

The paper considers a parallel electromagnetic oscillating circuit with a nonlinear inductance under conditions of excitation of ferroresonances. Physical mechanisms of dynamic self-regulation of the system of spin magnetic moments of ferromagnetic and ferrimagnetic dielectrics in the surrounding magnetic field are established. It is shown that upon entering the magnetic saturation regime, the effect of dynamic antiferroresonance is observed, due to the cyclic reversal of magnetization in the internal magnetic field. This effect has a collector character and corresponds to the maximum potential energy of the magnetic moment in the field and the antiparallel orientation of the moment with respect to the field. Such regimes are realized in thermodynamically non-equilibrium conditions and are correlated with unstable equilibrium positions of the corresponding mechanical analogues. The resulting forms of oscillations correlate with the dynamics of an inverted pendulum and have a significantly non-harmonic character. Autosynchronization of frequencies and nonlinear mixing of such forms with quasi-static modes imitating the time form of external excitation of oscillations were revealed. It is shown that the nonlinearity of ferromagnetic elements self-limits the height of resonance maxima. And on the other hand, it contributes to the cascade transport of energy by the spectrum of disturbances, which can have negative consequences when exciting low-frequency ferroresonances in power grids. The effect of dynamic antiferroresonance has the opposite direction to the known quasi-static behavior of the system of spin magnetic moments in an external field and must be taken into account when calculating and operating electrical systems with nonlinear inductances. Examples of collector self-oscillating modes similar in their physical mechanisms in nonlinear contact tribodynamics systems are given

Keywords: ferroresonance, oscillating circuit, magnetization dynamics, collector system, magnetic moment, contact tribodynamics

ESTABLISHING MECHANISMS FOR NONLINEAR COLLECTOR TRIBODYNAMICS OF MAGNETIZATION UNDER FERRORESONANCE REGIMES

Yuriy Zasp

PhD, Associate Professor

Department of Physics and Electrical Engineering*

Aleksandr Dykha

Corresponding author

Doctor of Technical Sciences, Professor, Head of Department

Department of Tribology, Automobiles and Materials Science*

E-mail: tribosenator@gmail.com

Serhii Matiukh

PhD, Associate Professor

Rector*

Maksym Dykha

PhD, Associate Professor

Department of Automation and Computer Integrated Technologies*

Oleksandr Lytvynov

PhD Student

Department of Tribology, Automobiles and Materials Science*

*Khmelnyskyi National University

Instituts'ka str., 11, Khmelnytskyi, Ukraine, 29016

Received date 29.02.2024

Accepted date 16.05.2024

Published date 28.06.2024

How to Cite: Zasp, Y., Dykha, A., Matiukh, S., Dykha, M., Lytvynov, O. (2024). Establishing mechanisms for nonlinear collector tribodynamics of magnetization under ferroresonance regimes. *Eastern-European Journal of Enterprise Technologies*, 3 (5 (129)), 15–24. <https://doi.org/10.15587/1729-4061.2024.304327>

1. Introduction

Research into nonlinear ferroresonance processes is stimulated primarily by the tasks related to the electric power industry in which abnormal ferroresonance overvoltage and overcurrent cause emergency shutdowns and damage to expensive equipment. Despite the wide range of numerical-analytical and graph-analytical methods for preventing this kind of regimes, the number of accidents in this field is constantly increasing, which is why further study of the physical mechanisms underlying ferroresonance excitations remains relevant. The complexity of power grids explains the formalization of their calculations based on Kirchhoff's quasi-static laws for electric and magnetic circuits. However,

these laws do not sufficiently take into account wave processes in nonlinear ferromagnetic elements, and the calculations are based on quasi-static magnetization curves and the corresponding Weber-ampere characteristics. In the context of Maxwell's equations, this approach corresponds to the choice of electric and magnetic field strengths as a basis. An alternative record of material relations for closing the system of Maxwell's equations due to the induction (and not due to the intensity) of the magnetic field is practically not used. Meanwhile, the very large indices of magnetic permeability of ferromagnets bring to the fore the induction of the magnetic field in the material, the nonlinear dynamics of which, in fact, predetermines the overvoltage and overcurrent induced. Therefore, research into the development of such an alternative is a relevant task.

The use of magnetic conductors in the power industry significantly reduces the dynamic range of ferroresonances – mainly to the industrial frequency (50 Hz) and its subharmonics (25 Hz, 16.7 Hz, etc.). At the same time, the widespread use of magnetic dielectrics in radio engineering transfers the phenomenon of ferroresonance to the range of sound and ultrasonic frequencies, in which the role of wave processes increases significantly. This range eventually borders on the ultra-high-frequency radio range of ferromagnetic and ferrimagnetic resonances, which are studied by radio spectroscopy methods. A known result of such studies is the cooperative nature of the precessional motion of solid-state spin magnetic moments in a strong permanent magnetic field. On the other hand, the cooperation of magnetic moments in a relatively weak alternating field of oscillating radio technical systems has not been sufficiently investigated. This actualizes the comparative analysis of magnetization dynamics of conducting and dielectric ferromagnets in an external as well as in an internal electromagnetic field. Features of dielectric ferro- and ferrimagnetic systems are an additional mesoscopic mechanism of polarization of the material caused by the accumulation of electric charges on structural inhomogeneities, as well as magnetostriction. These factors provide pronounced electromagnetic and acoustic emission upon excitation of ferroresonances, which must be used for diagnostics and experimental research of such resonances.

It is advisable to describe ferroresonance processes in electromechanical analogies with contact tribodynamics, in which contact hysteresis is an analog of magnetic hysteresis. At the same time, quasi-static neglect of wave processes leads to unexpected emergency excesses of dynamic contact reactions over quasi-static ones, crack formation and deep sub-contact damage of the material under resonant modes. The commonality of the mathematical description of electrical and mechanical oscillatory and wave processes actualizes this kind of comparative analysis.

Ferroresonances are formed under thermodynamically unbalanced conditions of balance (or imbalance) of energy pumping and energy dissipation. In this regard, when describing them, the terms “nonlinear dissipative system” and “non-equilibrium dissipative system” are usually used, emphasizing the scattering processes. At the same time, the alternative attraction of non-equilibrium systems to the accumulation (rather than dissipation) of the energy of the electromagnetic field and the energy of the magnetic moments in the field is not taken into account. Direct transfer of known tendencies of quasi-equilibrium systems to stable equilibrium states corresponding to the energy minimum is not always adequate under non-equilibrium conditions. In this regard, the problem of researching ferroresonance regimes, which are manifold in their content, is relevant, under which the system of spin magnetic moments of a ferromagnet goes to an unstable state of equilibrium, corresponding to the energy maximum in the surrounding field.

2. Literature review and problem statement

Works [1, 2] report the results of experimental studies of ferroresonance processes in industrial electrical networks and their components with nonlinear inductive elements. The authors proposed methods of multifactorial analysis and prediction of emergency-dangerous ferroresonance modes of

equipment operation. However, questions remain regarding the completeness of the description of magnetization dynamics under ferroresonance conditions because in these works, as in this field in general, the consideration is based on the intensity of the external magnetic field. An alternative approach based on the analysis of the influence of the induction of the internal magnetic field in ferromagnets on the dynamics of magnetization is not used here. This requires additional research.

In [3], the main attention is paid to subharmonic low-frequency resonances. These modes are characteristic of electromagnetic oscillating circuits with conducting magnets. At the same time, the regimes of superharmonic ferroresonances and basic resonances of the audio frequency range have not been studied in detail. Such studies should be conducted using magnetic dielectrics.

For the analysis of ferroresonance processes in the power industry, new methods have been tested using wavelet transformations of signals [4] and registration of accompanying magnetostrictive acoustic emission [5]. However, in the cited works, little attention is paid to the cascade transport of disturbance energy, which can have unexpected consequences in a wide range of frequencies. Relevant studies can be conducted on the basis of Fourier analysis of magnetostrictive acoustic emission under ferroresonance modes.

Numerical-analytical and graph-analytical methods of calculation and prediction of emergency-dangerous ferroresonance modes in electrical networks with nonlinear inductions are considered, in particular, in [6, 7]. Calculations are based on the polynomial dependence of the magnetizing current on the flux linkage, or hysteresis curves of magnetization, which are characteristic of the standard operating modes of voltage transformers, power transformers, and other devices of real industrial power networks. Such connection with in-kind data increases the practical value of the results of the cited works. Critically, one should note the a priori determination of the time forms of currents and voltages, usually expressed by harmonic and limited polyharmonic dependences. Meanwhile, under the real conditions of power grid operation, non-normative switching processes with a significantly non-harmonic form of disturbances initiate nonlinear ferroresonances, which have features of both forced and parametric oscillations. In this context, one should note the ability of open nonlinear systems for dynamic self-regulation of their parameters that is not a priori predictable – with the “goal” of the fastest energy pumping and winning in the competition of modes. This side of the problem of ferroresonance is not covered at all in the cited works, which focus mainly on the processes of electromagnetic energy dissipation.

To predict alternative (relative to the known) energy pumping channels of ferroresonances, it is worth investigating the dynamics of magnetization in the ferromagnet material under conditions of external electromagnetic influence. In power grids, the classical axial magnetization in magnetic conductors is implicitly postulated. However, this is not the only geometry of the internal magnetic field in the conditions of the already mentioned dynamic self-regulation of the system of spin magnetic moments in the surrounding field. This side of the problem, using the example of nanostructured systems, is highlighted in works on ultra-high-frequency ferromagnetic and ferrimagnetic resonance [8, 9], in which the precession of moments under conditions of a strong permanent magnetic field is analyzed. In the context of the results below, such a precession is only comparative

in view of the significant difference in the spatial and temporal scales of the studied systems. One should note the possibility (in addition to precession) of reversal of the cooperative magnetic moment in an external field, as well as the division of the system of spin magnetic moments of a ferromagnet into subsystems whose moments are mutually oriented in a certain way. Such processes in electrotechnical and radiotechnical nonlinear oscillating systems have been studied only within the framework of quasi-static magnetization curves, the parameters of which cannot be directly transferred to the self-organized dynamics of magnetic moments in an alternating electromagnetic field. In electric power systems using conductive ferromagnets, the magnetic induction rotor in the core material is largely determined by the conduction current density, which provides damping of magnetization turns at high frequencies. Therefore, the little-studied processes of reversal of magnetic moments in the surrounding field should be carried out using magnetic dielectrics, where the angular dependence of the induction is determined by the rotor magnetization and displacement currents in Maxwell's equations.

The specified dynamic self-regulation and self-organization of the system of spin magnetic moments of a ferromagnet in a relatively weak external field brings ferroresonance processes closer to self-oscillatory and self-wave processes. This prompts their consideration in electromechanical analogies with contact tribodynamics, in which the nonlinearity of the mechanical contact of deformable solid bodies causes complex coherent dynamics even in the simplest systems in terms of elemental composition [10]. Such analogies have deep historical roots, formalized in the Lagrange-Maxwell equations, applicable directly to linear systems. Their indirect application to nonlinear ferromagnetic and tribodynamic systems can be carried out with the introduction of conditionally equivalent parameters. However, the internal ideology of Lagrangian mechanics regarding the exclusion of reactions (bonds) makes such an approach auxiliary and illustrative since the main problem lies precisely in nonlinear contact reactions and ferromagnetic elements. Appropriate use of quasi-static hysteresis curves does not make it possible to solve it fully [10, 11]. Therefore, the main way is the experimental study of this kind of systems and the construction of mathematical models based on the results of measurements.

In the end, it is necessary to especially note the thermodynamic disequilibrium of the studied systems and the possibility of their collector directing to unstable equilibrium states, rather than dissipative relaxation to stable equilibrium. This side of the problem is not reflected at all in previous studies of ferroresonance and requires special attention.

Thus, for a comprehensive solution to the problem of ferroresonance and supplementing data on conducting ferromagnets in power systems, experimental studies of ferroresonance modes should be conducted. To this end, it is necessary to use magnetic dielectrics as part of the basic radio engineering RLC circuit. It is also necessary to reveal the peculiarities of such regimes with respect to self-oscillating and self-wave processes, dynamic self-regulation of the system of spin magnetic domains, and the collector increase of their energy, which may have emergency consequences.

3. The aim and objectives of the study

The purpose of our study is to establish the physical mechanisms of nonlinear tribodynamics of magnetization under ferroresonance modes of a parallel RLC circuit. This will make it possible to expand the possibilities of predicting emergency-dangerous ferroresonance modes in the power industry, as well as increase the reliability of using the phenomenon of ferromagnetic resonance in the field of thin magnetic films in radio electronics.

To achieve the goal, the following tasks were set:

- to carry out experimental measurements of resonant amplitude-frequency characteristics of an RLC circuit with a nonlinear ferromagnetic inductive element under the conditions of energy pumping by a source of alternating electric current;
- to register time forms of ferroresonance signals and forms of accompanying electromagnetic and acoustic magnetostrictive emission;
- to build a mathematical model describing the nonlinear dynamics of magnetization in the ferroresonance modes of a parallel RLC circuit;
- to reveal electromechanical analogies of the studied processes with collector resonance modes in nonlinear systems of contact tribodynamics.

4. The study materials and methods

The object of our research is a parallel RLC circuit with a nonlinear ferromagnetic (ferrimagnetic) core of an inductor, connected to a source of alternating electric current.

The main research hypothesis assumes that the behavior of the system of spin magnetic moments of a ferromagnet in an external alternating electromagnetic field has signs of self-organization and dynamic self-regulation. Such behavior cannot be fully described using the existing quasi-static magnetization curves and requires an experimental study with appropriate analysis of the results.

In the paper, assumptions are adopted regarding the collector nonequilibrium nature of ferroresonance modes (contrary to the currently dominant quasi-static dissipative models), as well as regarding electromechanical analogies of the studied processes with nonlinear resonance modes of contact tribosystems.

In the process of analyzing the results, simplified linearized models of nonlinear resonant modes are used involving the corresponding conditionally equivalent dynamic parameters.

In the scheme of the experimental circuit (Fig. 1), a capacitor of constant capacity (1 μ F or 2 μ F) and a non-ideal (internal resistance of 17 Ohm) non-linear inductance coil were used. The elements were connected according to the parallel circuit to an external alternating current generator of the audio frequency range, the type of FG-200 ASICO (India).

The advantage of this generator is the high stability of the output signal and the ability to adjust the internal resistance. The latter (5 k Ω) exceeded all characteristic resistances in the external circuit by at least an order of magnitude, which ensured operation under the current generator mode. The alternating voltage on the capacitor and inductance was measured by the digital voltmeter B7-16A, and the waveform was recorded by the os-

cilloscope OWON XDS2102A (China). The internal resistances of the voltmeter and oscilloscope (~1 MΩ) significantly exceeded all resistances in the system, which excluded their influence on the measurement results. The electrical capacity of the connecting cables, oscilloscope, and voltmeter was several orders of magnitude smaller than the capacity of the capacitor, which also excluded the corresponding uncontrolled nonlinearity factors.

Two inductors with different magneto-dielectric cores were used to study the influence of the magnetization saturation factor on the nature of oscillations. The toroid coil had a core made of sendust (85 % Fe, 9.5 % Si, 5.5 % Al) composite material with a relatively high saturation inductance (about 1.0 T). The solenoid coil was used with a MnZn-ferrite core, which was characterized by a relatively low saturation magnetization (about 0.3 T). The inductances of the coils and the capacities of the capacitors were selected in such a way as to ensure close values of the resonance frequencies at the limit of small excitations. To study the influence of the dissipation factor on the nature of oscillations, a cartridge of additional resistances was connected in series with the coil (Fig. 1).

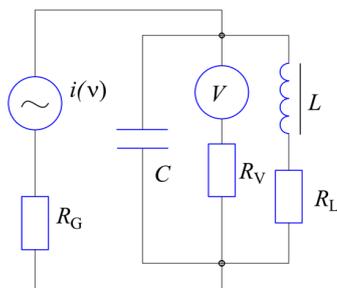


Fig. 1. Electrical diagram of an experimental RLC circuit

To analyze electromagnetic wave losses under resonance modes, electromagnetic wave detectors (remote inductors) connected to an oscilloscope were brought to the inductor coils at a distance of several centimeters. In parallel with this, acoustic emission analysis of magnetostriction signals during remagnetization of a ferromagnet was also carried out. The Audacity software package was used for recording and spectral analysis of acoustic signals.

5. Results of investigating the nonlinear dynamics of magnetization under the ferroresonance modes of a parallel RLC circuit

5.1. Results of experimental measurements of resonant amplitude-frequency characteristics of ferroresonant modes of a parallel oscillating circuit

Fig. 2 shows two families of resonance curves, obtained in the process of increasing the output power of the current generator, for each of the coils – with relatively large and relatively small non-linearity of the core material.

Unstable resonance modes, the transition through which had a jump-like character, are marked here with a dotted line. The high-frequency limit of the resonance curves was determined by the need to assign the main resonant mode, and the upper limit of the voltages was determined by the generator power limit.

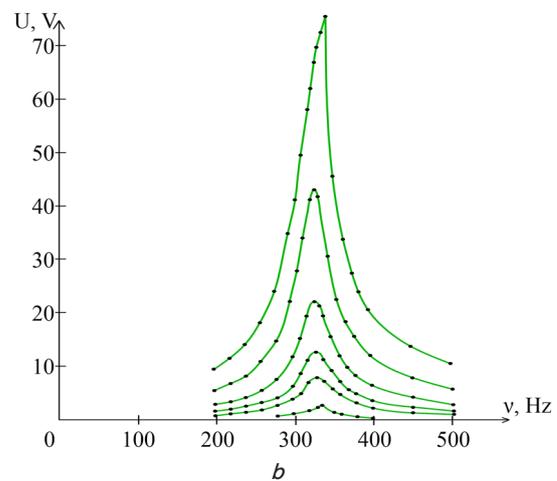
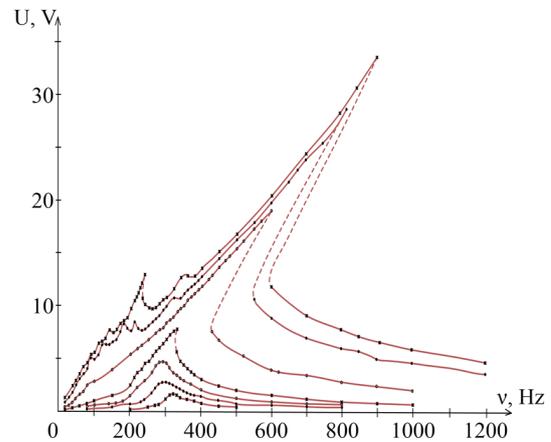


Fig. 2. Amplitude-frequency characteristics of ferroresonance oscillations in the circuits of the inductor: a – with a ferrite core; b – with a composite core

5.2. Results of registration of time forms of ferroresonance signals, as well as signals of accompanying electromagnetic and acoustic emission

Fig. 3 shows oscillograms of voltage signals on parallel sections of the circuit under the pre-resonance mode (~100 Hz), under the superharmonic resonance mode (~300 Hz), and under the basic resonance mode (~800–1000 Hz).

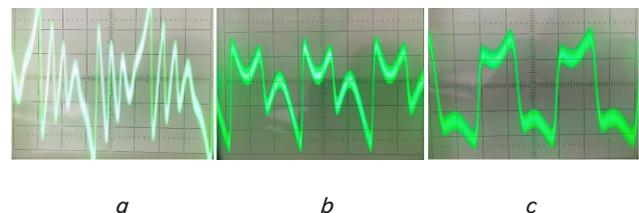


Fig. 3. Oscillograms of voltage signals on parallel sections of the circuit: a – under the pre-resonance mode; b – under the mode of superharmonic resonance; c – under the basic resonance mode

For comparison, Fig. 4 shows oscillograms of emission of electromagnetic waves under the same modes.

Fig. 5 shows a typical spectrum of magnetostrictive acoustic emission from a coil with a ferrite core under the basic resonance mode.

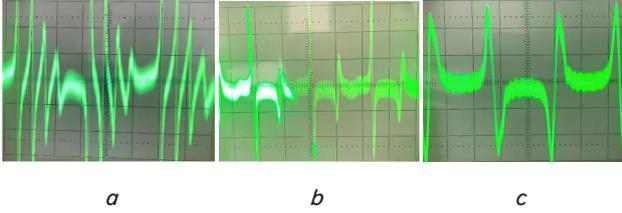


Fig. 4. Oscillograms of electromagnetic emission signals from the inductor coil: *a* – under the pre-resonance mode; *b* – under the mode of superharmonic resonance; *c* – under the basic resonance mode

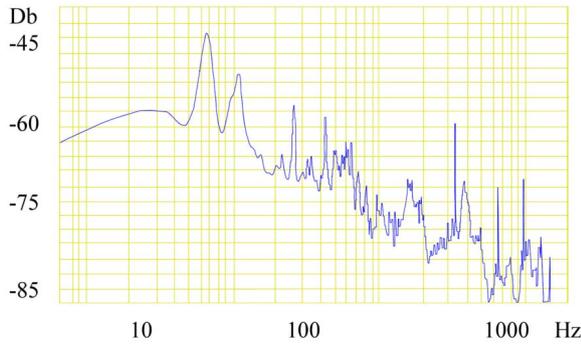


Fig. 5. Typical spectrum of magnetostrictive acoustic emission from a coil with a ferrite core under the basic resonance mode

Acoustic magnetostrictive emission from a coil with a composite magneto-dielectric core was audible only under the marginal amplitude resonance modes but was too weak for reliable technical registration and further analysis.

5.3. Mathematical model of nonlinear dynamics of magnetization under ferroresonance modes of a parallel RLC circuit

When constructing a mathematical model, the following features of the above experimental results should be taken into account. As can be seen from Fig. 2, the nature of the resonance curves significantly depends on the degree of nonlinearity, which is determined by the saturation limit of the ferromagnet. A characteristic feature of nonlinear modes is the direct slope of the frequency-dependent amplitude in the pre-resonance sections. For a weakly nonlinear composite ferromagnet, this kind of dependence is observed only at extremely high (for the used generator) excitation levels. Resonance characteristics have a mixed form of their skeletal curves, which qualitatively correlates with the quasi-static magnetization curve of a ferromagnet (Fig. 6) – with a transition from the initial nonlinear-soft to the next nonlinear-hard characteristic.

The time forms of voltage signals on circuit elements and electromagnetic emission signals have a quasi-harmonic form only for a weakly nonlinear ferromagnet. In the case of a ferrite core, the forms have a significantly different appearance (Fig. 3, 4).

We shall use the model method of electromechanical technologies in oscillating systems with variable parameters. When a capacitor with capacity *C* and a coil with inductance *L* are connected in parallel to an external alternating current circuit, we have the following analogies:

$$C \leftrightarrow m, \tag{1}$$

$$L^{-1} \leftrightarrow k, \tag{2}$$

$$\frac{di}{dt} \leftrightarrow Q(t), \tag{3}$$

where *m*, *k*, *Q(t)* are the mass, stiffness coefficient, and forcing force in the equivalent mechanical system, respectively. Under the condition of small dissipative losses, the resonant frequency is written here in a known way:

$$\omega_r^2 = (2\pi\nu_r)^2 \approx \frac{k}{m} = \frac{1}{LC}. \tag{4}$$

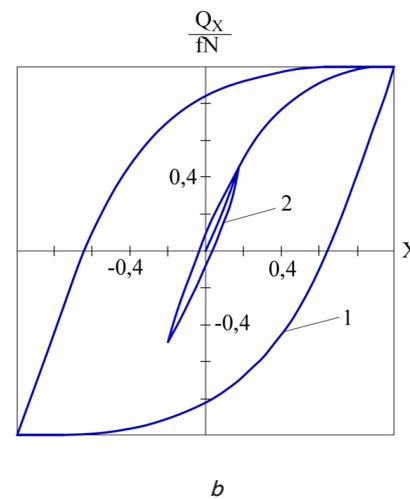
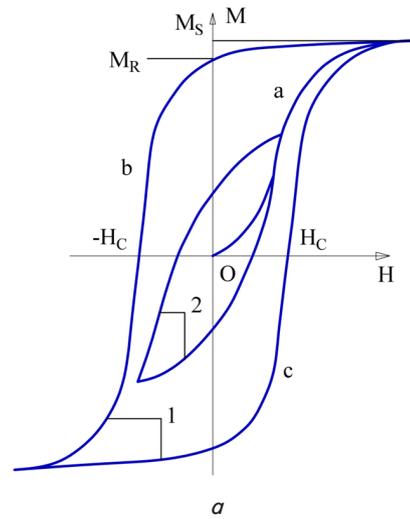


Fig. 6. Curves of quasi-static hysteresis: *a* – magnetic; *b* – contact

In such a model, the analog of the mechanical displacement *X* is the voltage *U* on the capacitor and the inductor. Under resonant modes, the current *I* through the coil significantly exceeds the current in the external power supply circuit as well (with a high *Q*-factor of the oscillating circuit). Under the conditions of resonant compensation of reactive resistances, we use the following ratio:

$$U_r = I_r R_L, \tag{5}$$

where U_r is the resonant voltage measured by a voltmeter, R_L is the total active resistance of the section with the inductor. Considering the inductance L as the proportionality factor between the magnetic flux coupling Ψ and the current strength I , we write Faraday's law in a system with a nonlinear inductance depending on the current strength and, accordingly, on time t :

$$\epsilon_s = -\frac{d(LI)}{dt} = -\left(L + I \frac{dL}{dI}\right) \frac{dI}{dt} = -\tilde{L} \frac{dI}{dt}, \tag{6}$$

where ϵ_s – e.r.s. of self-induction, \tilde{L} – nonlinear inductance:

$$\tilde{L} = L + I \frac{dL}{dI}. \tag{7}$$

As can be seen, at negative and sufficiently large values of the derivative in (7), the nonlinear inductance \tilde{L} becomes negative, which leads to a negative value of the square of the resonance frequency:

$$\omega_r^2 \approx \frac{1}{LC} \leq 0. \tag{8}$$

In electromechanical analogies, regimes of this kind correspond to the dynamics of an inverted pendulum near an unstable upper equilibrium position. The forms of oscillations in this case are essentially non-harmonic – they are stitched together from areas of exponential growth and decrease of voltage, which is clearly shown in Fig. 3, as well as on the corresponding oscillograms of electromagnetic emission from the coil (Fig. 4). The spectrum of magnetostrictive acoustic emission (Fig. 5) contains two clusters – low-frequency and high-frequency, corresponding to the slow movement of the conditional pendulum near the upper point of unstable equilibrium and the abrupt passage of the lower equilibrium point with the loss of part of the stored energy.

Equation (7) can be integrated under the condition of transition to the self-oscillating mode with reversal of the magnetic moment of the spin domains in the given magnetic field:

$$\tilde{L} = L + I \frac{dL}{dI} = -L, \tag{9}$$

$$L = \frac{K}{I^2}, \tag{10}$$

where K is a dimensional constant. Expression (9) in its physical meaning reflects dynamic antiferromagnetism – self-organized orientation of the system of spin magnetic moments in the direction opposite to the external magnetic field. This orientation corresponds to the non-equilibrium maximum (rather than the usual equilibrium minimum) of the magnetic moments in the external field. Under such conditions, as can be seen from (10), the inductance of the coil decreases inversely quadratically with respect to the current, which becomes possible with the transition to the magnetic saturation mode with the subsequent reversal of the magnetic moment.

Substitution (9), (10) in (8), taking into account (5), gives:

$$\omega_r^2 \approx -\frac{I_r^2}{KC} = -\frac{U_r^2}{R_L^2 KC}, \tag{11}$$

which reflects the direct proportionality of the resonance frequency and voltage under such modes, corresponding to the experimental data (Fig. 2).

According to experimental data (Fig. 2) and expression (8), the inductance is directly proportional to the magnetic permeability of the core. In this way, the formal dependence of nonlinear magnetic permeability $\tilde{\mu}$ (in relative terms) on resonant frequency, voltage, and current can be established:

$$\left| \frac{\tilde{\mu}}{\mu_0} \right| = \left| \left(\frac{\omega_r^0}{\omega_r} \right)^2 \right|, \tag{12}$$

where $\tilde{\mu}$ is the magnetic permeability at the limit of weak excitations, ω_r^0 is the cyclic resonance frequency at this limit.

Fig. 7 shows this kind of dependence in terms of the current corresponding to the active resistance R_L .

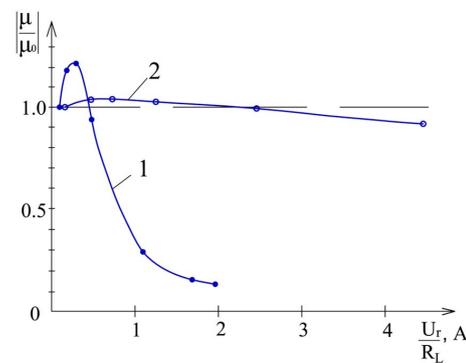


Fig. 7. Calculated dependences of the reduced magnetic permeability on the current in the coils: 1 – ferrite core; 2 – composite magnetolectric core

Qualitatively, the dependence $\tilde{\mu}(I)$ corresponds to a quasi-static loop of ferromagnetic hysteresis (Fig. 6). The descending part of this dependence agrees with (10) (in the case of a ferrite core) at the initial stage, but at higher excitation levels it is quantitatively approximated by an expression different from (10):

$$L = \frac{D}{I^{1.2}}, \tag{13}$$

$$\tilde{L} = -0.2L, \tag{14}$$

where D is the dimensional factor. This indicates the need to take into account dissipation processes in the dynamics of magnetization reversal.

These processes can be analyzed based on the known ratios regarding the Q-factor of the oscillating circuit:

$$Q = \frac{v_r}{\Delta v_{0.7}} \sqrt{\frac{L}{C}} = \frac{1}{\omega_r \cdot C \cdot R_L}, \tag{15}$$

where v_r , $\Delta v_{0.7}$, ω_r are the resonance frequency and standard frequency width of the maximum, as well as its cyclic resonance frequency.

It is advisable to supplement the above calculations with a description of the dynamics of magnetization in ferromagnets based on Maxwell's equations (SGS system):

$$\text{rot } \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \tag{16}$$

$$\text{div } \vec{B} = 0, \tag{17}$$

$$\text{rot } \vec{B} = \frac{4\pi}{c} (\vec{j} + c \cdot \text{rot } \vec{M}) + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}, \tag{18}$$

$$\text{div } \vec{D} = 4\pi\rho, \tag{19}$$

where \vec{E} , \vec{D} , \vec{B} is the intensity and induction of the electric field, as well as the induction of the magnetic field, respectively, \vec{M} is the magnetization of the core material, \vec{j} , ρ is the conduction current density and the volume density of extraneous electric charges in this material, c is the speed of light in a vacuum, t is time.

The material relations for closing the system (16) to (19) take the form:

$$\vec{D} = \vec{E} + 4\pi\vec{P}(\vec{E}), \tag{20}$$

$$\vec{H} = \vec{B} - 4\pi\vec{M}(\vec{B}), \tag{21}$$

where \vec{P} is the electrical polarization of the medium, \vec{H} is the intensity of the magnetic field in it. For the magnetic dielectrics studied here, the conduction current \vec{j} in (18) can be neglected, and therefore the magnetic induction rotor is determined by the magnetic current density:

$$\vec{j}_M = c \cdot \text{rot } \vec{M}(\vec{B}), \tag{22}$$

which directly reflects the rotations of the saturated magnetization of the ferromagnet not in the external field \vec{H} , but in the internal field \vec{B} . This form of description allows us to explain (at least on a qualitative level) the observed dynamics of magnetization under the non-equilibrium mode.

5.4. Results of registration of collector modes of motion in analog systems of nonlinear contact tribodynamics

Manifold modes of motion in analog tribocontact systems have already been demonstrated on the example of a simple auto-oscillating contact mechanical system in the form of a ring, or a disk, launched from the edge of the base plane under the mode of rotation and rotation with rolling. Fig. 8 shows acoustograms of this kind of processes, and Fig. 9 – a typical spectrum of accompanying acoustic emission.

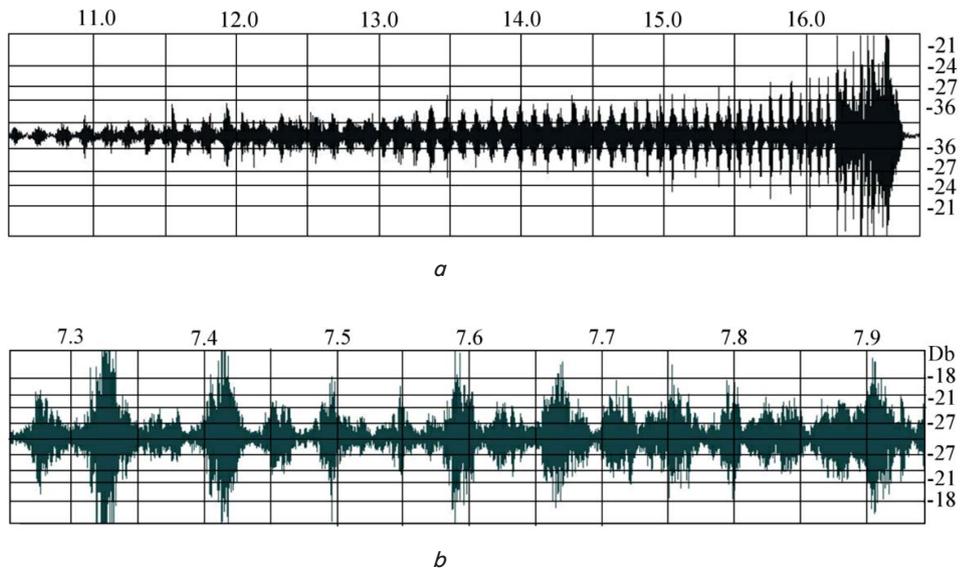


Fig. 8. Acoustograms of motion along the plane of the base under the rotation-roll mode: *a* – metal ring; *b* – mirror disk

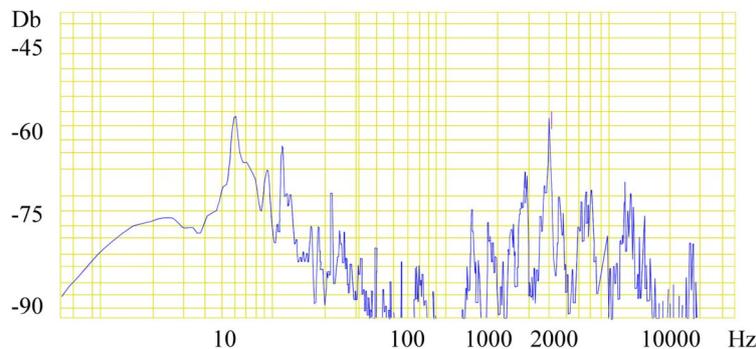


Fig. 9. The spectrum of the acoustic emission accompanying the movement of the mirror disk along the plane of the base

As can be seen, the movement of the rotor has a buffeting quasi-periodic character (in the absence of external cyclic excitation in this case), and the acoustic emission spectra, as in Fig. 5, contain two clusters. These results reflect the already mentioned mode of oscillation of a conventional inverted pendulum aimed at the maximum (and not at the minimum) of its energy in an external field – in this case, a gravitational field.

6. Discussion of results of investigating the nonlinear dynamics of magnetization under the ferroresonance modes of a parallel RLC circuit

A comparison of resonant amplitude-frequency characteristics (Fig. 2) and the results of model calculations (Fig. 7) confirms the leading role of magnetization saturation of the ferromagnet in the mechanisms of the transition to the nonlinear resonance regime. After all, a significant non-linearity of the rigid type in the characteristics (for ferrite cores) is observed directly at the calculated maximum for the relative magnetic permeability of the core. The transition through this maximum is accompanied by dynamic self-regulation of magnetization in the internal magnetic field. A feature of such a transition, which was not noted in other works on this topic, is a cyclic reversal of the collective magnetic moment in the surrounding field in the direction of an unstable state of equilibrium corresponding to a maximum (rather than a stable minimum) of energy. This is evidenced by the forms of electromagnetic emission from the core of the inductor, shown in Fig. 4. They have a significantly non-harmonic appearance and contain relaxation time segments in the direction of an unstable state of equilibrium. Inevitable and stimulated by variable external influence, the loss of such equilibrium generates pulsating induced currents in the oscillating circuit, the shape of which is significantly non-harmonic. At the same time, there remains the possibility of generating also traditional quasi-equilibrium modes imitating the temporal harmonic form of the excitation. Under the pre-resonance modes, the latter are suppressed by the high reactance of the circuit, however, superharmonic and fundamental resonance modes are characterized by self-synchronization of frequencies and nonlinear mixing of both modes. Such mixing in combination with the nonlinear transport of disturbance energy should be taken into account when designing and operating radio and electrical systems with nonlinear inductances.

The nature of dynamic self-regulation of magnetization under resonant modes depends significantly on the domain structure of the ferromagnet and the induction of its saturation, which directly follows from the comparison of ferrite and composite magnetic dielectrics – Fig. 2, 7. The damping properties of the latter, meanwhile, are insufficient to suppress resonant peaks. At the same time, an increase in the active resistance R_L already by several Ohms under resonant modes led to the breakdown of resonant maxima and jumps to low-amplitude resonant sections of the amplitude-frequency characteristics (for ferrites). Therefore, increasing the active resistance remains a simple and effective method of combating unwanted resonance modes: linear or non-linear. In this context, it is worth analyzing the Q-factor of this oscillating circuit under different modes. It is usually estimated by the relative width of resonance maxima according to (15). However, this expression does not take

into account a certain degree of coherence of the system of magnetic spin domains, which noticeably narrows the resonance lines towards saturation (Fig. 2). A similar narrowing of resonance lines is also observed in coherent systems of contact tribodynamics. At the same time, the nonlinearity of the system serves as an additional factor of restraining resonant amplitudes, but it leads to the cascade transport of energy by the spectrum of disturbances [10, 11]. This factor is of particular importance in terms of projecting the results obtained above (for magnetic dielectrics) to electrotechnical ferroresonance processes in magnetic conductors. In particular, low-frequency subharmonic resonances [3] can be excited by such transport from the side of uncontrolled high-frequency switching processes.

Continuing the line of electromechanical analogies, a comparison of ferromagnetic and contact hysteresis curves should be made (Fig. 6). Here, worth noting is a significant difference (the opposite result of the effect of nonlinear saturation in ferromagnetic systems and dry friction systems) taking into account that the equivalent mechanical stiffness, as already noted, is inversely and not directly proportional to the inductance. This predetermines the rigid character of nonlinearity in electromagnetic systems (with strong excitation (Fig. 2) and the soft character during tangential loading of mechanical contact [11, 12]. However, taking into account the normal components of contact reactions radically changes the situation, determining the mixed and rigid character of nonlinearities, similar to the results shown above [10].

The limitations of this work are the use of only magnetic dielectrics under ferroresonance modes, without researching the corresponding processes in magnetic conductors. In this connection, one should note the special role of the possible precession of magnetization in the internal magnetic field in the formation of low-frequency subharmonic ferroresonances, noted, for example, in [3]. This requires additional research because the currently available results from ferroresonance do not take into account the precession factor. On the other hand, the precession factor is considered to be the main factor in the ultra-high-frequency excitation of ferromagnetic and ferrimagnetic resonances in magnetic nanotechnologies using a permanent external magnetic field [8, 9]. In this field, in addition (before precession), one should take into account the possibility of self-wave reversal of the collective magnetic moment in the internal field of the environment, and not only in the external magnetic field. This will make it possible to separate the interfering collective effects and increase the reliability of the technological application of ferromagnetic resonance.

The disadvantage of our study is the use of purely laboratory equipment for the analysis of ferroresonance processes. In the future, this shortcoming can be eliminated via scientific cooperation with domestic and foreign specialists in the operation of industrial power networks, in which the problem of preventing emergency-dangerous ferroresonance modes is being solved.

Advancing our research may involve the construction of a more specific and adapted to practice mathematical model for describing the precession and reversal of magnetization of a ferromagnet in the surrounding field, based on Maxwell's equations. Difficulties of a mathematical and methodical nature on this path are the need to reformat the material relations necessary to close the system of Maxwell's equations. These ratios must be derived from the induction

of the internal magnetic field, and the dominant methods of describing ferroresonance processes directly or indirectly derive from the intensity of the external magnetic field.

7. Conclusions

1. It was established that the physical mechanism of dynamic self-regulation of the system of spin magnetic moments of magnetic dielectrics in the ferroresonance regimes of a parallel oscillating circuit is a cyclic reversal of the magnetic moment in the induced electromagnetic field. This mechanism is mathematically described in terms of the magnetic current density and the corresponding magnetization rotor in Maxwell's equations. At a qualitative level, it can be described through the introduction of a nonlinear inductance depending on the current strength. The main factor of this kind of self-regulation is the direction of the system of spin magnetic moments to an unstable state of equilibrium corresponding to the maximum (rather than the classical minimum) of energy in the external field. Cyclic loss of such equilibrium generates pulsating induced currents, which are not taken into account by traditional methods for calculating ferroresonance modes.

2. Our experimental studies revealed frequency self-synchronization and nonlinear mixing of two types of modes under ferroresonance conditions. Namely: the quasi-static mode, which imitates the time form of the external excitation of oscillations, and the dynamic mode, which corresponds to the unstable state of equilibrium of the magnetic moments. These processes lead to the cascade transport of energy by the spectrum of disturbances and determine the adaptability of disturbances to the external conditions of energy pumping. They should be taken into account when projecting the obtained results to electrotechnical ferroresonance processes in magnetic conductors, in which the cascade energy transfer of uncontrolled high-frequency switching biases initiates low-frequency subharmonic ferroresonances. Increasing the ohmic losses in the oscillating circuit remains a simple but effective method for suppressing unwanted ferroresonance modes. At the same time, one should take into account the effects of mode competition, which give preference to low-frequency modes with small losses for the dissipation of eddy current energy.

3. The mechanism of reversal of the collective magnetic moment in the internal field is mathematically described through the magnetic current density and the corresponding magnetization rotor in Maxwell's equations. At a qualitative level, it can be described through the introduction of a nonlinear inductance depending on the current strength. However, the last form of description, like the traditional methods in this field, is based on the given intensity of the external magnetic field and does not sufficiently take into account the existing factor of self-regulation of magnetization in the internal field of the environment.

4. It is advisable to conduct the analysis of ferroresonance processes in electromechanical analogies with nonlinear contact tribodynamics in the context of thermodynamic disequilibrium of such systems. This analysis reveals, in particular, coherent narrowing of resonance lines and non-equilibrium forms of self-oscillations that have a common time form in electromagnetic and mechanical contact systems.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Bakhor, Z., Yatseiko, A., Ferensovych, R. (2020). Assessment of ferroresonance processes in schemes of 6-35 kV electrical grids on the basis of reliability analysis. *Energy Engineering and Control Systems*, 6 (2), 137–145. <https://doi.org/10.23939/jeecs2020.02.137>
2. Akinci, T. C., Akgun, O., Yilmaz, M., Martinez-Morales, A. A. (2023). High Order Spectral Analysis of Ferroresonance Phenomena in Electric Power Systems. *IEEE Access*, 11, 61289–61297. <https://doi.org/10.1109/access.2023.3286817>
3. Olguín-Becerril, M. A., Angeles-Camacho, C., Fuerte-Esquivel, C. R. (2014). Ferroresonance in subharmonic 3rd mode in an inductive voltage transformer, a real case analysis. *International Journal of Electrical Power & Energy Systems*, 61, 318–325. <https://doi.org/10.1016/j.ijepes.2014.03.057>
4. Sharbain, H. A., Osman, A., El-Hag, A. (2017). Detection and identification of ferroresonance. 2017 7th International Conference on Modeling, Simulation, and Applied Optimization (ICMSAO). <https://doi.org/10.1109/icmsao.2017.7934904>
5. Martinez, R., Arroyo, A., Pigazo, A., Manana, M., Bayona, E., Azcondo, F. J. et al. (2022). Acoustic Noise-Based Detection of Ferroresonance Events in Isolated Neutral Power Systems with Inductive Voltage Transformers. *Sensors*, 23(1), 195. <https://doi.org/10.3390/s23010195>
6. Klimas, M., Majka, Ł. (2019). Enhancing the possibilities in visualisation of the ferroresonance phenomenon. *Poznan University of Technology Academic Journals. Electrical Engineering*, 98, 115–124. <https://doi.org/10.21008/j.1897-0737.2019.98.0010>
7. Solak, K., Rebizant, W., Kereit, M. (2020). Detection of Ferroresonance Oscillations in Medium Voltage Networks. *Energies*, 13 (16), 4129. <https://doi.org/10.3390/en13164129>

8. Schmool, D. S., Markó, D., Lin, K.-W., Hierro-Rodríguez, A., Quirós, C., Díaz, J. et al. (2021). Ferromagnetic Resonance Studies in Magnetic Nanosystems. *Magnetochemistry*, 7 (9), 126. <https://doi.org/10.3390/magnetochemistry7090126>
9. Abdel-hamed, A. M., M. EL-Shafhy, M., A. Badran, E. (2022). Elimination of ferroresonance in the distribution zone by high ohmic reactor-shunt limiter. *Indonesian Journal of Electrical Engineering and Computer Science*, 28 (3), 1286. <https://doi.org/10.11591/ijeecs.v28.i3.pp1286-1296>
10. Zaspá, Y., Dykha, A., Marchenko, D., Matiukh, S., Kukurudzyak, Y. (2020). Exchange interaction and models of contact generation of disturbances in tribosystems. *Eastern-European Journal of Enterprise Technologies*, 4 (5 (106)), 25–34. <https://doi.org/10.15587/1729-4061.2020.209927>
11. Dykha, A. V., Zaspá, Yu. P., Slashchuk, V. O. (2018). Triboacoustic Control of Fretting. *Journal of Friction and Wear*, 39 (2), 169–172. <https://doi.org/10.3103/s1068366618020046>
12. Dykha, A., Makovkin, O. (2019). Physical basis of contact mechanics of surfaces. *Journal of Physics: Conference Series*, 1172, 012003. <https://doi.org/10.1088/1742-6596/1172/1/012003>