illustrates the presence of arrays of normal stresses and coordinates of points of the punch contour:

```
(deftemplate point
    (slot x (type FLOAT))
    (slot y (type FLOAT))
    (slot stress (type FLOAT))
)

(deftemplate known-stress
    (slot stress (type FLOAT))
    (slot x (type FLOAT))
    (slot y (type FLOAT))
)

(deftemplate contour
    (slot points (type STRING) (default ""))
)
(defrule classify-contour
    (point (x ?x) (y ?y) (stress ?stress))
    (known-stress (stress ?known-stress) (x ?known-x) (y ?known-y))
    (test (<= (abs (- ?stress ?known-stress)) 0.1))
    =>
    (assert (contour (points (str-cat "(" ?known-x " " ?known-y ")"))))
)
(deffacts initial-known-stress
(known-stress (stress -14034000.0) (x -0.000718) (y -0.004951))
(known-stress (stress -18598000.0) (x -0.000125) (y -0.005012))
(known-stress (stress -17564000.0) (x -0.001298) (y -0.004838))
(known-stress (stress -17039000.0) (x -0.001892) (y -0.00465))
(known-stress (stress -14541000.0) (x -0.002472) (y -0.00436))
(known-stress (stress -14782000.0) (x -0.003051) (y -0.003969))
...
(known-stress (stress -15988000.0) (x -0.003601) (y -0.003477))
(known-stress (stress -16772000.0) (x -0.004079) (y -0.002898))
(known-stress (stress -16825000.0) (x -0.00447) (y -0.00226))
(known-stress (stress -17299000.0) (x -0.004702) (y -0.001681))
(known-stress (stress -17297000.0) (x -0.004876) (y -0.001102))
(known-stress (stress -13063000.0) (x -0.004977) (y -0.000522)).
```

The algorithm for solving the contact problem for the absolutely rigid punch-elastic half-space system can be represented by the following sequence of actions (Fig. 5, 6):

Step 0. Develop software for analysis and visualization of the analytical solution for pressing into the elastic half-space of stamps of various shapes close to ring, which was obtained earlier [11]. (The analytical solution is obtained separately for each problem).

Step 1. Create a project in ANSYS and the Static Structural module. In the Geometry subsection, create a three-dimensional model of the geometry of the half-space and punch.

Step 2. Carry out the process of generation and correction of the finite element mesh.

Step 3. If the finite-element mesh meets the conditions for qualitative partitioning, namely the fulfillment of the necessary conditions, then we proceed to step 4, if not, then we return to step 2.

Step 4. Calculation of the process of contact interaction of the punch and the elastic half-space by means of computer simulation.

Step 5. Check the reliability of the obtained data by comparing it with the results obtained earlier in step 0. If after evaluating the results it turns out that the error exceeds the permissible, then it is necessary to return to step 2, otherwise, go to step 6.

Step 6. Locate conditional sensors along the contact contour to simulate the process of reading information from field experiments.

Step 7. Read information about the stressed-strained state (SSS) of the system from conditional sensors.

Step 8. This step provides branching for the selection of the information transfer option. Transfer information about the stressed-strained state to the knowledge base of the expert system. If the user has chosen the automated option of data transfer to the knowledge base of the expert system, then go to step 9, otherwise – to step 10.

Step 9. Process the results of stress calculations using a specially developed software application, which is implemented in the C++ programming language and uses the

ANSYS API to obtain data from sensors. After completing the procedure, proceed to step 11.

Step 10. The data is exported to an Excel file for further analysis and processing using a js script and the formation of files for the knowledge base. A manual data transfer option has been added as an alternative to the automated option, which allows one to make changes to the data, or retrieve data from external sources outside of the ANSYS project.

Step 11. Data processing by an expert system. The solution of the infinite-dimensional optimization problem is reduced to a finite-dimensional one by approximating the vector function $U(x)$ using the finite element method. Determine the resulting vector by Newton's method. Save point coordinates.

Step 12. Sort the identified points in a circle to simplify the process of further visualization.

Step 13. If necessary, divide the initial collection of points into two collections for external and internal contours, respectively.

Step 14. Perform cubic spline interpolation for each collection.

Step 15. Output the value of the identified points. Visualize the outline of the punch.

Step 16. The end.

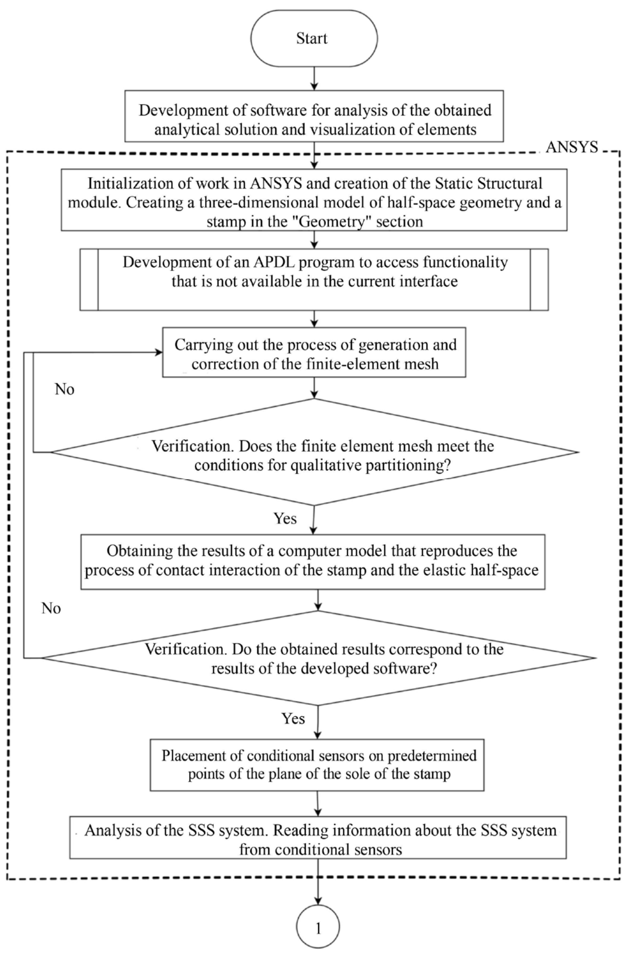


Fig. 5. Block diagram of the generalizing algorithm

The above block diagram (Fig. 5, 6) demonstrates the generalized algorithm for solving the contact problem for the absolutely rigid punch-elastic half-space system.

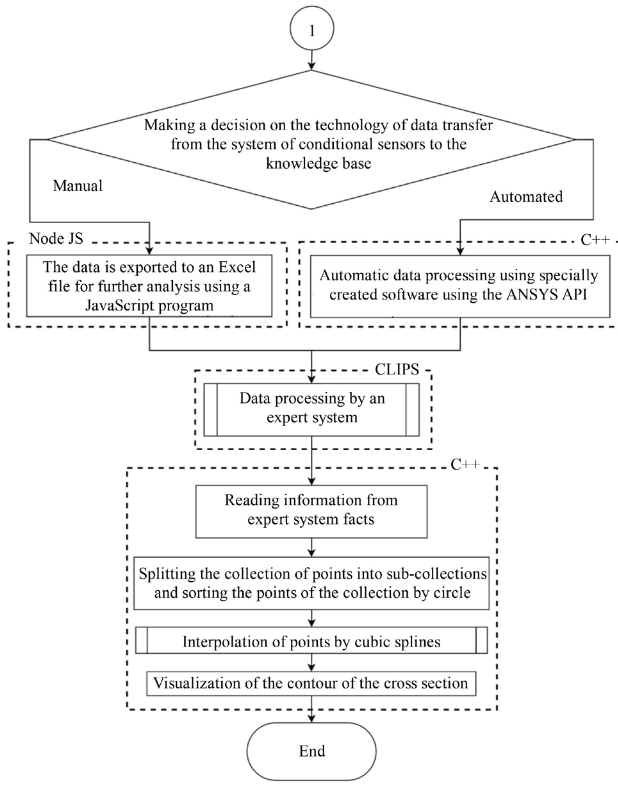


Fig. 6. Block diagram of the generalizing algorithm (continued)

5. 4. Determining the contour of the damaged form of the punch using the developed tools

Applying the algorithm described above, an example was considered – the problem of pressing into an elastic half-space a cylindrical absolutely rigid flat punch, the cross-section of which has an annular area in plan (Fig. 2). The analytical solution was obtained and the pattern of the stressed-strained state of the die-elastic half-space system was calculated [11]. A ring punch was considered for the test task.

The outer diameter of the die before damage was 100 mm, and the inner diameter was 40 mm. The material of the punch is steel: E=200000 MPa, μ=0.3, and for the elastic half-space – an isotropic material with the following parameters: E=180 MPa, μ=0.2. Conditional sensors were located evenly around the entire circle.

The simulation results are shown in Fig. 7 for the case when the stressed-strained state of the punch-elastic half-space system was established for the undamaged ring form of the punch.

The simulation results shown in Fig. 8 illustrate the situation where damage to the die shape has occurred by wear, for the case of one-sided damage from the same inner and outer sides of the ring. Wear-type damage occurred at a central angle of 60°. From the outer side of the ring, the depth of damage was 0.08 of the ring width, and from the inner side – 0.0625 of the ring width.

Information about normal stresses and displacements from conditional sensors located along the perimeter of the punch was submitted to the input of the expert system (Fig. 7).

For the represented 57 conditional sensors, after 450 calculations for various damage schemes (Fig. 7, 8), the following shape of the punch was obtained based on the data read from the sensors (Fig. 9).

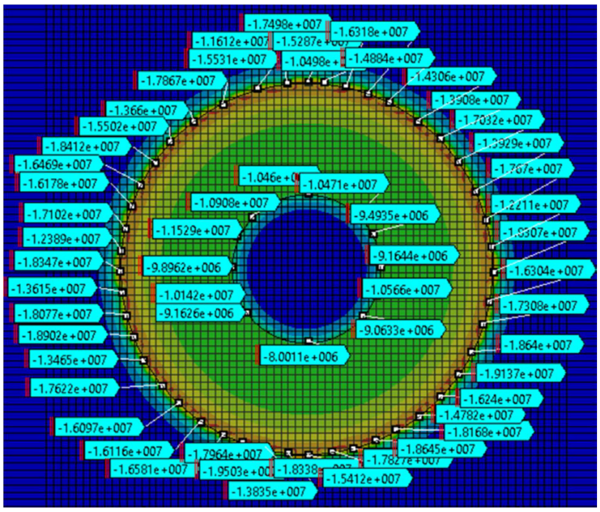


Fig. 7. Scheme of arrangement of conditional sensors for reading normal stresses before damage to the punch contour

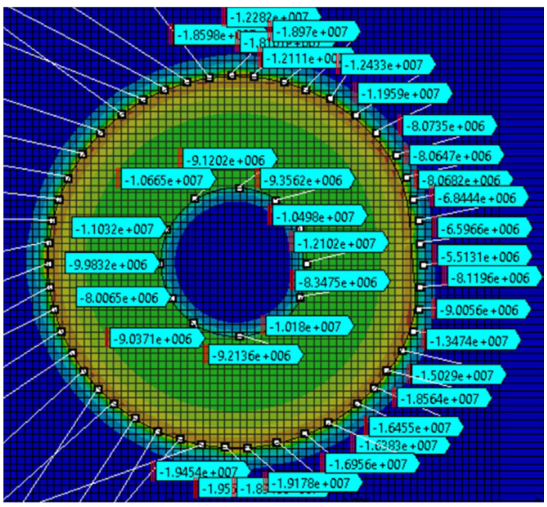


Fig. 8. Scheme of the location of conditional sensors for reading normal stresses after damage to the punch contour

Satisfactory results for the outer contour were obtained at a depth of 4–6 % of the ring thickness. It must be recognized that the reproduction of deeper damage to the shape of the punch requires an increase in the number of the experimental base. It should also be noted that the system more accurately determines the shape of the outer contour of the punch than the inner one. It can be assumed that this is caused, first of all, by the mutual influence of the points of the internal contour of the punch.

When conducting a number of numerical experiments, it was noticed that when the deformation of the contour occurs randomly and locally, such calculations give sufficiently good results. This approach makes it possible to take into account small damage on the contour (less than 1 % of the ring width) and reproduce such as single deep cracks (more than 10 % of the ring width).

6. Discussion of results of developing a generalized algorithm for modeling the contact interaction of a punch with an elastic half-space

Based on previous studies in the field of analytical methods for solving contact problems of the theory of elasticity [9–12, 18, 21], an analytical approach to solving problems of the punch-elastic half-space system was devised.

Finite-element models of the interaction of a flat absolutely rigid punch with an elastic half-space were built according to a certain rule for the formation of the cross-section of the punch, generalizing the approaches [26, 27] of searching for damage zones at contact sites (Fig. 5, 6). In fact, data arrays were built to study the damaged forms of the cross-sections of stamps, which made it possible to successfully restore the contours of flat stamps (Fig. 7–9).

A generalizing algorithm for solving contact problems has been developed, which uses existing software products ANSYS and CLIPS, as well as authentic software developments. It is this combination of an analytical approach and the use of computer capabilities that creates a single trajectory of constructing a solution to the contact problem for the die-elastic half-space system. This allows a holistic approach to the solution of complex practical problems based on a combination of numerical methods and existing software tools with analytical solutions, the experience of which can be found, for example, in works [20, 23, 24].

The advantages of this study in comparison with known ones are the combination of mathematical modeling and computer simulation for solving contact problems of the theory of elasticity.

The limitations of the study are the impossibility of applying the developed algorithm to the solution of contact problems of another class without an additional review of the analytical step

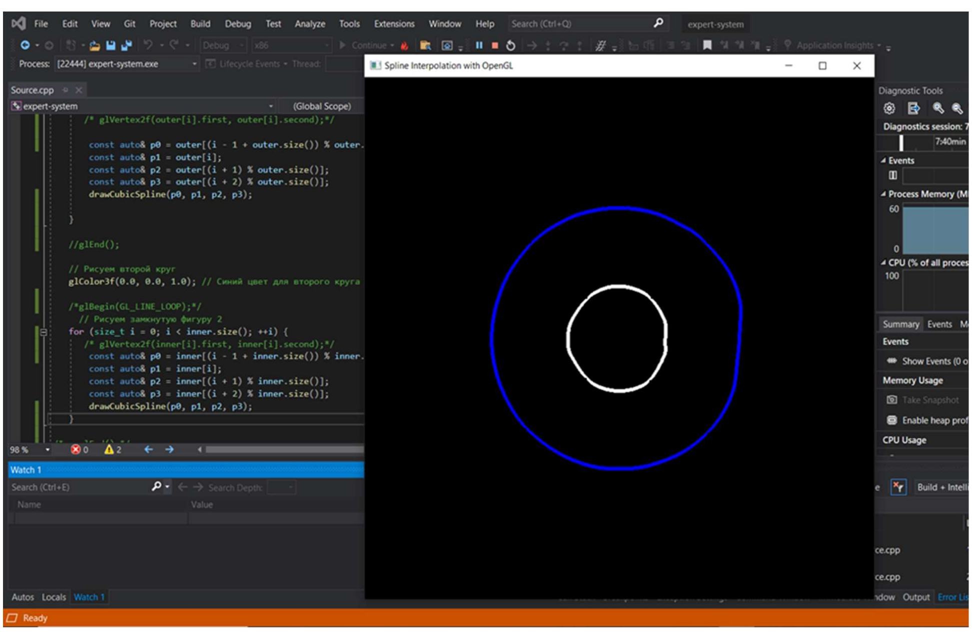


Fig. 9. Restored contour of the ring punch

of the algorithm. That is, there is a need to derive a new separate analytical solution. But it is quite possible to extend this approach, for example, to the problem of collinear interphase cracks with different electrical conditions on their banks [5]. The disadvantages of the proposed approach include the fact that the mathematical apparatus for solving contact problems is quite complex and it is not always possible to obtain an analytical solution for conducting comparative assessments [14, 15, 18]. But the proposed approach should be used to solve problems that make sense to break down into simpler components and find effective solutions. At each step, it is possible to improve and adapt the proposed approach, which will allow finding new ways to solve the problem [19].

In the same way, using a similar approach and already performed calculations, it is possible to search, for example, for areas of adhesion and sliding under a punch on the surface of an elastic half-space [23, 24], using already developed software and the functionality of the CLIPS software system.

Also, these results can be implemented in engineering strength calculations to increase the reliability and durability of structures or contribute to the saving of materials.

This work is a further advancement of research aimed at devising methods and algorithms for solving contact problems from the theory of elasticity.

7. Conclusions

1. To obtain an analytical solution, we have used a variant of the perturbation method, based on the expansion of the potential of a simple layer distributed over a doubly connected region by a small parameter. In this way, the problem of pressing a flat punch in the form of a non-circular ring was reduced to a sequence of problems for a punch in the form of a circular ring. That has made it possible to employ the well-known circular ring solution.

2. Finite-element models of the interaction of a flat absolutely rigid two-link in the plane of the die with an elastic half-space were built. This has made it possible to perform calculations for stamps of different configurations.

3. A generalized algorithm for identification of cross-sections of stamps was developed. Its feature is the possibility of application for solving the contact problem for the punch-elastic half-space system, with the introduction of CLIPS functionality, when the contact area is unknown in advance.

4. The contact problem for the punch-elastic half-space system has been solved; the contour of the damaged punch shape was determined using the developed tools. Based on this, it was found that the system more accurately determines the shape of the outer contour of the punch than the inner one. This is due primarily to the mutual influence of the points of the internal contour of the punch. It is noted that when the deformation of the contour occurs randomly and locally, such calculations give fairly good results. Thus, it becomes possible to take into account small damage on the contour, which is less than 1 % of the width of the ring, as well as reproduce single deep cracks, which exceed 10 % of the width of the ring.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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