The research focuses on multi-retail distribution with a strategic distribution network. Challenges include intense competition, logistical and transportation complexities requiring robust infrastructure like warehouses and efficient supply chain management, as well as operational inefficiencies and distribution costs. To address these issues, a model is applied to make strategic decisions, such as determining the necessary number of facilities to minimize total supply chain network operational costs and infrastructure for retail distribution. The outcome is a model that introduces a novel approach to enhancing supply chain efficiency and effectiveness from production to distribution stages, thereby reducing system costs, including ordering costs and inventory handling. Costs and loss costs resulting from remaining products produced can be minimized by considering networks, multiple suppliers, multiple warehouses, Distribution Centers (DC), multiple retailers, and multiple products, factoring in the distances between facilities in the network. Subsequently, comprehensive testing of inspection, distribution, and retail parameters is conducted, with a focus on specific periods and product types. When applying this model, certain characteristics need to be considered regarding the importance of selecting efficient suppliers of goods, such as procurement, performance improvement, and the number of supply chains and supply chain systems. This research introduces novelty in production methods that can lead to increased customer satisfaction, sales, market share, profit margins, more effective brand advertising, and revenue streams. In this process, the research undergoes a training, testing, and validation process in forming a strategic multi-retail distribution network model, spanning a total of 52 epochs. This process yields accuracy values for training at 90 %, testing at 92 %, and validation at 94 %.

Keywords: distribution, multi-retail, industry, infrastructure, mathematical models

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1. Introduction

In the current era of intense competition, producers are required to be more critical and reactive to existing changes, whether political, socio-cultural or economic. Clarity of strategy formulation is an important aspect of effective and efficient management in a multi-product retail context [1, 2]. Strategy is a formulated aspect that has a very broad basis regarding how a business is run to compete, to achieve a goal in multi-retail supply chain management [3]. The business world is now continuing to grow and develop. From various types of existing industries and markets. With increasingly rapid digitalization and the volume and type of data increasing every year, knowledge from big data, machine learning, deep learning, and the Internet of Things (IoT) makes data collection even easier [4, 5].

In the multi-retail context, there are data-based organizations, which are organizations that make data and analytics part of the company’s business itself, starting from strategy, operations, systems, processes and culture. Becoming a data-driven organization also means creating a mindset where data-driven analysis is applied by all levels of the organization as the basis of all business decisions.

In the multi-retail context, there are organizations or industries that provide data for analysis from the business side, starting from strategy, business operations, work procedures and work culture. Multi-retail industries that provide data can contribute to policy and play a role in all business decisions.

However, there are problems in transforming into a data-based organization, such as limited data sources and inadequate infrastructure [6, 7]. A very complex problem in decision-making is uncertainty. Uncertainty is a situation where a person does not know the expected results in the future due to not knowing the possibility of an expected event occurring. The existence of uncertainty will cause the author to face risks in the future. Therefore, with the uncertainty of the parameters that influence decisions, the author must use the right model to determine what decision the author will...
obtain and reduce the risks that will arise. The model that is discussed in this problem is a data-driven model where the model is based on data, where the model is built on the basis of connecting several system variables such as input, internal, and output variables with only limited assumptions about the system [8–15].

Therefore, in implementing multi-retail distribution, the strategy that becomes relevant by implementing Branch-and-Bound (BD) and Local Search (LR) accompanied by linear regression needs to be carried out for several reasons, such as the multi-retail industry having a very complex supply chain such as the large number of suppliers and sales who have different coordinate points, then from the supply and demand side changes in the business environment such as product demand, product availability and payment methods, which often change dynamically, then on the operational side there are often cost burdens that exceed the multi-retail industry budget so that. There are several reasons why the application of the BD method and linear regression is very necessary. Such studies provide results in the form of an optimal mathematical model that can be used in the multi-retail industry to make operational decisions so as to create operational efficiency.

2. Literature review and problem statement

The study [16] shows that the supply chain is a system for distributing goods and production services to customers. Where there are several problems faced by companies in the distribution system, companies are grappling with the complexity of predicting consumer needs and ensuring the timely provision of goods when needed, which reflects the challenges of the distribution system. In solving these problems, it will utilize mathematical models with heuristic formulas so that consumer needs are met. However, another problem arises, namely time inefficiency in distributing goods, so this becomes a deficiency in the heuristic formula.

The research [17] implemented Recurrent Neural Networks (RNN) equipped with a Long Short-Term Memory (LSTM) unit, which focuses on temporal wind speed data in predicting factory needs. This research utilizes the Global Energy Forecasting Competition data set, which attempts to create a model to predict factory needs within a certain time period. Research parameters focus on data related to the distribution used as needed. The problem in this research is to predict factory needs with the aim of forming a model based on past data so that by applying Recurrent Neural Networks it can overcome this problem. However, there are several weaknesses on the side of time inefficiency, this shall be discussed by future researchers.

In the research [18] related to the provision of distribution networks, to overcome the complexity of the distribution network, it is divided into several levels. The number of levels is determined based on the trade-off between problem complexity and distribution network integration. Then the entire network in the supply chain is divided into three subnetworks, namely: incoming network, distribution network and outgoing network. In general, the aim of designing a distribution network is to minimize total distribution costs so that retailer demand can be met without exceeding factory and distributor capacity. The problem in this research is dealing with complexity when distributing time levels so that a machining learning model can used, which can overcome this problem. However, there are problems in the level distribution process in terms of less than optimal time, which is a weakness in the model.

The work [19] discusses sustainable supply chain management by evaluating several environmental, social and economic performance parameters, which produces a trade-off model so that costs in a distributed system can be managed and can be met in accordance with sustainable supply goals. The problem in this case arises from the supply chain not synchronizing with environmental performance, resulting in several parameters being suboptimal in supply chain management. Therefore, a model is implemented to achieve sustainable supply. However, in this scenario, there is a longer and inefficient process in terms of time because parameters must be processed one by one.

The study [20] uses short-term techniques in implementing software reliability prediction models and adding layers of backpropagation and truncated normalization to the model to improve performance to solve damage prediction problems in software and hardware, which produces a model with an LSTM network approach so that it can perform good predictions compared to other neural network models and optimization techniques used so that when processing the model it will save time. However, this still has disadvantages in terms of being less efficient.

From the research [21] related to distributed logistics delivery, there has been a significant increase in several years, which has resulted in problems such as uncontrolled operational costs, uncontrolled goods affecting delivery operations, which will then be applied to learning-based machines. In solving this problem, an artificial neural network model is applied to manage operational and shipping costs, which then produces a model that can carry out supply chain management according to a distributed network so that operational costs and goods management can be controlled. However, this model has shortcomings in terms of time when carrying out processes on several parameters related to supply chain management and operations.

The work [22] applies artificial neural networks (ANN) to solve distributed management problems in logistics businesses, which will use several parameters related to distributed networks, including delivery distance, delivery time and operational costs. In this case, the ANN will provide a threshold value for the distributed network model so that all problems can be resolved. However, artificial neural networks have shortcomings so they need to be identified in each part, such as determining the parameters of demand, distribution and multi-retail networks, so it takes quite a long time to determine these parameters. Due to these shortcomings, a mathematical model is needed so that it can solve this problem.

3. The aim and objectives of the study

The aim of the study is to increase operational time in a strategic multi-retail supply chain. Thus, in multi-retail systems processing data and developing models to increase distribution system efficiency are found.

To achieve the aim, the following objectives were accomplished:

– to carry out logistic regression and bender decompositions based on predetermined variables;
– to select parameters for dynamic supplier selection optimization techniques.
4. Materials and methods

The object of the study is multi-retail distribution with a strategic distribution network. The hypothesis in the study is to test the impact of optimizing multiple retail efficiency decisions with a distributed network. This hypothesis will produce a multi-product retail optimization model. The problems in this research are high production costs, lack of stock visibility, inappropriate price selection and ineffective product placement in the distributed chain, all of which cause inefficiency in multi-product retail businesses that do not use a distributed network. This research uses hardware such as a laptop with Windows 10 specifications with a Core i5 processor and software such as Microsoft Word and Matlab. This research is designed according to Fig. 1, which consists of stages of implementing a retail efficiency decision optimization model, providing a solution to resolving problems related to retail products and distributed networks. Then, Matlab software is employed to analyze and test the created model. The obtained results are based on simulation analysis conducted to understand how the process model operates within a distributed network. The following architecture in Fig. 1 explains the stages of the distributed retail network model.

![Fig. 1. Validating process](image)

Fig. 1 shows Multi-retail Suppliers, a challenge faced when manufacturers are involved in a multi-product procurement process with multiple suppliers over time. The demand for products received by manufacturers fluctuates over time, and suppliers are geographically dispersed, requiring transportation to deliver products to production locations. This approach will explore the frequency approach, which utilizes probability theory when samples are available to ascertain a probability distribution, and degree of confidence theory, which is applied when no samples are available, peering using fuzzy variables. Fuzzy variables involve estimates made through a pooling function specified by the decision maker. When an optimization problem involves fuzzy variables or parameters, fuzzy programming is used to find the solution. In this case, we will handle suppliers in the context of multi-product procurement from distributed suppliers and then consider various purchasing scenarios and management permits, especially in demand forecasting. This approach contributes to improved decision-making in dynamic supplier selection processes, providing valuable insights for optimizing procurement strategies in changing and uncertain environments.

5. Results of optimizing multi-retail efficiency decisions with distribution networks

5.1. Results of determining the critical factors that influence supplier selection dynamic environment

The supplier performance indicators of each criterion are obtained from evaluating the data that has been in the company and the calculation formula is Price offers that enter at the most competitive price will get a high value. Determination of supplier performance indicator weights of the criteria that have been carried out is the initial stage for determining the weighting of each criterion of supplier performance:

a) value range;

b) weight range;

c) assessment decision.

From the results of the decisions obtained, multi-retail companies such as Indomaret will take action based on supplier performance evaluation decisions. If the decision remains a supplier, if there is a purchase order, the supplier will still be given an order, but if the decision is removed from the list of selected suppliers, the supplier gets removed and is not given another order; if the supplier still wants to become a partner, they are treated like a new supplier, and an assessment is conducted as if they were newly entered.

This analysis result proposed the integration of Analytic Network Process (ANP) and multi-period multi-objective mixed-integer linear programming (MOMILP). In practical scenarios, manufacturers often find it challenging to compete effectively with their competitors when dealing with unreliable suppliers in terms of quality, delivery, capacity, and other factors. Furthermore, a mixed-integer linear programming (MILP) model for multi-product and multi-period inventory lot sizing with a supplier selection problem is used.

Transportation costs play a crucial role in procurement decisions, and factors like splitting orders among multiple suppliers can lead to smaller delivery quantities and subsequently higher transportation costs. Thus, transportation cost management is key to improving procurement efficiency. However, a supplier selection model has been developed that explicitly considers transportation costs. Some researchers [20, 21] have proposed procurement models for single products, involving multiple suppliers and multiple periods.

Remarkably, there is limited research in the literature such as that conducted by [19] on the supplier selection problem calculating transportation costs in the context of multi-product procurement from multiple suppliers over several periods. The so-called DSSP proposes a multi-objective mixed integer non-linear program (MOMINLP) for multi-supplier lot size problems involving multiple products such as food products and non-food products. A mixed-integer non-linear program (MINLP) was developed to address the dynamic supplier selection problem (DSSP). A detailed explanation of the data parameters of this dataset can be seen in Table 1. The dataset was obtained from multi-retail data from 2020 January to 2023 December.

In this model where the government subsidizes the transport fuel price by 0.59, the price after subsidization is (1−0.59)×$1.02/liter=0.42/liter. Therefore, it is assumed that TPL uses diesel trucks for delivery with fuel consumption=39.5 liters/100 km (γ = 0.63369 liters/mile).
Verification is done to ensure the program is made without errors and can be run perfectly. This is done by looking at the resulting program, if there is an error marked with an error sign (error). The program code designed in Matlab does not show any errors, so the model can be verified. If the program does not show any errors, the next step is to run the model and then validate the model. This research presents a class of fuzzy modeling using Matlab. Fig. 2, 3 showcases the graph of buyer and vendor inventory results.

**Table 1**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Item</th>
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<tr>
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<td>item/years</td>
</tr>
<tr>
<td>$D$</td>
<td>7</td>
<td>item/weeks</td>
</tr>
<tr>
<td>$A_0$</td>
<td>30</td>
<td>$</td>
</tr>
<tr>
<td>$S_0$</td>
<td>3,600</td>
<td>$</td>
</tr>
<tr>
<td>$h_b$</td>
<td>45</td>
<td>$/items/years</td>
</tr>
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<td>$h_v$</td>
<td>38</td>
<td>$/items/years</td>
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<tr>
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<td></td>
</tr>
<tr>
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<td>100</td>
<td>$/items</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>100</td>
<td>$/items</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.25</td>
<td></td>
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<td>$</td>
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<tr>
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<td>Days</td>
</tr>
<tr>
<td>$\pi_e$</td>
<td>30</td>
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</tr>
</tbody>
</table>

In Fig.3, there are 4 labels, namely wholesalers, retailers, distributors, and multi-retail industries serving as the sales places for multi-retailers. The variables utilized are product availability and customer demand, measured in units of pcs (unit usage). In Fig. 3, graphical results are depicted, including Simulation models and Dynamic Supplier Decisions, as well as Supply Allocation Problems. Model validation is a process to find out whether the optimization model created is in accordance with the objectives to be achieved. One method of validation can be done by comparing vendor and buyer inventory patterns with vendor and buyer inventory patterns. The graph of the buyer inventory simulation results can be seen in Fig. 2. The buyer inventory pattern in the graph below displays a pattern that is similar to the inventory pattern. Then in Fig. 3, it can be seen that the vendor inventory pattern in the graph is similar to the vendor inventory pattern.

### 5.2. Model results with dynamic supplier selection optimization techniques

In the model results with dynamic supplier selection optimization techniques, several parameters are employed to dynamically meet suppliers, utilizing variables as decision-makers to formulate the following formula:

$$\text{Minimize } Z=Z_1+Z_2+Z_3+Z_4+Z_5+Z_6+Z_7+Z_8.$$  \hspace{1cm} (1)

The objective function (1) aims to minimize procurement costs, which encompass eight components:

a) purchase costs;

b) transportation costs;

c) order costs;

d) contract costs;

e) holding costs;

f) shortage costs;

g) penalty for defective products;

h) penalty for delays.

Buyers seek to minimize this objective function while adhering to the following constraints.

Based on the model design that has been made, Bayesian optimization is carried out with a total of 100 iterations where the decision variables in this model are the number of production lots ($Q$), the number of production batches ($m$) and the value of $k$. Before optimization, the initial condition of the production lot is 250 units with the number of batches of 1 and the value of $k$ is 0. Then optimization is carried out with p-bounds as a decision variable, with the aim of minimizing the joint total expected cost value. However, the Bayesian optimization programming code used, in general, uses the maximization function so that the model formula designed is given the addition of negative values at the beginning of the formula. So that the optimal result is obtained with the minimization function.

After running Bayesian optimization on the laptop, the results of minimizing the JTEC value in 100 iterations with various random numbers from the decision variables tried by the system were obtained. Based on the optimization results, the most optimal decision variable is the lot size ($Q$) of 785.46 units with a production batch number ($m$) of 2 times and a $k$ value of 1,226, where the lowest JTEC value in this scenario 1 model is $125,995.637.

The model optimization results display the lowest combined cost. Where in this model it is assumed that there is an investment in the form of improving the quality of the
production system. One of the efforts that can be made by vendors in this investment is to hold workshops or special training to improve the production capabilities of workers. Other quality improvement investments can be made by replacing machines or production equipment with better quality. This solution can be done if most of the defective products are caused by poor-quality machines. So it would be better before investing in quality improvement, the company is expected to first analyze what factors lead to the emergence of defective products, then adjust to what quality improvements are more appropriate. So that the probability of uncontrolled production processes can be reduced to 0.00004925.

After the vendor is able to control the production process better, then the vendor and buyer can return to a compromise regarding the backorder price discount, ordering cost \( (A) \) and production setup cost \( (S) \). This is because with better production conditions, it is possible for vendors and buyers to reduce the policies of the three cost criteria. Better production quality helps vendors to be more effective in conducting production setup and production results also become better and on time.

There is a multi-product retail store problem defined where the problem based on the multi-product retail optimization model is how to determine the best decisions in stock procurement and product allocation in several retail stores to maximize profits while considering the uncertainty of product demand, procurement costs, and storage costs.

Uncertainty in product demand can be caused by factors such as economic fluctuations, seasonal trends, or changes in consumer behavior. Meanwhile, product procurement and storage costs can vary depending on several factors, such as inventory levels and order times. In a robust multi-product retail optimization model, decisions must include risk-taking and adaptive strategy selection. This is done by optimizing decisions in the worst-case scenario in the face of uncertainty. Thus, the model allows for an overall optimal decision, unaffected by uncertainties and risks that may occur in the future.

The model requires historical data on product demand, procurement costs, and storage costs, as well as information on uncertain factors such as expected future demand and possible fluctuations in raw material prices. By using optimization algorithms, the model can generate the best solution for managing stock and product allocation in various retail stores. A retail company that sells daily necessities such as food, beverages, and household items wants to optimize the procurement of stock and product allocation across its multiple retail stores. The company also wants to consider uncertainties in product demand, procurement costs, and storage costs. The following are the results of modeling using Matlab software in Fig. 4.

The basic mathematical model of multi-product decision optimization usually consists of the following variables and parameters:

1. Decision Variables: This is the variable that must be generated by the optimization model. In this case, the decision variable can be the production quantity of each product.
2. Cost Parameters: The production cost of each product.
3. Demand Parameters: Demand for each product to be produced.

The basic mathematical model for multi-product decision optimization can be formulated as follows:

Minimize \[ Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n. \]

Subject to:

\[ a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1; \]
\[ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2; \]
\[ \vdots \]
\[ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m; \]
\[ x_1, x_2, \ldots, x_n \geq 0. \]

Fig. 4. Modeling decision
There are performance validation results based on the number of resources produced in the multi-tailer distribution chain using material, time, and shelf life variables as shown in Fig. 5 below.

![Fig. 5. Performance model](image)

In Fig. 5, the model performance of a multi-retail distributed network shows that the model will use a number of epochs of 52 with each training, testing and validation process. In the testing process, the testing data graphic model shows good performance where the testing data lines process horizontally. This research will go through a training, testing and validation process in forming a strategic multi-retail network distribution model with a number of epochs of 52, which produces an accuracy value for training of 90%, an accuracy value for testing of 92% and an accuracy value for validation of 94%.

6. Discussion of multi-retail efficiency decision optimization with distribution networks

As a result of this research, a multi-retail efficiency decision optimization model with a distribution network was proposed and applied. The model will perform processes such as data cleaning, data understanding, and performance evaluation. The model results will use the parameter data in Table 1, which will then produce a graph of multi-retail sales in the industry, which has experienced an increase in strategic distribution. It contains customer and trader variables. The results shown in Fig. 3 include the trader, wholesaler and retailer variables, respectively. Each is utilized to fulfill product availability and customer demand, resulting in a graph depicting increasing product demand. The results of the model appear in Fig. 4, indicating a decision model that determines production quantities based on cost parameters, demand parameters, and capacity parameters. Meanwhile, in Fig. 5, the performance of the multi-retail distributed network model displays the model's performance horizontally. The model formed will solve problems such as predicting goods with a combination of variables that produce maximum profits and minimize losses. Models that consider predictions of future product demand with higher accuracy and optimal solutions and models that can predict future product demand produce higher accuracy and optimize production with optimal solutions. In solving this problem, we will use an algorithm where the search direction along the active boundary surface is characterized as being in the Z matrix range that is orthogonal to the normal boundary matrix. The results obtained can increase time efficiency and effectiveness in the logistics business, where in the research conducted [20, 21] there were time efficiency problems. This research uses the Branch-and-Bound (BD) and Local Search (LR) methods to improve the initial solution. The scope of application of the results is implemented in multi-retail industrial companies, such as Indomaret, to facilitate rapid business growth. Consequently, the conditions for applying the results are obtained to ensure time efficiency. This involves the effective utilization of the Branch-and-Bound (BD) and Local Search (LR) methods, as well as linear regression, by the model.

There is a peculiarity in the use of the proposed method. This research obtains results with a different level of novelty compared to previous studies such as those carried out in [22], which uses conventional methods and formulations such as ordinary distribution formulations, which still use measurements based on manual data so that they are not effective in optimizing multi-retail networks. However, this research applies models and supporting parameters for multi-retail distributed networks in an automated manner, which will increase efficiency and effectiveness in the logistics business.

However, in this research there are limitations such as input data processing, which must be done first because it does not use a data warehouse system so decisions regarding data input and stock allocation must be made carefully. Then an evaluation is carried out to ensure that the decisions taken remain profitable even though there are uncertainties and risks in the future. Then, after identifying the weaknesses in this research, new developments are planned, including the utilization of a combination of machine learning and neural network techniques. This approach enables methodological combinations and the creation of new models aimed at addressing existing weaknesses effectively.

7. Conclusions

1. This research produces an optimization model in a multi-retail network by utilizing logistic regression, which can solve problems in the logistics business process where goods to be sent from the sender’s address to the delivery destination do not experience problems in terms of time and cost. So the combination of Bender Decomposition can maximize the profits of operations so that they can compete effectively and efficiently. This research will go through a training, testing and validation process in forming a strategic multi-retail network distribution model with a number of epochs of 52, which produces an accuracy value for training of 90%, an accuracy value for testing of 92% and an accuracy value for validation of 94%.

2. The model optimization results show the lowest combined costs. Where in this model it is assumed that there is investment in the form of improving the quality of the production system. One effort that can be made is the application of the Branch-and-Bound (BD), Local Search (LR) and linear regression methods, which can improve production quality. This allows analyzing factors that cause operational inefficiencies and the possibility of uncontrolled production processes.
Control processes

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The study was conducted without financial support.

Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors have used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

References