

*The object of this study is the process of end milling, taking into account the discontinuity of the process, simultaneous cutting with several flutes arranged in a spiral, tool runout, and feedback in the elastic machining system, in particular, for the depth of cutting. The subject of the study is the cutting force and identification of its empirical model. During identification, the cutting force coefficient is automatically determined when matching the theoretical and experimental oscillograms of the cutting force component. The reported results related to forecasting the cutting force at end milling are based on a mechanistic approach and involve the process modeling method for forecasting. The simulation uses an algorithm for representing the interaction of the cutter flutes workpiece engagement, based on the scan of the cutter according to the rotation angle coordinate. The algorithm makes it possible to identify empirical coefficients and exponents of the cutting force model based on experimental oscillograms of cutting force components. The built model is implemented in an application program and owing to the representation of the machining system in the form of a closed structural diagram, it allows predicting the elastic displacement, which will determine the actual cutting depth. The developed program under an interactive mode using digital files of experimental cutting force components makes it possible to perform model identification and predict cutting force components with an error of 4.6%. The adequacy of the algorithms was confirmed by measuring the profile of the machined surface in the places where the cutting mode changed with the feed stopped. The developed simulation algorithm makes it possible to take into account the simultaneous cutting by several flutes arranged in a spiral, the runout of the tool, and the feedback in the elastic machining system, in particular, the depth of cutting*

**Keywords:** cutting force, end milling, digital simulation, empirical model identification

UDC 621.924.1

DOI: 10.15587/1729-4061.2024.303791

# FORECASTING THE CUTTING FORCE IN END MILLING

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Received date 04.03.2024

**How to Cite:** Petrakov, Y., Ohrimenko, O., Gladskiy, M. (2024). Forecasting the cutting force in end milling. *Eastern-European*

Accepted date 09.05.2024

*Journal of Enterprise Technologies*, 3 (1 (129)), 80–87. <https://doi.org/10.15587/1729-4061.2024.303791>

Published date 28.06.2024

## 1. Introduction

In mechanical engineering, in particular in the aerospace industry, openwork parts with curvilinear guides are widely used, for the manufacture of which there is no alternative to machining by end milling methods on computer numeric control (CNC) machines [1]. To ensure maximum productivity with strict ensuring high requirements for accuracy and surface quality of parts, especially when machining parts in low-rigidity systems, the most important task is to predict the cutting force [2].

The cutting force at milling depends on many factors, which can be conventionally divided into geometric and physical. Geometric factors are determined by the scheme of cutting allowance, where each tooth of the mill cuts a layer of material in accordance with the front angle of the cutting wedge, cutting depth, cutting width and cutting mode. The cutting mode, in combination with the speed of rotation and feed, predetermines the feed per tooth, which correlates with the thickness of the undeformed chip. Given that, depending on the machining width and the number of cutter teeth, several cutting cutter teeth may be present at the same time during the cutting process, which determines the degree of unevenness of the process in terms of cutting force. The unevenness of the process is also influenced by such a geometric parameter as the runout of mill due to the uneven height of the teeth, as well as some deviation from the alignment of the mill and the spindle of the machine. In addition, geometric factors that will affect the process include the direction of feed and cut-

ting speed – up or down milling. First of all, physical factors should include the parameters of the strength of the material being machined, its tendency to plasticity and destruction. Therefore, it can be argued that the accuracy of forecasting the cutting force at milling will depend entirely on the number of factors taken into account, as well as on the adequacy of taking into account their interaction.

A special role belongs to all these factors, which in their combination predetermine the thickness of the undeformed chip. This parameter of the cutting process, as a rule, is the basis of the model, which is based on the mechanistic approach, which is recognized as the most informative.

When determining the cutting force, it is necessary to take into account that it acts in an elastic machining system, and therefore the specified machining parameters are deviate due to the action of elastic deformations. Such a process reflects the closedness of the machining system, in which there is natural feedback on all coordinates. It is especially important to take into account the cutting force in machining systems of low rigidity, where, in addition to significant distortions of the surface shape, there is a significant danger of the occurrence of vibrations.

Thus, forecasting the cutting force is a rather difficult task, the solution of which imposes certain limitations on the results of the milling process, both productivity and quality of machining. The use of CAM systems when preparing a control program for a CNC machine does not solve the problem and does not allow designing a process that is optimal in

terms of productivity. Also, the tool manufacturer's recommendations are of a general nature, as they fundamentally cannot take into account the real parameters of the elastic machining system where the process will be performed.

Therefore, devising effective methods for forecasting the cutting force based on the most complete mathematical models and the development of digital tools simulation is an urgent task related to the general problem of end milling optimization.

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## 2. Literature review and problem statement

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The cutting force is the most important energy indicator of the end milling operation, therefore optimization of the process according to the criterion of the minimum processing time with compliance all requirements for accuracy and quality is not possible without determining this characteristic. A review of the scientific literature, for example [1, 2], shows that a significant amount of research was aimed at modeling the cutting force during peripheral milling by determining the cutting force coefficients. One way or another, such models show the connection of the cutting force with the thickness of the undeformed chip precisely through the coefficients of the cutting force.

Many models based on a mechanistic approach have been developed [3]. The vast majority of models are focused on the use of numerical methods. Since the milling process is characterized by a continuous change in the thickness of the cutting chip, which makes it quasi-stationary, the determination of the cutting force coefficients is somewhat complicated. A new model for estimating the cutting force during peripheral milling is proposed in [4]. It is claimed that the novelty of the model relates to the use of the average chip thickness over the entire working length of the cutting flute. Therefore, it is expected to be more efficient in the computational sense compared to traditional models where the milling cutter is divided into thin disks in the axial direction. Comparison with real processes still does not give an adequate answer to predicting the cutting force during simultaneous cutting with several cutter flutes.

An accurate model for predicting the local milling force is key to reliable modeling of fluctuations in the cutting force and its effect on the shape of the machined surface [5]. However, accounting for such changes in milling force remains a serious problem due to changes in instantaneous chip thickness and inconsistent cutting ratios when milling conditions change. Thus, the general milling force model with coefficients calculated on the basis of average forces based on experimental results may compromise reliability when applied to different milling conditions.

The coefficients of the cutting force are related to the density cutting force, according to which it is recommended to conduct simulations in different configurations of milling processes [6]. Various procedures used to predict the specific cutting force based on the results of experimental studies were analyzed. A research method is proposed that makes it possible to estimate the density cutting force based on information from the machine tool control system. The density cutting force is calculated as the ratio of the material removal rate and the power measured on the machine spindle. This approach can only be effective when milling surfaces, the contour of which is outlined by guides of variable curvature, since for such an operation there is no direct relationship between feed, cutting speed, and instantaneous thickness of the

cutting chip. Therefore, it can be expected that the proposed method based on the analysis of the material removal rate gives adequate results for such operations.

However, building a milling force prediction model is quite a difficult task since the force is the result of multi-parameter relationships of the technological machining system. In general, on the basis of the analysis of many works in the field of aviation engineering, research into the force model is distinguished based on three methods: empirical dependences, finite element methods, and a model based on the instantaneous undeformed chip thickness [7].

The empirical model is based on the study of the cutting force as a function of the cutting speed, the feed per tooth, as well as the axial and radial cutting depth according to the traditional methods of cutting theory. Then such a model acquires a final empirical formula, for example, according to Taguchi's orthogonal experiment. Such a model requires the definition and selection of a large amount of experimental data. The disadvantage of the proposed empirical model, as well as of such a method in general, is that it ignores the complex trajectory of the cutter flute movement, especially when machining curved surfaces. In addition, the milling force predicted by the empirical model cannot take into account the errors caused by the tool runout and the closedness of the elastic machining system.

The finite element model must be integrated with the software and requires refinement by the difference between the program data and the real processes with material removal as a result of the cutting process. Simulation must involve, for example, such common programs as Abaqus/Explicit, Deform, AdvantEdge, Ansys/LS-DYNA, etc., which inhibits the use in mechanical engineering.

It is claimed that the model based on the instantaneous undeformed chip thickness during cutting is the most widespread. Although when determining such an important parameter, analytical dependences are proposed that do not reflect the natural feedback of the machining system. A method of identifying cutting force coefficients based on measurements as a function of constant or average cutting thickness is noteworthy. It is noted that in the process of milling, the bending deformation of the tool would be fixed on the surface of the workpiece, which gives another possible way for determining the coefficient of cutting force [7].

It is known that the machining accuracy during end milling is affected by the elastic displacement of the machining system under the action of the cutting force. In [8], a new method of analyzing the elastic displacement of the workpiece during end milling is proposed. The advantage of the method is the possibility of combining it with geometric modeling of milling based on voxels, which is used to predict the cutting force. To predict the error of the shape of the machined surface, the stiffness of the tool is taken into account first of all, with some corrections for additional elastic displacement of the entire machining system. To calculate the amount of elastic displacement of the tool, the force load is taken as a point force. However, the overall balance does not take into account the feedback of the machining elastic system.

Much attention is paid to the compensation of errors caused by elastic displacement of the machining system. An automatic control system is proposed, which makes it possible to maintain the deviation of the tool at a given reference value, using an automatic control circuit with a PI controller [9]. The complexity of the proposed system will require significant modernization of existing machine tool equip-

ment. Another method of error compensation offers the use of combined control based on a priori and a posteriori information at the same time [10]. To compensate for such errors, it is proposed to use modeling using the data of measurements of the pre-processed contour and the model of the cutting force during end milling, which is based on the characteristic of the analog of the allowance removal speed along the milling path.

A new cutting simulator for end-milling operations is proposed, which takes into account the static deviation of the tool [11]. In the model, the undeformed chip thickness used to estimate the cutting force is calculated based on the static deflection of the system, which consists of the elastic displacements of the tool and tool mandrel caused by the cutting force. An iterative simulation algorithm is built into the simulator, which, as the authors show, slows down the modeling process. Here, it would be possible to use a system approach to the representation of the technological machining system in the form of closed feedback modules, which makes it possible to obtain an adequate solution in one iteration [12].

Analysis of the cutting force plays a key role in the study of the dynamics of the cutting process. The production of parts of stamps and molds, miniature tools becomes possible with the use of micro milling. One of the main problems in this process is elastic displacement in the machining system [13]. To predict the cutting force, the milling cutter is divided into separate disks and the elementary forces acting on each disk are calculated. The tool in the model is represented as a two-dimensional Timoshenko beam element and is calculated by constructing a stiffness matrix. However, when determining the effect of force on elastic shifts, the closedness of the cutting process due to the elastic machining system is not taken into account, which significantly reduces the adequacy of the model.

The use of iterative modeling methods significantly inhibits the effectiveness of simulation, and therefore there are attempts to eliminate such a shortcoming due to the parallel use of a graphics processor in the traditional method [14]. However, the feedback of the real machining system is again replaced by an iterative calculation until the process converges on the magnitude of the elastic displacement.

Prediction of cutting force and tool deviations for general-purpose end mills, taking into account tool runout, accurate trochoidal trajectory, and deviation feedback is reported in [15]. The radial runout of the tool is simulated by the different height of the different cutter teeth. A new methodology is proposed, which is based on the use of an iterative simulation algorithm that uses initial data on the process and cutting mode. Here, as well as in previous studies, it is desirable to turn to the system approach of representing the machining system in the form of a closed structural diagram. So, with the certainty of the proposed parameters, together, in this case, it will also give the possibility of direct calculation without using an iterative algorithm.

Our review of the literature [1–15] demonstrates that despite the large number of studies on the topic, the issue of devising a comprehensive approach to predicting the cutting force and elastic displacement during end milling remains unresolved. Namely, the issue of representing a mechanistic approach taking into account the closedness of the machining system by the depth of cutting and specifying the coefficients of the cutting force according to the experimental results of the study has not been resolved. Such problems should be solved in the application program that automates the determination of the power characteristics of the end milling process.

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### 3. The aim and objectives of the study

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The purpose of our work is to devise a new procedure to forecast the cutting force and elastic displacement in the machining system at end milling, which will enable an adequate determination of force limitations when solving the problem of process optimization according to the criterion of maximum productivity while ensuring the needed quality.

To achieve the goal, it is necessary to solve the following tasks:

- to build a mathematical model of the end milling process, taking into account the spiral arrangement of the flutes, the runout of the tool, and the type of machining – up or down milling;
- in the mathematical model, when determining the parameters of the allowance layer, which is used to predict the cutting force, take into account the elastic displacement of the closed machining system;
- to experimentally confirm the devised methodology and software for predicting the cutting force in end milling.

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### 4. The study materials and methods

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The object of research is the end milling process.

During the research, a hypothesis was implemented about the cutting force, taking into account the discontinuity of the process, simultaneous cutting with flutes arranged in a spiral. Tool beating and feedback in the elastic machining system are also taken into account, in particular by the depth of cut.

A mechanistic approach was used to build the model, and when taking into account the feedback from the elastic displacement and the correction of the force coefficient, the results of experimental studies and the representation of the machining system in the form of a closed structural diagram were applied. This approach is implemented in an application program that functions according to the developed algorithm, which involves the use of oscillograms of the cutting force in digital format.

The adequacy of the proposed solutions and forecasting results have been confirmed by the results of field experiments on the shape of the machined surface with measurement of the consequences of elastic displacement on the machined surface. In the course of the research, a three-component dynamometer MCS 10-005-3C was used, which was connected to ClipX BM40 amplifiers – all made by HBM (Germany) and corresponding programs for machining signals and their representation in digital form.

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### 5. Results of investigating the cutting force in the milling process

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#### 5.1. Mathematical model of the end milling process

The mathematical model of the cutting force at end milling was built on the basis of a mechanistic approach, which is based on the determination of the geometric parameters of the undeformed chip, which is cut by each tooth along its cutting length. As a basis for modeling the allowance cutting process, it is advisable to use geometric ratios based on the scan of the cutter blades along the axial coordinate [16], which will determine the cutting thickness on each elementary section of the spiral tooth of the cutter along the cutting length. The cutting force is determined by a dependence

that has proven effective in dynamic calculations of the end milling process [12]:

$$F = \sum_j^z \sum_{i=1}^n C_p (a_i)^{ka} V^{kv} \delta b_i, \quad (1)$$

where  $z$  is the number of cutter teeth,  $C_p$ ,  $ka$  are empirical power coefficients and degree index,  $a$  is the thickness of the cut by the elementary section of the cutter tooth blade width  $\delta b$ ,  $n$  is the number of sections by the cutting width ( $n=B/\delta b$ , where  $B$  is the total width cutting).

As the practice of measuring the components of the cutting force shows, it is advisable to adjust the cutting force depending on the cutting speed. Such a dependence at milling is also noted in papers in which the cutting force is related to the ratio of the reference speed to the chip speed [17]. Similar dependences, but through correction factors, are also given in metal cutting handbooks [18]. Therefore, in formula (1), a multiplier representing the power dependence on the cutting speed  $V$  with the power index  $kv < 0$  is entered. The cutting speed (m/min) is determined from the well-known formula:  $V = \pi D n / 1,000$ , where  $D$  is the cutter diameter (mm),  $n$  is the cutter rotation frequency (rpm).

Calculations are performed according to an algorithm that involves digital modeling for each elementary step as a function of the angle of rotation of the milling cutter. At the same time, the movement per modeling step is performed according to the length of the cutting flute, and the cutting thickness is determined from the formula:

$$a_i = f_t \sin(\varphi_i), \quad (2)$$

where  $f_t$  is the feed per tooth of the cutter ( $f_t = f / (zS)$ , where  $f$  is the feed,  $z$  is the number of teeth of the mill,  $S$  is the spindle speed),  $\varphi_i$  is the polar cutting angle in the elementary section, along which the computational movement is performed.

The algorithm also implies determining the components of the cutting force at each step along the movement along the of the cutting flute:

$$\begin{cases} (F_x)_j = \sum_{i=1}^n C_p (a_i)^{ka} V^{kv} \delta b_i \cos(\varphi_i + \gamma), \\ (F_y)_j = \sum_{i=1}^n C_p (a_i)^{ka} V^{kv} \delta b_i \sin(\varphi_i + \gamma), \end{cases} \quad (3)$$

where  $\gamma$  is the front angle on the cutter flute.

The final values of the cutting force and its components are determined as the sum of the calculated values for each cutting flute when the angle of rotation of the milling cutter is changed per simulation step. The digital simulation algorithm is implemented in the application program, the main interface of which is shown in Fig. 1.

The interface displays the state of the calculation process at the end of the simulation of the accompanying end milling of a part made from Steel 40 by milling cutter T5K10,  $\varnothing 12$  mm, 4 teeth, spiral flute inclination  $45^\circ$ , front cutting angle  $10^\circ$ .

The end milling process is simulated with a milling cutter runout of 0.01 mm, the cutting mode is recorded in the corresponding interface windows. As a result of the operation of the algorithm, plots of the components of the cutting force appear on the oscilloscope screen, as well as the average force (327 N), calculated by the component  $F_y$  – line 1. The plots of the components represent the behavior of the ma-

chining system for two rotations of the milling cutter –  $720^\circ$  along the abscissa axis. On the screen of the oscilloscope, one can observe the change in the peak values of the plots as a result of the simulation of the milling cutter.

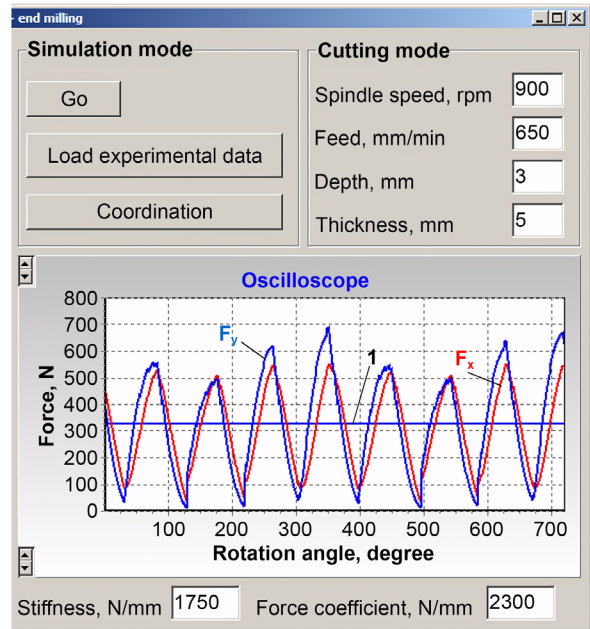


Fig. 1. Main interface of the application program: 1 – the average value of the component  $F_y$

### 5. 2. Elastic machining system

Since actual machining takes place in a loop machining system with feedback in the direction of the Y axis, that is, in the depth of cutting, the program provides for taking into account such a process according to the simplified scheme shown in Fig. 2. The simplification consists in ignoring cutting along the trail, which is implemented according to the scheme reported in [12] and reflects the dynamics of the process with the occurrence of regenerative oscillations. In this case, the first contour of such a scheme is sufficient, which reflects the negative feedback on the depth of cutting. That is, an increase in the component  $F_y$  of the cutting force will provoke a decrease in the actual cutting depth  $H_a$  as a result of elastic displacement, which, in turn, will cause a decrease in the cutting force and so on.

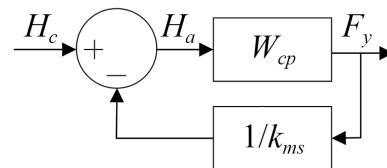


Fig. 2. Structure of the cutting process in a closed elastic system

According to the rules of transformation of structural schemes, based on Fig. 2, one can determine the actual cutting depth  $H_a$ :

$$H_a = \frac{H_c}{1 + W_{cp} / k_{ms}}, \quad (4)$$

where  $H_c$  is the commanded cutting depth,  $W_{cp}$  is the transfer function of the cutting process according to the model imple-

mented in the application program (Fig. 1),  $k_{ms}$  is the stiffness of the machining system in the direction of the Y coordinate.

**5. 3. Experimental verification of research results**

Experimental studies were carried out on a milling machine TM 0p with a CNC made by the HAAS company (Fig. 3). A square plate with a thickness of 5 mm, fixed on the MCS 10-005-3C dynamometer table, was used as a blank.

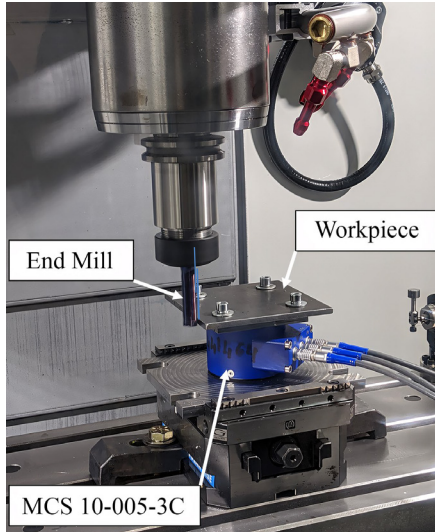


Fig. 3. Experimental bench

Milling was performed according to a special control program while moving along the perimeter of the workpiece with a cutting depth of 3 mm on all sides. The machining scheme implemented down milling, on each side there was a change in the cutting mode with feed stops for 1 s. In this way, 12 sections of the workpiece marked with the corresponding numbers were machined in one experiment (Fig. 4).

The purpose of our experimental research is to test the built mathematical model of the cutting force and verify the methodology for specifying the empirical coefficients of the cutting force based on the results of measurements of the components of the cutting force under different modes.

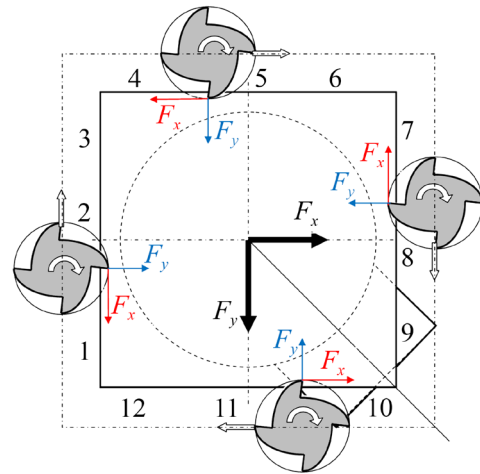


Fig. 4. Schematics of milling

During milling, the oscillogram of the cutting force components was stored in the computer in the form of digital files. Fig. 5 shows a visualization of the process, in which one can observe the entire cycle of the experiment and the location of the oscillograms in accordance with the scheme for determining the sign of the action of the force components on the dynamometer. The positive direction of force along the coordinate axes is indicated by arrows inside the dynamometer table. It can be seen that three experiments under different cutting modes were performed on each side of the workpiece, and their oscillograms are marked with the same numbers as on the workpiece in Fig. 4.

The resolution of the received files, as evidenced by the enlarged fragments, makes it possible to use them to verify the dependence of the cutting force in the developed software in order to identify the initially accepted empirical coefficients. Fig. 5 shows enlarged oscillograms of cutting force components during milling with a feed of 300 mm/min and the following spindle speeds: section 1 – 420 rpm; section 2 – 560 rpm; and section 3 – 650 rpm.

The empirical model (3), which is the basis of the developed simulation algorithm, takes into account the influence of only such parameters of the cutting mode as feed per tooth, depth, width, and speed of cutting. Therefore, its identification cannot be based on the shape of the cutting force change signal.

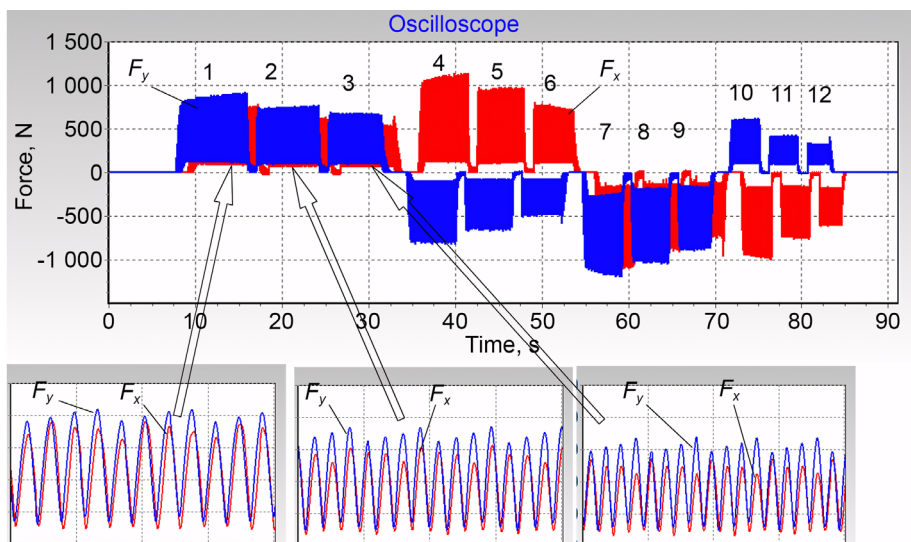


Fig. 5. Oscillograms of cutting force components

Thus, to determine (or correct) the empirical coefficients of the model, it is necessary to focus on the average value of the cutting force for a certain milling cycle. Such an algorithm was included in the developed application program in which the primary coefficient of empirical dependence is corrected (Fig. 6).

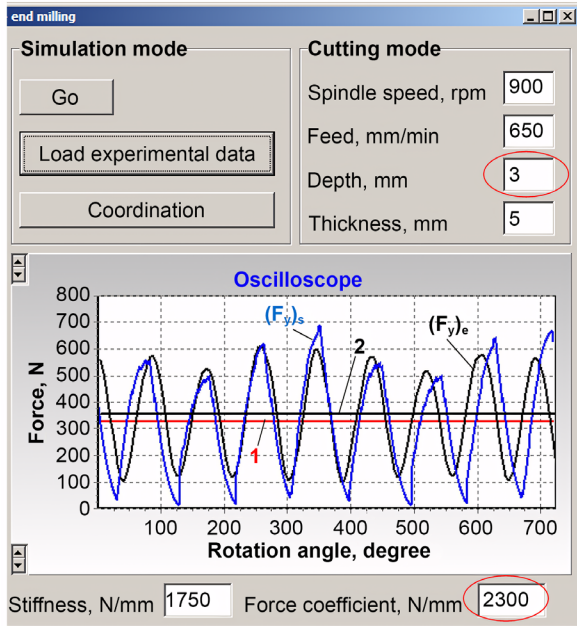


Fig. 6. State of the interface when loading the experimental file: 1 – average value of the component  $(F_y)_s$ ; 2 – average value of the component  $(F_y)_e$

In Fig. 6, in the oscilloscope window, the line  $(F_y)_s$  represents the simulated (as before in Fig. 1) component of the cutting force, line 1 is its average value, line  $(F_y)_e$  is the oscillogram of the experimental file, line 2 is its average value. In the windows surrounded by ellipses – the primary value of the coefficient  $C_p$  of the cutting force, which was determined according to the reference data [17] and the commanded cutting depth  $H_c$ . It can be seen that the adopted empirical coefficient and indicators of theoretical dependence do not match the experimental results.

After activating the coordination algorithm («Coordination» button – Fig. 7), the cutting force coefficient changes and the actual cutting depth is determined. The actual cutting depth is determined taking into account the feedback of the elastic machining system in accordance with the diagram in Fig. 2. The shape of the theoretically modeled plot of the change in the cutting force component automatically matches the experimental one. Comparison of Fig. 7, 5 shows that the matching algorithm performs matching based on the average value of the  $F_y$  component of the cutting force. In addition, the actual cutting depth  $H_a$  was calculated from formula (4) taking into account the elastic loop of the machining system. Alignment was performed according to the  $F_y$  component of the cutting force since it has the greatest influence on the elastic displacement of the machining system in the direction of the cutting depth.

Thus, the developed software product could be used as a tool for evaluating the results from determining the empirical dependence of the cutting force with further application, for example, of Taguchi's orthogonal experiment [7].

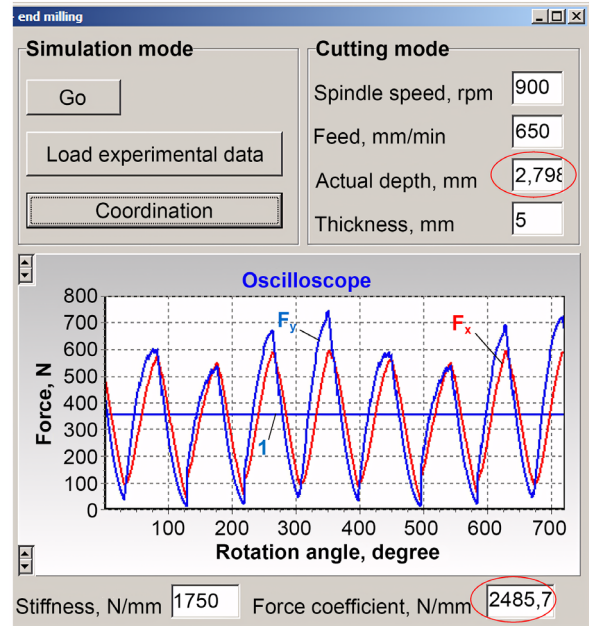


Fig. 7. Program interface when matching: 1 – average value of the  $F_y$  component of the cutting force

As a result of the search under various cutting modes, the following coefficients and degree indicators were obtained:  $C_p=2500, ka=0.9, kv=-0.15$ . A comparison of the experimental and predicted values of the component of the cutting force is given in Table 1. The maximum deviation is 6.4 %, which indicates the sufficient adequacy of the devised forecasting procedure, which is based on modeling the cutting force at end milling.

Table 1

Forecasting results

No.	S, rpm	$f, \text{ mm/min}$	$f_t, \text{ mm}$	V, m/min	$(F_y)_e, \text{ H}$	$(F_y)_s, \text{ H}$
7	450	400	0.222	16.9	474.6	475.2
8	600	400	0.167	22.6	367.8	351.3
9	800	400	0.125	30.2	270.7	259.7
10	900	650	0.181	33.9	353.2	355.2
11	1,300	650	0.125	49.0	249.1	241.5
12	1,650	650	0.098	62.2	201.8	188.9

In the process of milling, the elastic displacement of the machining system according to the stiffness distribution will mainly occur as a bending of the milling cutter under the action of the cutting force and must be fixed on the surface of the workpiece being processed. The milling of the workpiece took place in certain areas when the cutting mode was changed with stops of the cutter feed without turning off the spindle speed. Therefore, in these places, it is possible to record the change in the surface relief, which gives another possible way of determining the cutting force coefficient.

Taking into account the ratio of the geometric dimensions of the machining system, the elastic displacement can be determined from the known dependence [19], as the deflection of the milling cutter, the stiffness of which is:

$$k_{ms} = \frac{3EJ}{L^3}, \tag{5}$$

where  $E$  is the modulus of elasticity,  $J$  is the moment of inertia of the section,  $L$  is the length according to the diagram in Fig. 8.

The moment of inertia can be defined as for a circular cross-section with an attenuation factor due to flutes. After some calculations and coordination, the stiffness of the machining system in the experiment was determined as 1750 N/mm.

Fig. 8 shows a photograph of a section machined with a stop, demonstrating the hole that was formed by cutting without feed when the elastic system straightens.

Thus, the depth of the hole and the difference between the levels of the surfaces will correspond to the elastic deviations due to the action of the components of the cutting force:

$$\delta_1 = \frac{(P_y)_{10}}{k_{ms}} = \frac{353.2}{1,750} = 0.202 \text{ mm},$$

$$\delta_2 = \frac{(P_y)_{11}}{k_{ms}} = \frac{249.1}{1,750} = 0.142 \text{ mm}, \tag{6}$$

where  $(P_y)_{10}$ ,  $(P_y)_{11}$  are the components of the cutting forces at respective sections (Fig. 5 and Table 1).

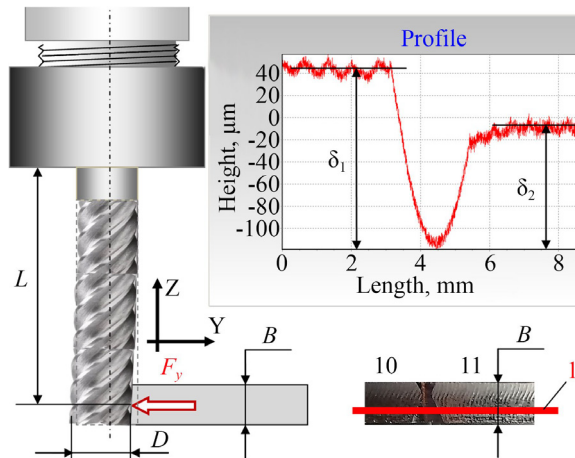


Fig. 8. Deflection of the milling cutter and profile of the groove: 1 – profile measurement line on the digital profilograph

Our calculated values generally correspond to the experimental measurements, which also indicates the adequacy of the proposed model and the developed algorithm. The conical shape of the hole is determined by the elastic deflection of the milling cutter, which forms an inclination of the machined surface to the Z axis.

### 6. Discussion of results from forecasting the cutting force at end milling

Our paper reports the results of forecasting the cutting force at end milling, which are based on a mechanistic approach and use the process modeling method for forecasting. The simulation uses an algorithm to represent the cutter flutes workpiece engagement, which is based on the scan of the cutter flutes according to the rotation angle coordinate. The developed application program advances the simulation algorithm reported in [16] towards its use in the identification of empirical coefficients and indicators of the cutting force model based on experimental oscillograms of the components of the cutting force.

When studying the end milling process, the empirical model should be used as the primary component for determining the cutting force at milling as it inherently ignores the complex trajectory of the cutter teeth during cutting. Therefore, the developed modeling and matching algorithm creates real conditions for its application in practice.

In order to achieve adequacy when trying to take into account the feedback operating in a real elastic machining system, researchers [8, 14] are forced to use iterative calculation methods. Iterations are performed when approaching the desired result by a certain criterion, for example, by stopping the change in elastic shear after the next iteration. Representation of the machining system in the form of a structure closed through loop of negative feedback (Fig. 2) made it possible to avoid the use of the iterative method in its modeling.

On all the oscillograms (Fig. 5), it is possible to observe the impact of the milling cutter, as well as the change in the average level of the components of the cutting force when the cutting mode is changed. The directions of the component vectors are also identified in accordance with the coordinate system of the measuring dynamometer.

In the studies by many authors [7, 8], it is noted that in the process of milling, the bending deformation of the tool will be fixed on the surface of the workpiece, which gives another possible way of determining the cutting force coefficient. This result was used in our study when measuring the profile between two adjacent sections of the machined contour (Fig. 8). Since the transition to another cutting mode occurred with a feed stop, where, due to the continuation of cutting, the elastic system straightened and a cutting flute was formed on the surface, by which the  $F_y$  component of the cutting force can be identified. The match between the results of the profile measurements and the values of the measured components of the cutting force confirms the adequacy of the model.

It has been convincingly proven that the use of simulation results and experimental findings makes it possible to identify the cutting force model with a sufficient accuracy of 6.4 % (Table 1).

Our research is currently limited to the identification of cutting force mainly during peripheral milling of flat surfaces. When applying the method to processes of milling surfaces with curved guides, it is necessary to take into account the change in the speed of removal of the allowance along the form-forming coordinate. This is likely the advancement of our study. The positive aspects of this research have a drawback, which is the need to conduct field experiments using expensive measuring equipment.

### 7. Conclusions

1. The constructed mathematical model of the cutting force and its components represents the cutter flutes workpiece engagement on the flute scan by the coordinate of the rotation angle and takes into account the runout of the tool. It uses simulation based on a mechanistic model and is the basis of a developed applied forecasting program, which is a tool of a technologist-programmer.

2. Taking into account in the algorithm the elastic negative feedback of the real machining system, in which the digital modeling procedure was used as the transfer function of the direct channel, made it possible to calculate the actual cutting depth.

3. The developed application program allows specifying the empirical coefficient of the cutting force based on experimental results. In one click, the theoretically modeled plot of the cutting force component is aligned with the experimental digital file and the new value of the cutting force coefficient is calculated. Our results confirm the adequacy of modeling procedures.

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#### Conflicts of interest

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The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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#### Funding

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The study was conducted without financial support.

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#### Data availability

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All data are available in the main text of the manuscript.

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#### Use of artificial intelligence

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The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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