***This paper investigates the structural changes in the turbulent motion of an incompressible fluid in the hydrodynamic entrance region of plane-parallel pressure motion. Movement in pressure hydromechanical systems usually occurs in a turbulent regime. Studying the patterns of changes in hydrodynamic parameters under conditions of stationary turbulent pressure motion in the inlet region is a very urgent task. The study was carried out on the basis of boundary layer equations. Taking into account the dependence of changes in the kinematic viscosity coefficient that occur between layers of fluid, a boundary value problem was formed. Analytical solutions have been obtained that make it possible to obtain patterns of changes in velocity and pressure in any effective flow section. Based on the general conclusions of the study, solutions were found for two cases:***

***a) the velocity of the fluid entering the cylindrical pipe is constant;***

***b) the velocity of the incoming fluid has a parabolic distribution.***

***For these cases, using computer analysis of the data obtained, general graphs of velocity changes were constructed in various sections along the hydrodynamic entrance region. These graphs, which display the change in velocity along the entire length of the inlet, make it possible to obtain the velocity of fluid movement at any point along the inlet length and estimate the length of the transition zone. The results obtained are among the least studied issues of classical fluid mechanics and are of important theoretical interest. The results obtained are applicable for the correct construction of the hydrodynamic entrance region of machinery. A calculation formula has been obtained to determine the length of the hydrodynamic inlet region***

***Keywords: plane-parallel motion, hydrodynamic entrance region, turbulent motion, viscous fluid, velocity distribution***

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# IDENTIFYING SOME REGULARITIES OF THE TURBULENT STEADY-STATE PLANE-PARALLEL MOTION OF INCOMPRESSIBLE FLUID AT THE ENTRANCE LENGTH

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## 1. Introduction

A fluid serves as the working medium in a variety of hydraulic automation systems, transferring power to the executive body. Turbulent fluid flow may occur in pressure systems. There are different transition sections in pressure systems where structural alterations in the flow's hydrodynamic properties take place. One major avenue to enhance machine tool designs is the study of turbulent stationary motion in the transition zones of channels containing fluid under pressure. The correctness of research findings determines the stability and accuracy of control and regulation systems. Thus, it is imperative and very practical to analyze the structural changes in turbulent flows in the inlet area.

Studies of velocity distribution patterns in the transition areas of closed beds (entrance and exit regions, sudden expansion, narrowing, etc.) show that the particles near the stationary walls perform a decelerating motion, and the particles near the axis gain acceleration and perform an accelerating motion. Due to these two phenomena, a rearrangement of the velocity field occurs at the transition sites, resulting in a change in the velocity distribution pattern. The transition zone is the zone following the inlet cut, where the velocity distribution pattern of the incoming fluid is rearranged to the velocity distribution pattern of the stabilized closed bed zone. The accuracy of the results of the inspection of the transition site of the entrance cut is determined by the problems of the precise construction of the fluid channels of the machinery, which can provide guarantees for the clear and stable operation of the control and regulation systems. In this regard, the discussed problem is topical and has an important practical significance.

The most important problem of fluid movement research is the construction of a mathematical model of the given physical phenomenon, the results of which determine the applicability limits of the selected calculation method. It is very important that the built model more accurately describes the ongoing hydromechanical phenomena and, at the same time, provides the possibility of obtaining analytical solutions.

The investigation of turbulent stationary motion in the transition areas of pressurized fluid channels is one of the main ways to improve the structures of machine tools. The precise and stable operation of the management and regulation systems depends on the accuracy of the research results.

The study of turbulent stationary hydromechanical phenomena in the transition areas of pressure pipes is one of the most complex problems of hydromechanics, where the change of quantities, in addition to time, also depends on the coordinates of the point. Therefore, studies to identify patterns of changes in the hydrodynamic parameters in case of a turbulent plane-parallel flow in the hydrodynamic entrance region are relevant.

## 2. Literature review and problem statement

Studies of hydrodynamic phenomena in the transition area of the inlet shear were mainly carried out under conditions of laminar motion. Many theoretical and approximate calculation methods have been developed. At the base of each calculation method are conclusions about the nature of the movement, with which theoretical research and a summary of the results are carried out.

In work [1], the problem of laminar flow of a viscous incompressible fluid at the inlet section of a plane-parallel pressure motion is considered. The problem is solved with the help of integration of approximating Navier-Stokes equations at constantly distributed inlet velocities. According to the results of the study, the regularities of velocity and pressure changes along the transitional section were revealed. However, the solutions obtained are not acceptable for turbulent flows.

The study of the patterns of change in the hydrodynamic parameters under the conditions of non-stationary flow at the entry of the cylindrical pipe and the initial arbitrary distribution of velocities in the entry section was conducted based on the boundary layer equations [2]. The change of hydrodynamic parameters of the flow also occurs on the sections of the sudden expansion of the live section. Conducted research [3] regularity of changes in hydrodynamic parameters in the area of sudden expansion of the live section of the plane-parallel pressure movement based on the equations of the boundary layer. A method [4] of identifying the regularities of changes in hydrodynamic parameters of the flow at the transition section is developed, which allows to obtain a velocity profile in any cross section based on the results of the deformation of the velocity fields under common initial and boundary conditions. These works investigated the patterns of changes in the hydrodynamic parameters of a viscous fluid during laminar motion, which narrows the scope of their application.

In work [5], the problem of pulsating flow of a viscous fluid at the inlet of a round pipe was solved on the basis of approximate equations. A comparison of the results of theoretical and experimental studies of the pulsation motion of a viscous fluid is given in [6].

The problem of the entrance section of a round pipe with a suddenly applied velocity at the entrance of the pipe was solved in [7] with the help of the hypothesis of self-modeling of the velocity profiles in the boundary layer and the impulse equation. A similar problem for the suddenly applied velocity at the inlet at small Reynolds numbers was solved by numerical integration of the Navier-Stokes equation in work [8]. In conditions of periodic disturbance, a thin boundary layer at the inlet of a round pipe was investigated with the help of linear approximations. However, the boundary layer is considered on a flat plate, which reduces the accuracy of the results.

In work [9], the problem of laminar unsteady flow of a viscous fluid in axisymmetric pipes based on changes in viscosity and pressure gradient is considered. The proposed method and obtained solutions reveal regularities of hydrodynamic flow parameters taking into account viscosity variability and can be applied, in particular, to Newtonian fluid. The proposed methodology and obtained solutions allow to reveal regularities of hydrodynamic flow parameters taking into account viscosity variability, in particular, it can also be applied to Newtonian fluid [10].

In work [11], an analytical solution to the equation of motion for unsteady motion of fluid in round pipes is presented, in which an arbitrary change in kinematic viscosity in time is allowed. Velocity and flow are expressed in the form of a series of Bessel and Kelvin functions of a radial variable, while the dependence on time is expressed in the form of a Fourier series. Analytical solution for velocity is compared with direct numerical solution of equation of motion.

Gradual increase in flow rate causes non-periodic non-stationary flow. Analysis of flow stability depending on the ratio of the current flow to the established laminar flow is carried out in work [12]. As a result, the conditions for the stability of the unsteady flow in the pipe are obtained. The mentioned researches are mainly related to the interpretation of the phenomena taking place at the entrance of the pipe. However, hydrodynamic parameter rearrangement phenomena also occur in other transition sites of the pipe, on which researches are scarce. At the site of sudden expansion of the cut ($D/d$=4). the flow lines were constructed by numerical integration of the flow equations of the plastic fluid, the velocity and pressure changes in the axial direction were determined [13]. Under conditions of sudden, symmetric and asymmetric expansion of the shear, quantitative estimates of members of the Nave-Stokes equations were carried out, and the resulting nonlinear inhomogeneous differential equations were integrated numerically [14]. The results of the analysis were compared with the results of the experiments. Remarkable experimental studies in the region of sudden shear dilatation were performed in [15]. For that purpose, a test rig was built and the sites of sudden shear expansion were tested for the cases $d/D$=0.22;0.5;0.85. The investigations were carried out under Newtonian and non-Newtonian fluid conditions.

In stationary motion conditions, the patterns of viscous fluid motion in the transition area of the plane parallel motion inlet shear, with general boundary conditions, were studied [16]. Velocity and pressure change patterns were obtained, graphs of their change were constructed, and the length of the transition area was determined. However, the movement of the fluid in the transition areas is often non-stationary, so the study of the mentioned problem under the conditions of non-stationary motion is important, which has important practical significance.

Taking into account the dependence of turbulent stresses on a stationary wall, a study was carried out to identify the structural state of an incompressible flow [17]. In [18], an experimental study was carried out to identify the velocity component normal to the wall. It was revealed that not all values of turbulent stresses lead to an asymptotic state of velocities towards a stationary wall. In [19], an analysis of the study of the transition period and turbulence over the past thirty years is presented. Despite best efforts, some inevitable omissions will continue to be explored as turbulence research deepens. The influence of the pressure gradient on the universal logarithmic law that determines the average velocity profile was studied in [20].

Based on the results of a literature review on the study of the input region of plane-parallel pressure motion, it was revealed that the change in the viscosity coefficient is not taken into account. This formulation of the problem makes it necessary to study the input region, where the movement is laminar. However, the study of the input region of plane-parallel turbulent pressure motion requires taking into account the change in the viscosity coefficient.

Based on the results of the study, it is necessary to reveal the behavior of hydrodynamic parameters' change in the transition region in case of turbulent pressure flow and develop a method of hydraulic calculation of the entrance area of plane-parallel channels.

## 3. The aim and objectives of the study

The aim of the study is to reveal regularities of changes in the hydrodynamic parameters of a viscous fluid at the entrance region of the plane-parallel pressure motion during stationary turbulent motions of incompressible fluid. This will make it possible to correctly design plane-parallel hydraulic channels of various mechanisms and machines.

To achieve this aim, the following objectives are accomplished:

– to formulate the boundary value problem and determine the initial and boundary conditions and develop a method for solving the boundary value problem and reveal the regularity of changes in the hydrodynamic parameters of the turbulent stationary flow of a viscous fluid at the inlet section of the plane-parallel pressure motion;

– to plot graphs of axial velocity change along the length of the entrance region and identify the conditions for determining the length of the inlet section of the plane-parallel thrust movement in stationary turbulent flow.

## 4. Materials and methods

### 4. 1. Object and hypothesis of the study

The object of study is the hydrodynamic entrance region of plane-parallel pressure flow.

The main hypothesis of the study is that the coefficient of turbulent viscosity is taking into account according to the Boussinesq hypothesis. Thus, the turbulent viscosity coefficient change was taken into consideration in studying structural changes in turbulent plane-parallel flow.

The turbulent motion of an incompressible fluid at the entrance region of a flat channel has been studied. The studies were carried out on the basis of simplified Navier-Stokes equations taking into account the coefficient of turbulent viscosity. A boundary value problem has been formulated and an integration technique has been developed. To obtain numerical results, computer calculations were carried out.

### 4. 2. Choosing a calculation scheme

The tangential stresses arising in the turbulent flow are determined by the kinematic coefficient of turbulent viscosity. According to Boussinesq [22], the kinematic coefficient of viscosity is directly proportional to the distance from the stationary wall (Fig. 1). In this case, the turbulent tangential stresses are determined by the following formula:

$$\tau = -\rho\varepsilon \frac{du}{dy}, \tag{1}$$

where $\varepsilon$ is the kinematic coefficient of turbulent viscosity, which is directly proportional to the distance from the stationary wall [23]:

$$\varepsilon = ny, \tag{2}$$

where $n$ is a ratio coefficient that has a velocity dimensionality.

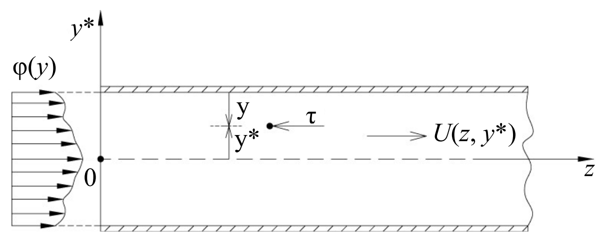


Fig. 1. Entrance length study scheme

In the transition region, the velocity of the particle changes in two directions: in the direction of the $oz$ axis of motion and in the direction of the $oy$ axis perpendicular to the axis of motion. Therefore, the average velocity of the turbulent current in the transition area depends on the $z$ and $y$ coordinates of the point: To solve the problem, the velocity distribution pattern in the inlet section is given in the form of an arbitrary function: $U=\varphi(y)$.

The mathematical model of the problem leads to the integration of the boundary layer equations [23].

But for the boundary layer integration of equations obtained by due to the presence of a non-linear term is associated with certain mathematical complications, so linear approximation was made [1] after which the differential equation of motion takes the following form:

$$U_o \cdot \frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{1}{\rho}\frac{\partial \tau}{\partial y}, \tag{3}$$

where $y$ coordinate is calculated from the fixed wall of the pipe.

Taking into account relations (1), (2), the last equation will take the following form:

$$U_o \cdot \frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} - n\left(y\frac{\partial^2 u}{y^2} + \frac{\partial u}{\partial y}\right), \tag{4}$$

$$0 \le y \le h.$$

The boundary conditions of eq. (4) integration will be:

$$u(z,0) = 0, \quad u(0,y)| = \phi(y). \tag{5}$$

The equation of indivisibility in this case will be:

$$\frac{\partial u}{\partial z} + \frac{\partial u}{\partial y} = 0, \tag{6}$$

where $U_0$ is the characteristic velocity of the section, which is equal to the average velocity of the effective cross-section:

$$U_o = \frac{1}{h}\int_0^{+h} \phi(y)\,\mathrm{d}y. \qquad (7)$$

It is assumed that the constant change in pressure gradient depends only on the $z$ coordinate of the path. It means that the pressures at all points of the live section have the same values, therefore:

$$-\frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial z} = f(z). \qquad (8)$$

Considering eq. (8), eq. (4) will take the following form:

$$U_o\frac{\partial u}{\partial z} = f(z) - n\left(y\frac{\partial^2 u}{\partial \mathrm{y}^2} + \frac{\partial u}{\partial y}\right). \qquad (9)$$

The boundary conditions of eq. (9) integration will be:

$$u(z,0)=0, \quad u(0,y)|=\phi(y), \qquad (10)$$

$$u \to u', \text{ when } z \to \infty,$$

where $u'$ is the averaged velocity of the turbulent motion at the point in the stabilized region.

In the stabilized region $\frac{\partial u}{\partial z}=0,$ and $u'$ velocity will be determined from the following equation:

$$\frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial z} = -n\left(\frac{\partial u'}{\partial y} + y\frac{\partial^2 u'}{\partial y^2}\right), \qquad (11)$$

and the tangential stresses will depend on the change in pressure as follows:

$$\frac{\partial \mathrm{P}}{\partial z} = \frac{\partial \tau}{\partial y^*}. \qquad (12)$$

The gradient of pressure change in the direction of movement in a stabilized turbulent flow remains constant, therefore from equation (12) it is possible to get that the tangential stresses in the live section are distributed according to the linear law, therefore:

$$\tau = \tau_0\frac{y^*}{h} = \tau_0\frac{h-y}{h}, \qquad (13)$$

where $\tau_0$ is the tangential stress on the fixed wall.

From (1), (2) and (11) let's get the velocity distribution pattern in the effective cross-section in the established turbulent flow [22]:

$$u' = \frac{U_*^2}{n\cdot h}(h\cdot \ln y - y) + C, \qquad (14)$$

where $y$ is the distance of the point from the fixed wall (Fig. 1), $U_*$ is the dynamic velocity near the stationary wall:

$$U_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{\partial \mathrm{P}}{\partial z}\cdot\frac{h}{\rho}}.$$

The integration constant $C$ needs to be determined from the velocity condition of the fluid moving on a stationary wall, where the particles stick to the wall due to the viscosity of the fluid and are stationary. However, from the logarithmic law of velocity distribution (14) it follows that the velocity near the stationary wall must be infinitely large, which does not correspond to reality. Therefore, the value of the integration constant $C$ is determined not from the condition of a stationary wall, where the velocity due to particle adhesion is equal to zero, but from the condition of being at a certain distance from the wall, where the laminar boundary layer existing near the wall transitions to a turbulent boundary layer. Therefore, the value of the integration constant $C$ is determined at some distance from the wall under the condition that the laminar and turbulent tangential stresses are equal [22].

Eq. (14) can be transformed into the following form:

$$\frac{u'}{U_*} = A\left[\ln\frac{y}{h} - \frac{y}{h}\right] + C. \qquad (15)$$

This logarithmic law of velocity distribution contains two constant coefficients, the values of which are determined experimentally [23]. The value of coefficient $A$ does not depend on the properties of the stationary wall, its type of roughness, but depends on the degree of turbulence.

Let's import dimensionless variables:

$$\frac{u}{U_0} = U(z,y), \quad \frac{P}{P_0} = \bar{P}, \quad \frac{y}{h} = x, \quad \frac{z}{h} = \sigma. \qquad (16)$$

(9) will take the following form:

$$\frac{\partial U}{\partial \sigma} = -f(\sigma) - \frac{n}{U_0}\left(x\frac{\partial^2 U}{\partial x^2} + \frac{\partial U}{\partial x}\right). \qquad (17)$$

The boundary conditions for the integration of the equation (17) will be:

$$U(\sigma,0)=0, \quad \sigma>0,$$

$$U(0,x)=\phi(x), \quad 0\le x\le 1. \qquad (18)$$

Let's look for the solution of the eq. (17) in the form of two additions, the first of which takes into account the influence of the pressure change and the initial distribution of velocities in the inlet section, and the second is the change of the velocity in the stabilized area from the pressure gradient:

$$U(\sigma,x) = U_1(\sigma,x) + u'(x), \qquad (19)$$

where $U_1(\sigma, x)$ is the partial solution of the inhomogeneous differential equation in the case of boundary conditions, and $u'(x)$ is determined by eq. (15):

$$\frac{\partial U_1}{\partial \sigma} = -f(\sigma) - \frac{n}{U_0}\left(x\frac{\partial^2 U_1}{\partial x^2} + \frac{\partial U_1}{\partial x}\right). \qquad (20)$$

Let's look for the general solution of eq. (20) in the form of a sum:

$$U_1(\sigma,x) = \sum_{k=1}^{\infty} C_k(\sigma)\cdot J_1\left(\lambda_k\sqrt{x}\right), \qquad (21)$$

where $C_k(\sigma)$ are the unknown coefficients depending on the variable, $J_1\left(\lambda_k\sqrt{x}\right)$ are the Bessel functions of the first order of the first genus, $\lambda_k$ ($k$=1, 2...) are the systematic constant coefficients to be determined.

Eq. (21) is the general solution of the inhomogeneous eq. (20). The eigenvalues of the equation are determined from the condition that at the center of plane parallel motion, which is the axis of symmetry of the range of motion, the velocity has a maximum value, therefore $\left.\frac{\partial U(\sigma,x)}{\partial x}\right|_{x=1}=0,$ as a result of which, from eq. (21), let's get the characteristic equation of the problem $J_1'(\lambda_k)=0,$ the positive roots $\lambda_k$ of which will be the eigenvalues.

Eq. (21) satisfies all boundary conditions of eq. (20). To determine the unknown $C_k(\sigma)$ coefficients, the value of the function $U_1(\sigma, x)$ is substituted from eq. (21) into eq. (20):

$$\sum_{k=1}^{\infty} C_k'(\sigma)\cdot J_1\left(\lambda_k\sqrt{x}\right)= \\ =f(\sigma)-\frac{n}{U_0}\sum_{k=1}^{\infty} C_k(\sigma)\cdot\left[\lambda_k^2 x J_1''\left(\lambda_k\sqrt{x}\right)+J_1'\left(\lambda_k\sqrt{x}\right)\right]. \tag{22}$$

Taking into account that [24]:

$$\lambda_k^2 x J_1''\left(\lambda_k\sqrt{x}\right)+J_1'\left(\lambda_k\sqrt{x}\right)=-\frac{1}{4}\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k\sqrt{x}\right), \tag{23}$$

will get:

$$\sum_{k=1}^{\infty} C_k'(\sigma)\cdot J_1\left(\lambda_k\sqrt{x}\right)= \\ =f(\sigma)-\frac{n}{U_0}\sum_{k=1}^{\infty} C_k(\sigma)\cdot\frac{1}{4}\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k\sqrt{x}\right). \tag{24}$$

Considering eq. (15), (21), the general solution of the problem will be:

$$U(\sigma,x)=\sum_{k=1}^{\infty} C_k(\sigma)\cdot J_1\left(\lambda_k\sqrt{x}\right)+AU_*\left[\ln x-x\right]+B_*. \tag{25}$$

To determine the values of the $C_k(\sigma)$ coefficient, the solution of eq. (25) is substituted into the eq. (17) and there is:

$$\sum_{k=1}^{\infty} C_k'(\sigma)\cdot J_1\left(\lambda_k\sqrt{x}\right)= \\ =f(\sigma)-\frac{n}{U_0}\sum_{k=1}^{\infty} C_k(\sigma)\cdot\frac{1}{4}\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k\sqrt{x}\right)- \\ -\frac{nAU_*}{U_0}. \tag{26}$$

The term $F(\sigma)=f(\sigma)-\frac{nAU_*}{U_0}$ of eq. (26) is transformed into a series according to the eigenfunctions:

$$F(\sigma)=\sum_{k=1}^{\infty} a_k(\sigma)J_1\left(\lambda_k\sqrt{x}\right), \tag{27}$$

as a result of which eq. (26) will take the following form:

$$\sum_{k=1}^{\infty} C_k'(\sigma)\cdot J_1\left(\lambda_k\sqrt{x}\right)=\sum_{k=1}^{\infty} a_k(\sigma)J_1\left(\lambda_k\sqrt{x}\right)- \\ -\frac{n}{4U_0}\sum_{k=1}^{\infty} C_k(\sigma)\cdot\left(1-\frac{1}{\lambda_k^2 x}\right)J_1^2\left(\lambda_k\sqrt{x}\right)\lambda_k^2. \tag{28}$$

Multiplying both sides of eq. (28) by $J_1\left(\lambda_m\sqrt{x}\right)dx$ and integrating in [0; 1} interval and taking into consideration the orthogonality condition [24] of eigenfunctions:

$$\int_0^1 J_1\left(\lambda_k\sqrt{x}\right)\cdot J_1\left(\lambda_m\sqrt{x}\right)\mathrm{d}x= \\ =\begin{cases} 0 \gg \tilde{n}\mu\ k\neq m, \\ \int_0^1 J_1^2\left(\lambda_k\sqrt{x}\right)\mathrm{d}x=\frac{\lambda_k^2-1}{\lambda_k^2}J_1^2(\lambda_k), \gg \tilde{n}\mu\ k=m, \end{cases} \tag{29}$$

there is:

$$a_k(\sigma)=C_k'(\sigma)+\frac{n}{4U_0}\frac{\lambda_k^2 L_2(\lambda_k)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}C_k(\sigma). \tag{30}$$

Denoting $\alpha=\frac{n}{4U_0},$ $\beta_k=\frac{\lambda_k^2 L_2(\lambda_k)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}$ eq. (30) can be transformed as:

$$C_k'(\sigma)+\alpha\beta_k C_k(\sigma)=a_k(\sigma). \tag{31}$$

Since:

$$L_2(\lambda_k)=\int_0^1\left(1-\frac{1}{\lambda_k^2 x}\right)\int_0^1 J_1^2\left(\lambda_k\sqrt{x}\right)\mathrm{d}x= \\ =\frac{1}{\lambda_k^4}\left[\left(\lambda_k^4+1\right)J_1^2(\lambda_k)-\lambda_k^2\right], \tag{32}$$

and get:

$$\beta_k=\frac{\left(\lambda_k^4+1\right)J_1^2(\lambda_k)-\lambda_k^2}{\lambda_k^2\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}. \tag{33}$$

Solution of eq. (31) according to [4] will be:

$$C_k(\sigma)=\exp(-\alpha\beta_k\sigma)\int_0^{\sigma} a_k(\tau)\exp(\alpha\beta_k\tau)\mathrm{d}\tau+ \\ +C_k\exp(-\alpha\beta_k\sigma). \tag{34}$$

$a_k(\tau)$ coefficients are determined from eq. (27):

$$\int_0^1 F(\sigma)J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x=\frac{\lambda_k^2-1}{\lambda_k^2}J_1^2(\lambda_k)a_k(\sigma), \tag{35}$$

from which:

$$a_k(\sigma)=\frac{\lambda_k^2 L_1(\lambda_k)F(\sigma)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}, \tag{36}$$

where:

$$L_1(\lambda_k)=\int_0^1 J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x.$$

Having the values of the $a_k$ coefficients, let's get the values of the $C_k(\sigma)$ coefficients from eq. (34):

$$C_k(\sigma)=\exp(-\alpha\beta_k\sigma)\frac{\lambda_k^2 L_1(\lambda_k)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}\times \\ \times\int_0^{\sigma}\exp(\alpha\beta_k\tau)F(\sigma)\mathrm{d}\tau+C_k\exp(-\alpha\beta_k\sigma). \tag{37}$$

When $f(\sigma)=C_0=\text{const},$ then:

$$F(\sigma)=C_0-\frac{nAU_*}{U_0}=B_0=\text{const},$$

therefore:

$$\int_0^\sigma \exp(\alpha\beta_k\tau)F(\sigma)\mathrm{d}\tau=$$
$$=\int_0^\sigma B_0\exp(\alpha\beta_k\tau)\mathrm{d}\tau=\frac{B_0}{\alpha\beta_k}\left(\exp(\alpha\beta_k\sigma)-1\right).$$

Substituting the value of the last expression into eq. (37):

$$C_k(\sigma)=\frac{B_0\lambda_k^2L_1(\lambda_k)}{\alpha\beta_k\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}\left(1-\exp(-\alpha\beta_k\sigma)\right)+$$
$$+C_k\exp(-\alpha\beta_k\sigma). \tag{38}$$

Substituting these values of $C_k(\sigma)$ coefficients into eq. (25), let's get the solution of the problem:

$$U(\sigma,x)=$$
$$=\sum_{k=1}^{\infty}\left\{\begin{array}{l}\dfrac{B_0\lambda_k^2L_1(\lambda_k)}{\alpha\beta_k\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}\left(1-\exp(-\alpha\beta_k\sigma)\right)+\\ +C_k\exp(-\alpha\beta_k\sigma)\end{array}\right\}\times$$
$$\times J_1\left(\lambda_k\sqrt{x}\right)+AU_*\left[\ln x-x\right]+B. \tag{39}$$

The values of the $C_k$ coefficients are determined from the boundary condition (18) $U_0(0,x)=\varphi(x)$ therefore:

$$\phi(x)=\sum_{k=1}^{\infty}C_kJ_1\left(\lambda_k\sqrt{x}\right)+AU_*\left[\ln x-x\right]+B. \tag{40}$$

Multiplying both sides of the above equation by $J_1\left(\lambda_k\sqrt{x}\right)$ and integrating in [0;1] interval:

$$\int_0^1\phi(x)J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x=\frac{\lambda_k^2-1}{\lambda_k^2}J_k^2(\lambda_k)C_k+$$
$$+\int_0^1\left[AU_*(\ln x-x)+B\right]J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x,$$

from this:

$$C_k=\frac{\lambda_k^2L_3(\lambda_k)}{\left(\lambda_k^2-1\right)J_k^2(\lambda_k)}-\left[\frac{AU_*\lambda_k^2L_0(\lambda_k)+B\lambda_k^2L_1(\lambda_k)}{\left(\lambda_k^2-1\right)J_k^2(\lambda_k)}\right]. \tag{41}$$

Substituting this value of the coefficient in eq. (39):

$$U(\sigma,x)=$$
$$=\sum_{k=1}^{\infty}\left\{\begin{array}{l}\dfrac{B_0L_1(\lambda_k)}{\alpha\beta_k}\left(1-\exp(-\alpha\beta_k\sigma)\right)+\\ \left[\left[L_3(\lambda_k)-\begin{bmatrix}AU_*L_0(\lambda_k)+\\+BL_1(\lambda_k)\end{bmatrix}\exp(-\alpha\beta_k\sigma)\right]\right]\end{array}\right\}\times$$
$$\times\frac{\lambda_k^2J_1\left(\lambda_k\sqrt{x}\right)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}+AU_*\left[\ln x-x\right]+B, \tag{42}$$

where:

$$L_0=\int_0^1(\ln x-x)J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x. \tag{43}$$

This regularity of velocity variation in the entrance transition area makes it possible to obtain the variation of velocities according to the section and according to the length in any section of the transition area.

## 5. Research results of structural changes in the hydrodynamic parameters of a turbulent flow at the entrance region of a plane-parallel pressure motion

### 5. 1. Results of the decision on structural studies of the initial section of plane-parallel pressure motion

Having a general solution to the problem, now let's consider the following specific cases, the first case, when $F(\sigma)=B_0=\text{const}$, $\phi(x)=A_0=\text{const}$, then:

$$U(\sigma,x)=$$
$$=\sum_{k=1}^{\infty}\left\{\begin{array}{l}\dfrac{B_0L_1(\lambda_k)}{\alpha\beta_k}\left(1-\exp(-\alpha\beta_k\sigma)\right)+\\ \left[\left[A_0L_1-\begin{pmatrix}AU_*L_0(\lambda_k)+\\+BL_1(\lambda_k)\end{pmatrix}\right]\exp(-\alpha\beta_k\sigma)\right]\end{array}\right\}\times$$
$$\times\frac{\lambda_k^2J_1\left(\lambda_k\sqrt{x}\right)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}+AU_*\left[\ln x-x\right]+B, \tag{44}$$

the second case, when $F(\sigma)=B_0=\text{const}$, $\phi(x)=A_0x^2$ therefore:

$$C_k=\frac{A_0\lambda_k}{\left(\lambda_k^2-1\right)J_k^2(\lambda_k)}\int_0^1x^2J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x-$$
$$-\left[\frac{AU_*L_0(\lambda_k)+BL_1(\lambda_k)}{\left(\lambda_k^2-1\right)J_k^2(\lambda_k)}\right]. \tag{45}$$

Since:

$$L_4=\int_0^1x^2J_1\left(\lambda_k\sqrt{x}\right)\mathrm{d}x=$$
$$=\frac{1}{7}F\left[\left\{\frac{2}{7}\right\},\left\{2,\frac{9}{2}\right\},-\frac{\lambda_k^2}{4}\right]\lambda_k,$$

there is:

$$U(\sigma,x)=$$
$$=\sum_{k=1}^{\infty}\left\{\begin{array}{l}\dfrac{B_0L_1(\lambda_k)}{\alpha\beta_k}\left(1-\exp(-\alpha\beta_k\sigma)\right)+\\ \left[\left[A_0L_4-\begin{pmatrix}AU_*L_0(\lambda_k)+\\+BL_1(\lambda_k)\end{pmatrix}\right]\exp(-\alpha\beta_k\sigma)\right]\end{array}\right\}\times$$
$$\times\frac{\lambda_k^2J_1\left(\lambda_k\sqrt{x}\right)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}+AU_*\left[\ln x-x\right]+B. \tag{46}$$

Based on the results of the boundary value problem integration, regularities of changes in the hydrodynamic parameters of the incompressible fluid in the entrance region of plane-parallel pressure flow (44), (46) were obtained. Numerical calculations were performed and graphs of changes in axial velocities along the cross-section and along the length of the entrance region were plotted.

**5. 2. Plotting of the graphs of changes in the turbulent flow's hydrodynamic parameters in the entrance region**

**5. 2. 1. Graphs of structural changes in the hydrodynamic parameters of a turbulent flow at the inlet section of plane-parallel pressure motion taking place in the first case**

To visualize the patterns of changes in axial velocity along the cross section and along the length of the transition section, depending on the initial distribution of velocities, graphs of their changes were constructed.

Graphs (Fig. 2, 3) of structural changes at the entrance region of the pressure turbulent flow at $F(\sigma)=B_0$=const, $\varphi(x)=A_0$=const have been plotted following the eq. (44).

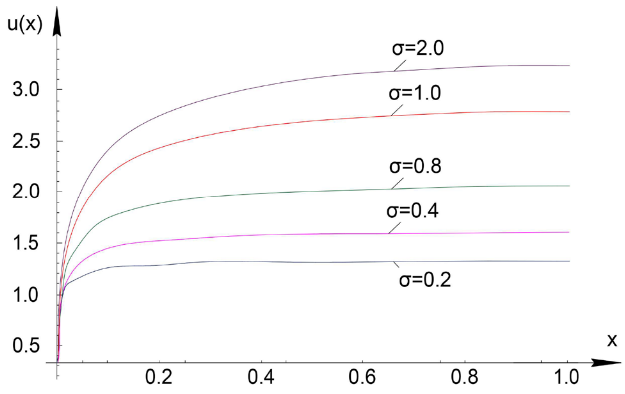


Fig. 2. Variation of axial velocities in the transversal cross-section ($x$) along the length ($\sigma$) and the hydrodynamic entrance region when $AU_*$=0.01; $\varphi(x)=A_0$=1; $B_0$=25; $\alpha$=5

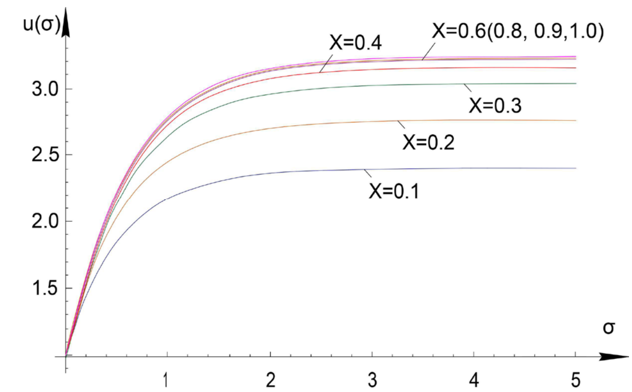


Fig. 3. Variation of axial velocities along ($\sigma$) of the hydrodynamic entrance length when $AU_*$=0.01; $A_0$=1; $B_0$=5; $\alpha$=1

From graphs shown in Fig. 2, 3 the length of the hydrodynamic entrance, where occurs redistribution of velocity, is obtained $\sigma$=2.3 and $z$=2.3$h$.

**5. 2. 2. Graphs of structural changes in the hydrodynamic parameters of a turbulent flow at the inlet section of plane-parallel pressure motion taking place in the second case**

Graphs (Fig. 4, 5) of structural changes at the entrance region of the pressure turbulent flow at $F(\sigma)=B_0$=const, $\varphi(x)=Ax^2$ have been plotted following the eq. (46).

The hydrodynamic entrance length according to graphs (Fig. 4, 5) of parabolic distribution of initial velocities in the hydrodynamic entrance region will be $\sigma$=1.2 and $z$=1.2$h$.

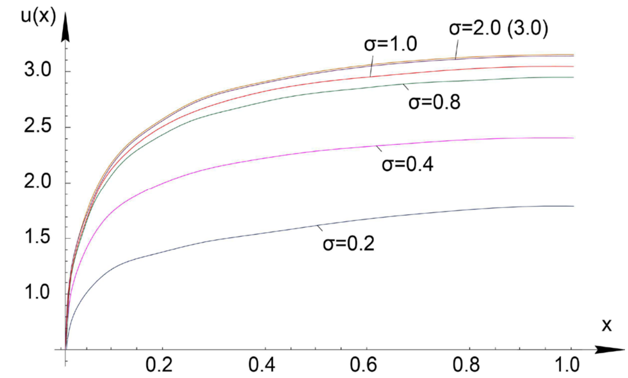


Fig. 4. Variation of axial velocities in the transversal cross-section ($x$) along the length ($\sigma$) and the hydrodynamic entrance region when $AU_*$=0.1; $\varphi(x)=A_0x^2$; $A_0$=1; $B_0$=2.5; $\alpha$=2

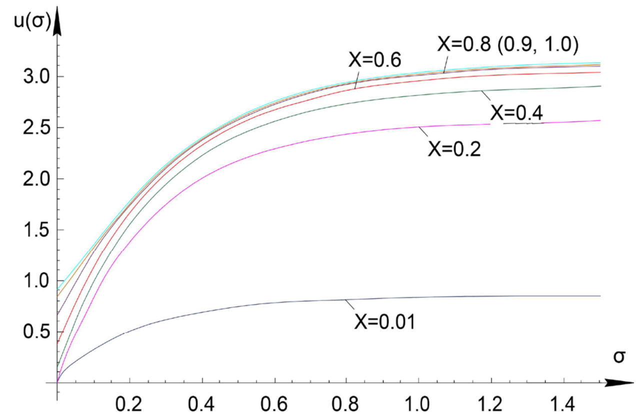


Fig. 5. Variation of axial velocities along ($\sigma$) of the hydrodynamic entrance length when $AU_*$=0.1; $\varphi(x)=A_0x^2$; $A_0$=1; $B_0$=2,5; $\alpha$=2

## 6. Discussion of the research results on structural changes in hydrodynamic parameters in the hydrodynamic entrance region of plane-parallel pressure motion

The peculiarity of the study is that it takes into account changes in the coefficient of turbulent viscosity depending on the distance of the fixed wall. Based on the results of the research obtained, it is possible to correctly design similar channels of various hydromechanical equipment.

The accuracy of the correspondence between the calculated data and the field data depends on the accuracy of the hypothesis of taking into account the coefficient of turbulent viscosity. The proposed solution corresponds to the hypothesis of turbulent viscosity varying linearly with the distance of the stationary wall. In this case, the calculated data can provide an exact coincidence with the field data at Reynolds number up to 20.000, which is a limitation of its applicability. Further improvement of this study is associated with clarifying the calculated dependence of turbulent shear stress on coordinates. However, when refining the calculated dependence, one must proceed from the assumption that the formulated boundary value problem can be integrated and the regularities of structural changes in hydrodynamic parameters can be obtained.

A boundary value problem of turbulent plane-parallel pressure fluid motion, which has great practical significance, has been formulated. As a result of solving the boundary value problem, formulas for the distribution of axial velocities and pressure along the length of the inlet section of plane-parallel pressure motion during turbulent flow of a viscous incompressible fluid were obtained. The studies were carried out with a uniform and parabolic velocity distribution at the inlet sections.

Graphs constructed by computer calculation using formulas (44), (46) demonstrate the development of the process at the inlet section of plane-parallel pressure motion. Analysis of the results of the numerical calculation and the resulting graphs (Fig. 2–5) showed that the degree of development of the process depends on the pressure gradient and the initial velocity distribution in the inlet section. The flow of a viscous fluid in the inlet transition section under turbulent motion is unstable. The shape of the velocity distribution diagrams at each fixed section is stable. It changes along the length of the transition section (Fig. 2–5) due to the deformation of the distribution diagrams and the influence of the pressure gradient. Outside the transition section, the velocity distribution diagrams remain unchanged. The study of the development of viscous fluid flow in the inlet region of plane-parallel pressure motion was carried out using the Boussinesq hypothesis on the distribution of turbulent shear stresses.

Based on the integration results, approximate results were obtained. However, the accuracy of integrating results in engineering calculations is quite sufficient. The results of this study can contribute to improving the design changes in the transition sections of hydraulic systems of various mechanisms and machines, which will lead to an increase in their reliable operation.

Based on the relevance of the problem, further development is associated with clarifying the length of the transition section and design changes in the input area of the cylindrical channel. Analysis of the results of the numerical calculation and the resulting graphs determined the length of the input section. The condition of the input region is the coincidence of the numerical values of the velocities at each fixed point of the section.

The deviation of axial velocities in the transition section at $y$=1 should not exceed 1 % of the unsteady velocity of the stabilized section. Based on this condition, the length of the transition section was obtained, which has important practical applications in the design of various hydraulic automation systems. The length of the hydrodynamic entrance region in the case of a uniform distribution of velocity in the inlet sections is $z=2.3h$. This indicator in case of laminar movement is $z=0.103h\mathrm{Re}$, which indicates a strict reduction in the length of the transition region.

This study was carried out on the basis of the Boussinesq hypothesis about the kinematic coefficient of turbulent viscosity, which is acceptable at the Reynolds number Re≤20000. This limitation narrows the applicability of this study results.

The disadvantage of this study is related to the limited scope of application of the accepted kinematic coefficient of turbulent viscosity.

Improvement of this study is associated with clarifying the dependence of the kinematic coefficient of turbulent viscosity on the point location, which is acceptable under developed turbulent conditions.

## 7. Conclusions

1. As a result of solving the formulated boundary value problem, patterns of structural changes in a turbulent plane-parallel flow were revealed, in particular patterns of changes in axial velocities and pressure along the length of the inlet section, with uniform and parabolic distributions of initial velocity values. The peculiarity of this study is that the connection between the coefficient of turbulent kinematic viscosity and the distance of the stationary channel wall is taken into account. The obtained patterns of structural parameters of the flow along the length of the inlet section make it possible to identify the nature of the movement.

2. Graphs of changes in dimensionless hydrodynamic parameters of the flow were constructed for uniform and parabolic distributions of initial velocities at the entrance to the pipe. The results obtained make it possible to identify the influence of pipe and fluid parameters on changes in the parameters of the initial section. Thanks to the universality of the obtained graphs, it is possible to draw a conclusion about the nature and degree of change in hydrodynamic parameters at the entrance region of turbulent plane-parallel pressure flow. The length of the hydrodynamic entrance region was determined for a uniform ($z=2.3h$) and parabolic ($z=1.2h$) distribution of initial velocities, which is important information when designing entrance region of pressure hydraulic systems depending on the regime of flow. The correct design of a hydraulic system guarantees controlled flow regime.

## Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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## Data availability

Manuscript has no associated data.

## Use of artificial intelligence

The authors have used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

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