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OPTIMIZATION OF CROSS-SECTIONAL DIMENSIONS OF CASTELLATED BEAMS WITH HEXAGONAL OPENINGS

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The object of this study is a castellated beam, in which the web openings have the shape of a regular hexagon. The beam is examined to find optimal cross-sectional dimensions. The optimization task is stated as the task of finding the optimal profile numbers for top and bottom Tees of the beam and the optimal width of the web opening while ensuring the required load-carrying capacity of the beam. Minimization of the volume of the beam material was considered as an optimality criterion. The stated optimization problem was solved using the exhaustive search method. For an assortment of normal I-beams with parallel flanges, castellated beams were obtained with optimal cross-sectional dimensions depending on the steel grade, the beam span, and the magnitude of transverse uniformly distributed load. The optimization calculations proved that it was possible to increase the elastic section modulus of the beam to 35.48...50 % through the use of a castellated web. Castellated beams with optimal cross-sectional dimensions at the same load-carrying capacity are characterized by lower steel consumption (up to 23.19 %) compared to I-beams with a solid web. Analysis of the results has made it possible to devise recommendations for the optimal distribution of material in the cross-sections of such beams. The results are valid only for the assortment of normal I-beam profiles and only for the case of uniformly distributed load acting on the beam when the compressed beam flange is laterally restrained from the bending plane and the beam web has openings in the form of regular hexagons. It is under such conditions that the reported results can be implemented in practice both at the stage of selecting cross-sections of the studied class of structures, and at the development of effective assortments of castellated beams

Keywords: castellated beam, hexagonal openings, optimal design, mixed variables, exhaustive search

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1. Introduction

Modern rolling I-beams with parallel faces of the flanges, including wide flanges, up to 1 m high, provide the ability to cover spans of 13...15 m in the presence of significant loads. The specific labor intensity of their production in terms of basic operations is 2...2.5 times lower than in similar trusses. However, material consumption is 1.5 times higher than in trusses.

As a result of the search for ways to increase the efficiency of rolled I-beams, an original structural form occurred – an I-beam with holes in the web, in which the material is concentrated closer to the flanges. In the technical literature, it received several names – I-beam with a perforated web, I-beam with a developed section, lattice I-beam. Hereafter, we shall use the term "castellated beams", which is used in design standards, in particular, DBN V.2.6-198:2014 "Steel structures. Design norms".

The effectiveness of the castellated beam compared to the original I-beam is explained by the fact that its height is approximately 1.4...1.5 times greater, the web thickness is 1/75...1/95 of the height. Castellated beams provide 20...30 % material savings compared to rolled I-beams and are cheaper by 10...18 %. They are 25–35 % more efficient than welded I-beams due to the reduction in processing operations and the amount of welding [1].

Castellated beams made of two steel grades are lighter by 34–39 % and cheaper by 16–20 % compared to hot-rolled I-beams with solid web made of one steel grade, with the same load-carrying capacity. These positive qualities combined with compactness, transportability, and the possibility of using highly automated manufacturing make them competitive even compared to lattice structures [2, 3].

On the one hand, increasing the load-carrying capacity of the beam for bending requires developing the cross-section of the beam in height as much as possible by forming holes in the beam (hexagonal cutouts) of the largest possible size. However, too large holes in the beam web lead to premature exhaustion of its bearing capacity under the condition of ensuring strength under normal stresses at the design points

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at the top and bottom of the hole. At the same time, normal stresses occur in the extreme fibers of the cross-section of the beam, which are much smaller than the yield strength of steel. With this in mind, scientific research into the optimal design of castellated beam structures is considered relevant. Since castellated beams actually expand the scope of application of hot-rolled I-beam profiles, the results of such studies will be demanded by manufacturers of such profiles.

2. Literature review and problem statement

Applied tasks of optimal design of building structures are often stated as problems of searching for unknown parameters of the structure, which ensure the extreme value of the determined optimality criterion in the search space delineated by a set of given constraints. The mathematical model of parametric optimization problems includes a set of design variables, an objective function, as well as constraints, which in general reflect nonlinear relationships between them [4].

A large body of research tackles the problem of finding the optimal dimensions of cross-sections of castellated beams [5, 6]. As part of the design variables, both the dimensions of the rolled profiles from which the beam is made, as well as the dimensions and pitch of the holes in the beam web [7] were considered. The first group of variables varies discretely, according to a defined set of possible values (assortment of rolling profiles). The presence of discrete design variables caused researchers to choose meta-heuristic optimization methods for solving the optimization problem of castellated beams [8]. At the same time, convergence to the global optimum is not always achieved, which required researchers to devise a significant number of modifications to meta-heuristic optimization methods.

Work [9] provides an overview of various statements of optimization problems for the considered class of structures and optimization algorithms based on meta-heuristic methods. At the same time, the desired parameters are set both on numerical sets and on finite sets of an arbitrary nature. The search strategy in such algorithms is based on the calculation and comparison of the values of some evaluation function of project solutions at the points of the search space under consideration. At the same time, requirements regarding unimodality, continuity, and differentiability of such a function are not put forward. This makes it possible to use meta-heuristic methods for a wide class of functions of the optimality criterion and constraints of the mathematical model, including for functions that do not have an analytical description.

In paper [10], which considers the problem of finding optimal cross-sectional dimensions of a castellated beam, the authors used a hybrid optimization algorithm. This made it possible to avoid obtaining local optima during the optimization search. Work [11] reports a new meta-heuristic optimization method based on colliding bodies, which is used to solve problems of optimizing the dimensions of the cross-section of a castellated beam. The authors developed its modification, which improves the speed of convergence to the optimal solution and reduces the number of static analyses of the structure during the search for the optimal point. For the considered class of problems, the particle swarm method and its modifications were proposed in papers [12, 13], which allowed the authors to obtain a better convergence to the optimum. At the same time, only the height of the opening, the angle of inclination of the cutting line to the axis of the beam, and the distance between the openings were considered as design variables of the optimization problem [13]. And the dimensions of rolled I-beam profiles, from which a castellated beam was made, were taken as fixed and did not change during the search for the optimum point. Paper [14] considers the problem of minimizing the cost of manufacturing a castellated beam structure using the charged system search algorithm. At the same time, design code constraints are considered in the mathematical model of the optimization problem. For the class of problems under consideration, the gray wolf pack optimization algorithm was also successfully applied in [15]. However, the authors of works [14, 15] did not consider castellated beams of monosymmetric cross-section, when in the compressed zone the cross-section elements of the beam have a greater thickness (for them, a more powerful rolled I-beam is used) than in the tensioned zone. Castellated beams of a monosymmetric cross-section are characterized by a higher load-carrying capacity provided that the local stability of the cross-section elements is ensured.

In addition, when applying the meta-heuristic methods listed above, despite their high efficiency and productivity, the authors obtained design solutions for structures that are only close to optimum [16]. This allows us to state that it is expedient to conduct a study aimed at finding a global optimum for problems of optimizing castellated beams, stated in a mixed space of design variables.

On the other hand, a critical review of the works proved that the issue related to determining the optimal size ratio and geometric characteristics of cross-sections of castellated beams remained unresolved. Given this, it is advisable to analyze the obtained optimal solutions and develop recommendations for the optimal distribution of material in cross-sections of castellated beams.

3. The aim and objectives of the study

The purpose of our study is to develop a procedure for optimizing castellated beams and to study the properties of such beams with optimal parameters. This will make it possible to compile recommendations for designers regarding the optimal distribution of material in cross-sections of castellated beams.

To achieve the goal, the following tasks were set:

– to state the optimization problem of castellated beams in the presence of continuous and discrete design variables and propose a method for its solution;

– to solve the problems of optimizing the cross-sections dimensions of castellated beams for different steel grades and different distributed loads on the beams;

– to develop recommendations for the optimal distribution of material in cross-sections of castellated beams.

4. The study materials and methods

The object of our research is castellated beams, which are investigated for the purpose of finding the optimal design solution in a mixed (continuous and discrete) space of variables. At the same time, the number of rolled I-beam profiles from the specified range of I-beams and the width of the hexagonal opening in the beam web are considered as design variables.

The presence of holes in the web causes a changed pattern of the stress state in the beam web. While the distribution of normal stresses σ_x (directed along the axis of the beam) in the beam flanges in the middle of the opening is close to linear (Fig. 1, cross section 1-1), then in the corner zones of the openings the normal stress diagrams σ_x are curvilinear, which is due to the concentration of stresses (Fig. 1, cross-section 2-2). Some curvilinearity of the diagram of normal stresses σ_x is also observed in the web-post zone (Fig. 1, cross-section 3-3). At the same time, normal stresses σ_y appear in the butt weld of the beam web-post, oriented perpendicular to the axis of the beam, which are also caused by the concentration of stresses in the vicinity near the holes (Fig. 1, cross-section 4-4). The influence of stress concentrators on the load-carrying capacity of castellated beams is in most cases covered by reserves of plasticity of the material. However, under cyclic and shock loads and impacts, especially under conditions of low temperatures, when the development of plastic deformations is complicated, cracks may appear in the corners of the holes due to the existing stress concentrators [17].

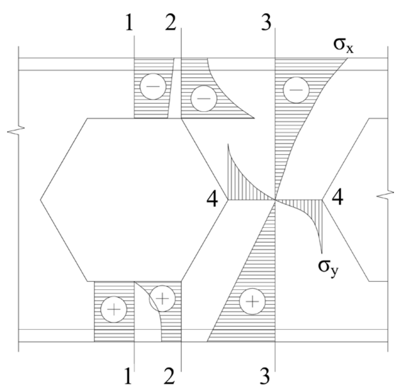

Fig. 1. Stress distribution on the section of a castellated beam

The design cross-section of a castellated beam is two T-sections beams (the upper and lower flanges of the beam), the joint operation of which is enabled by the existing web-post between the holes. Top and bottom Tees of the castellated beam, located within the opening, work both on the action of the bending moment in the beam, which occurs under the action of the transverse load, and on the action of shear forces, which cause additional bending of the flanges. At the same time, the limit state of the beam Tees is characterized by a significant development of plastic deformations, which permeate almost its entire cross-section in the corners of the openings. The beam web-post behaves mainly in shear, and its load-carrying capacity is determined by local buckling. In the limit state, the cross-sectional elements (web/flange) of the compressed Tee of the beam may also subjected to local buckling.

There are several approaches to the calculation of castellated beams, from simple engineering calculation models in the elastic region without taking into account stress concentrators near the holes to complicated elastic computation models based on the finite element method [18]. There were also attempts to evaluate the load-carrying capacity of castellated beams according to the limit equilibrium criterion [19] or according to the criterion of limited plastic deformations [20].

A castellated beam, according to its calculation scheme, is many times a statically indeterminate system. This structural form occupies an intermediate place between a continuous beam and a truss without diagonals (Vierendeel girder). However, the performed theoretical and experimental studies showed that without a significant decrease in the accuracy of the calculation, the castellated beam can be considered a regular Vierendeel girder. Such a girder consists of horizontal members the beam flanges of T-section, and vertical members – beam web-posts. At the same time, it is assumed that there are points with zero moments in the middle of the web-posts and on the sections of the Tees in the sections in the middle of the opening. Thus, it is possible to imagine that at these points there are conventional hinges in which only shear and axial forces act. Such a model of a castellated beam made it possible to devise a method of approximate calculation of its load-carrying capacity by the Vierendeel method, which is used in the design standards of these structures – DBN V.2.6-198:2014 “Steel structures. Design norms” and EN 1993-1-13:2024 “Eurocode 3 – Design of steel structures – Part 1-13: Beams with large web openings”.

Thus, the current work uses an approximate estimation of the load-carrying capacity of a castellated beam by the Vierendeel method. It is assumed that the material of the beam works in the zone of elastic deformations of steel, excluding the area in the vicinity of the web opening, where plastic deformations can develop. At the same time, the physical and mechanical properties of the beam material were assumed to be the same in all directions. Deformations in the fibers of the beam cross-sections were assumed to be much smaller than its dimensions. When the beam is deformed under load, the hypothesis of plane sections is fulfilled.

In our work, the problem of optimal design of castellated beams is stated as a problem of nonlinear programming in the presence of design variables of continuous and discrete types. To solve such a problem, the method of exhaustive search is used, the choice of which is determined by the presence of a mixed search space. The software implementation of the exhaustive search method for the optimization problem of a castellated beam is performed in the MS Visual Studio 2022 environment by the C# language.

5. Results of optimization of cross-sectional dimensions of castellated beams

5. 1. Statement of the optimization problem

On the one hand, increasing the load-carrying capacity of the beam for bending requires developing the beam cross-section in height as much as possible by forming openings in the beam (hexagonal cutouts) of the maximum possible size. However, too large openings in the beam web lead to premature exhaustion of its load-carrying capacity under the condition of ensuring strength under normal stresses at the design points at the top and bottom of the opening. At the same time, normal stresses occur in the extreme fibers of the beam cross-section, which are much smaller than the yield strength of steel. Given this, it is considered appropriate to state the task of optimizing the cross-section dimensions of the beams under study.

The task to optimize cross-section dimensions of a castellated beam of a monosymmetric cross-section was stated as the search for the optimal number of profiles for the upper and bottom Tees and the optimal width of the web opening. At the same time, as part of the system of constraints, the conditions for ensuring the necessary load-carrying capacity of the beam in accordance with the design code requirements were considered. The criterion of optimality was the minimum volume of the material from which the beam was made. The following were considered as the initial data for performing the optimization calculation:

1) the predefined range of I-beam hot-rolled profiles, from which profile numbers were chosen for the upper and lower flanges of the beam;

2) steel grade from which the beam was made;

3) beam span;

4) the shape of the opening is a regular hexagon;

5) beam load scheme – the load applied to the upper flange of the beam as uniformly distributed along the entire beam span;

6) the value of the design loads for verifications of the load-carrying capacity of the beam according to the ultimate and serviceability limit states.

The stated optimization problem was represented in terms of a nonlinear programming problem [21] as a search for unknown values of the variable parameters of the beam structural member:

$$\vec{X}=\{X_\iota\}^T,\quad \iota=\overline{1,N_X}; \tag{1}$$

which provide the lowest value of the optimality criterion:

$$f^*=f\left(\vec{X}^*\right)=\min_{\vec{X}\in\Im_N} f\left(\vec{X}\right); \tag{2}$$

in the area of admissible decisions, which is defined by the system of constraints:

$$\varphi\left(\vec{X}\right)=\left\{\phi_\eta\left(\vec{X}\right)\le 0 \mid \eta=\overline{1,N_{IC}}\right\}; \tag{3}$$

where $\vec{X}$ is a vector of design variables (unknown design parameters); N_X – the total number of design variables; f, φ_η – some functions of the vector argument; $\vec{X}^*$ – optimal design solution of the beam or optimum point (vector of optimal values of variable parameters of the structure); f^* is the optimal value of the objective function (criterion of optimality); N_{IC} is the number of constraints-inequalities $\varphi_\eta\left(\vec{X}\right)$, which determine the area of admissible design solutions $\Im$.

The vector of design variables $\vec{X}$ includes parametric design variables of continuous (real) and discrete (integer) types [22]. The width of the hexagonal opening a in the beam web was considered as a parametric continuous variable that varies continuously within some defined interval of possible values. Parametric discrete variables vary according to a defined finite set of possible values. As such variables, the ordinal numbers (indexes) of the profiles in the defined (considered) range of hot-rolled I-beam profiles were considered, namely N_1 – for the top Tee and N_2 – for the bottom Tee:

$$\vec{X}=\{N_1,\ N_2,\ a\}^T. \tag{4}$$

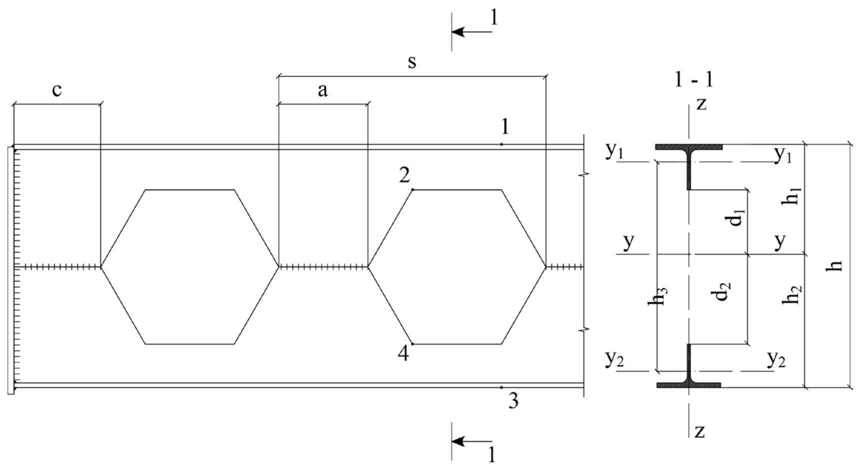

Fig. 2. Diagram of the castellated beam section

In the design cross-section of a castellated beam, the material is concentrated on the line of the beam flanges and represents two T-shaped sections that behave together (Fig. 2). The total height of the calculated beam cross-section h and the height of the top $h_{\perp,1}$ and bottom $h_{\perp,2}$ Tees of the beam depend on the vector of design variables $\vec{X}=\{N_1,\ N_2,\ a\}^T$ and are considered as state variables. Their value is determined by the section height of the original I-beams with variable numbers N_1 and N_2, which are used, respectively, for the top and bottom Tees of the beam, and the variable width a of the hexagonal opening.

The system of constraints (3) of the mathematical model involved strength checks of the castellated beam according to the level of normal stresses, formulated for the cross-sections located in the middle of each hexagonal opening. At the same time, the number of openings n in the form of regular hexagon with side a was calculated as a integer from dividing the constant span of the beam L by the hole pitch $s=3a$.

The design constraints-inequalities were formulated for four design cross-section points located at a distance of $0.5a$ from the middle of the hole under consideration (Fig. 2):

– at design point 1 (according to the level of maximum normal stresses occurring in the extreme fiber of the upper flange of the beam):

$$\frac{1}{R_{y1}\gamma_c}\left(\frac{M_{y,j}h_1}{I_{y,\Sigma,net}}+\frac{Q_{z,j}a}{2W_{y,1,\max}}\cdot\frac{I_{y,1}}{I_{y,1}+I_{y,2}}\right)-1\le 0; \tag{5}$$

– at design point 2 (according to the level of normal stresses occurring at the top of the hole):

$$\frac{\gamma_u}{R_{u1}\gamma_c}\left(\frac{M_{y,j}d_1}{I_{y,\Sigma,net}}+\frac{Q_{z,j}a}{2W_{y,1,\min}}\cdot\frac{I_{y,1}}{I_{y,1}+I_{y,2}}\right)-1\le 0; \tag{6}$$

– at design point 3 (according to the level of maximum normal stresses occurring in the extreme fiber of the lower flange of the beam):

$$\frac{1}{R_{y2}\gamma_c}\left(\frac{M_{y,j}h_2}{I_{y,\Sigma,net}}+\frac{Q_{z,j}a}{2W_{y,2,\max}}\cdot\frac{I_{y,2}}{I_{y,1}+I_{y,2}}\right)-1\le 0; \tag{7}$$

– at design point 4 (according to the level of normal stresses occurring at the bottom of the hole):

$$\frac{\gamma_u}{R_{u2}\gamma_c}\left(\frac{M_{y,j}d_2}{I_{y,\Sigma,net}}+\frac{Q_{z,j}a}{2W_{y,2,\min}}\cdot\frac{I_{y,2}}{I_{y,1}+I_{y,2}}\right)-1\le 0; \tag{8}$$

here $I_{y,1}$ and $I_{y,2}$ are the second moments of area of the upper and lower T-sections of the beam relative to its own axis of inertia parallel to the flanges; $I_{y,\Sigma,net}$ – second moment of area of the beam cross-section minus the opening relative to the y-y bending axis, which is perpendicular to the beam web; $W_{y,1,\max}$ and $W_{y,1,\min}$ are, respectively, the largest and smallest elastic section modulus of the upper T-beam section; $W_{y,2,\max}$ and $W_{y,2,\min}$ are, respectively, the largest and smallest elastic section modulus of the lower T-beam section; R_{y1}, R_{u1} – design resistance of rolled steel for the upper T-shaped section of the beam; R_{y2}, R_{u2} – design resistance of rolled steel for the lower T-shaped section of the beam; γ_c is the coefficient of beam operating conditions; $M_{y,j}$ and $Q_{z,j}$ are the bending moment and shear force that occur in the jth design section of the beam and are calculated depending on the location of this section and the scheme and magnitude of the external load; d_1, d_2 – distances from the center of mass of the beam cross-section to the hole edge, respectively, along the top and bottom of the hole; h_1, h_2 are the distances from the center of mass of the beam cross section to the extreme fibers of the beam along the upper and lower flange, respectively (Fig. 2).

The system of constraints (3) also included strength checks of a castellated beam based on the level of shear stresses, which were formulated as:

$$\frac{Q_{\sup}s}{0.58at_{w1}h_3R_{y1}\gamma_c}-1\leq 0; \tag{9}$$

$$\frac{Q_{\sup}s}{0.58at_{w2}h_3R_{y2}\gamma_c}-1\leq 0, \tag{10}$$

where t_{w1} and t_{w2} are web thicknesses of rolled I-beams, from which the upper and lower flanges of the castellated beam are made, respectively, depending on the variable numbers of profiles N_1 and N_2; h_3 is the distance between the centers of mass of the upper and bottom Tees of the beam (Fig. 2), which depends on the vector of design variables $\vec{X}=\{N_1,\ N_2,\ a\}^T$; $Q_{\sup}$ is the shear force acting in the cross section of the beam located at a distance $\ell=c+2.5a$ from the support, here c is the width of the end-post or distance from the support section of the beam to the edge of the first hole (Fig. 2), the value c was calculated depending on the variable a of the opening width, the variable number of holes n and the constant span of the beam L.

The local buckling verification of the web of the upper compressed Tee in the area of the opening was formulated in accordance with the design code requirements DBN V.2.6-198:2014 “Steel structures. Design norms” as:

$$1.0-\frac{b_{f1}}{h_{w1}}\leq 0; \tag{11}$$

$$\frac{b_{f1}}{2h_{w1}}-1\leq 0; \tag{12}$$

$$\frac{h_{w1}}{0.498\left(1+0.25\sqrt{2-\dfrac{b_{f1}}{h_{w1}}}\right)t_{w1}}\sqrt{\frac{R_{y1}}{E}}-1\leq 0, \tag{13}$$

where h_{w1} is the design height of the top Tee web of the upper (compressed) zone, $h_{w1}=h_{\perp,1}-t_{f1}-r_1$; b_{f1}, t_{f1} and r_1 are, respectively, the flange width, the flange thickness, and the radius of the flange-to-web junction of the top Tee of the beam, which depend on the variable number of the profile N_1 chosen for it.

The local buckling checks of cross-section elements, as regulated by the codes, are derived from the assumption that each cross-section element is considered as a long thin plate properly restrained along one of the free edges. Such supporting of the plates is provided by adjacent cross-section elements. In view of this, conditions (11) and (12) must be fulfilled for the T-shaped cross-section. Otherwise, it would mean that either the width of the flange is insufficient to provide adequate web restraining, or the web height is insufficient to provide adequate support for the flange.

Checking the end-post local stability of a beam end-post with a perforated web is performed for unevenness in accordance with DBN B.2.6-198:2014 “Steel structures. Design norms:

$$\frac{h_{ef}}{2.5t_{w,\min}}\sqrt{\frac{R_{y,\max}}{E}}-1\leq 0, \tag{14}$$

here h_{ef} is the design height of the web section of the beam web-post, which is calculated as: $h_{ef}=h-t_{f1}-r_1-t_{f2}-r_2$; h is the full height of the developed section of the castellated beam, which depends on the design variables vector; $t_{w,\min}$ is the smaller of the web thicknesses of rolled I-beams, which are used to manufacture the top and bottom Tees, $t_{w,\min}=\min\{t_{w1},\ t_{w2}\}$; $R_{y,\max}$ is the larger of the design resistances of the material of rolled I-beams, $R_{y,\max}=\max\{R_{y1},\ R_{y2}\}$. Inequality (14) was not included in the system of constraints (3) in the mathematical model. In the event that the condition of ensuring local stability of the beam web-post (14) is not fulfilled, the beam web is strengthened with one-sided stiffeners, arranged in a checkerboard pattern in each web-post of the beam.

It should be noted that the verification of the lateral-torsional buckling of the castellated beam was also not considered as part of the system of constraints (3). It is assumed that the compressed flange of a castellated beam is restrained by the necessary number of shear ties, which prevent horizontal displacements of the compressed flange from the beam bending plane.

As part of the system of constraints (3), the stiffness limitation of a castellated beam was considered at the level of the maximum deflection in the middle of the beam span, which was stated as for a beam loaded with a uniformly distributed load:

$$\frac{5}{384}\times\frac{q_{sls}L^4}{0.95EI_{y,\Sigma,net}f_{ult}}-1\leq 0, \tag{15}$$

here E is the modulus of elasticity of steel, $E=2.06\cdot 10^5$ MPa; f_{ult} – the limit deflection of the beam, which was taken in accordance with the requirements of DSTU B V.1.2-3:2006 as $f_{ult}=L/250$; 0.95 is a coefficient that makes it possible to take into account some increase in the deflection of the beam due to the shear deformations of the web and the bending of the top and bottom Tees of the beam [23].

The optimality criterion (2) considered was the volume of material required for the manufacture of a castellated beam:

$$f^*=f\left(\vec{X}^*\right)=\min_{\vec{X}\in\Im_N}V\left(N_1,N_2,a\right). \tag{16}$$

Finally, the task of optimizing the cross-section dimensions of a castellated beam was stated as the search for the optimal numbers of profiles N_1 and N_2 for the beam top and bottom Tees, as well as the optimal width a of the web opening. At the same time, the criterion of optimality considered was the minimum volume of material (16). As part of the system of constraints, the conditions for ensuring the necessary load-carrying capacity of the beam were considered in terms of strength (5) to (8), local stability (11) to (13), and stiffness (15).

It should be noted that as part of the mathematical model of the stated optimization problem, it would also be possible to consider additional constraints that reflect the structural requirements for the construction of such beams [24]. These conditions can easily be expressed in the form of constraints on the upper and/or lower bounds for the design variables values.

The dimensionality of the parametric optimization problem consisted of 3 design variables and 8 constraints in the form of inequalities. Taking into account the small size of the stated parametric optimization problem, it was solved by the method of exhaustive search (full search) using software developed in the C# language in the MS Visual Studio 2022 environment [25].

5. 2. Results of optimizing the cross-section sizes of castellated beams for different classes of steel and load

As a result of the optimization calculation, castellated beams were obtained with optimal cross-sectional dimensions depending on the steel grade, the beam span and the magnitude of uniformly distributed load acting on the beam. The search for optimal dimensions of a castellated beam was carried out for an assortment of normal hot-rolled I-beams with parallel flanges according to GOST 26020-83. At the same time, beam spans of 12 m, 15 m, and 18 m were considered at different values of transverse uniformly distributed load and different steel grades. The results of the optimization calculation of a castellated beam with a span of 18 m for steel grades C235, C295, and C345 are given in Tables 1–3.

Table 1

Results of optimizing the cross-section dimensions of a castellated beam with a span of 18 m and grade of steel C235

Uniform load q_{uls}, kN/m	The optimal profile number for beam flanges		Optimal dimensions		I-beam with a solid web of the required load-carrying capacity	Efficiency of web openings application, %
	Top N_1	Bottom N_2	Opening width a, mm	Beam height h, mm		
5	45B1	40B1	225	612	50B2	21.942
10	50B1	50B1	260	717	60B1	17.220
15	55B1	55B1	290	798	70B1	17.369
20	60B1	60B1	325	874	70B2	12.791
25	70B1	60B2	365	960	80B1	7.129
30	70B2	70B2	396	1,040	90B1	10.015
40	80B1	80B1	443	1,175	90B2	7.440
50	90B1	90B1	496	1,323	100B2	9.704
55	90B1	90B1	480	1,309	100B3	18.728
65	100B1	100B1	536	1,454	100B4	9.556
70	100B3	100B1	540	1,466	–	–
75	100B2	100B2	533	1,460	–	–
80	100B3	100B2	500	1,436	–	–
85	100B3	100B3	520	1,456	–	–
90	100B4	100B3	500	1,443	–	–
95	100B4	100B4	516	1,460	–	–

Table 2

Results of optimizing the cross-section dimensions of a castellated beam with a span of 18 m and grade of steel C295

Uniform load q_{uls}, kN/m	The optimal profile number for beam flanges		Optimal dimensions		I-beam with a solid web of the required load-carrying capacity	Efficiency of web openings application, %
	Top N_1	Bottom N_2	Opening width a, mm	Beam height h, mm		
5	45B1	40B1	225	612	50B2	21.94244
10	50B1	50B1	260	717	60B1	17.21991
15	60B1	50B1	325	824	70B1	17.82288
20	60B1	55B1	325	851	70B2	19.64315
25	60B2	60B2	325	878	80B1	14.69566
30	70B1	60B2	366	961	80B2	16.68309
35	70B1	70B1	391	1,030	90B1	18.46128
40	70B2	70B2	383	1,029	90B1	10.47703
45	80B1	70B2	406	1,096	90B2	13.22791
50	80B1	80B1	437	1,169	100B1	14.4004
55	80B2	80B2	431	1,171	100B1	6.048207
65	90B1	90B1	484	1,312	100B2	9.966547
75	90B2	90B2	472	1,309	100B3	7.814641
80	100B1	90B2	488	1,368	100B4	13.941
85	100B1	100B1	520	1,440	100B4	10.0247
90	100B1	100B1	509	1,431	–	–
95	100B2	100B2	523	1,451	–	–
100	100B2	100B2	513	1,442	–	–
105	100B3	100B3	515	1,452	–	–
110	100B3	100B3	506	1,444	–	–
115	100B3	100B3	495	1,435	–	–
120	100B4	100B4	507	1,452	–	–
125	100B4	100B4	497	1,443	–	–

Table 3

Results of optimizing the cross-section dimensions of a castellated beam with a span of 18 m and grade of steel C345

Uniform load q_{uls}, kN/m	The optimal profile number for beam flanges		Optimal dimensions		I-beam with a solid web of the required load-carrying capacity	Efficiency of web openings application, %
	Top N_1	Bottom N_2	Opening width a, mm	Beam height h, mm		
5	45B1	40B1	225	612	50B2	21.94244
10	50B1	50B1	260	717	60B1	17.21991
15	60B1	50B1	325	824	70B1	17.82288
20	60B1	55B1	325	851	70B2	19.64315
25	70B1	55B1	382	950	80B1	17.11609
30	70B1	60B1	377	968	80B2	19.51083
35	70B1	70B1	404	1,041	90B1	18.0199
40	70B2	70B1	391	1,033	90B1	14.39022
45	80B1	70B2	419	1,107	90B2	12.74789
50	80B1	70B2	404	1,094	100B1	19.62236
55	80B1	80B1	437	1,169	100B1	14.4004
60	90B1	80B1	466	1,246	100B1	8.331669
65	90B1	80B2	453	1,238	100B2	14.37432
70	90B1	90B1	487	1,315	100B2	9.900444
75	90B1	90B1	475	1,304	100B3	18.82932
80	100B1	90B1	504	1,378	100B3	8.692654
85	100B1	90B2	494	1,373	100B4	13.76608
90	100B1	100B1	528	1,447	100B4	9.790021
95	100B1	100B1	517	1,438	–	–
100	100B2	100B2	532	1,459	–	–
105	100B2	100B2	522	1,450	–	–
110	100B3	100B2	518	1,451	–	–
115	100B3	100B3	516	1,453	–	–
120	100B3	100B3	507	1,445	–	–
125	100B4	100B4	518	1,462	–	–
130	100B4	100B4	510	1,455	–	–
135	100B4	100B4	503	1,449	–	–

beam was solved. After all, increasing the load-carrying capacity of the beam for bending requires developing the section of the beam in height as much as possible by forming web openings in the beam of the maximum possible size. At the same time, too large web openings in the beam lead to the premature exhaustion of its load-carrying capacity under the condition of ensuring strength under normal stresses at the design points at the top and bottom of the opening, while the extreme fibers of the beam section remain understressed.

Our results of optimization calculations of castellated beams have made it possible to plot the dependence of the optimal width of the web opening in the beam depending on the ratio of the maximum bending moment occurring in the beam to the yield strength of steel. Thus, Fig. 6–8 show plots of the optimal width of the opening in the beam web for different beam spans (12 m, 15 m, and 18 m) and different steel grades (C235, C255, C295, C325, and C375).

Tables 1–3 demonstrate that the use of a web openings expands the scope of application of hot-rolled I-beams. The technology of manufacturing castellated beams makes it possible to increase the beam section modulus to 35.48...50 %. At the same time, the effectiveness of the use of the web openings, which is estimated by the volume of material required for the manufacture of the beam, reaches 23.19 %.

The above dependences of the optimal height of the castellated beam (Fig. 3–5) and the optimal width of the web opening (Fig. 6–8) on the required elastic section modulus of the beam cross section can serve as recommendations for designers. According to these plots, it is possible to select the dimensions of the cross-sections of castellated beams.

5. 3. Recommendations for the optimal distribution of material in cross-sections of castellated beams

Our results of the optimization calculations have made it possible to plot the dependence of the optimal height of the beam depending on the ratio of the maximum bending moment occurring in the beam to the yield point of steel (required elastic section modulus). Fig. 3–5 show plots of the optimal height of a castellated beam for different beam spans (12 m, 15 m, and 18 m) and different steel grades (C235, C255, C295, C325, and C375).

In addition, during the optimization calculations, the problem of finding the optimal width of the web opening in the

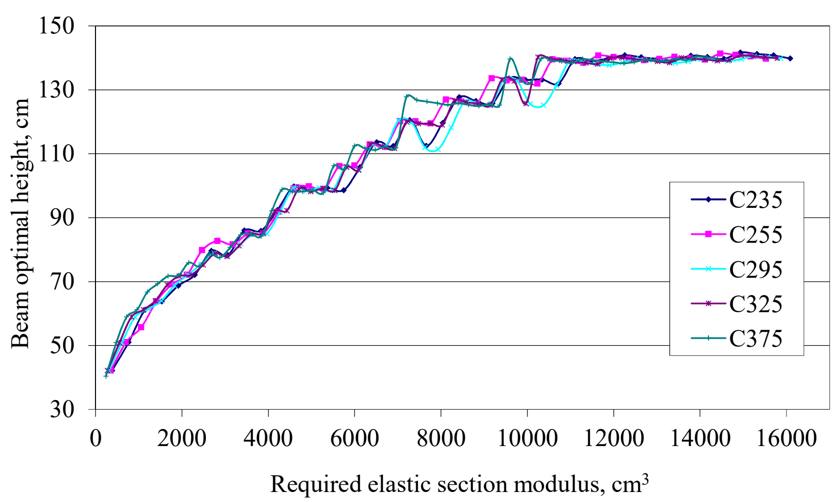

Fig. 3. Plotting the optimal height of a castellated beam with a span of 12 m depending on the required elastic section modulus of the beam

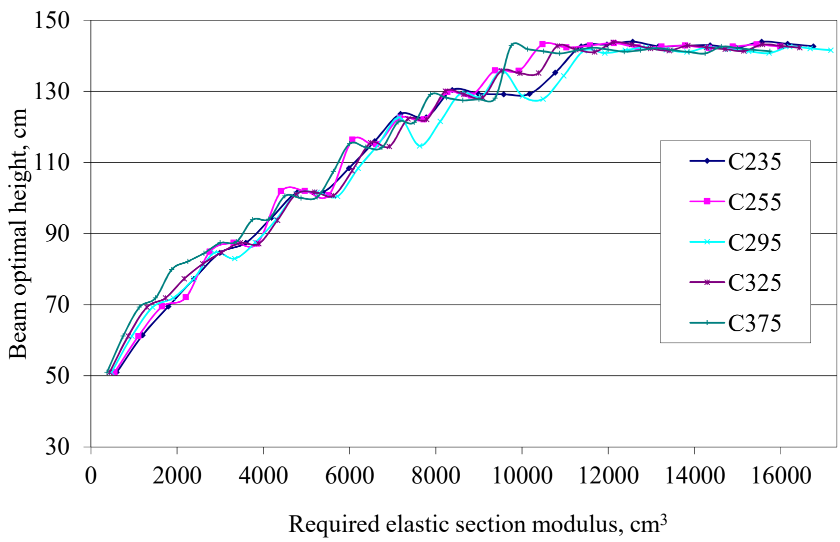

Fig. 4. Plotting the optimal height of a castellated beam with a span of 15 m depending on the required elastic section modulus of the beam

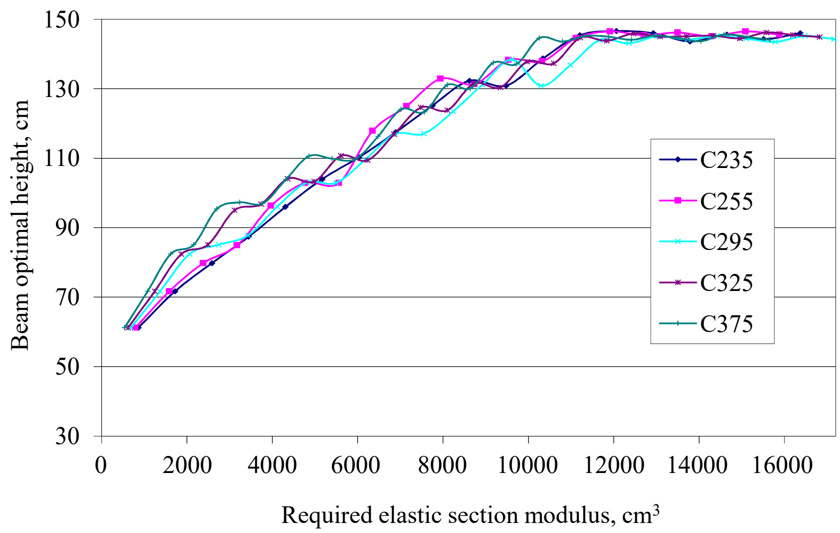

Fig. 5. Plotting the optimal height of a castellated beam with a span of 18 m depending on the required elastic section modulus of the beam

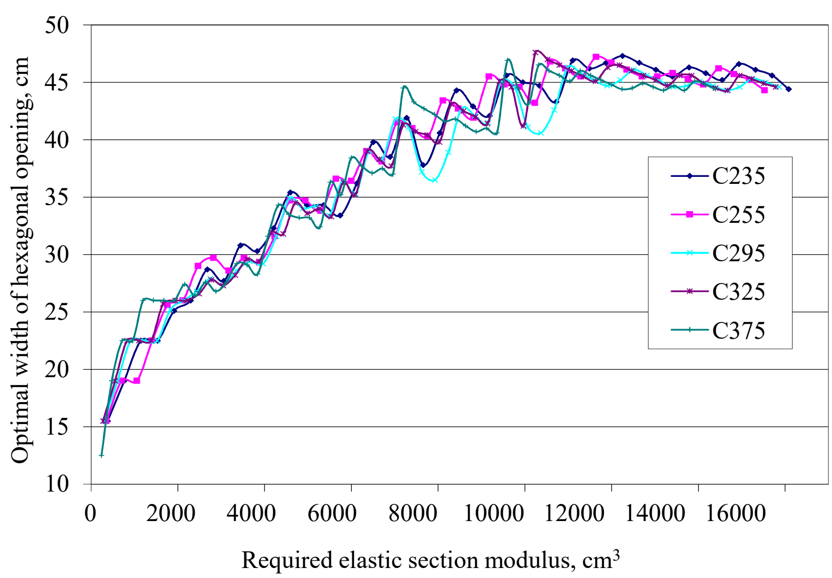

Fig. 6. Plotting the optimal width of the hexagonal opening for a castellated beam with a span of 12 m depending on the required elastic section modulus of the beam

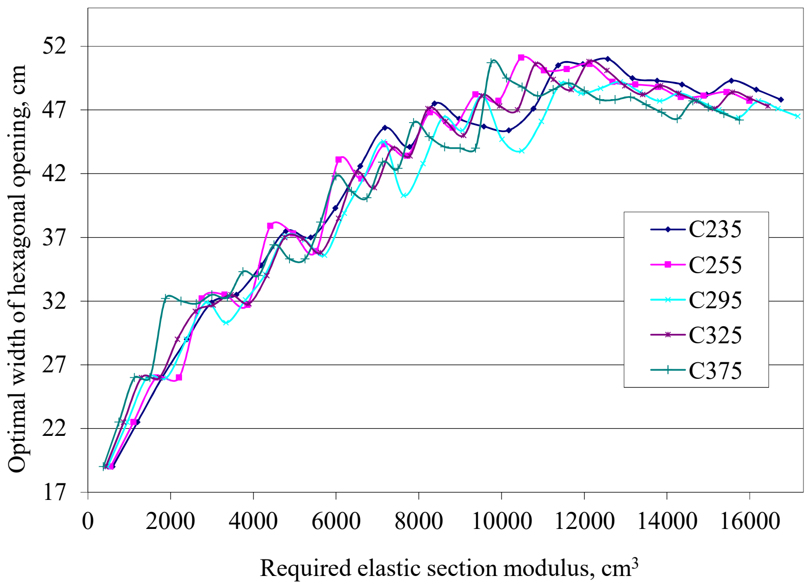

Fig. 7. Plotting the optimal width of the hexagonal opening for a castellated beam with a span of 15 m depending on the required elastic section modulus of the beam

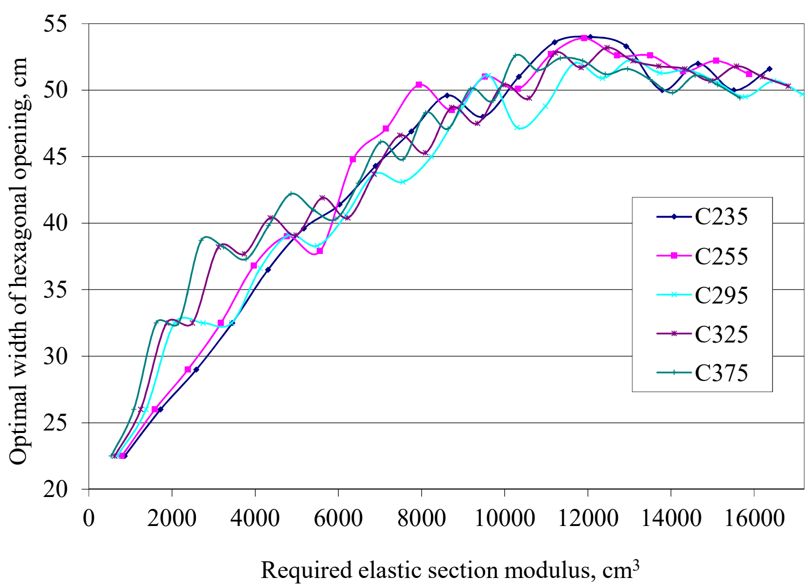

Fig. 8. Plotting the optimal width of the hexagonal opening for a castellated beam with a span of 18 m depending on the required elastic section modulus of the beam

The peculiarity of the proposed approach in comparison with existing ones is the simultaneous variation in the search for the optimum point of both the discrete cross-section sizes of rolled I-beam profiles from which the beam is made, and the continuous size of the width of the hexagonal opening in the beam web. Thus, unlike [13], in which only the height of the opening, the angle of inclination of the cutting line to the axis of the beam, and the distance between the openings were considered as design variables of the optimization problem, the vector of design variables with variables of discrete and continuous types proposed in this work makes it possible to obtain a better design beam solution.

It should be noted that it is not possible to compare our results of optimization calculations with the optimal design solutions of castellated beams known and described in the literature since the latter were obtained for other design conditions (other calculation schemes and other design standards). However, the difference of this work compared to similar well-known studies (such as, for example, [15]) is obtaining the dependences of the optimal height of the castellated beam and the optimal width of the web opening depending on the ratio of the maximum bending moment in the beam to the yield strength of steel. The specified dependences are of practical value and could be used in engineering design practice to select cross-sectional dimensions of castellated beams.

It should be noted that the reported results regarding the optimal distribution of material in cross-sections of castellated beams (Tables 1–3, Fig. 4–9) have certain application limits. In particular, they are valid only for the range of normal I-beam profiles and only for the case of an uniformly distributed load acting on the beam when the compressed beam flange is sufficiently restrained from the bending plane and the beam web has openings in the form of regular hexagons. It is under such conditions that our results can be implemented in practice both at the stage of selecting cross-sections of the studied class of structures, and at the development of effective assortments of castellated beams. However, the proposed methodology for finding the optimal dimensions of the cross-sections of castellated beams could be applied both for other conditions of loading and lateral restraining of beams, and for other assortments of I-beam profiles.

6. Discussion of results of cross-sectional size optimization of a castellated beam

Our results of the optimization calculations (Tables 1–3) are primarily explained by the type of cross-section of the structural member and its stress-strain state. This obviously causes the presence of stress concentration in the corners of the openings in the beam web and affects the obtained optimal dimensions of the opening width. The reliability of the optimization results of cross-section dimensions for castellated beams (Tables 1–3) is confirmed by the following:

– the rigor and correctness of the mathematical model of the optimization problem of the investigated class of structures;

– the stability of the resulting numerical solutions in relation to the initial data and the analysis of convergence of the iterative search process.

7. Conclusion

1. We have stated and solved the problem of finding the optimal cross-section dimensions of castellated beams. In this case, the vector of design variables contained variables of continuous and discrete types, and the system of problem

constraints included load-carrying capacity constraints in accordance with the code-based requirements.

2. As a result of the optimization calculation, castellated beams were obtained with optimal cross-sectional dimensions depending on the steel grade, the beam span, and the magnitude of uniformly distributed load. Our optimization calculations proved that the elastic section modulus of the beam could be increased to 35.48...50.0 % due to the use of a openings in the web. Castellated beams with optimal cross-sectional dimensions at the same load-carrying capacity are characterized by lower steel consumption (up to 23.19 %) compared to I-beams with a solid web.

3. Recommendations for the optimal distribution of material in cross-sections of castellated beams have been devised. These recommendations are represented in the form of plots of the optimal beam height and the optimal width of the web opening of the beam depending on the ratio of the maximum bending moment occurring in the beam to the yield strength of the steel. In this case, the average ratio of the optimal width of the hexagonal opening in the beam web to the optimal height of the beam was 0.34...0.37. The average ratio of the optimal beam height to the optimal beam web thickness was 80.95...84.1.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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