This paper considers a heat conduction process for an isotropic medium with local external and internal thermal heating. It was necessary to construct linear and non-linear mathematical models for determining the temperature field, and consequently, for the analysis of temperature regimes in these heat-active environments. To solve the linear boundary value problems and the resulting linearized boundary value problems with respect to the Kirchhoff transformation, the Henkel integral transformation method was used, as a result of which the analytical solutions to these problems were obtained. For a heat-sensitive environment, as an example, a linear dependence of the coefficient of thermal conductivity of the structural material of the structure on temperature, which is often used in many practical problems, was chosen. As a result, analytical relations for determining the temperature distribution in this environment were established. To determine the numerical values of the temperature and analyze the heat exchange processes in the given structure, caused by the external heat load, a geometric image of the temperature distribution was constructed depending on spatial coordinates. The resulting linear and non-linear mathematical models testify to their adequacy to the real physical process. They make it possible to analyze heat-active media regarding their thermal resistance. As a result, it becomes possible to increase it and protect it from overheating, which can cause the destruction of not only individual nodes and their elements but the entire structure as well

Keywords: temperature field, thermal conductivity of material, convective heat exchange, heat flow, thermal resistance of structures

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CONSTRUCTING MATHEMATICAL MODELS OF THERMAL CONDUCTIVITY IN INDIVIDUAL ELEMENTS AND UNITS OF ELECTRONIC DEVICES AT LOCAL HEATING CONSIDERING THERMOSENSITIVITY

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1. Introduction

Modern advances in digital technology have led to a significant increase in the power and functionality of digital devices. Processors, microcontrollers, graphics cards, and other electronic components are becoming larger and more powerful, which poses serious challenges for thermal management. High levels of heat generation lead to significant temperature gradients, which cause unwanted overheating, reduced performance, and reduced lifespan of digital devices. Temperature fields in structural heat-active assemblies and elements of digital devices are one of the important reasons that lead to this. For example, a large number of electronic components placed on a limited board area creates a large difference in heat transfer and heat generation capacity. This creates non-uniform thermal fields that require detailed analysis and optimization to enable stable and reliable device operation. Research shows that thermal management is a critical aspect of achieving optimal performance and longevity of digital devices. Modern modeling and simulation technologies make it possible to analyze temperature fields in detail, evaluate the influence of various factors and devise effective cooling strategies. Heat sources in digital devices contain electronic components that consume power and generate heat during their operation. These sources can be of various types, in particular, processors, graphics processors, memory, logic gates, etc. Each of these components has its own power, which is determined depending on the operating frequency, voltage, and other functional parameters. To simulate heat sources in digital devices, thermal component models are designed. These models contain information about the thermal properties of the components, in particular, power, thermal resistance, and temperature coefficient of resistance. Such models make it possible to analyze the thermal effects of individual components and their interaction in the system. In addition to modeling heat sources, an important aspect is the placement of components on the board. The distance between components and their placement can affect heat distribution in an electronic device. Optimal placement reduces thermal interaction and enables more efficient heat transfer. Some digital devices contain dynamic heat sources that depend on operating conditions and tasks. For example, processors can change their frequency and power depending on the load. Modeling of dynamic heat sources requires taking these changes into account and analyzing the impact on temperature fields. For numerical modeling of temperature fields, it is necessary to take into account heat sources in the heat conduction equations. This requires the introduction of terms, which reflect the thermal generation in the relevant regions of the device. At the same time, the constant or variable nature of heat sources should be taken into account. In modern mechanical engineering, precision instrument manufacturing, and automobile manufacturing, in particular, for the mechanisms of automated control over production processes, separate nodes of structures and their elements are widely used in the form of spatial thermally active structures with internal and external local heating. The design and development of such mechanisms involves not only expanding their capabilities and improving operational characteristics but also ensuring stable operation, high reliability, and thermal stability. The increase in the capacities of such mechanisms and their integration into the system significantly complicates the problem of thermal resistance to thermal loads of their structures, which partially or completely fail due to thermal overloads.

The first step in the analysis of temperature regimes is to determine the thermal power of the device. Thermal power reflects the amount of energy that a device emits or gives to the environment, which is achieved by measuring the current and voltage that pass through the device, as well as using software tools for modeling thermal processes. The second step is to determine the thermal resistance of the device, which the device inflicts on the heat flow. This is achieved by measuring temperature gradients at different points of the device. The third step is to determine the temperature regime of the device. The temperature mode displays the temperature of the device after a certain period of operation. This is achieved by measuring the temperature of the device after a long time of operation or using software tools for modeling thermal processes. Based on the results of analysis of thermal modes, decisions are made regarding the installation of additional measures to prevent overheating of the device. One of the ways to reduce the temperature of the device is to use fans or heat sinks to remove heat from its individual components and elements. Materials with high thermal conductivity are also used to reduce the temperature of the device. An important aspect of the analysis of temperature regimes is the consideration of environmental factors, such as temperature, humidity, and atmospheric pressure. These factors can affect the temperature regime of the device and the need to use additional measures to reduce the temperature. In modern digital devices, temperature regimes play an important role in ensuring their stable and reliable operation. As a result, when designing and manufacturing digital devices, it is necessary to take into account the operating temperature conditions in order to prevent their overheating.

Therefore, it is a relevant task to carry out studies on the construction of mathematical models for the determination and analysis of temperature fields in individual elements and nodes of electronic devices during local heating, taking into account thermal sensitivity.

2. Literature review and problem statement

Determination of temperature regimes in both homogeneous and heterogeneous structures attracts the attention of many researchers. Temperature plays an important role in determining the physical and chemical characteristics of materials. This effect becomes particularly significant when there are significant temperature fluctuations, as observed in heat conduction processes. Temperature differences lead to certain changes in material properties, which makes it difficult to determine the distribution of temperature and thermal stress. As a result, determining the thermoelastic state of structures becomes much more difficult.

In [1], the thermoelastic problem of an elliptical cavity in an infinite medium was investigated using the generalized complex variable method. As a result of the analysis of the thermoelastic state of the medium, the temperature dependence of the coefficient of thermal conductivity, modulus of elasticity, and coefficient of thermal expansion are taken into account. Analytical expressions for temperature, heat flow, and thermoelastic fields were built taking into account these dependences. There are no studies on the influence of temperature dependence of thermoelastic parameters on temperature distribution, heat flow, and thermoelastic fields.

Analytical solutions of the distribution of temperature, displacements, and stresses in layered rectangular plates with a simple support, which are subjected to thermomechanical loads, are given in [2]. The material properties of the layers depend on the temperature. As a result of the analysis, it was found that it is not possible to determine the analytical solution of the boundary value problem for local thermomechanical loads.

Paper [3] investigated the thermoelastic parameters of functionally graded porous plates with different material distribution and found that thermal stresses are more sensitive to material distribution than temperature and deformations. It was established that the unsolved problem is the lack of an analytical solution of the boundary value problem for the structure under consideration, which would make it possible to determine the influence of thermal stresses on the distribution of material in the environment.

In work [4], research is aimed at determining the influence of the temperature dependence of material properties and indicators of the compositional gradient in functionally graded rectangular plates in relation to temperature, deformations, and stresses. It was found that the indicators of the composite gradient were determined by a numerical method, which does not allow the computer algorithm to automatically form composite materials.

The solution for the steady-state reaction of thick cylinders subjected to pressure and external heat flow on the inner surface is given in [5]. The influence of the temperature gradient on the deformations of the environment is not taken into account, which significantly worsens the accuracy of the model.

A thermal analysis of cylinders of different thickness, made of functionally graded materials, which are under the influence of non-uniform heat flows concentrated on the inner and outer layers [6, 7], was performed. The above studies do not make it possible to analyze the thermal state of the cylinders for local thermal disturbance.

Work [8] gives the solution to the non-stationary problem of thermal conductivity and thermoelasticity for functional gradient thick spheres. Thermophysical and thermoelastic parameters of materials, with the exception of Poisson's ratio, are arbitrary functions of the radial coordinate. The axisymmetric stationary problem of thermal conductivity and thermoelasticity for hollow functional gradient regions relative to the heat source was considered. The boundary value problem is simplified since the temperature distribution is determined only by one spatial coordinate.

Thermal modeling of electronic devices is one of the most important tools for assessing their reliability under various operating modes. In [9], a thermal model of electronic devices is presented, which is based on experimental temperature measurement data obtained by an infrared camera. These data are input to the mathematical model built, which is based on the method of finite differences and some known physical dependences. The model constructed was verified by comparing simulation data with experimental data. It can be used to study the thermal behavior of the device under various operating conditions. The temperature distribution is determined experimentally, which introduces an error into the developed mathematical model based on the finite difference method. Consequently, the results contain significant errors.

In most portable electronic devices, in addition to the temperature of several heat sources, i.e., the junction temperature, the body temperature, i.e., the skin temperature, must also be monitored to protect the user. Thus, building a compact device-level thermal model to predict skin temperature will not only improve the efficiency of thermal design at an early stage but also help devise a model-based temperature control strategy. In paper [10], dynamic compact thermal models of two portable electronic devices, including a smartphone and a laptop, were built based on the convolution method. Under the assumption of linear time invariance of the system, the skin temperature for the two test devices can be quickly determined after the step response of each heat source is obtained. The constructed model is experimental and does not allow determining the temperature regimes for more than two portable electronic devices.

The increase in specific power of electronic devices, due to high performance and miniaturization requirements, has prompted researchers to search for new and alternative methods of temperature control. Most electronic devices are frequently subjected to high frequency power cycles. Cooling systems must be able to manage transient thermal profiles to delay the temperature response and reduce temperature gradients within the device that can lead to thermal stresses. In the long run, this can lead to the failure of the electronic device. The integration of phase change materials (PCM) in heatsinks for electronic devices represents an interesting technical system to increase the thermal inertia of the cooling system while providing a more stable operating temperature in the electronic components. Paper [11] discusses recent research trends in this field, with a special focus on electric batteries, power electronics, and portable device applications. In the studies, the value of the working temperature is determined experimentally. The errors contained in these values significantly affect the efficient operation of electronic device components.

Much of the effort in electronics thermal management has focused on developing cooling solutions that provide steady-state operation. However, electronic devices are increasingly used in applications with time-varying workloads. These include microprocessors (especially those used in portable devices), power electronic devices, and arrays of powerful semiconductor laser diodes. Transient solutions for temperature management are becoming essential to enable the performance and reliability of such devices. New requirements for temperature control in transient processes are defined in [12], and cooling recommendations described in the literature for such applications are given, focused on the time scales of the thermal response. Management of temperature regimes is carried out experimentally, which significantly limits the establishment of optimal values of the temperature field for the effective functioning of electronic devices.

Paper [13] analyzed the features of the temperature field distribution and the reaction of the heat-conducting material as a function of its polishing parameters. Polishing of rails is widely used as a technique for re-profiling the surfaces of rails in case of wear, as well as for eliminating missing damage. However, polishing can lead to burn-out of the surface and the formation of a white etching layer (WEL). Taking into account the position of the rail surface, the result of the study was the construction of an analytical thermal model based on an unevenly distributed heat source for predicting the temperature field during the polishing process. The temperature as a result of the rail polishing experiment was measured using special thermocouples. At the same time, the reaction of the rail material was analyzed in detail from the point of view of surface heating and the white etching layer. The results show that at a polishing temperature of about 400 °C, WEL starts to appear on the rail surface. Remains of austenite were found on the polished surfaces of the rails, which indicates the existence of martensite as a result of the effect of a combination of thermal and mechanical interactions. In order to describe the relationship between polishing temperature, surface burn-out and WEL, suitable diagrams have been built for use in real production. To obtain a high-quality surface of the rails during polishing, it would be advisable to construct a mathematical model of the heat conduction process, which would significantly increase the accuracy of determining temperature gradients.

The features of temperature field inversion of heat source systems (Temperature Field Inversion of Heat-Source System, TFI-HSS) through neural networks with physical information are considered. It was found that the inversion of the temperature field of heat source systems with limited observations is important for monitoring the performance of the system. Although some methods, in particular interpolation, have been proposed to solve the TFI-HSS problems, when using such methods, the interaction between data constraints and physical constraints is ignored, which causes their low calculation accuracy. A method of inversion of the temperature field based on a neural network was developed to perform the TFI-HSS task, a method for selecting the position of observations based on the number of conditions of the matrix of coefficients for choosing the optimal position of observations for noise was devised. For the TFI-HSS problem, the PINN-TFI method allows encoding the constraint terms into the loss function, and the problem formulated in this way turns into a loss function minimization problem. At the same time, it was found that noise observations significantly affect the reconstruction performance of the

PINN-TFI method. To reduce the need for noise observations, it is recommended to use the CMCN-PSO method to find optimal positions, in which the number of observation conditions is used to estimate certain positions. The results demonstrate that the use of the PINN-TFI method makes it possible to significantly increase the accuracy of forecasting, and when applying the CMCN-PSO method, a more convenient technique for obtaining a reliable temperature field can be found [14]. The results contain large errors, which prompts the construction of mathematical models for determining temperature gradients using modern analytical and numerical methods.

The authors of [15] report the results of an experimental and numerical study of the reconstruction of the temperature field based on acoustic tomography. They argue that obtaining a high-quality measurement of the temperature distribution is crucial for optimal control over the combustion process in the boiler. At the same time, acoustic tomography (AT) is used to measure the temperature distribution by multi-beam acoustic time of flight (TOF). The reconstructed model and the TOF measurement model are crucial for the practical application of AT measurement. The temperature field is represented in the form of a reconstruction model based on the approximation of the radial basis function with polynomial reproduction for solving the inverse problem. In such a reconstructed model, the effect of refraction of sound wave paths in a non-uniform temperature field is taken into account. To improve the quality of temperature field reconstruction and protect it from noise, the Truncated Singular Value Decomposition Reconstruction (TSVDR) method was used. In addition, the generalized cross-correlation with the second correlation is applied to estimate TOF in order to effectively avoid interference from noise. Numerical modeling and experimental studies were carried out to evaluate the effectiveness of the given method of temperature field reconstruction. The results indicate that the model developed by the authors, taking into account the refraction effect, makes it possible to reconstruct the temperature distribution with higher accuracy and better anti-noise ability compared to other existing methods. Experimental results were compared with thermocouple measurements. It was found that taking into account the refraction effect leads to better reconstruction characteristics. The value of the working temperature was obtained experimentally. Despite taking into account the effect of refraction, these values contain errors. Research results will be more effective if a mathematical model for determining the temperature distribution in the environment is developed.

Existing methods have been improved and new approaches have been devised to construct mathematical models that allow analyzing heat exchange in piecewise homogeneous media [16, 17]. Planar and spatial models of heat exchange are given, in which the differential equations contain coefficients dependent on the thermophysical properties of the phases and the geometric structure. Approaches for determining analytical and analytical-numerical solutions of boundary value problems of thermal conductivity are presented in [18]. Heat exchange processes occurring in homogeneous and layered structures with foreign inclusions of canonical form were analyzed in [19, 20]. In those works, the models in which local heating is taken into account remained little studied. The use of classical analytical and numerical methods does not make it possible to effectively take into account the local thermal heating of individual elements and nodes of electronic device structures. Therefore, a technique for building mathematical models of thermal conductivity, which take into account external and internal local heating of media, is given.

3. The aim and objectives of the study

The purpose of our study is to construct linear and nonlinear mathematical models of thermal conductivity for isotropic spatial heat-active media that are subject to internal and external local heating. As a result, there is an opportunity to increase the accuracy of determining temperature fields, which will further affect the effectiveness of methods for designing modern electronic devices.

To achieve this goal, the following tasks must be solved:

– to build a linear mathematical model for determining the temperature field with a locally concentrated heat flow;

– to construct a non-linear mathematical model for determining the temperature field with a locally concentrated heat flow;

– to build a linear mathematical model for determining the temperature field with internal heating of structures;

– to construct a non-linear mathematical model for determining the temperature field with internal heating of thermosensitive structures.

4. The study materials and methods

The object of our research is the heat conduction process for isotropic media heated by internal and external heat sources.

Hypothesis: the study was carried out within the framework of the classical theory of thermal conductivity.

Accepted assumptions and simplifications in the research process: media are not anisotropic, i. e., when their thermophysical parameters change in spatial directions and the heat conduction process is stationary since the change in the temperature field is determined only by spatial coordinates.

The theory of generalized functions was used to build linear and nonlinear mathematical models for determining the temperature field and analyzing temperature regimes in spatial environments with internal and external thermal heating. This makes it possible to effectively describe the local concentration of heat sources, which leads to the solution of boundary value problems of heat conduction, which contain differential equations and boundary conditions with a singular right-hand side. The Kirchhoff transformation was used to linearize nonlinear mathematical models of thermal conductivity.

An isotropic layer assigned to the cylindrical coordinate system ($Or\varphi z$) is considered. On the boundary surface of the layer $L_+=\{(r,\ \varphi,\ h)\colon 0\le r\le R,\ 0\le\varphi\le 2p\}$ the given structure is heated by a concentrated heat flux, the surface density of which is q_0=const, and on the other surface of the layer $L_-=\{(r,\ \varphi,\ -l)\colon 0\le r<\infty,\ 0\le\varphi\le 2p\}$ convective heat exchange occurs with the environment with a constant temperature t_c=const according to Newton's law (Fig. 1).

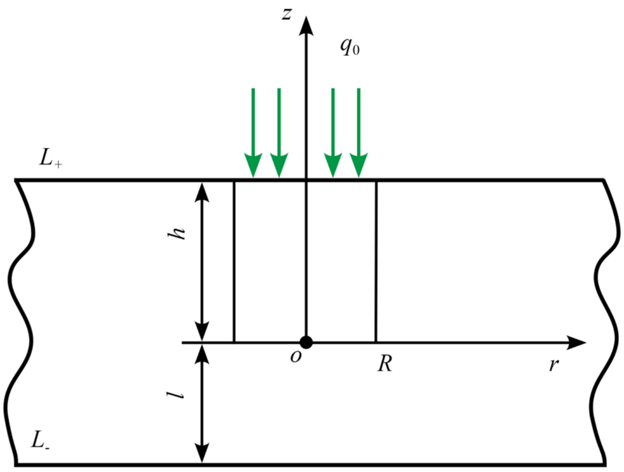


Fig. 1. Section of an isotropic layer with a plane φ=0, which is heated by a heat flow

In the given structure, it is required to determine the temperature distribution $t\ (r,\ z)$ according to the spatial coordinates r, z, which is obtained by solving the heat conduction equation [16–20]:

$$\frac{1}{r}\mathrm{div}\left[r\mathrm{grad}T(r,z)\right]=0, \tag{1}$$

with boundary conditions containing a discontinuous right part:

$$\left.\frac{\partial T(r,z)}{\partial z}\right|_{z=-l}=\frac{\alpha}{\lambda}T(r,z)\Big|_{z=-l},$$

$$\left.\frac{\partial T(r,z)}{\partial z}\right|_{z=h}=\frac{q_0}{\lambda}S_-(R-r), \tag{2}$$

where $T(r,\ z)=t(r,z)-t_c$; λ – thermal conductivity coefficient of the layer;

α – coefficient of heat transfer from the boundary surface of the layer L_-;

$$S_\pm(\zeta)=\begin{cases}1, & \zeta>0,\\ 0.5\mp0.5, & \zeta=0,\\ 0, & \zeta<0;\end{cases}$$

where $S_\pm(\zeta)$ are asymmetric unit functions [16–20].

In the case of constructing a nonlinear mathematical model for determining the temperature field with a locally concentrated heat flow, the case when the isotropic layer is thermosensitive (thermophysical parameters depend on temperature) is considered. Taking into account the thermal sensitivity, the temperature distribution $t(r,\ z)$ according to the spatial coordinates r, z in the given structure is obtained by solving the nonlinear heat conduction equation [16–20]:

$$\frac{1}{r}\mathrm{div}\left[r\lambda(t)\mathrm{grad}t(r,z)\right]=0, \tag{3}$$

with boundary conditions containing a discontinuous right part:

$$t(r,z)\big|_{r\to\infty}=0,\quad \left.\frac{\partial t(r,z)}{\partial r}\right|_{r\to\infty}=0,$$

$$\left.\frac{\partial t(r,z)}{\partial z}\right|_{z=-l}=0,\quad \lambda(t)\left.\frac{\partial t(r,z)}{\partial z}\right|_{z=h}=q_0S_-(R-r), \tag{4}$$

where $\lambda(t)$ is the coefficient of thermal conductivity of the thermosensitive layer.

An isotropic layer assigned to the cylindrical coordinate system $(Orjz)$ is considered in the region $\Omega_0=\{(R,\varphi,h):0\le\varphi\le2\pi\}$, which has uniformly distributed internal heat sources with a specific power q_0=const. On the boundary surface of the layer $L_+=\{(r,\ \varphi,\ h)\text{: } 0\le r<\infty,\ 0\le\varphi\le2p\}$ there is convective heat exchange with the environment with a constant temperature t_c=const according to Newton's law, and the other surface of the layer $L_-=\{(r,\ \varphi,\ -l)\text{: } 0\le r<\infty,\ 0\le\varphi\le2p\}$ is thermally insulated (Fig. 2).

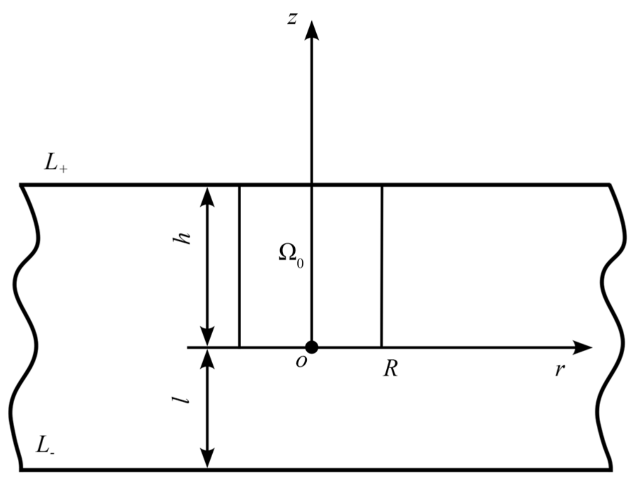


Fig. 2. Section of an isotropic layer with a plane φ=0, which is heated by internal heat sources concentrated in the region Ω_0

In the given structure, it is necessary to determine the temperature distribution $t(r,\ z)$ according to the spatial coordinates r, z, which is obtained by solving the thermal conductivity equation with a discontinuous right-hand side [16–20]:

$$\frac{1}{r}\mathrm{div}\left[r\mathrm{grad}T(r,z)\right]=-\frac{q_0}{\lambda}S_-(R-r)S_-(z), \tag{5}$$

and boundary conditions:

$$\left.\frac{\partial T(r,z)}{\partial z}\right|_{z=h}=\frac{\alpha}{\lambda}T(r,z)\Big|_{z=h},\quad \left.\frac{\partial T(r,z)}{\partial z}\right|_{z=-l}=0. \tag{6}$$

In the case of building a nonlinear mathematical model for determining the temperature field with internal heating of thermosensitive structures, the case when the isotropic layer is thermosensitive was considered. Then the temperature distribution $t(r,\ z)$ along the spatial coordinates r, z in the given structure for this case is obtained by solving the nonlinear heat conduction equation with a discontinuous right-hand side [16–20]:

$$\frac{1}{r}\operatorname{div}\left[r\lambda(t)\operatorname{grad}t(r,z)\right]=-q_0S_-(R-r)S_-(z), \qquad (7)$$

and boundary conditions:

$$t(r,z)\big|_{r\to\infty}=0,\ \left.\frac{\partial t(r,z)}{\partial r}\right|_{r\to\infty}=0,\ \left.\frac{\partial t(r,z)}{\partial z}\right|_{z=-l}=0, \qquad (8)$$

$$\lambda(t)\left.\frac{\partial t(r,z)}{\partial z}\right|_{z=h}=\alpha\left(t(r,z)\big|_{z=h}-t_c\right). \qquad (9)$$

5. Results of research into the process of constructing mathematical models of thermal conductivity for isotropic spatial heat-active media

5. 1. Linear mathematical model for determining the temperature field with locally concentrated heat flow

Henkel's integral transformation along the r coordinate was applied to equation (1) and boundary conditions (2), and as a result, a second-order ordinary differential equation with constant coefficients was built:

$$\frac{d^2\bar{T}}{dz^2}-\xi^2\bar{T}=0, \qquad (10)$$

and boundary conditions:

$$\left.\frac{d\bar{T}(z)}{dz}\right|_{z=h}=\frac{q_0R}{\lambda\xi}J_1(R\xi),\ \left.\frac{d\bar{T}(z)}{dz}\right|_{z=-l}=\frac{\alpha}{\lambda}\bar{T}(z)\Big|_{z=-l}, \qquad (11)$$

where:

$$\bar{T}(z)=\int_0^\infty rJ_0(r\xi)T(r,z)\mathrm{d}r,$$

– function transformant $T(r, z)$;

$$J_v(x)=\sum_{n=0}^{\infty}(-1)^n\frac{(x/2)^{v+2n}}{n!(v+n)!}.$$

– Bessel function of the first kind of the ν-th order; ξ is a parameter of Henkel's integral transformation.

The general solution to equation (10) is defined as:

$$\bar{T}(z)=c_1e^{\xi z}+c_2e^{-\xi z},$$

where the integration constants c_1 and c_2 are found using the boundary conditions (11). As a result, it is obtained:

$$\bar{T}(z)=\frac{q_0RE(z)}{\lambda\xi^2E}J_1(R\xi). \qquad (12)$$

Here:

$$E(z)=(\lambda\xi-\alpha)e^{-\xi(z+l)}+(\lambda\xi+\alpha)e^{\xi(z+l)},$$

$$E=(\lambda\xi+\alpha)e^{\xi(h+l)}-(\lambda\xi-\alpha)e^{-\xi(h+l)}.$$

The inverse Henkel integral transformation was applied to relation (12) and the following result was obtained:

$$T(r,z)=\int_0^\infty \xi J_0(r\xi)\bar{T}(z)\mathrm{d}\xi. \qquad (13)$$

As a result, the desired temperature field in the layer caused by external local heating (heat flow is concentrated on the boundary surface in the local area) is expressed by formula (13), from which the temperature value at an arbitrary point of the structure can be obtained.

5. 2. Nonlinear mathematical model for determining the temperature field with locally concentrated heat flow

The Kirchhoff transformation is considered:

$$\vartheta(r,z)=\frac{1}{\lambda^0}\int_0^{t(r,z)}\lambda(\zeta)\mathrm{d}\zeta, \qquad (14)$$

where λ^0 is the reference thermal conductivity coefficient of the layer material.

Expression (14) was differentiated in terms of the variables r and z, and as a result, we obtained:

$$\lambda^0\frac{\partial\vartheta(r,z)}{\partial r}=\lambda(t)\frac{\partial t(r,z)}{\partial r},\lambda^0\frac{\partial\vartheta(r,z)}{\partial z}=\lambda(t)\frac{\partial t(r,z)}{\partial z}. \qquad (15)$$

Taking into account expressions (15), the original equation (3) will take the following form for the function $\vartheta(r, z)$:

$$\Delta\vartheta=0, \qquad (16)$$

where $\Delta=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}$ is the Laplace operator in the cy lindrical coordinate system.

Boundary conditions (4) using relation (14) are transformed to the following form:

$$\vartheta(r,z)|r\to\infty=0,$$

$$\left.\frac{\partial\vartheta(r,z)}{\partial r}\right|r\to\infty=0,$$

$$\left.\frac{\partial\vartheta(r,z)}{\partial z}\right|\&_{z=-l}=0,$$

$$\left.\frac{\partial\vartheta(r,z)}{\partial z}\right|\&_{z=h}=\frac{q_0}{\lambda^0}S_-(R-r). \qquad (17)$$

Using the Kirchhoff transformation (14) made it possible to reduce the nonlinear boundary value problem (3), (4) to a linearized differential equation with partial derivatives of the second order (16) and boundary conditions with a discontinuous right-hand side (17).

Henkel's integral transformation along the r coordinate was applied to equation (16) and boundary conditions (17), resulting in a second-order ordinary differential equation with constant coefficients:

$$\frac{d^2\bar{\vartheta}}{dz^2}-\xi^2\bar{\vartheta}=0, \qquad (18)$$

and boundary conditions:

$$\left.\frac{d\bar{\vartheta}(z)}{dz}\right|\&_{z=-l}=0,\left.\frac{d\bar{\vartheta}(z)}{dz}\right|\&_{z=h}=\frac{Rq_0}{\lambda^0\xi}J_1(R\xi), \qquad (19)$$

where:

$$\bar{\vartheta}(z)=\int_0^\infty r\vartheta(r,z)J_0(r\xi)\mathrm{d}r,$$

– the transformant of the function $\vartheta(r,z)$.

The general solution to equation (18) is defined as:

$$\bar{\vartheta}(z)=c_1e^{\xi z}+c_2e^{-\xi z},$$

and using boundary conditions (19), the constants of integration c_1 and c_2 are found. As a result, the solution to problem (18), (19) was obtained:

$$\bar{\vartheta}(z)=\frac{Rq_0}{\lambda^0\xi^2}J_1(R\xi)\frac{\mathrm{ch}\xi(z+l)}{\mathrm{sh}\xi(h+l)}. \quad (20)$$

The inverse Henkel integral transformation was applied to relation (20) and as a result the expression for the linearizing function $\vartheta(r,z)$ was determined in the following form:

$$\vartheta(r,z)=\int_0^\infty \xi J_0(r\xi)\bar{\vartheta}(z)\mathrm{d}\xi. \quad (21)$$

The desired temperature field $t(r,z)$ for the given structure can be obtained using the resulting nonlinear algebraic equation using relations (14), (21), after substituting specific expressions of the dependence of the coefficient of thermal conductivity of the structural material of the layer on temperature.

5. 3. Internal heating of structures and a linear mathematical model for determining the temperature field

Henkel's integral transformation along the r coordinate is applied to equation (5) and boundary conditions (6). As a result, a transition to an ordinary differential equation of the second order with constant coefficients and a discontinuous right-hand side was performed:

$$\frac{d^2\bar{T}}{dz^2}-\xi^2\bar{T}=-\frac{Rq_0}{\lambda\xi}J_1(R\xi)S_-(z), \quad (22)$$

and boundary conditions:

$$\left.\frac{d\bar{T}(z)}{dz}\right|_{z=-l}=0,\quad \left.\frac{d\bar{T}(z)}{dz}\right|_{z=h}=\frac{\alpha}{\lambda}\bar{T}(z)\Big|_{z=h}. \quad (23)$$

The general solution to equation (22) is determined by the method of variation of constants:

$$\bar{T}(z)=c_1e^{\xi z}+c_2e^{-\xi z}-\frac{Rq_0}{\lambda\xi^3}J_1(R\xi)(\mathrm{ch}\xi z-1)S_-(z).$$

Using boundary value conditions (23), a partial solution to problem (22), (23) is obtained in the form:

$$\bar{T}(z)=-\frac{Rq_0}{\xi^3}J_1(R\xi)B(z), \quad (24)$$

where:

$$B(z)=(\mathrm{ch}\xi z-1)S_-(z)+BE(z),$$

$$B=\frac{\lambda\xi\mathrm{sh}\xi h-\alpha(\mathrm{ch}\xi h-1)}{(\lambda\xi-\alpha)e^{\xi(2l+h)}-(\lambda\xi+\alpha)e^{-\xi h}},$$

$$E(z)=e^{\xi(z+2l)}+e^{-\xi z}.$$

The inverse integral Henkel transform is applied to relation (24) and the following is obtained:

$$T(r,z)=\int_0^\infty \xi J_0(r\xi)\bar{T}(z)\mathrm{d}\xi. \quad (25)$$

As a result, the desired temperature field in the layer caused by internal local heating (internal heat sources are concentrated in the volume of the cylinder Ω_0) is expressed by formula (25), from which the temperature value at an arbitrary point of the structure can be obtained.

5. 4. Internal heating of thermosensitive structures and a nonlinear mathematical model for determining the temperature field

Taking into account expressions (14) and (15), the original equation (7) is transformed into the following form:

$$\Delta\vartheta=-\frac{q_0}{\lambda^0}S_-(R-r)S_-(z). \quad (26)$$

and boundary conditions (8), (9) according to relation (9) are transformed as follows:

$$\vartheta(r,z)\big|_{r\to\infty}=0,\quad \left.\frac{\partial\vartheta(r,z)}{\partial r}\right|_{r\to\infty}=0,$$

$$\left.\frac{\partial\vartheta(r,z)}{\partial z}\right|_{\&_{z=-l}}=0, \quad (27)$$

$$\left.\frac{\partial\vartheta(r,z)}{\partial z}\right|_{\&_{z=h}}=\frac{\alpha}{\lambda^0}\left(t\big|_{\&_{z=h}}-t_c\right). \quad (28)$$

Using the Kirchhoff transformation (14) made it possible to reduce the nonlinear boundary value problem (7) to (9) to a linearized second-order partial differential equation with a discontinuous right-hand side (26), linearized boundary conditions (27), and a partially linearized boundary condition (28).

$t(r,h)$ is approximated by a piecewise constant function:

$$t(r,h)=t_1+\sum_{l=1}^{m-1}(t_{l+1}-t_l)S_-(r-r_l), \quad (29)$$

where $r_l\in(0;\infty)$; $r_1\le r_2\le\ldots\le r_{l-1}$; l is the number of interval divisions $(0;R^*)$; $t_l\left(l=\overline{1,m}\right)$ – unknown approximate temperature values $t(r,h)$; R^* is the value of the radial coordinate for which the temperature value $t(r,h)$ almost reaches the ambient temperature t_c (it is found from the corresponding linear boundary value problem).

A linear boundary condition was obtained as a result of substituting expression (29) into relation (28):

$$\left.\frac{\partial\vartheta}{\partial z}\right|_{\&_{z=h}}=\frac{\alpha}{\lambda^0}\left[t_1+\sum_{i=1}^{m-1}(t_{i+1}-t_i)S_-(r-r_i)-t_c\right]. \quad (30)$$

Henkel's integral transformation along the r coordinate was applied to equation (26) and boundary conditions (27), (30) and as a result a second-order ordinary differential equation with constant coefficients with a discontinuous right-hand side was obtained:

$$\frac{d^2\bar{\vartheta}}{dz^2}-\xi^2\bar{\vartheta}=-\frac{Rq_0}{\lambda^0\xi}J_1(R\xi)S_-(z), \tag{31}$$

and boundary conditions:

$$\left.\frac{d\bar{\vartheta}}{dz}\right|_{\&_{z=-l}}=0,\quad \left.\frac{d\bar{\vartheta}}{dz}\right|_{\&_{z=h}}=\frac{\alpha A}{\lambda^0\xi}, \tag{32}$$

where:

$$A=(t_m-t_c)\delta_+(\xi)-\sum_{i=1}^{m-1}r_iJ_1(r_i\xi)(t_{i+1}-t_i),$$

$$\delta_\pm(\zeta)=\frac{dS_\pm(\zeta)}{d\zeta}\cdot$$

– asymmetric Dirac delta functions [16–20].

The general solution to equation (31) is defined as:

$$\bar{\vartheta}(z)=c_1e^{\xi z}+c_2e^{-\xi z}-\frac{Rq_0}{\lambda^0\xi^3}J_1(R\xi)(\mathrm{ch}\xi z-1)S_-(z),$$

and using the boundary conditions (32) the constants of integration c_1 and c_2 are found. As a result, the solution to problem (31), (32) is obtained:

$$\bar{\vartheta}(z)=\frac{1}{\xi}\left[\alpha AP(z)-RJ_1(R\xi)\frac{q_0}{\lambda^0\xi^2}A(z)\right], \tag{33}$$

where:

$$A(z)=P(z)\mathrm{sh}\xi h-(1-\mathrm{ch}\xi z)S_-(z),$$

$$P(z)=\frac{\mathrm{ch}\xi(z+l)}{\mathrm{sh}\xi(h+l)}.$$

The inverse Henkel integral transformation was applied to relation (33) and the expression for the Kirchhoff function $\vartheta(r, z)$ was determined in the following form:

$$\vartheta(r,z)=\int_0^\infty \xi J_0(r\xi)\bar{\vartheta}(z)\mathrm{d}\xi. \tag{34}$$

After substituting the expression of the temperature dependence of the coefficient of thermal conductivity of the material of the layer into ratios (9), (34), after performing certain transformations, it is possible to obtain a system of nonlinear algebraic equations for determining the unknown approximate values t_i $\left(i=\overline{1,m}\right)$ of the temperature $t(r, h)$ at the boundary surface of the layer.

The desired temperature field $t(r, z)$ for the given structure can be determined using the obtained nonlinear algebraic equation applying relations (14), (34), after substituting into them a specific expression of the dependence of the coefficient of thermal conductivity of the structural material of the layer on temperature.

The dependence of the thermal conductivity coefficient on temperature was considered in the form of a ratio:

$$\lambda=\lambda^0(1-kt), \tag{35}$$

where k is the temperature coefficient of thermal conductivity of the material of the layer.

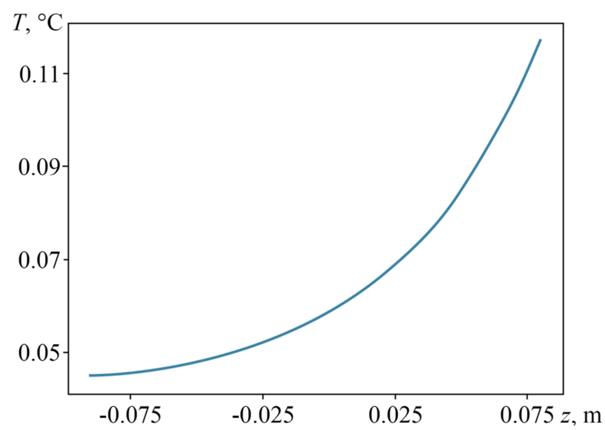
Taking into account expressions (9), (16), (34), and (35), the relationship for determining the temperature $t(r, z)$ in the layer region is obtained:

$$t(r,z)=\frac{1}{k}\left(1-\sqrt{1-2k\vartheta(r,z)}\right). \tag{36}$$

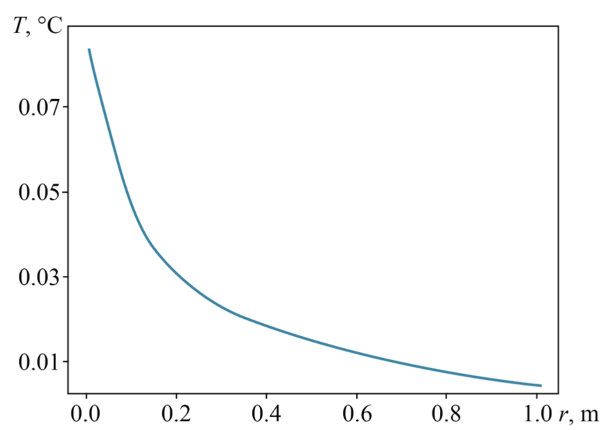
Formula (36) fully describes the temperature field in the thermosensitive layer both during internal local heating and external heating.

According to formulas (6) and (36), numerical calculations of the temperature field were performed and its behavior in the environment was given depending on the spatial radial r and axial z coordinates for the following initial data: q_0=200 W/m^2; l=0.1 m; h=0.075 m; R=0.05 m; α=17.64 W/(m^2·degree); the material of the medium is silicon, for which the coefficient of thermal conductivity λ=67.9 W/(degree·m) at a temperature of 27 °C (Fig. 3). For a heat-sensitive environment, the material of which is silicon, in the temperature range [0 °C; 1127 °C] relation (35), as a partial case, as a result of the performed interpolation will be as follows:

$$\lambda(t)=67.9\ \mathrm{W}/(\mathrm{degree}\cdot\mathrm{m})(1-0.0005\,\mathrm{t}/\mathrm{degree}). \tag{37}$$



a



b

Fig. 3. Dependence of temperature $T(r, z)$ on: *a* – axial z for r=0.05; *b* – radial r for z=0.0375 coordinates

The results show that the temperature, as a function of spatial coordinates, is smooth and monotonic, which confirms the adequacy of the constructed mathematical models to the real physical process.

The results obtained for the selected material (silicon) for the linear dependence of the coefficient of thermal conductivity on temperature are almost no different from the results obtained for a constant coefficient of thermal conductivity. Their insignificant difference is explained by the fact that the value of the temperature coefficient of thermal conductivity for silicon in the given temperature range, as shown by relation (37), is small enough.

6. Discussion of results of investigating the mathematical models built for determining temperature fields in spatial environments

The boundary value problems of thermal conductivity are stated according to how a real physical process is carried out in the given environments. As a result, differential equations of heat conduction and boundary conditions rigorously describe mathematical models of the stationary process of heat conduction, which correspond to the corresponding physical models. The shape of the curves in Fig. 3, which are built on the basis of numerical values of temperature as a function of spatial coordinates, obtained using analytical solutions of boundary value problems, testifies to the correspondence of the results to the physical process. This is confirmed by the smoothness of the temperature function along the spatial coordinates and the fulfillment of the given boundary conditions on the boundary surfaces of the medium.

In our studies, the theory of generalized functions was used, which made it possible to effectively describe local temperature disturbances, as a result of which the resulting differential equations and boundary conditions contain discontinuous right-hand parts. For the complete linearization of the nonlinear boundary value problem (7) to (9), one of the approaches is given, in particular, the piecewise linear approximation of the temperature on the boundary surface of the medium by the spatial coordinate relation (29). This approach made it possible to effectively obtain the linear boundary value problem (26), (27), (30) for determining the linearizing function, which subsequently makes it possible to obtain the temperature distribution according to formula (36) for a certain dependence of the thermal conductivity coefficient of the medium material on temperature.

It should be noted that the above cited works did not consider the local heating process, which is important to consider in modern mobile digital devices, when individual nodes and their elements are small enough, but emit significant thermal energy. The use of generalized functions for the construction of the above mathematical models made it possible to investigate the effect of local heating on the temperature distribution in the environment by spatial coordinates.

Our studies consider only the stationary process of heat conduction; the studies were performed for homogeneous media. Consequently, such studies can be continued for heterogeneous media, for non-stationary heat conduction processes, as well as for anisotropic media.

Since in the architecture of modern electronic devices individual thermally active nodes and their individual elements are concentrated in the form of composite materials, consequently there is a need to build linear and nonlinear mathematical models of the heat conduction process for the media of the composite structure. As a result, the given mathematical models of heat conduction are simplified, but they make it possible to construct more complex mathematical models of the heat conduction process for composite media based on them.

On the basis of our analytical solutions to both linear and nonlinear boundary value problems of heat transfer, it is proposed to develop computational algorithms and software tools for their numerical implementation. This will make it possible to carry out research for a number of materials used in the process of designing digital electronic devices, regarding the effect of their thermal sensitivity on the temperature distribution.

It is proposed to take into account the thermal sensitivity of structural materials, which significantly complicates the process of solving the relevant nonlinear boundary value problems of thermal conductivity. The sought-after solutions of these problems describe the temperature behavior as a function of spatial coordinates somewhat more adequately to the real physical process.

7. Conclusions

1. A linear mathematical model for determining the temperature field, and subsequently for the analysis of thermal regimes in the structures of electronic devices as a result of their heating by a locally concentrated heat flow, has been constructed. An analytical solution to the boundary value problem was obtained in the form of a improper integral, and based on this, the behavior of temperature as a function of spatial coordinates in the given structure was determined graphically.

2. A nonlinear mathematical model was built for determining the temperature field, and subsequently for the analysis of thermal regimes in thermosensitive structures of electronic devices as a result of their heating by a locally concentrated heat flow. The linearization of this model was performed and the analytical solution to the boundary value problem with respect to the linearizing function in the form of an improper integral was obtained. With the use of this solution and the given dependence of the coefficient of thermal conductivity of the medium material on temperature, it became possible to derive a nonlinear algebraic equation for determining the temperature field. The case of linear dependence of the coefficient of thermal conductivity for the selected medium material (silicon), obtained by the method of interpolation, was considered.

3. A linear mathematical model for determining the temperature field, and subsequently for the analysis of thermal regimes in the structures of electronic devices as a result of their heating by locally concentrated internal heat sources, has been constructed. An analytical solution to a linear boundary value problem in the form of a improper integral was obtained, which can be used to develop computational algorithms, software tools for determining the behavior of temperature as a function of spatial coordinates in the given environment.

4. A nonlinear mathematical model was built for determining the temperature field, and subsequently for the analysis of thermal regimes in thermosensitive structures of electronic devices due to their heating by locally concentrat-

ed internal heat sources. An analytical solution to the nonlinear boundary value problem was obtained in the form of a improper integral, using which and the given dependence of the thermal conductivity coefficient of the medium material on the temperature, the desired temperature field can be determined. As a result of the linearization of the nonlinear boundary value problem using the Kirchhoff transformation, it was not possible to linearize one of the boundary conditions. In this regard, the temperature on the boundary surface of the medium was approximated by one of the spatial coordinates by a piecewise constant function, and as a result, fully linearized boundary conditions were obtained.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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