

The object of this study is the pneumatic system in the spring suspension of rolling stock under conditions of high-speed movement from 170 to 250 km/h.

Based on the thermodynamic model of the pneumatic spring suspension system, the influence of volume of the additional tank and the initial pressure in the pneumatic spring on the nature of change in the spring's dynamic stiffness, energy losses, and damping coefficient was studied.

Based on the built "force-deformation" dependences for the pneumatic spring, it was established that the change in the volume of an additional tank has a slight effect on the deformation of the pneumatic spring at different speeds of the high-speed rolling stock.

It was established that in the range of rolling stock speeds of 170–250 km/h, the diameter of the connecting pipeline is 30 mm, and the volume of the additional tank is from 30 to 60 l, the maximum change in the dynamic stiffness of the pneumatic spring up to 15.5 % occurs at the pressure in the spring of 6.5 bar.

Dependences of energy loss and damping coefficient during the operating cycle of the pneumatic spring suspension system were derived. It was established that an increase in the volume of the additional tank and the initial pressure in the pneumatic spring leads to an increase in the energy loss during the operation cycle of the pneumatic system. The maximum values of the damping coefficient over the entire considered range of variable parameters are 1.16–1.29.

It was established that with the volume of the additional tank in the range from 30 to 50 liters, the maximum values of the damping coefficient are observed at a diameter of the connecting pipeline of 25 mm and a speed of movement from 200 to 250 km/h. And with an additional tank volume of 60 liters – with a diameter of 30 mm and a speed of 170 to 250 km/h

Keywords: pneumatic spring suspension system, spring stiffness, damping coefficient, railroad rolling stock

DETERMINING THE EFFECT OF ADDITIONAL TANK VOLUME AND AIR PRESSURE IN THE SPRING ON THE DYNAMIC INDICATORS OF A PNEUMATIC SYSTEM OF SPRING SUSPENSION IN HIGH-SPEED RAILROAD ROLLING STOCK

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1. Introduction

The pneumatic spring suspension system is one of the components in the carriage part of modern high-speed rolling stock [1]. The use of a pneumatic system of spring suspension between the body and the bogie makes it possible to reduce the dynamic forces that arise during the interaction of the rolling stock with the rail track, and to enable the dispersion (dissipation) of energy in the process of movement [2–5].

Currently, modern high-speed electric trains are in operation on railroad tracks: EKr-1 "Tarpan", HRCS2 "Hyundai Rotem", and the diesel trains DPKr-2 and DPKr-3. A pneumatic spring suspension system with different volumes of an additional tank is used as the second stage of spring suspension (Fig. 1).

Principal structural elements of such systems are a pneumatic spring of the diaphragm type, which can work in both vertical and horizontal longitudinal and transverse directions, an additional tank, and a connecting pipeline. The connecting pipe provides the flow of air from the air spring to the additional tank and vice versa, which enables the system to provide energy dissipation. Depending on the vertical load, the amount of pressure in the pneumatic spring varies within 5–7 bar, which corresponds to the static and dynamic operating conditions of the rolling stock. Also, a characteristic feature of the pneumatic system of spring suspension for rolling stock is the different volume of the additional tank, the size of which on EKr-1 "Tarpan" electric trains and DPKr-2 diesel trains is 38 liters, and on DPKr-3 diesel trains – 55 l.

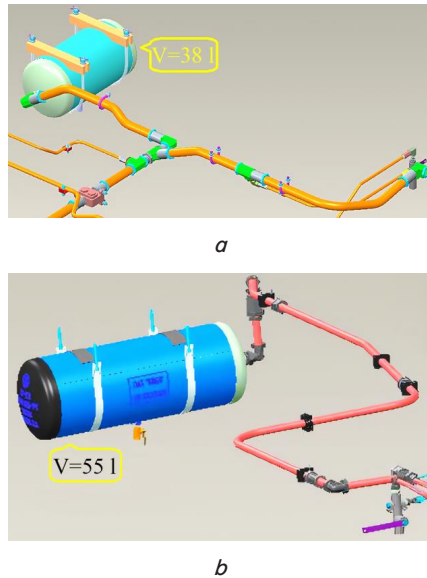


Fig. 1. Additional tank and connecting pipelines for modern railroad rolling stock: *a* – DPKr-2; *b* – DPKr-3

The basic dynamic indicators of the pneumatic spring suspension system for railroad rolling stock are the dynamic stiffness of the pneumatic spring and the damping coefficient during the work cycle. The values of the specified indicators affect the level of dynamic forces in the links between the structural elements of the rolling stock, which in turn affect the level of indicators of the dynamics and safety of the movement of the rolling stock [6, 7]. Therefore, at the stage of designing the pneumatic spring suspension system, designers try to find the optimal parameters of the pneumatic spring suspension system. One of them is the stiffness of the air spring and the damping coefficient per cycle of the air spring suspension system. At the same time, the change in the pressure in the system and the volume of the additional tank have a significant influence on the change in the dynamic indicators of the high-speed rolling stock.

Conducting studies into the influence of changes in air pressure in the system and the volume of the additional tank on the stiffness and damping parameters of the pneumatic spring will allow further evaluation of the influence of these parameters on the dynamics of the rolling stock and traffic safety indicators under conditions of high-speed traffic. Correct modeling of the operation of the pneumatic spring suspension system at the stage of designing high-speed rolling stock will allow improving the traffic safety indicators of transport carriages by choosing the optimal parameters for the spring suspension system. Therefore, theoretical studies on the operation of the pneumatic spring suspension system under its various parameters and operating conditions are an urgent task of scientific research.

2. Literature review and problem statement

Over the last 50 years, the dynamic patterns of pneumatic springs have been studied in detail. Vertical models of the behavior of pneumatic springs can be categorized into equivalent mechanical and thermodynamic models. Typical equivalent mechanical models include the “Nishimura model” [8], the “Simpack model” [9], the “Vampire model” [10], and the “Berg model” [11]. According to Sayyaadi and

Shokouhi [12], the frictional part of the “Berg model” was replaced by a simplified viscoelastic model.

However, in the model by Nishimura [8], the nonlinear pattern of the pneumatic spring is observed only at high values of viscous damping, which is typical for a small length of the connecting pipeline. Also, in the models by Nishimura and Simpack [9], the inertia of the air mass in the connecting pipeline between the pneumatic spring and the additional tank is not taken into account. Vampire [10] and Berg [11] models are non-linear and can describe the dynamic behavior of a pneumatic spring in vertical and horizontal directions. The main difference between them is the value of the nonlinearity coefficient, which is taken as 2.0 and 1.8, respectively.

However, to determine the characteristics of the pneumatic spring, authors recommend determining individual parameters based on the analysis of experimental results, which requires access to sophisticated measuring equipment, as well as conducting tests for each new model of the pneumatic spring.

Typical thermodynamic models include two main elements [12]: a model of an air spring and an additional reservoir [13–15] and a model of a pipeline connecting the air spring to an additional reservoir [16–18].

In [13], a dimensionless model of a pneumatic suspension with an additional tank is given, based on which structural considerations are proposed for the selection of components of the pneumatic system. However, the influence of the volume of the additional tank and the pressure in the air spring on the dynamic performance of the air spring suspension system has not been considered and evaluated.

In [14], the authors presented an analytical model of a pneumatic spring based on experimental characteristics. It is shown that the dynamic behavior of the pneumatic system can be improved due to the correct selection of the volumes of the pneumatic spring and the additional tank. However, the study of the laws of influence of the volume of the additional tank and the pressure in the pneumatic spring on the dynamic stiffness of the pneumatic spring and the damping coefficient was not carried out.

It should be noted that the behavior of the suspension is largely influenced by the polytropic index. This issue was considered in works [15, 16], in which heat transfer between pneumatic chambers and the atmosphere was taken into account, and the rate of heat flow was taken to be proportional to the temperature difference. It was established that the dynamic stiffness for the adiabatic case is greater compared to the isothermal case and represents a resonance effect for frequencies close to 15 Hz. However, the influence of the speed regimes of rolling stock on the dynamic parameters of the pneumatic spring suspension system was not investigated in the cited works.

In [17], the influence of the operation of the pneumatic spring suspension system on the criteria of comfort and the force of the “wheel-rail” contact was investigated.

In [18], the influence of the diameter and length of the connecting pipeline on the stiffness of the pneumatic spring, energy loss, and damping coefficient during the operating cycle of the pneumatic spring suspension system was investigated. However, the study of the influence of the volume of the additional tank and the pressure in the pneumatic spring on the dynamic parameters of the pneumatic spring suspension system was not conducted.

The authors of [19] developed a model of a pneumatic spring as a dynamic system with three phase coordinates:

pressure in the cylinder and additional tank, mass of air in the cylinder. The influence of the pneumatic resistance of the throttle element on the elastic and damping properties of the pneumatic spring was also analyzed. However, setting the optimal parameters of the pneumatic system depending on the pneumatic resistance of the throttle element does not fully correspond to the operating conditions of the pneumatic spring suspension system. Due to the fact that there is a possibility of changing the initial pressure in the pneumatic spring and the volume of the additional tank, which was not reflected in the work.

In [20, 21], a vertical dynamic model of a pneumatic spring was built, based on the equations of thermodynamics and hydrodynamics, and the geometric parameters were determined by an approximate analytical method. The authors noted that in the absence of an additional tank, the vertical stiffness of the pneumatic spring decreases intensively when its volume increases to 30 l and stabilizes when the volume increases further. When an additional tank with a volume of 50 liters is connected to the air spring, the vertical stiffness of the air spring depends less on its volume.

However, there is no study of the regularities of the damping coefficient change during the operation cycle of the pneumatic spring suspension system depending on the volume of the additional tank and the initial pressure in the pneumatic spring.

In [22], a dynamic model of the pneumatic spring was developed, taking into account thermodynamic processes in the pneumatic spring suspension system, effective friction, and viscoelastic damping of the pneumatic spring shell. With the help of the proposed model, a study of the amplitude- and frequency-dependent properties of the pneumatic spring-connecting pipeline-additional tank system was carried out.

Paper [23] reports an indirect method of implementing a polytropic process by means of modifying the ratio of pressure and volume obtained as a result of simulating an isothermal process. The dependences of the vertical stiffness of the pneumatic spring on the frequency of the disturbance force and the vertical load at different values of the volume of the pneumatic spring were constructed and analyzed.

In [24], the influence of the parameters of the secondary suspension of the railroad vehicle on the performance of the pneumatic spring was investigated and discussed. The volume of the additional tank, the length and diameter of the connecting pipeline were taken into account. It was noted that when the volume of the additional tank is changed to zero, the pneumatic spring behaves as a simple stiffness component, and the angle of loss of reaction forces is zero for all excitation frequencies. This indicates the absence of damping properties in the pneumatic spring suspension system. It was found that increasing the volume of the additional tank leads to a decrease in the dynamic stiffness of the system at low frequencies and an increase at high frequencies.

However, in works [22–24] there are no studies into the regularities of the damping coefficient change during the operation cycle of the pneumatic spring suspension system depending on the volume of the additional tank and the initial pressure in the pneumatic spring.

In [25], the dynamic behavior of a pneumatic spring in the frequency range up to 400 Hz is considered. It is noted that the ability of a pneumatic spring to withstand an axial load mainly depends on its effective area and on air pressure, which in turn depend on the vertical deformation of the pneumatic spring. It was emphasized that often when mod-

eling the dynamic behavior of an air spring, the force that occurs during the deformation of its shell is neglected [13, 15], which cannot be allowed with a small and relatively rigid design of an air spring [26]. Taking into account the above features, the authors obtained the dependence of the dynamic stiffness of the pneumatic spring on the ratio of the volume of the additional tank to the volume of the pneumatic spring at its nominal height of 90 mm.

In [17], a comparison of alternative approaches to the mathematical modeling of the pneumatic spring suspension system was performed and their influence on the dynamic behavior of the rolling stock as a whole was evaluated. It is found that the lateral deformation and the roll deformation do not lead to significant fluctuations of the air pressure in the middle of the air spring, and the response in the vertical direction can be considered independent.

In [27], a study of the behavior of the dynamic characteristics of the pneumatic spring suspension system was conducted using six basic, most common, mathematical models. One of the models is directly related to the appearance of the pneumatic system (thermodynamic model). The other five models are equivalent mechanical models: the Vampire model, the Berg model, the linear Nishimura model (linear damping), the nonlinear Nishimura model with quadratic damping, and the spring model that is parallel to the damper. It was noted that the thermodynamic model, Vampire and Berg models also take into account the inertial action of the air mass in the connecting pipe between the air spring and the auxiliary tank. It is concluded that for frequencies above 2 Hz it is important to consider nonlinear viscous damping, quadratic or a slightly lower exponent. It has been determined that for an air spring suspension system with a connecting pipe length of more than 2 m, the Nishimura linear model and the spring model that is parallel to the damper should not be used to study the system characteristics at frequencies higher than 2–3 Hz.

So, after analyzing research works [8–28], it should be emphasized that currently a wide range of studies of the dynamic behavior of the pneumatic spring suspension system have been performed, without taking into account the complex influence of its components on the operation of the pneumatic spring. The task of establishing the regularities of changes in the dynamic parameters of the pneumatic spring depending on the volume of the additional tank and the initial pressure in the pneumatic spring under conditions of high-speed train movement of 170 to 250 km/h remains unresolved. Also, no studies were conducted on the patterns of changes in dynamic stiffness and damping coefficient under conditions of high-speed movement of railroad rolling stock from 170 to 250 km/h.

3. The aim and objectives of the study

The purpose of our work is to establish the patterns of changes in the dynamic parameters of the pneumatic spring depending on the volume of the additional tank and the initial pressure in the pneumatic spring under conditions of high-speed movement of trains from 170 to 250 km/h. This will make it possible to determine the dynamic stiffness and damping coefficient of the pneumatic spring, taking into account changes in pressure in the spring, the volume of the additional tank, and with additional consideration of changes in driving speeds from 170 to 250 km/h.

To achieve the specified goal, the following tasks must be performed:

- to build the “force-deformation” dependences of the pneumatic spring at different speed modes of the rolling stock, volumes of the additional tank and initial pressures in the pneumatic spring suspension system;
- to evaluate the dynamic stiffness of the pneumatic spring;
- to evaluate the energy losses and the damping coefficient during the cycle of operation of the pneumatic spring suspension system of the rolling stock under conditions of high-speed movement.

4. The study materials and methods

4.1. The object and hypothesis of the study

The object of our research is the pneumatic system of spring suspension for rolling stock under conditions of high-speed movement from 170 to 250 km/h.

The main hypothesis of the research assumes that the dynamic parameters of the pneumatic spring suspension system are determined by “force-deformation” dependences. They are built based on the dynamic “rolling stock-track” model. It takes into account variable changes in the volume of the additional tank, the pressure in the pneumatic spring and the speed of movement. This makes it possible to establish the patterns of changes in dynamic stiffness and damping coefficient under conditions of high-speed movement of rolling stock at speeds from 170 to 250 km/h.

Based on [20, 21], it was established that the additional tank has the property of adjusting the vertical stiffness of the pneumatic spring. However, in order to achieve the greatest damping effect, it is necessary to provide a rational ratio between the volume of the additional tank and the pneumatic spring, as a result, the ratio between the volumes takes the form:

$$V_2 > \frac{n \cdot P_1 \cdot R_0 \cdot A_{ef}}{\alpha(P_1 - P_a)} \cdot V_1, \tag{1}$$

where n is the polytropic coefficient; V_1, V_2 – volume of pneumatic spring and additional tank, respectively, m^3 ; P_1 – pressure in the pneumatic spring, Pa; A_{ef} – effective area of the pneumatic spring, m^2 ; R_0 is the distance between the point of contact of the upper arc of the circle with the upper plate and the axis of symmetry of the pneumatic spring; $m, N/m$; P_a – atmospheric pressure, Pa; α – geometric coefficient.

4.2. Thermodynamic model of the pneumatic spring suspension system

It is assumed that air under normal conditions is approximately an ideal gas. The ideal gas equation was used to determine the pressure change in the pneumatic spring suspension system:

$$PV = mRT. \tag{2}$$

Differentiating equation (2) by time, the following is obtained:

$$\begin{aligned} \frac{dP(t)}{dt}V(t) + \frac{dV(t)}{dt}P(t) = \\ = \frac{dm(t)}{dt}RT(t) + \frac{dT(t)}{dt}m(t)R. \end{aligned} \tag{3}$$

Then the equation for determining the change in the amount of pressure in the air spring takes the form:

$$\dot{P}(t) = -\dot{h}(t)\frac{P(t)}{h(t)} + \dot{m}(t)\frac{RT(t)}{h(t)A_1} + \dot{T}(t)\frac{m(t)R}{h(t)A_1}, \tag{4}$$

where P, V, T are the pressure, volume, and temperature of the working body of the pneumatic spring, Pa, m^3, K , respectively; m is the mass of air, kg; R is the universal gas constant, $J/(kg \cdot K)$; h is the current height of the pneumatic spring, m; A_1 is the effective area of the pneumatic spring, m^2 .

Structurally, the pneumatic spring suspension system involves the connection of a pneumatic spring with an additional tank through a connecting pipeline. This makes it possible, based on the Bernoulli equation, to determine the speed of the air flow through the connecting pipeline with certain geometric parameters according to the formula:

$$V(t) = \sqrt{\frac{2\Delta P(t)}{\rho(t) \cdot \left(f \frac{l}{d} + K_s + K_p\right)}}, \tag{5}$$

where ρ is the density of the working body, kg/m^3 ; l, d – length and diameter of the connecting pipeline, m; K_s – compression loss coefficient; K_p – expansion loss coefficient; f is the coefficient of friction along the length (Darcy coefficient).

Knowing the value of the average speed of the air flow, the cross-sectional area of the flow in the connecting pipeline and the density of the working fluid, the equation for determining the mass flow rate will take the following form:

$$\begin{aligned} \dot{m}(t) = \frac{\left(\frac{\pi}{4}\right) d^2 \sqrt{2 \cdot \rho(t) \cdot |P_1(t) - P_2(t)|}}{\sqrt{f \frac{l}{d} + K_s + K_p}} \times \\ \times \text{sign}(P_1(t) - P_2(t)). \end{aligned} \tag{6}$$

Based on the first law of thermodynamics, the relationship between the internal energy of the body, the amount of heat transferred to the body, and the work performed is established. This makes it possible to take into account the transfer of internal energy between the pneumatic spring and the additional tank according to the formula:

$$\Delta U(t) = \Delta Q(t) - \Delta W(t) + \Delta E(t), \tag{7}$$

where U is the internal energy of the air spring, J; Q – heat transfer, W; W – completed work, J; E is the transfer of internal energy between the pneumatic spring and the additional tank, J.

4.3. Structural-logical scheme for determining the main dynamic parameters of the pneumatic spring suspension system

A procedure for determining the main dynamic parameters of the pneumatic spring suspension system is given in the form of a structural-logical diagram (Fig. 2).

At the initial stage, it is necessary to set the initial data. Further, to take into account the peculiarities of the design of the rolling stock and its movement along the railroad track, a calculation scheme of the dynamic model “rolling stock – track” was built (Fig. 3).

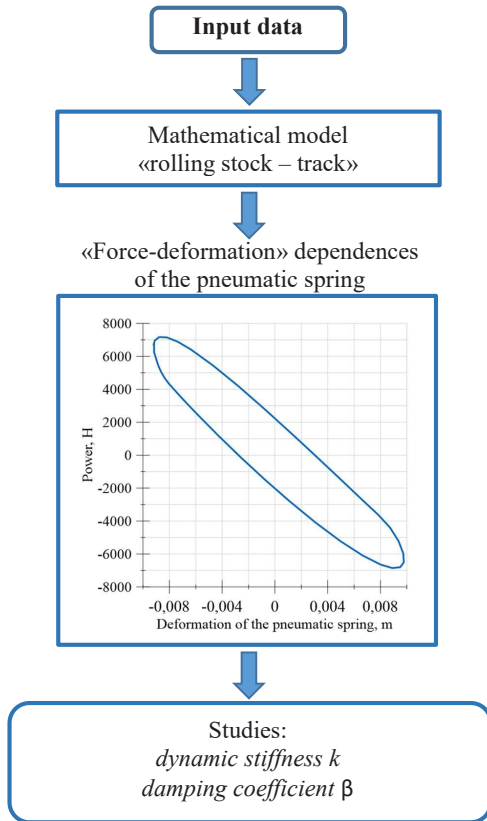


Fig. 2. Structural-logical scheme for determining the main dynamic parameters of the pneumatic spring suspension system

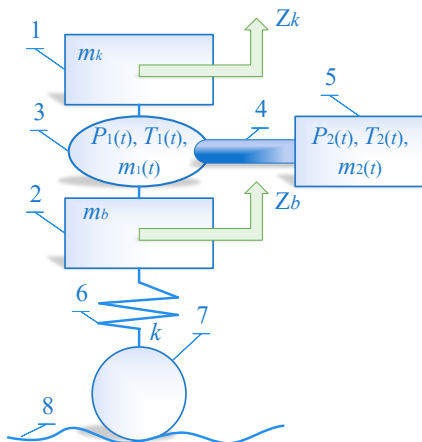


Fig. 3. Calculation scheme of the dynamic model “rolling stock – track”: 1 – body; 2 – bogie; 3 – pneumatic spring; 4 – connecting pipeline; 5 – additional tank; 6 – spring of the box stage of the spring suspension; 7 – a wheel of a wheelset; 8 – vertical irregularities of the rail track

Features in the calculation scheme of the dynamic model “rolling stock – track” are the location of the pneumatic spring suspension system (pneumatic spring 3, connecting pipeline 4, and additional tank 5) between body 1 and bogie 2 of the vehicle. Its joint movement along the rail track is taken into account under the assumption that the wheelset and the mass of the track interacting with it move continuously.

Using the Euler-D'Alembert principle and the principle of freeing a rigid body from ties, a system of differential equations is obtained:

$$\begin{cases} m_k \cdot \ddot{z}_k - P_1(t) \cdot A_1 + m_k \cdot g = 0, \\ m_b \cdot \ddot{z}_b + P_1(t) \cdot A_1 + k \cdot \left(z_b - H \cdot \sin\left(\frac{2\pi \cdot v \cdot t}{L}\right) \right) = 0, \end{cases} \quad (8)$$

where m_k is the mass of the body reduced to one spring, kg; m_b – weight of the bogie reduced to one spring, kg; z_k , z_b – vertical movement of the body and bogie, respectively, m; k is the equivalent stiffness of the spring suspension stage, N/m; H, L – amplitude and length of the vertical sinusoidal unevenness on the rail track, m; g – acceleration of free fall, m/s²; v – speed of movement, km/h; t – movement time, sec.

The obtained numerical solution to the system of equations (8) using the MATLAB application program package makes it possible to construct the “force-deformation” dependence of the pneumatic spring. Using it, the parameters of the dynamic stiffness of the pneumatic spring and the damping coefficient of the pneumatic spring suspension system are set. Dynamic stiffness is determined from the formula:

$$k = \frac{F_{\max}}{\Delta_{\max}}, \quad (9)$$

where F_{\max} is the force amplitude, kN; Δ_{\max} is the deformation amplitude, m.

The damping coefficient, which takes into account energy losses during the operating cycle of the pneumatic spring suspension system, is determined from the following formula:

$$\beta = \frac{E}{F_{\max} \cdot \Delta_{\max}}, \quad (10)$$

where E is energy dissipation per cycle, J.

So, using the above sequence of operations, the parameters of the pneumatic spring suspension system can be determined and analyzed, using the initial data given in Table 1.

Table 1

Initial data for mathematical modeling of the pneumatic spring suspension system operation

Name	Designation	Value
Initial pressure in the pneumatic spring, bar	P_{10}	5.0–7.0
Effective area of the pneumatic spring, m ²	A_1	0.2419
The initial height of the air spring, m	h_0	0.214
Geometrical parameters of the connecting pipeline:		
– length, m	l	4.0
– diameter, mm	d	25.0–35.0
Compression loss ratio	K_s	0.5
Expansion loss ratio	K_p	1.0
The volume of the additional tank, l	V_2	30–60
Parameters of geometric irregularity on the track:		
– length, m	L	10
– amplitude, mm	H	15
Specific heat:		
– at constant volume, J/(kg·K)	c_v	718
– at constant pressure, J/(kg·K)	c_p	1005

It should be noted that the initial pressure of the pneumatic spring suspension system is adopted as a variable

parameter in the range from 5 to 7 bar since the load on the pneumatic spring may change during rolling stock operation.

5. Studying the main indicators of the pneumatic spring suspension system for high-speed rolling stock

5.1. Analysis of “force-deformation” dependences of a pneumatic spring at different speed modes of rolling stock

According to the results of the calculations carried out according to the structural-logical scheme described above, the “force-deformation” dependence of the pneumatic spring was constructed at different speeds of the vehicle, varying in the range from 170 to 250 km/h (Fig. 4, 5). At the same time, the volume of the additional tank was assumed equal to 30, 40, 50, or 60 liters, and the initial pressure in the pneumatic spring varied from 5.0 to 7.0 bar.

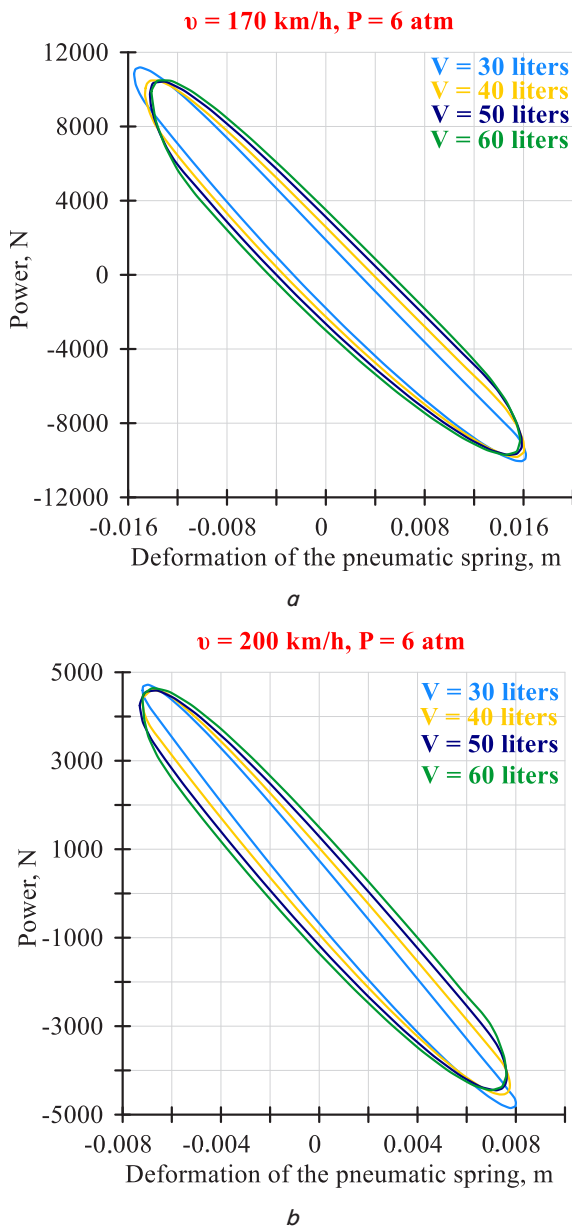


Fig. 4. The “force-deformation” dependences of the pneumatic spring under different speed modes of the rolling stock depending on change in the volume of the additional tank: *a* – 170 km/h; *b* – 200 km/h

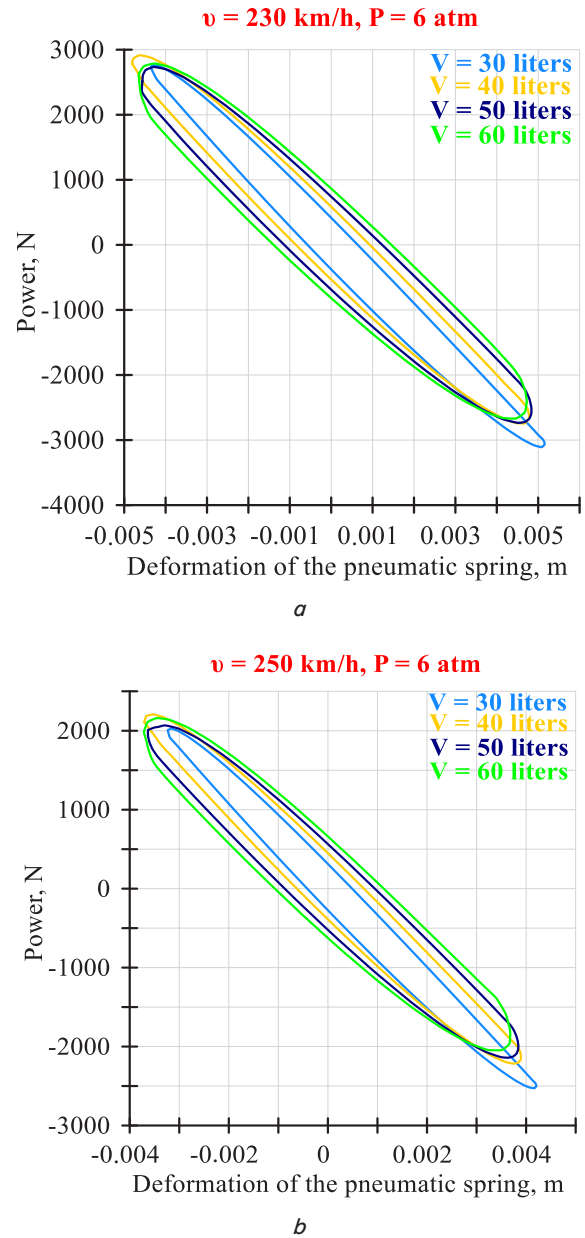


Fig. 5. The “force-deformation” dependences of the pneumatic spring under different speed modes of the rolling stock depending on change in the volume of the additional tank: *a* – 230 km/h; *b* – 250 km/h

Fig. 4, 5 demonstrate that the change in the speed of the vehicle significantly affects the magnitude of the vertical force acting on the pneumatic spring. Thus, at a driving speed of 170 km/h (which corresponds to a disturbance frequency of 4 Hz), the maximum force reaches 11.2 kN, at 200 km/h (5.6 Hz) – 4.7 kN, at 230 km/h (6.4 Hz) – 2.9 kN, and at 250 km/h (6.9 Hz) – 2.2 kN. Therefore, an increase in speed from 170 to 250 km/h leads to a decrease in vertical force by 5 times. Accordingly, the deformations of the pneumatic spring are reduced from 16.2 mm at a speed of 170 km/h to 4.2 mm at a speed of 250 km/h. The results can be explained by the fact that when the speed of the rolling stock increases, the frequency of the kinematic disturbance increases and the ratio between the natural and forced oscillation frequencies of the spring changes.

An increase in the volume of the additional tank with other constant parameters leads to a decrease in the force in the pneumatic spring. But this influence is insignificant. Thus, at a driving speed of 170 km/h, an increase in the volume of the additional tank from 30 l to 60 l leads to a decrease in force from 11184 N to 10465 N (by 6.4 %), and at a speed of 250 km/h – from 2207 to 2026 H (a decrease by 8.2 %).

Fig. 6 shows the “force-deformation” dependence of the pneumatic spring at a speed of 200 km/h and various initial pressures. Pneumatic systems with volumes of additional tanks of 30 and 60 liters were considered. In both cases, an increase in the initial pressure in the air spring led to an increase in the forces and deformations of the spring. Thus, the initial pressure changes from 5 to 7 bar caused an increase in force by 58.5 % in the case when the volume of the additional tank was 30 L, and by 55.2 % when the volume of the additional tank was 60 L. The increase in deformation was relatively small and its value was 5.8 % and 7.0 %, respectively, for the systems with additional tanks of 30 L and 60 L.

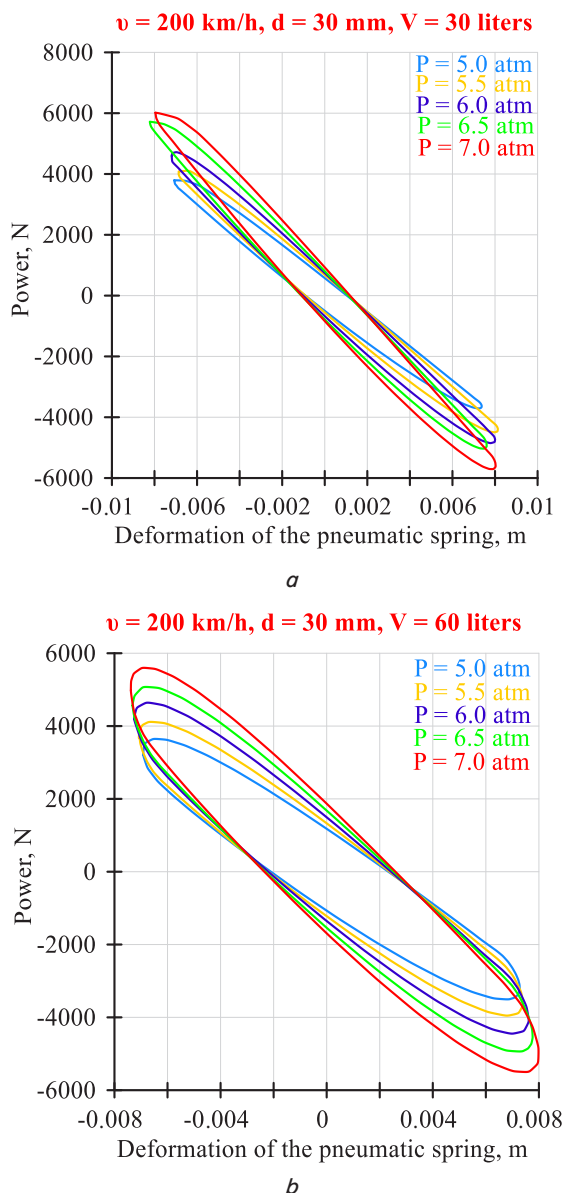


Fig. 6. The “force-deformation” dependence of the pneumatic spring when the initial air pressure and the volume of the additional tank change: *a* – $V=30 \text{ l}$; *b* – $V=60 \text{ l}$

Analysis of the dependences shown in Fig. 4–6 revealed that with a given sinusoidal unevenness of the rail track, the “force-deformation” dependences of the pneumatic springs have the shape of ellipses. This indicates the cyclical nature of the movement and dissipation of energy during the oscillation cycle of the system. Therefore, according to formula (9), it is possible to estimate the vibration damping coefficient. When the initial pressure in the spring changes (Fig. 6), the angle of inclination of the major axis of the ellipse to the abscissa axis (axis of deformation of the pneumatic spring) changes, which, according to formula (8), indicates a change in the dynamic stiffness of the spring. Therefore, based on the resulting dependences, it is possible to determine the dynamic stiffness and damping coefficient of the pneumatic spring suspension system. As well as analyze the effect on these characteristics of the studied factors: speed of movement, initial pressure in the spring, and the volume of the additional tank.

5. 2. Studying the dynamic stiffness of the pneumatic spring

Fig. 7, 8 show the dependence of the dynamic stiffness of the pneumatic spring on the volume of the additional tank and the initial pressure in the spring. The studies were performed at different speeds of rolling stock (170, 200, 230, and 250 km/h).

Fig. 7, 8 demonstrate that an increase in the value of the initial pressure in the pneumatic spring suspension system leads to an increase in the dynamic stiffness of the pneumatic spring. This happens at all considered values of the speed of the rolling stock and any volume of the additional tank. Increasing the volume of the additional tank, on the contrary, leads to a decrease in the dynamic stiffness of the spring. Moreover, the nature of change in stiffness depends on the speed of movement. Thus, at speeds of 230 and 250 km/h, the dynamic stiffness significantly decreases when the volume of the additional tank increases from 30 to 50 l (by 14 % and 16 %, respectively, at an initial pressure of 7 bar), and when the volume is further increased may even grow. At a driving speed of 170 km/h, the change in volume leads to both an increase and a decrease in dynamic stiffness, but the overall fluctuation of stiffness values does not exceed 7 %. In general, it can be noted that the dynamic stiffness of the pneumatic spring depends significantly on the frequency of disturbance and the initial pressure in the spring, but it depends less on the volume of the additional tank.

Table 2 gives changes in the maximum values of the dynamic stiffness of the pneumatic spring, expressed as a percentage, with an increase in the initial pressure in the spring from 5.0 to 7.0 bar in the entire considered range of movement speeds and volumes of the spring’s additional reservoir. An increase in the speed of the rolling stock mainly leads to a decrease in the differences in the values of dynamic stiffness. But these changes are insignificant and do not exceed 5 %, except for the additional tank with a volume of 50 l, for which the difference is 11.3 %. In almost all considered cases, a change in the initial pressure in the spring from 5.0 to 7.0 bar leads to a change in dynamic stiffness by a value close to 30 %.

Table 3 gives changes in the maximum values of the dynamic stiffness of the air spring, expressed as a percentage, when the volume of the additional tank increases

from 30 l to 60 l in the entire considered range of movement speeds and initial pressure in the air spring. Table 3 demonstrates that an increase in the speed of movement leads to an increase in the effect of the change in the volume of the additional tank on the change in the value of the dynamic stiffness. Thus, at a speed of 170 km/h, the change in stiffness does not exceed 7 %, and, at 250 km/h, it reaches 15.7 %.

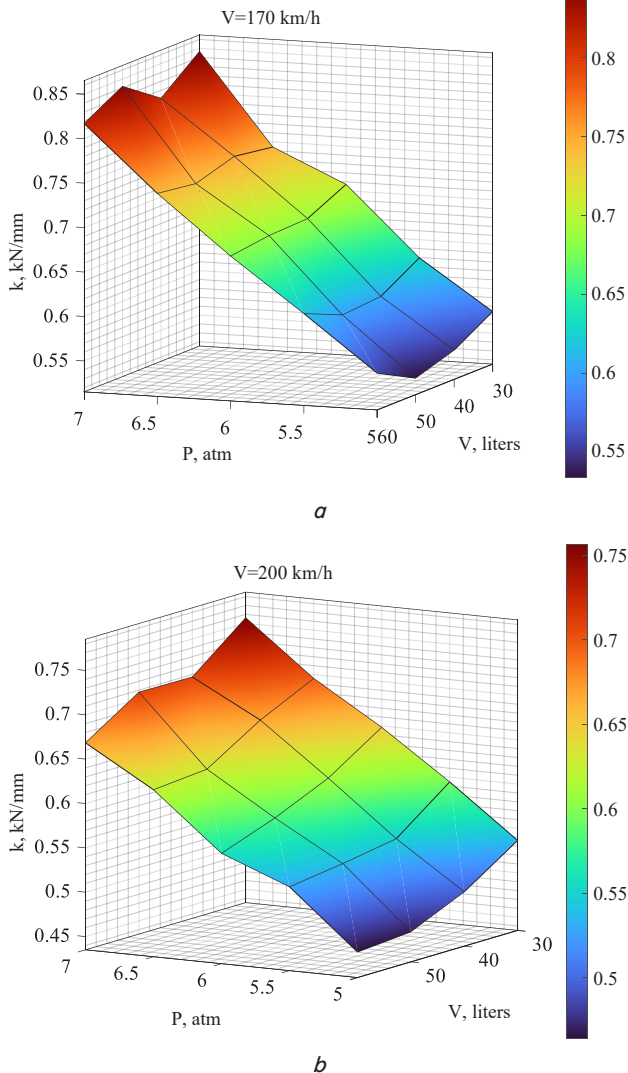


Fig. 7. Dynamic stiffness of the air spring depending on change in the volume of the additional tank and the initial pressure in the air spring: a – 170 km/h; b – 200 km/h

It was not possible to establish the deterministic influence of the initial pressure on the change in dynamic stiffness.

It follows from Tables 2, 3 that changes in the value of dynamic stiffness due to changes in the initial pressure in the spring significantly exceed the changes in stiffness caused by the change in the volume of the additional tank (36.6 % and 15.7 %, respectively, if we compare the maximum differences).

In addition to the volume of the additional tank, the diameter and length of the connecting pipeline can affect the value of the dynamic stiffness. In work [18] it was shown that the influence of the diameter of the connecting

pipeline is dominant. The results of changing the dynamic stiffness of the pneumatic spring when the diameter of the connecting pipeline changes from 25 to 35 mm are shown in Fig. 9 and Fig. 10.

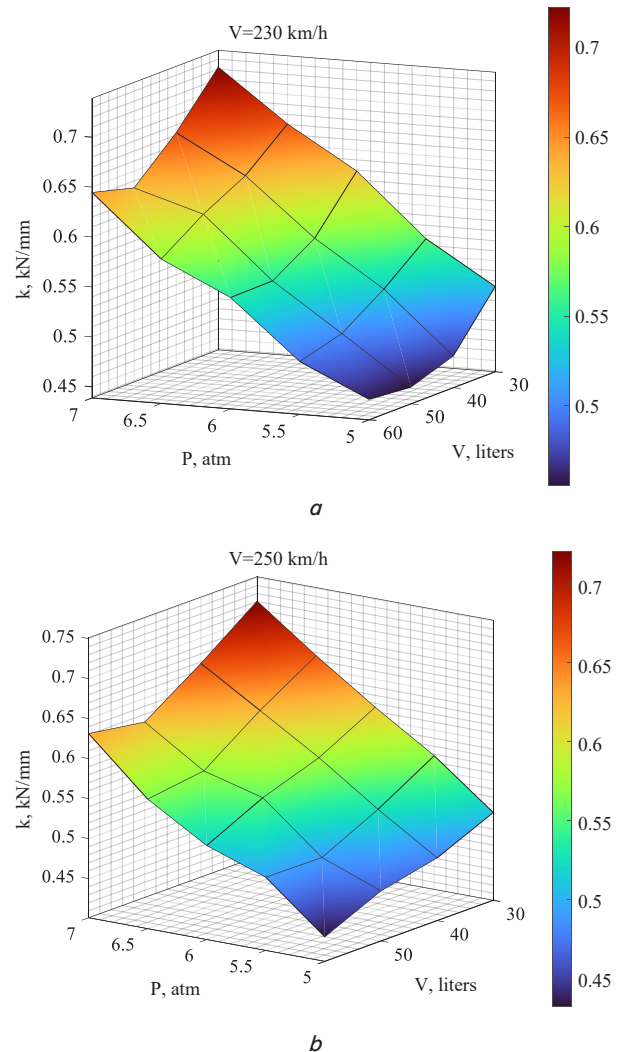


Fig. 8. Dynamic stiffness of the pneumatic spring depending on change in the volume of the additional tank and the initial pressure in the pneumatic spring: a – 230 km/h; b – 250 km/h

Table 2

Change in the maximum values of the dynamic stiffness of the pneumatic spring when the initial pressure in the pneumatic spring changes within 5–7 bar

Volume of additional tank, liters	Speed, km/h			
	170	200	230	250
30	32.16 %	29.05 %	27.41 %	29.11 %
40	32.43 %	29.81 %	30.01 %	28.23 %
50	36.58 %	33.72 %	28.04 %	25.25 %
60	31.88 %	30.58 %	28.73 %	31.62 %

It follows from Fig. 9, 10 that reducing the diameter of the connecting pipeline contributes to increasing the dynamic stiffness of the pneumatic spring. The maximum value of it is: at a speed of 170 km/h – 0.88 kN/mm,

at 200 km/h – 0.74 kN/mm, at 230 km/h – 0.69 kN/mm, and at 250 km/h – 0.69 kN/mm.

Table 3

Change in the maximum values of the dynamic stiffness of the pneumatic spring when changing the volume of the additional tank within the range of 30÷60 liters

Initial pressure in the air spring, bar	Speed, km/h			
	170	200	230	250
5.0	7.05 %	13.52 %	13.08 %	15.48 %
5.5	4.74 %	10.94 %	13.22 %	13.44 %
6.0	4.11 %	13.71 %	12.48 %	15.69 %
6.5	1.80 %	10.40 %	13.01 %	15.46 %
7.0	4.19 %	11.61 %	12.32 %	13.91 %

It is observed that the diameter of the connecting pipeline increases at speeds of 170÷250 km/h and the volume of the additional tank of 30÷60 liters leads to an increase in the difference in the maximum values of dynamic stiffness, which is: at a speed of 170 km/h – from 4.9 % to 17.8 %, at 200 km/h – from 3.0 % to 21.3 %, at 230 km/h – from 3.2 % to 23.7 %, and at 250 km/h – from 5.2 % to 23.8 %.

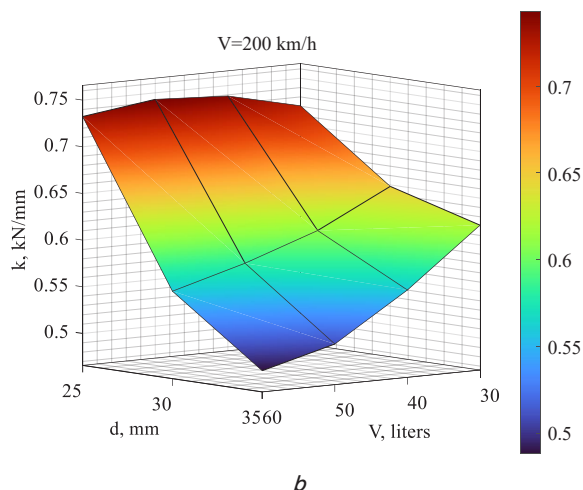
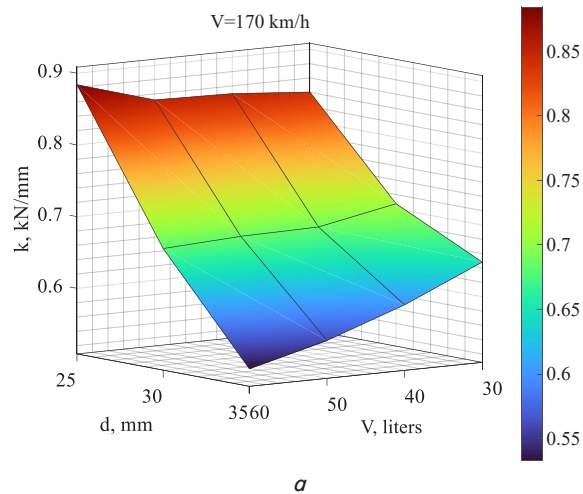


Fig. 9. Dynamic stiffness of the pneumatic spring depending on change in the volume of the additional tank and the diameter of the connecting pipeline: *a* – 170 km/h; *b* – 200 km/h

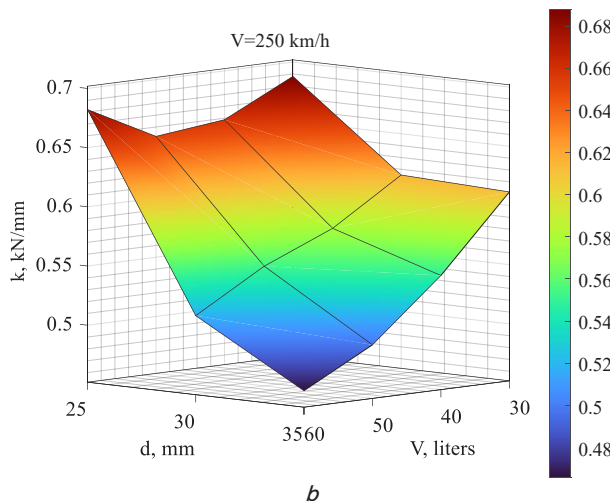
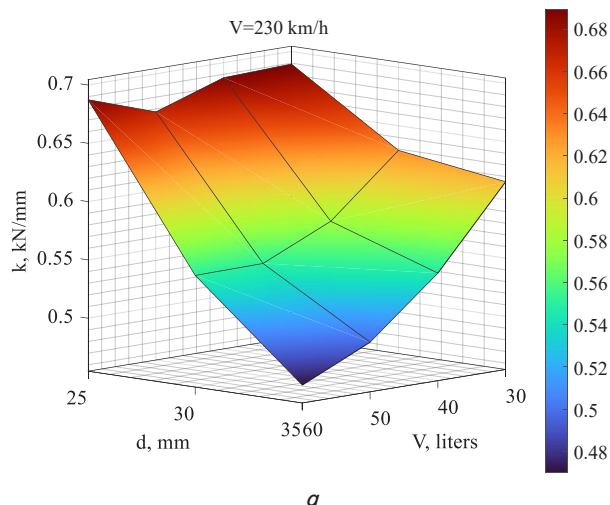


Fig. 10. Dynamic stiffness of the pneumatic spring depending on change in the volume of the additional tank and the diameter of the connecting pipeline: *a* – 230 km/h; *b* – 250 km/h

5.3. Studying the nature of changes in the damping coefficient and energy loss during the operating cycle of the pneumatic spring suspension system

It is known that during the operation of the rolling stock, fluctuations of the main components of its carriage part occur, which is primarily related to the design features and technical condition of the rail track and running parts of the rolling stock. This contributes to the emergence of additional dynamic forces, which in this case deform the pneumatic spring. As a result, air flows from the air spring into the additional tank through the connecting pipe, which leads to energy loss during the operation cycle of the air spring suspension system.

Using the “force-deformation” dependences of the pneumatic spring, histograms of the amount of energy loss during the operation cycle of the pneumatic spring suspension system were constructed with an additional tank volume of 30 l (Fig. 11, 12) and 60 l (Fig. 13, 14).

Analysis of Fig. 11–14 revealed that an increase in the volume of the additional tank and the initial pressure in the air spring leads to an increase in the energy loss during the operating cycle of the air spring suspension system. Energy losses also increase when the speed of the rolling stock decreases. This is logically related to the fact that at lower speeds of movement, greater forces act on the air spring, and it is more deformed.

An increase in the initial pressure in the pneumatic spring from 5.0 to 7.0 bar leads to an increase in energy losses. In percentage: at a speed of 170 km/h – 147 % (for an additional tank with a volume of 30 liters) and 101 % (60 liters); at 200 km/h – 92 % (30 l) and 75 % (60 l), at 230 km/h – 84 % (30 l) and 59 % (60 l), at 250 km/h – 63 % (30 l) and 47 % (60 l). It can also be seen that in the spring suspension system with an additional tank volume of 30 l, energy losses are almost half as much as with an additional tank volume of 60 l. But the effect of the initial pressure change on the amount of losses with an additional tank volume of 30 liters is greater than with 60 liters.

The damping coefficient is used to quantify the damping properties of the pneumatic spring suspension system. The nature of change in the damping coefficient depending on the volume of the additional tank, the initial pressure in the pneumatic spring, and the diameter of the connecting pipeline is shown in Fig. 15–18.

For all calculated cases, the damping coefficient of the pneumatic spring varied in the range from 0.3 to 1.3. The maximum value of the damping coefficient with a connecting pipeline diameter of 30 mm is: at a speed of 170 km/h – 1.24; at 200 km/h – 1.29; at 230 km/h – 1.16; at 250 km/h – 1.18.

The largest values of the coefficient were observed at the maximum volume of the additional tank of 60 l in the conducted study, which is consistent with the data obtained for energy losses during the pneumatic system operation cycle.

It was established that the change in the initial pressure in the pneumatic spring affects the change in the maximum values of the damping coefficient of the pneumatic spring suspension system, which is: at a driving speed of 170 km/h – 13 %; at 200 km/h – 17 %; at 230 km/h – 33 %, and at 250 km/h – 39 %.

When the pneumatic spring is deformed, air flows into the additional tank through the connecting pipeline, which is accompanied by energy losses. Therefore, it is important to consider exactly how the damping coefficient of the pneumatic spring suspension system will change depending on the change in the volume of the additional tank and the diameter of the connecting pipeline. The results of the relevant study are shown in Fig. 17, 18.

It was established that with the volume of the additional tank in the range from 30 to 50 liters, the maximum values of the damping coefficient are observed at a diameter of the connecting pipeline of 25 mm and a speed of movement from 200 to 250 km/h. With an additional tank volume of 60 liters – with a diameter of 30 mm and a speed of 170 to 250 km/h.

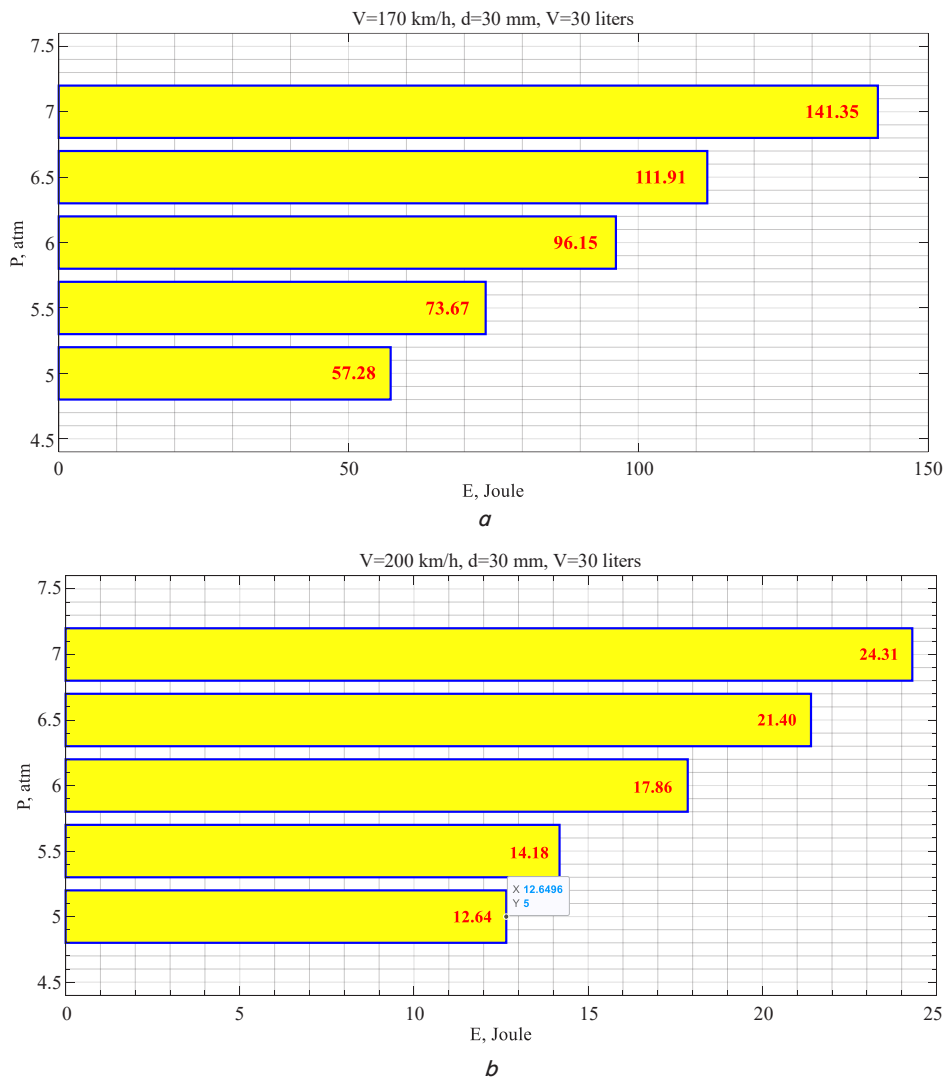


Fig. 11. The amount of energy loss during the operation cycle of the pneumatic spring suspension system with an additional tank volume of 30 l: a – 170 km/h; b – 200 km/h

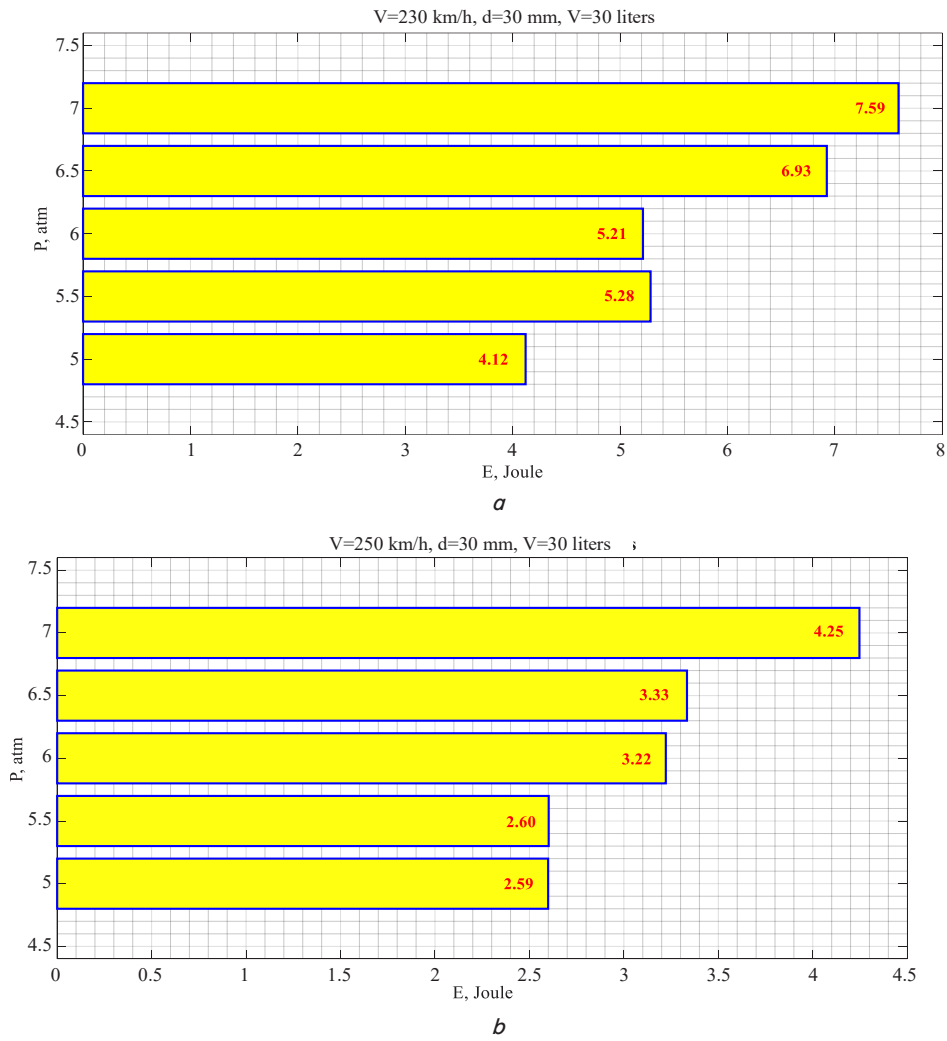


Fig. 12. The amount of energy loss per cycle of operation of the pneumatic spring suspension system with an additional tank volume of 30 l: *a* – 230 km/h; *b* – 250 km/h

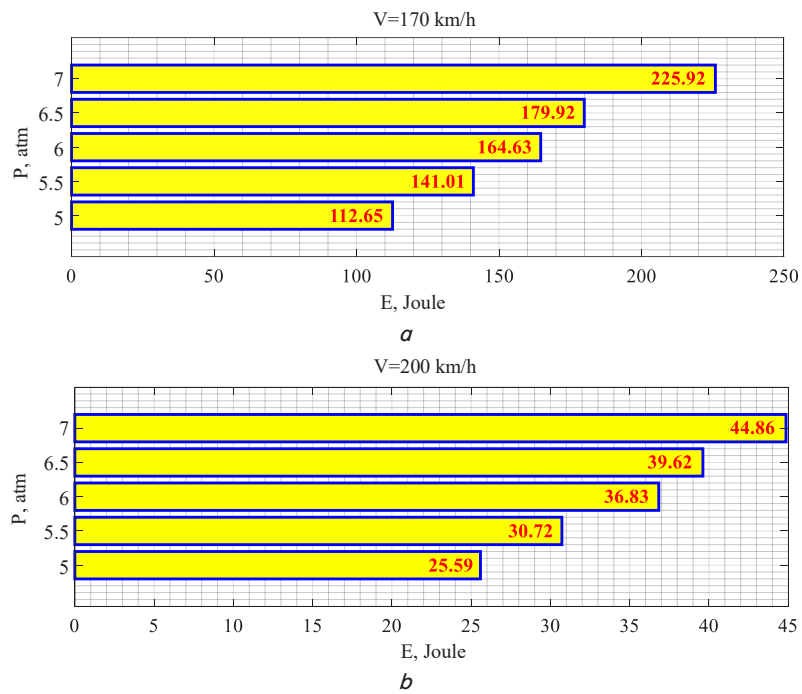


Fig. 13. The amount of energy loss per cycle of operation of the pneumatic spring suspension system with an additional tank volume of 60 l: *a* – 170 km/h; *b* – 200 km/h

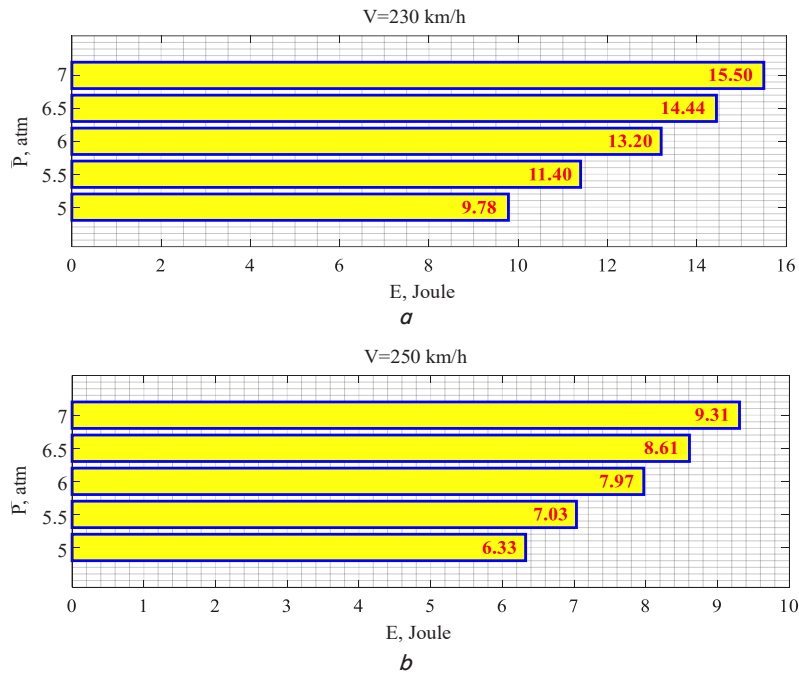


Fig. 14. The amount of energy loss per cycle of operation of the pneumatic spring suspension system with an additional tank volume of 60 l: *a* – 230 km/h; *b* – 250 km/h

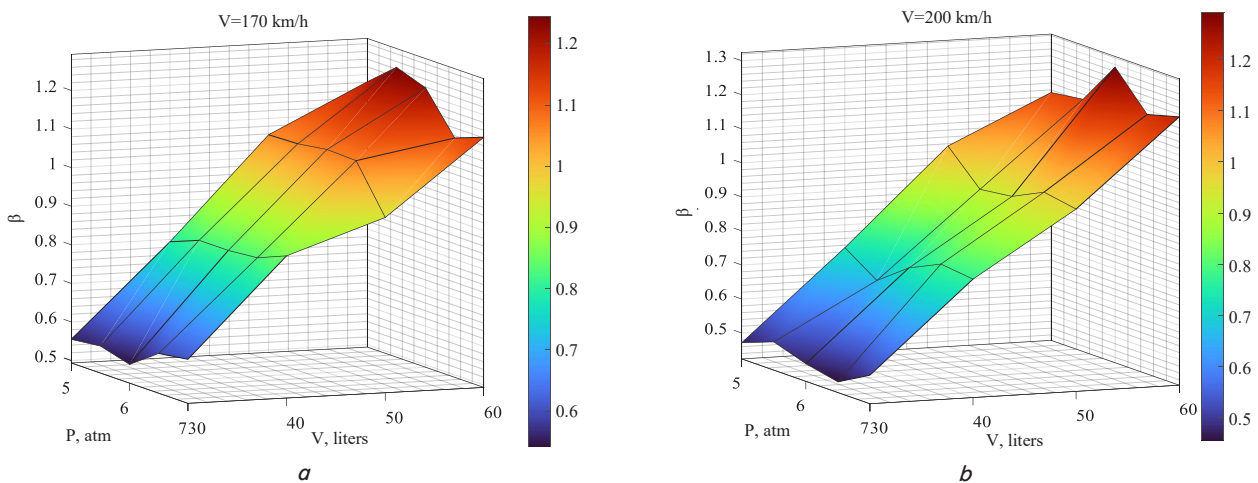


Fig. 15. Dependence of the damping coefficient of the pneumatic spring suspension system on a change in the volume of an additional tank and the initial pressure in a pneumatic spring with a diameter of the connecting pipeline of 30 mm: *a* – 170 km/h; *b* – 200 km/h

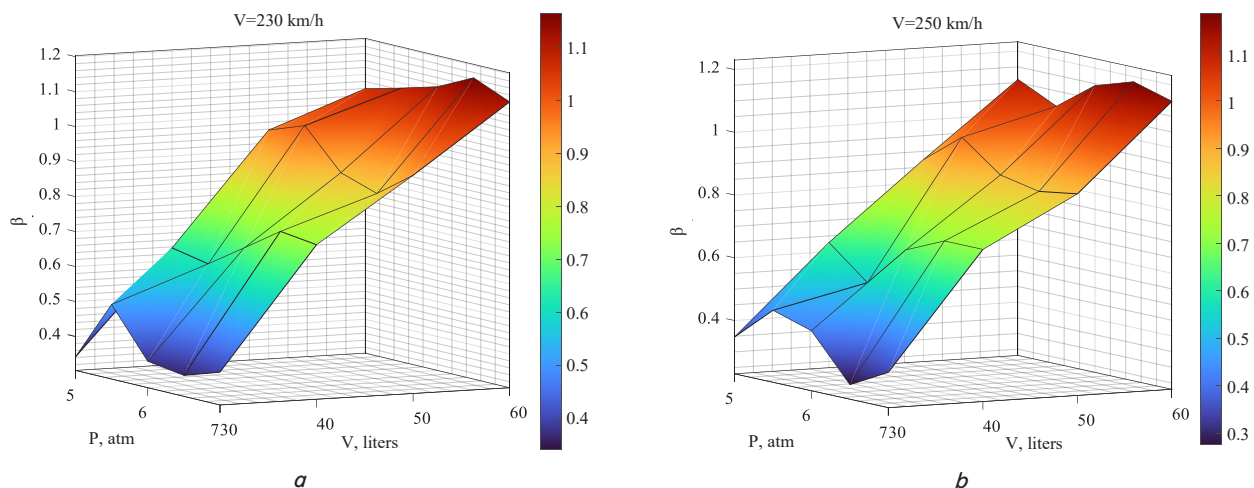


Fig. 16. Dependence of the damping coefficient of the pneumatic spring suspension system on a change in the volume of an additional tank and the initial pressure in a pneumatic spring with a diameter of the connecting pipeline of 30 mm: *a* – 230 km/h; *b* – 250 km/h

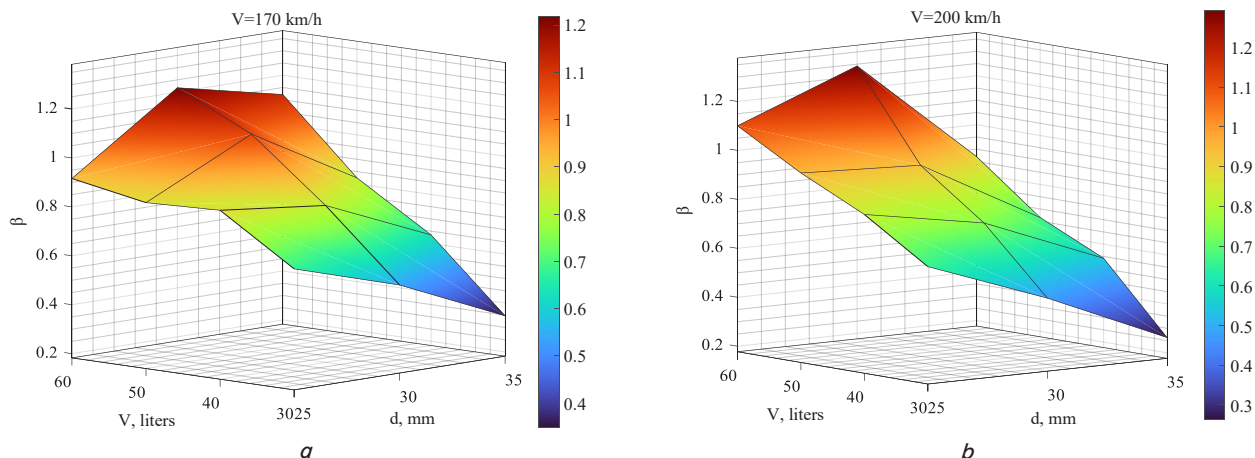


Fig. 17. Dependence of the damping coefficient of the pneumatic spring suspension system on a change in the volume of an additional tank and the diameter of a connecting pipeline at an initial pressure of 6 bar: *a* – 170 km/h; *b* – 200 km/h

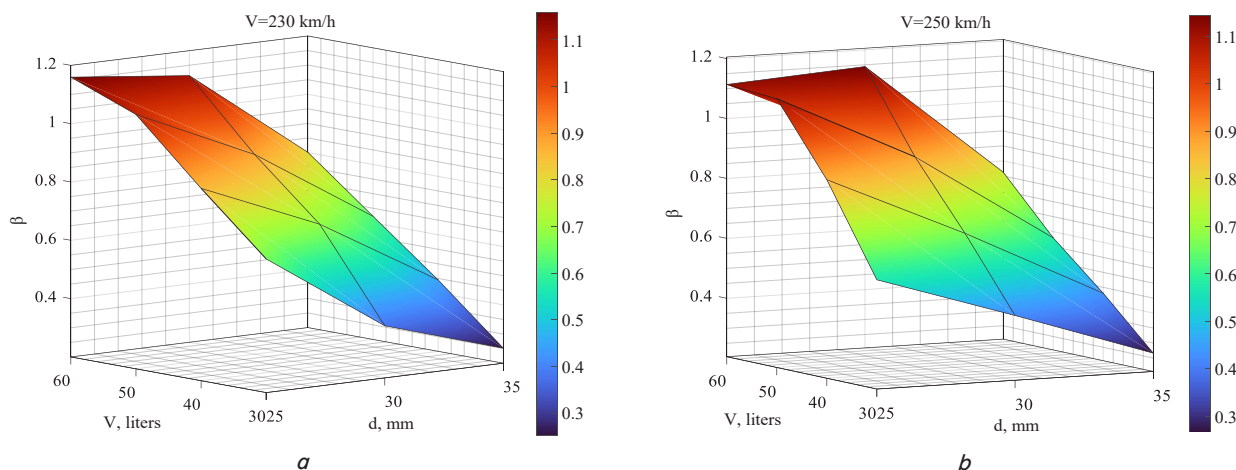


Fig. 18. Dependence of the damping coefficient of the pneumatic spring suspension system on a change in the volume of an additional tank and the diameter of a connecting pipeline at an initial pressure of 6 bar: *a* – 230 km/h; *b* – 250 km/h

6. Discussion of results of investigating the patterns of changes in the dynamic parameters of a pneumatic spring suspension system

Under conditions of high-speed movement of rolling stock at speeds of 170–250 km/h, a study of the patterns of changes in the dynamic parameters of the pneumatic spring suspension system was conducted. In this case, the influence of the volume of the additional tank and the initial pressure in the pneumatic spring on the dynamic stiffness of the spring, energy loss, and damping coefficient during the operating cycle of the pneumatic spring suspension system is taken into account.

The results of our studies confirm that the deformation of the pneumatic spring depends significantly on the frequency of disturbance (which depends on the speed of movement) and much less on the volume of the additional tank. It should be noted that a noticeable decrease in the amount of deformation of the pneumatic spring with an increase in the speed of the rolling stock can be explained by a mismatch in the frequencies of natural and forced oscillations. It depends on the given initial conditions and the nature of the unevenness of the rail track.

Analysis of changes in the dynamic stiffness of the pneumatic spring (Fig. 7, 8) reveals that with an increase in the initial pressure (from 5 to 7 bar) in the pneumatic spring suspension system, the dynamic stiffness of the pneumatic spring increases. This happens at any speed of the rolling stock (from 170 to 250 km/h)

and any volume of the additional tank (from 30 to 60 l). The increase in dynamic stiffness of the spring is due to the increase in the initial pressure in the pneumatic system, which leads to an increase in the force that must be applied to deform the pneumatic spring within the accepted limit deformations.

It is also shown that an increase in the movement speed leads to an increase in the effect of the change in the volume of the additional tank on the change in the value of the dynamic stiffness. Thus, at a speed of 170 km/h, the change in stiffness does not exceed 7%, and at 250 km/h, it reaches 15.7% (Table 3). A change in the initial pressure at all considered speeds of movement and volumes of additional tanks leads to a change in the dynamic stiffness of the spring by approximately 30% (Table 2). Therefore, it follows from the performed calculations that the change in the value of the dynamic stiffness is due to the change in the initial pressure in the spring. This significantly exceeds the change in stiffness due to the change in the volume of the additional tank (36.6% and 15.7% at maximum values, respectively).

It was established that the energy losses during the operation cycle of the pneumatic spring suspension system depend on the volume of the additional tank and the initial pressure in the pneumatic spring and increase as they increase. The maximum values of the damping coefficient in the entire considered range of variable parameters are 1.16–1.29. The largest values of the coefficient are observed at the maximum volume of the additional tank of 60 l in the conducted study, which is consistent

with the data obtained for energy losses during the pneumatic system operation cycle.

The results of our research made it possible to establish the influence of the volume of the additional tank and the initial pressure in the pneumatic spring on the nature of the change in the dynamic stiffness of the pneumatic spring and the damping coefficient of the pneumatic spring suspension system. This complements the studies reported in the reviewed works [8–28].

The practical significance of the results relates to the possibility of their application by engineers and researchers in the study of the impact of the pneumatic spring suspension system on dynamic and traffic safety indicators. They could also be used to determine the permissible speeds of railroad rolling stock depending on operating conditions.

One of the limitations of our research is that the dynamic parameters (stiffness and damping coefficient) of the pneumatic spring suspension system were obtained with the parameters of only one unevenness on the rail track. However, in [29–33] it is stated that the disruption of the ballast of the railroad track and vertical irregularities, which are related to the structural and operational features of the work of the track and crossings, affect the dynamic behavior of the rolling stock of the railroad. The disadvantages of our study are the lack of experimental data on the dynamic parameters of the pneumatic spring. The further development of this direction of scientific research is the study of the influence of various types of rail track irregularities on the dynamic parameters of the pneumatic spring suspension system. As well as conducting experimental studies of the pneumatic spring under the operational conditions of the rolling stock. In the future, this will make it possible to establish conditions for the safe movement of high-speed rolling stock and to reasonably choose the optimal design parameters for the pneumatic spring suspension system.

7. Conclusions

1. It was established that with a given sinusoidal vertical unevenness of the rail track, the force in the pneumatic spring and the amount of deformation of the spring differ significantly at different speeds of the carriage. Thus, at a speed of 170 km/h, the maximum force reaches 11.2 kN, at 200 km/h – 4.7 kN, at 230 km/h – 2.9 kN, and at 250 km/h – 2.2 kN. Therefore, an increase in speed from 170 to 250 km/h leads to a decrease in vertical force by 5 times. Accordingly, the deformations of the pneumatic spring are reduced from 16.2 mm at a speed of 170 km/h to 4.2 mm at a speed of 250 km/h. Increasing the volume of the additional tank from 30 l to 60 l with other constant system parameters leads to a decrease in vertical force by 6.4 % at a speed of 170 km/h and 8.2 % at a speed of 250 km/h. A change in the initial pressure in the air spring from 5 to 7 bar at a carriage speed of 200 km/h causes an increase in force by 58.5 %. This happens when the volume of the additional tank is 30 l, and by 55.2 % when the additional tank is 60 l. The increase in deformation is relatively small and its value is 5.8 % and 7.0 %, respectively, for systems with additional tanks of 30 L and 60 L.

2. An increase in the value of the initial pressure in the pneumatic spring suspension system from 5 bar to 7 bar leads to an increase in the dynamic stiffness of the pneumatic spring. This happens for all considered values of the speed of the carriage and any volume of the additional tank. Thus, with an additional tank volume of 30 l, the dynamic stiffness increases by 40.9 % at a speed of 170 km/h and by 41.1 % at a speed of 250 km/h. An increase in the volume of the additional tank mainly leads to a

decrease in the dynamic stiffness of the spring. But the nature of this dependence is influenced by the speed of the carriage. In general, fluctuations in stiffness values depending on the volume of the additional tank do not exceed 7 %. Changes in the value of dynamic stiffness due to changes in initial pressure in the spring significantly exceed changes in stiffness caused by changes in the volume of the additional tank (36.6 % and 15.7 %, respectively, when comparing the maximum differences).

3. Energy losses per cycle of operation of the air spring in the system with an additional tank volume of 60 L are significantly higher than with an additional tank of 30 L. Thus, at a speed of 170 km/h, the average value of losses for the first system is 72 % higher than for the second, and at a speed of 250 km/h, this difference is 145 %. An increase in the initial pressure in the air spring also leads to an increase in energy losses. For example, for a system with an additional 60 l tank at a speed of 170 km/h, this increase is 101 %, and at a speed of 250 km/h – 47 %. For a system with an additional tank of 60 l, the corresponding values are 146 % and 64 %. Therefore, the influence of the change in the initial pressure on the volume of losses with an additional tank volume of 30 liters is greater than for 60 liters. The damping coefficient of the pneumatic spring for all cases considered is in the range from 0.3 to 1.3. The highest values of the coefficient were observed at the maximum volume of the additional tank in the conducted study of 60 l and were 1.24 and 1.18 at speeds of 170 km/h and 250 km/h, respectively. It was found that with the volume of the additional tank in the range from 30 to 50 l, the maximum values of the damping coefficient are observed at a diameter of the connecting pipeline of 25 mm and a speed of movement from 200 to 250 km/h. And with an additional tank volume of 60 liters, they occur with a diameter of 30 mm and a speed of 170 to 250 km/h.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The manuscript has associated data in the data warehouse.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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