In this research, by using the finite element method, the effect of five parameters (density of a liquid mineral fertilizer ($\rho$), its temperature ($T$), tank wall thickness ($L$), spacing of stiffeners ($K$) and stiffeners height ($h$)) on the strength of standard polyethylene rotomolded tanks used for storage of liquid mineral fertilizers (LMF) was studied. Using the Taguchi method, it was found that these parameters are ranked according to the degree of their influence (in decreasing order) on: maximum stresses ($\rho > L > h > T > K$), maximum stresses in the tank walls ($\rho > L > K > T > h$) and deformations of the tank ($D_X/D_Y: \rho > L > h > T > K$ and for $D_Z: \rho > L > h > K > T$).

Validation of the FEM strength calculations was carried out, which showed satisfactory convergence of the calculated and experimental values. Generalized equations are derived that describe the effect of all five studied parameters on $P$, $PW$ and tank deformations (along the X, Y and Z axes).

On the basis of the derived equations, a nomogram has been constructed, which makes it possible to choose the optimal wall thickness that will correspond to the LMF density and storage temperature. Applying the optimal wall thickness ensures a guaranteed service life of at least 50 years, minimizing the risk of environmental accidents caused by tank failure and the release of LMF and associated toxic substances into groundwater. This research offers valuable insights for designing safer and more durable storage tanks for liquid mineral fertilizers. As an optimal design of the tank for storing the most common fertilizer UAN-32 (Urea Ammonium Nitrate, 32 % nitrogen), with a density of 1.32 g/cm$^3$ and at storage temperatures up to 40 °C, the following values of structural parameters are recommended: $L=10$ mm, $K=38$ mm, and $h=4$ mm.

Keywords: polyethylene tanks, finite element method, deformations, strength calculation, Taguchi method

1. Introduction

Currently, farmers in most countries of the world use liquid mineral fertilizers (LMF) when growing various crops (wheat, potatoes, etc.). For their storage, as a rule, plastic tanks (with a volume of 4,500 to 20,000 liters) manufactured by rotational molding are used [1, 2]. However, they are usually used for up to 7 years, and then they are destroyed (thin plastic walls crack in places of maximum stress). This problem is acute because currently toxic pesticides are often added to LMF; when the tank is destroyed, they contaminate the fertile soil layer [3, 4]. It should also be noted that such toxic substances can poison groundwater [5, 6]. The destruction of these tanks (wall cracking) is mainly caused by two reasons: high stresses in their walls (which exceed the allowable ones, providing long-term hydrostatic strength, determined by the standard ISO 9080:2003 Plastics piping and ducting systems – Determination of the long-term hydrostatic strength of thermoplastics materials in pipe form by extrapolation) and internal defects in their walls (microbubbles and thermal destruction of the material) [7]. In addition, the density of LMF ranges from 1.1 to 1.9 g/cm$^3$, which is significantly higher than the density of water (1 g/cm$^3$), and some unscrupulous manufacturers of tanks make them to store water (saving plastic on the wall thickness), and then sell to farmers for the storage of LMF.
In India, they tried to solve this problem by introducing a mandatory standard, which strictly regulates the wall thickness of tanks. However, practice shows that this is not enough. In addition to these standards (IS 12701 (1996); rotational moulded polyethylene water storage tanks), there are other methods for calculating the stresses of thin-walled PE tanks made by rotational molding. One of such methods is the momentless theory of shells [8–10], however, it does not take into account the influence of torsional and bending moments, as well as transverse forces of the stress-strain state on the strength of tanks. Also, as an example, we can cite the methodology presented in the American standard ASTM D1998-06. «Standard Specification for Polyethylene Upright Storage Tanks». This technique has proven effective and adequate over a long period of time, but it does not consider such important tank geometry parameters as the height of the stiffeners, their spacing, and others.

These parameters must be taken into account when optimizing the tank design, due to their significant influence on deformation and maximum stresses. Such miscalculations are unacceptable in production conditions, since this reduces the life of the polyethylene tank and causes premature cracking, which in turn can harm the environment due to the stored LMF. In addition to environmental damage, the reputation of the manufacturing company will also suffer. Therefore, the selection of accurate calculation methods and optimal models for optimizing the design of polyethylene tanks manufactured by rotational molding is relevant. The results of these studies will be useful in production, since based on them, design engineers (technologists) will be able to select optimal design parameters, depending on temperature conditions, as well as the density of LMF, thereby optimizing the strength of the polyethylene tank and ensuring the required service life.

2. Literature review and problem statement

FEM is used to simulate various technological operations of rotational molding [11–13]. In [11], a nonlinear axisymmetric FEM model for heat transfer and powder deposition in rotational molding is presented. The model uses the Lagrange-Eulerian method to track the gradual growth of the plastic layer. The results obtained using this approach compare well with previously used one-dimensional models and experimental data. In [12], FEM is used to simulate the rotational molding process, including the multi-layer sliding model, phase change and distortion, which allows the analysis of complex physical processes. In [13], this method is used to analyze the contact fatigue of support roller in rotational molding equipment, which can predict potential problems with the durability of equipment and propose solutions to eliminate them. The FEM method is actively used to model rotomolded products and their properties [11–13]. However, there are no examples of using FEM to study the joint effect of such parameters as wall thickness, spacing of stiffeners and height of stiffeners on the strength of tanks.

Examples of the use of the finite element method in calculating the strength of PE tanks are the German standard DVS 2205-1-2015 Calculation of tanks and apparatus made of thermoplastics – Characteristic values, as well as the technique described in [14]. This study proposed a computer model based on FEM and substantiated critical loads on the tank walls. Load simulation is made by applying force to the nodes of the upper plates of the tank wall. This made it possible to compare the obtained form of buckling with the theoretical form under axial compression. The DVS 2205 standard provides tank and reservoir calculations based on FEM. This type of analysis allows you to predict the behavior of tanks in a real environment and during operation by virtual simulation and testing of CAD models. Neither the work [14] nor the DVS 2205-1-2015 standard considers the geometric parameters of the stiffeners as parameters for optimizing the design of the PE tank.

In [15–18], the FEM method is used to study stresses and deformations in plastic tanks made by rotary molding. In [15], the FEM method in the ANSYS program studied tanks made by rotary molding from two materials: polypropylene and HDPE. It was found that these materials, due to their different mechanical properties (density, Young’s modulus, Poisson’s ratio, yield strength and tensile strength), provide various deformations and stresses in the manufacture of tanks. In [16], two-layer tanks made by rotary molding were studied by the FEM method. Three types of polyethylene (LLDPE, LDPE, HDPE) were used as optimization parameters for each of the two layers. The authors found that the optimal material for the manufacture of double-layer tanks with a volume of 5,000 liters is a combination of LLDPE (outer layer) and HDPE (inner layer). In [17], the strength characteristics of polyethylene tanks made by rotary molding were studied. Using the engineering theory of bending of inhomogeneous layered walls, the bending stiffness values of the walls of underground tanks of various designs made of LLDPE have been determined. The optimal design of the tank walls was determined by the FEM method, where the two outer layers are made of MICROLEX RM 1242 WT LLDPE, and the inner layer is represented by a foamed polyethylene structure. The optimal ratio of the outer layer thickness to the total wall thickness is 18.9 %. In [18], using FEM and experimental results of a typical accelerated flow test, conclusions were drawn about the long-term operation characteristics of products made by rotary molding from two materials LLDPE and polypropylene. In the above-mentioned works [15–18], the authors did not optimize the tank structures (wall thickness and geometric parameters of stiffeners), except [17], where the optimal wall parameters (ratio of layers thicknesses) of an underground tank were determined. However, as the experience of rotary molding shows, the strength of tanks can be significantly influenced by various geometric parameters (wall thickness, pitch and height of stiffeners).

An option to overcome existing difficulties is to use, along with the Finite Element Method, appropriate mathematical optimization methods, such as the probabilistic deterministic planning (PDP) method and the Taguchi method [19–21], which are able to determine the optimal geometric parameters of tanks with a minimum number of experiments. The Taguchi method is a very popular optimization tool in various fields of science research. The PDP method [22, 23] is less popular worldwide and applied mostly for mathematical modeling, though could be used for optimization as well. For example, in [19], thanks to the Taguchi method, the composition of silicone enamel was optimized. This method was also used to identify the most influential factors and achieve the desired product quality. Thus, in [20], the bending strength of hybrid composite materials was studied. In [21], the Taguchi method was applied to study the machinability of aluminum metal matrix composites reinforced with copper. In [22], the PDP method was used to mathematically
simulate the effect of surfactants on the dispersion of acrylic resins used to coat oil well equipment. The resulting mathematical model allowed determining the optimal values of technological parameters to improve the quality of the coating. In the study [23], the PDP method was used to mathematically simulate the effect of surfactants on the wetting of titanium dioxide in alkyd paints and varnish materials. This method helped to determine optimal conditions for improving wetting and coating properties. In the above works [19–23], the Taguchi method and the PDP method were not used in conjunction with FEM and the effect of input parameters on output ones was determined during active field experiments at the physical and chemical level. Physical and chemical field experiments often turn out to be much more expensive than computer simulations using FEM. Thus, using FEM together with such methods as Taguchi and PDP allows studying the joint effect of such parameters as wall thickness, spacing of stiffeners and height of stiffeners on the strength of plastic tanks without large expenditure of resources (including time).

The feasibility of using the Taguchi method and PDP in this study is due to two reasons. The first reason is that the vast majority of manufacturers of storage tanks for LMF do not have specialists able to carry out the FEM calculation of tanks. The second reason is the high labor intensity of performing FEM calculations in CAD programs due to the need to build 3D models of PE tanks. For example, taking into account three parameters (varying on four levels), such as wall thickness, spacing of stiffeners and stiffeners height, 64 models will need to be built. Therefore, by deriving equations and nomograms that describe the mutual influence of these parameters on tank strength, this research empowers engineers in plastic tank production to select designs that optimize structural strength and ensure a longer operational lifespan.

3. The aim and objectives of the study

The aim of the study is to optimize the design of a polyethylene tank for storing liquid mineral fertilizers using the Taguchi method, which will minimize the environmental risks associated with LMF storage by increasing the durability of tanks (up to 50 years).

To achieve this aim, the following objectives are accomplished:
- to carry out strength calculations of a standard polyethylene tank using FEM;
- to validate the results of the FEM calculation for the strength of tanks;
- to identify by the Taguchi method the most significant parameters affecting strength and deformation, as well as the optimal values of the input parameters;
- to develop a mathematical model of the influence of input parameters on maximum stresses and deformations in the tank under study (including its walls).

4. Materials and methods

4.1. The object and hypothesis of the study

The object of the study is a standard tank with a volume of 10 m³ for storing LMF (Fig. 1).

The hypothesis of the study: FEM calculations together with the use of Taguchi and PDP methods make it possible to optimize the design of a polyethylene tank taking into account operating conditions.

Tank material – high-density polyethylene (HDPE) Lupolen 4021 KRM with the following mechanical properties: density – 939.5 kg/m³, modulus of elasticity – 750 MPa, yield strength – 19 MPa (variable parameter depending on the temperature of the stored fertilizer), Poisson’s ratio – 0.45, the typical value for this class of polymers is accepted. The tank diameter is 2,200 mm, height is 2,920 mm.

4.2. Experimental design

In this paper, optimization was carried out in two ways. The first method is the Taguchi method, which is widely known and popular in the international scientific community. The second method is the method of probabilistic deterministic planning (PDP), which has found application in the works of scientists from the countries of the former USSR. The PDP method is rather a method of mathematical modeling, which, among other purposes, can be used for optimization. The Taguchi method and the PDP method have certain similarities and some differences. Both methods include the following steps [22]:

1. Selection of input parameters (factors) affecting the process under study and their levels of variation. In our case, it is necessary to obtain a PE tank with minimum values of such output parameters as: maximum stress in the tank (\(P, \text{MPa}\)), maximum stress in the tank walls (\(PW, \text{MPa}\)) and maximum deformations (modulo) along the X, Y and Z axes (\(DX, DY, \text{and} \ DZ, \text{mm}\)).

During the research, PE tanks were designed in which the following were varied:
1) fluid density (hereinafter \(\rho\)) from 1.00 to 1.90 g/cm³;
2) fluid temperature (hereinafter \(T\)) from 20 to 60 °C;
3) wall thickness (hereinafter \(L\)) from 7 to 10 mm;
4) spacing of stiffeners (hereinafter \(K\)) from 32 to 56 mm;
5) stiffeners height (hereinafter \(h\)) from 2 to 8 mm.

Table 1 shows the factors under consideration and their corresponding levels [22].

<table>
<thead>
<tr>
<th>Table 1: Factors under consideration and their levels</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factor</strong></td>
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<tr>
<td>Fluid density ((\rho))</td>
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<tr>
<td>Fluid temperature ((T))</td>
</tr>
<tr>
<td>Wall thickness ((L))</td>
</tr>
<tr>
<td>Spacing of stiffeners ((K))</td>
</tr>
<tr>
<td>Stiffeners height ((h))</td>
</tr>
</tbody>
</table>

4.3. The object and hypothesis of the study

The object of the study is a standard tank with a volume of 10 m³ for storing LMF (Fig. 1).

The hypothesis of the study: FEM calculations together with the use of Taguchi and PDP methods make it possible to optimize the design of a polyethylene tank taking into account operating conditions.
by using special formulas to dimensionless signal-to-noise method, the values of the output parameters are normalized presented in kind using SI units of measurement, in the Taguchi appears. In the PDP method, the output parameters are pre developed plan-matrix and determining the numerical values of the response function (output parameter). At this stage, the number of experiments is 25. In our case, five factors varied on four levels (Tables 2, 3).

2. Conducting an experiment plan in the form of a plan-matrix consisting of m columns corresponding to the number of input parameters (factors), and n rows corresponding to the number of variations of the specified levels (numerical values) of factors. To ensure the orthogonality of the matrix plan, each level of one input parameter is set only once with each level of another input parameter. The matrix plan is usually based on a Latin or Greek-Latin square. The number of factors should be no more than one more than the number of levels of variation, i.e. with the number of levels equal to 3, the influence of no more than 4 factors can be studied. In this case, the total number of experiments is equal to the square of the number of levels, i.e. if there are 3 levels, then the number of experiments is 9, if the number of levels is 5, then the number of experiments is 25. In our case, five factors varied on four levels (Tables 2, 3).

3. Conducting an active experiment according to the developed plan-matrix and determining the numerical values of the response function (output parameter). At this stage, the first difference between the Taguchi and PDP methods appears. In the PDP method, the output parameters are presented in kind using SI units of measurement, in the Taguchi method, the values of the output parameters are normalized by using special formulas to dimensionless signal-to-noise ratios. In the Taguchi method, three different functions are used to find the signal-to-noise ratio (S/N): «the-smaller-the-better» (SB), «the-larger-the-better» (LB), or «nominal-the-best» (NB). For each of the output parameters, one of three equations (SB, LB or NB) can be used, depending on the optimization objective function (1)–(3) [22]:

\[
S_{N_S} = \frac{1}{n} \sum_{i=1}^{n} y_i
\]

\[
S_{N_L} = -10 \log_{10} \left( \frac{1}{n} \sum_{i=1}^{n} y_i \right)
\]

\[
S_{N_N} = -10 \log_{10} \left( \frac{1}{n} \sum_{i=1}^{n} (y_i - y_0)^2 \right)
\]

where \( n \) is the number of test runs, \( y_i \) is the measured values of the output parameter, and \( y_0 \) is the desired nominal value of the output parameter.

The highest signal-to-noise ratio ensures an optimal level of quality for each output parameter with minimal variance. In this study, the L16 matrix was used, and for the Taguchi method, the signal-to-noise ratio (S/N) was used as a characteristic of the selection quality. To minimize the values of the output parameters, we used the SB («the-smaller-the-better») function (1) [23].

The plan-matrix of experiments and its results (output parameters) in normal form are shown in Table 2. The S/N ratios for the output parameters were calculated according to (1), as shown in Table 3.

4. Sampling each response function for each level of each factor and plotting the corresponding partial dependencies. This stage is completely the same for the Taguchi method and the PDP method, taking into account that at the previous stage the response functions for these methods were defined differently.

Table 1

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
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<td>( \rho, \text{g/cm}^3 )</td>
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<td>1.70</td>
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<tr>
<td>( T, ^\circ \text{C} )</td>
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<td>60</td>
</tr>
<tr>
<td>( L, \text{mm} )</td>
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<td>9</td>
<td>10</td>
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<td>( h, \text{mm} )</td>
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</table>

Table 2

<table>
<thead>
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<th>Experiment</th>
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<th>Output parameters</th>
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<td>( T, ^\circ \text{C} )</td>
</tr>
<tr>
<td>1</td>
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</tr>
<tr>
<td>16</td>
<td>1.90</td>
<td>60</td>
</tr>
</tbody>
</table>
5. Determining the optimal values of the factors for each of the response functions based on graphs of partial dependencies. At this stage, the response functions presented in kind are evaluated in the PDP method. Accordingly, both the minimum and maximum on the graph can be optimal, depending on the optimization goals. In the Taguchi method, the response functions are presented in a normalized form, so the optimum value corresponds to the maximum on the graph of the partial dependence.

The final stage of optimization by the Taguchi method is to determine for each response function the most influential factors (ranking of factors), the change of which in the studied range of variation has the greatest change in the value of the response function. The final stage of the PDP method is the approximation of graphically represented partial dependencies by mathematical equations and the construction of a multifactorial mathematical model based on them for each of the response functions.

To derive a multifactorial statistical mathematical model of the effect of input parameters on each output parameter, (4) was used:

$$ Y_s = \prod_{i=1}^{n} Y_i, $$

where $Y_s$ is the generalized equation; $Y_i$ is the partial function; $\prod_{i=1}^{n} Y_i$ is the product of all partial functions; $n$ is the number of partial functions equal to the number of input parameters; and $Y_{i_0}$ is the total average of all the considered values of the generalized function to a degree one less than the number of partial functions.

The reliability of the obtained mathematical model is determined by calculating the coefficient of nonlinear multiple correlation [7]:

$$ R = \sqrt{1 - \frac{(n-1) \sum_{i=1}^{n} (y_i - \bar{y})^2}{(n-p-1) \sum_{i=1}^{n} (y_i - \bar{y})^2}}, $$

where $n$ is the number of experiments; $p$ is the number of input (independent) parameters; $i$ is the serial number of the experiment; $y_i$ is the actual value of the output parameter in the $i$ experiment; $\bar{y}$ is the calculated value of the output parameter, calculated using a multi-factor mathematical model, for the conditions (values of input parameters) of the $i$ experiment; and $\bar{y}$ is the average value of the actual value of the output parameter for all $n$ experiments (the general average).

### 4.3. Method of calculating the strength properties of tanks in the Femlab package with the Simcenter Nastran solver

The method of calculating the strength properties of the tank includes the following seven steps (a detailed description is provided in Supplementary materials 1):

- import of the required model geometry into Parasolid;
- creation of all materials used in the calculation with their physical and mechanical properties (density, modulus of elasticity, yield strength, Poisson’s ratio);
- construction of a finite element grid based on edited geometry;
- setting the load (hydrostatic pressure and own weight of the tank) and their impact directions;
- fixation of the lodgment nodes in all degrees of freedom;
- calculation of the finite element model in the Simcenter Nastran solver;
- analysis of the results under static load.
5. Results of research on optimizing the design of a polyethylene tank

5.1. Results of calculation for the strength of tanks

The results of the tank strength FEM calculation are shown in Table 2, columns 7 to 11 (‘Output parameters’) and in Fig. 2 (experiment 1 Table 2). The results of other experiment 2–16 Table 2 calculations are given in Supplementary materials 2.

The results of the FEM strength calculation of a standard 10 m$^3$ polyethylene tank in the Simcenter Femap with Nastran program (at a liquid fertilizer density of 1,000 g/cm$^3$, a temperature of 20 $^\circ$C, a wall thickness of 7 mm, a spacing of stiffeners of 32 mm and a stiffeners height of 2 mm).

5.2. Validation of the results of the finite element method calculation for the strength of tanks

To confirm the correctness of calculations for the strength of tanks, we carried out their validation. Validation consisted in the FEM strength calculation of the existing (standard) tank design with the determination of its wall deformations. At the same time, in the calculation, the density of the liquid mineral fertilizer was indicated equal to 1 g/cm$^2$ and the ambient temperature was 30 $^\circ$C. Then, water (with a density of 1 g/cm$^3$) was poured into an existing (standard) tank with a volume of 10 m$^3$. After holding for 2 hours at an ambient temperature of 30±1 $^\circ$C, the deformations of its walls were determined with a standard measuring tape with accuracy class 2 according to state standard GOST 7502-98 Metal measuring tapes. Specifications (Fig. 3).

---

**Fig. 2.** Results of the FEM calculation: a – stresses in the tank; b – stresses in the most loaded places; c – deformations along the X and Y axes; d – deformations along the Z axis

**Fig. 3.** Validation of the FEM calculation: a, c, d – the process of measuring the perimeter of the tank; b – marking the measurement points
As a result, the following 3D models (Fig. 4) of a standard 10 m³ polyethylene tank (at a density of liquid mineral fertilizer 1000 g/cm³, temperature 30 °C, wall thickness 9 mm, spacing of stiffeners 32 mm and stiffeners height 4 mm) and its filling factor 100 % were built.

Using FEM, the change in the radius of the empty (Fig. 5) and full tank (Fig. 6) in each of the 43 sections was calculated. Deformations obtained by measurements (when filling the tank with water) correlate with the calculated values obtained by FEM (Fig. 4). The difference is no more than ±3 %. We consider this error to be acceptable.

Therefore, the performed tank strength FEM calculations in Simcenter Femap with Nastran are correct (Fig. 5, 6).

5.3. Results of optimization of the PE tank parameters by the Taguchi method

The average values of the signal-to-noise ratio for each input parameter level for $P$ are shown in Table 4, for $PW$ – in Table 5, for $DX$ and $DY$ – in Table 6, and for $DZ$ – in Table 7.

The effects of the input parameters on the S/N ratio of the output parameters are shown in Fig. 7, $a$–$e$.

![Fig. 4. Validation of the FEM calculation:](image)

**Fig. 4. Validation of the FEM calculation:**

$a$ – the view of the real tank after filling it with water; $b$ – 3D model of the tank deformation after filling it with water, calculated in the Femap program

![Fig. 5. Graphs showing the distribution of the radius values of the empty tank in each of its 43 sections](image)

**Fig. 5. Graphs showing the distribution of the radius values of the empty tank in each of its 43 sections**

![Fig. 6. Graphs showing the distribution of the radius values of the tank filled with water in each of its 43 sections](image)

**Fig. 6. Graphs showing the distribution of the radius values of the tank filled with water in each of its 43 sections**

### Table 4

<table>
<thead>
<tr>
<th>Level</th>
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### Table 5

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### Table 6

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### Table 7

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According to Tables 4–7, it is possible to determine the parameters that most affect the maximum stresses and deformations of the tank when storing various LMF in it. Using the graphical dependencies shown in Fig. 6, it is possible to determine the optimal values of the input parameters.

5.4. Influence of input parameters on maximum stresses and deformations of PE tanks

Fig. 8, a–e shows the influence of \( p, T, K, L, h \) on the maximum stresses \((P\) and \( PW)\). Fig. 9, a–e demonstrates the influence of the above parameters on the deformations of a PE tank \((DX, DY, DZ)\).

Using the obtained partial dependencies, shown in Fig. 8, 9, multifactor mathematical models (6)–(10) were built based on the generalized equation (4):

\[
P = \frac{(12.211 - p \cdot 1.864)(0.0673 - T + 13.535)(-2.1955 - L + 34.87)}{(-0.0075 - K^2 + 0.7338 - K - 0.9997)(-0.2197 - h^2 + 2.3641 - h + 10.979)} - 16.21^4
\]

\[
PW = \frac{(3.6798 - p \cdot 0.2967)(0.0119 + T + 4.6781)(-0.4393 - L + 8.883)}{(-0.0013 - K^2 + 0.1325 - K + 1.8991)(-0.052 - h^2 + 0.5347 - h + 4.0369)} - 5.15^4
\]

where \( P \) – maximum stress in the tank, MPa; \( PW \) – maximum stress in the tank walls, MPa; \( DX, DY, \) and \( DZ \) – maximum deformations (modulo) along the \( X, Y \) and \( Z \) axes, respectively; \( p \) – fluid density, \( \text{g/cm}^3 \); \( T \) – fluid temperature, \( ^\circ\text{C} \); \( L \) – wall thickness, mm; \( K \) – spacing of stiffeners, mm; \( h \) – stiffeners height, mm.

The reliability of the obtained mathematical models was estimated by calculating the coefficients of nonlinear multiple correlation. The minimum coefficient of nonlinear multiple correlation among the proposed mathematical models is 0.953.

This nomogram is intended for a standard tank with a volume of 10,000 liters (used by most of manufacturers) with values \( K = 38 \text{ mm} \) and \( h = 4 \text{ mm} \) (Fig. 10).
This nomogram will allow specialists without experience in calculations and design of tanks using the FEM method to choose the most optimal wall thickness of a PE tank for storing liquid fertilizers, depending on the requirements of farmers (fertilizer density and temperature), while ensuring the necessary guaranteed service life of the tank.

6. Discussion of the results of standard PE tank design optimization

Based on the analysis of the results obtained and graphically presented in Fig. 8, 9, the following features can be distinguished:

- with an increase in the density of the liquid \( p \) from 1.0 to 1.9 g/cm\(^3\), the maximum stress in the tank \( P \) increases approximately by 2.2 times (from 9.84 to 21.50 MPa), and the maximum stress in the tank walls \( PW \) – by about 2 times (from 3.30 to 6.73 MPa) (Fig. 8, a);
- with an increase in temperature from 20 °C to 60 °C, the maximum stress in the tank \( P \) increases by 21 % (from 14.56 to 15.78 MPa), and the maximum stress in the tank walls \( PW \) – by 12 % (from 4.84 to 5.42 MPa) (Fig. 8, b);
- with an increase in the tank wall thickness \( L \) from 7 to 10 mm, the maximum stress in the tank \( P \) decreases by 33 % (from 19.09 to 12.75 MPa), and the maximum stress in the tank walls \( PW \) – by 22 % (from 5.73 to 4.46 MPa) (Fig. 8, c);
- in the range of spacing of stiffeners values \( K \) from 32 to 56 mm, an optimum \( P = 14.89 \text{ MPa}, PW = 4.85 \text{ MPa} \) is observed at \( K = 32 \text{ mm} \) (Fig. 8, d);
- considering the effect of the stiffeners height \( h \) on the maximum stresses in the tank \( P \) and in the tank walls \( PW \), the minimum values \( P = 14.81 \text{ MPa}, PW = 4.90 \text{ MPa} \) are observed at \( h = 2 \text{ mm} \) (Fig. 8, e);
- with an increase in the density of the liquid \( p \) from 1 to 1.9 g/cm\(^3\), the deformation along the \( X \) and \( Y \) axes \( (DX/ DY) \) increases by approximately 2.2 times (from 7.72 to 16.90 mm), and along the \( Z \) axis \( (DZ) \) – by about 2.1 times (from 9.08 to 7.21 mm) (Fig. 8, e).

As a result of the calculations, it was found that the maximum stress in the tank \( P \) is influenced by all the studied parameters from Tables 4–7. However, the most significant parameter is the liquid density \( p \) (Table 4, Fig. 8, a). All studied parameters according to the degree of influence on \( P \) can be arranged in a row (in decreasing order): \( p \geq L > h > T > K \).

For the maximum stress in the walls \( PW \), the most significant parameter of influence is also \( p \) (Table 5, Fig. 8, a). However, there are changes in the series of significance of the influence of the studied parameters, i.e. parameter \( K \) (spacing of stiffeners) (Fig. 8, d) became more significant in comparison with \( T \) (temperature) (Fig. 8, b) and \( h \) (stiffeners height) (Fig. 8, e).

The influence of the geometric parameters of the stiffeners (height and pitch of the stiffeners) on the strength of a standard polyethylene tank has been determined. Such studies have not previously been presented in the scientific literature.
It has been shown that the strength of a standard polyethylene tank manufactured by rotational molding is, to a large extent, determined by the wall thickness. This correlates with the work [17], which shows that to ensure the long-term stability of a polyethylene tank, it is necessary to guarantee the required bending rigidity of the walls. Guaranteed bending rigidity for tank walls can be achieved by wall thickness, the use of stiffeners or the use of foam structures.

The deformation of the tanks along the X and Y axes is mainly determined by two parameters – the density of LMF (ρ) (Table 6, Fig. 9, a) and the wall thickness (L) (Table 6, Fig. 9, c).

The studied parameters have different degrees of influence on deformations and can be arranged in a row (in order of decreasing influence): \( p > L > h > T > K \). However, there is one difference – the influence of the spacing of stiffeners K (Fig. 9, d) becomes more significant for the Z axis in comparison with the temperature of the medium (T) (Fig. 9, b).

Preliminary calculations based on the proposed nomogram (Fig 10) will ensure minimum stresses in the walls of tanks manufactured by rotational molding. As a result, the risks of environmental disasters caused by the ingress of liquid mineral fertilizers and toxic pesticides dissolved in them into groundwater during the destruction of PE tanks are minimized.

The results of these calculations are consistent with the IS 12701 (1996) standard: «Rotational moulded polyethylene water storage tanks: Sanitary Appliances and Water Fittings». According to Table 1, the minimum wall thickness for a 10,000-liter tank with an LMF density of 1 g/cm\(^3\) should be 11.5 mm. And according to our nomogram, at a temperature of 60 °C (standard for India) and a density of 1 g/cm\(^3\), the wall thickness should be at least 11.2 mm. The deviation is 2.6 %.

The results of this study are applicable to standard polyethylene tanks for storing liquid mineral fertilizers with a density of up to 1.9 g/cm\(^3\). However, in agronomic practice, liquid mineral fertilizers with a density of over 1.9 g/cm\(^3\) are sometimes used, mainly presented in the form of suspensions. The derived mathematical dependencies and nomogram are not applicable for the design of tanks for their storage.

Due to the complexity of the problem of taking into account the joint effect of storage time, chemical composition and temperature of liquid mineral fertilizers on the strength of rotational polyethylene (caused by its degradation), a logical continuation of the work is to conduct additional studies (taking into account the contribution of storage time, chemical composition and temperature of liquid mineral fertilizers to the degradation of PE).

7. Conclusions

1. In the Simcenter Femap program, calculations of the strength of a standard polyethylene tank with a volume of 10 m\(^3\) were made and analyzed. The modeling took into account the density of liquid fertilizers (1,000, 1,320, 1,700, 1,900 kg/m\(^3\)), ambient temperature (20, 33, 46, 60 °C), wall thickness (7–10 mm), distance between stiffeners (32, 48, 56 mm), as well as the height of the stiffeners (2–8 mm). It was revealed that the maximum stresses in the tank structure (\( P = 27.7 \) MPa) were identified with the following combination of the studied parameters (\( L = 7 \) mm, \( K = 48 \) mm, \( h = 4 \) mm, \( p = 1.9 \) g/cm\(^3\) and \( T = 60 \) °C). The minimum stresses (\( P = 8.44 \) MPa), characteristic of the tank design, were identified with the following combination of the studied parameters (\( L = 7 \) mm, \( K = 32 \) mm, \( h = 2 \) mm, \( p = 1.00 \) g/cm\(^3\) and \( T = 20 \) °C).

2. Using FEM, the effect of five parameters (density of liquid mineral fertilizer (\( p \)), its temperature (\( T \)), tank wall thickness (\( L \)), spacing of stiffeners (\( K \)) and stiffeners height (\( h \))) on the strength of a standard polyethylene storage tank for liquid mineral fertilizers was studied. Using the Taguchi method, these parameters were ranked according to the degree of their influence (in decreasing order) on: maximum stresses (\( p > L > h > T > K \)), maximum stresses in the tank walls (\( p > L > K > T > h \) and deformations of the tank (for \( DX/DY: p > L > h > T > K \) and for \( DZ: p > L > h > K > T \)).

It is shown that an increase in liquid density (\( p \)) and temperature causes an increase in maximum stress in the tank (\( P \)) and stress in the tank walls (\( PW \)). An increase in \( p \) (from 1 to \( 1.9 \) g/cm\(^3\)) causes an increase in \( P \) by \( 116 \) %, and in \( PW \) by \( 103 \) %, while with an increase in temperature from 20 °C to 60 °C, \( P \) increases by 21 %, and \( PW \) – by 12 %. An increase in the tank wall thickness (\( L \)) causes a decrease in stress; thus, when increasing \( L \) from 7 to 10 mm, \( P \) decreases by 33 %, and \( PW \) – by 22 %. It was found that the minimum values of the stresses \( P \) and \( PW \) are observed at the values of \( K = 32 \) mm, and \( h = 2 \) mm.

An increase in \( p \) and temperature causes an increase in deformations in the walls (\( DX/DY, DZ \)). An increase in \( p \) (from 1 to \( 1.9 \) g/cm\(^3\)) causes an increase in \( DX/DY \) by \( 119 \) %, \( DZ \) – by \( 112 \) %, and with an increase in temperature from 20 °C to 60 °C, \( DX/DY \) increases by \( 20 \) %, and \( DZ \) – by 13 %. An increase in the tank wall thickness (\( L \)) causes a decrease in deformations along the spatial coordinate axes, so when increasing \( L \) from 7 to 10 mm, \( DX/DY \) decreases by 33 %, and \( DZ \) – by 23 %. It was found that the minimum values of deformations (\( DX/DY, DZ \)) are observed at a value of \( K = 32 \) mm, and \( h = 2 \) mm.

3. Using the method of probabilistically deterministic planning, generalized equations are derived that describe the effect of all five studied parameters (\( p, L, h, T \) and \( K \)) on maximum stresses, maximum stresses in the tank walls and deformations of tanks (along the three axes \( X, Y \) and \( Z \)). On the basis of the derived equations, a nomogram was constructed, which allows specialists without FEM design skills to choose the optimal wall thickness of polyethylene tanks (\( L \)) for storing LMF, taking into account operating conditions (\( p, T \)). At the same time, the minimum values of stresses in the tank walls are provided, which increases the operational life of the product to the required value (up to 50 years). As a result, the risks of environmental disasters caused by the ingress of liquid mineral fertilizers and toxic pesticides dissolved in them into groundwater during cracking of rotomolded tanks are minimized.

4. As an optimal design of the tank for storing the most common fertilizer UAN-32 (Urea Ammonium Nitrate, 32 % nitrogen), with a density of 1.32 g/cm\(^3\) and at storage temperatures of no more than 40 °C, the following values of structural parameters are recommended: \( L = 10 \) mm, \( K = 38 \) mm, and \( h = 4 \) mm. Multifactor mathematical models based on the results of engineering FEM calculations were derived. The basis of each mathematical model is the product of partial dependencies expressed by polynomials of the first and second order. This presentation format is intuitive for process engineers due to its simplicity and the possibility to calculate optimal values of output parameters using an ordinary calculator.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal,
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**Data availability**

The manuscript has data included as electronic supplementary material.

**References**