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DESIGNING A HELICAL KNIFE FOR A SHREDDING DRUM USING A SWEEP SURFACE

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The object of this paper is a helical blade in a shredding drum from a sweep surface. Such drums are used in harvesters for crushing plant mass. If the flat blades are installed on the drum, cutting of the plant mass occurs simultaneously along the entire length of the blade. This could cause a pulsating dynamic load. If a flat knife with a straight blade is installed at an angle to the axis of the drum, then the distances from it to the points of the blade will be different, as well as the cutting conditions along the blade. The elliptical shape enables the same distance from the axis of rotation to the points of the blade, but this does not solve the problem. Many short flat knives with a straight blade can be mounted on the drum, placing them in such a way that the time between the individual knives is minimized. However, all these disadvantages can be eliminated by a helical knife with a blade in the form of a helical line.

The design of a helical knife from an unfolding helicoid has been considered. In differential geometry, the bending of unfolded surfaces of zero thickness is considered. Bending of the workpiece into a finished product occurs with minimal plastic deformations, the magnitude of which depends on the thickness of the workpiece sheet. The methods of differential geometry of unfolding surfaces were applied to the analytical description of the surface of the helical knife.

The parametric equations of the unfolding helicoid were derived according to the given structural parameters of the knife in space and on the plane. That has made it possible to mathematically describe the contour lines that cut the knife from the surface and on its sweep. Formulae for calculating a flat workpiece through the structural parameters of the knife have been derived. Thus, with the specified structural parameters of the knife $R=0.25$ m, $\tau=20°$, $\varphi=65°$, according to the resulting formula, we find the radius of the knife blade on a flat workpiece: $R_0=4.8$ m

Keywords: unfolding helicoid, flat workpiece, return edge, helical knife, shredding drum

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1. Introduction

Shredding drums are used in forage harvesters. Flat or screw-shaped knives are attached to them. Flat knives are easy to manufacture but they cannot provide the grinding quality of screw-type ones [1]. During drum operation, it is necessary to ensure its uniform load. With helical knives, this happens due to the fact that after the end of the cutting process with one knife, the next knife immediately begins work. If a flat knife is fixed on the drum so that its blade forms a certain angle with the counter-cutting plate by analogy with a helical one, certain problems arise. Firstly, the quality of cutting along the length will not be of the same quality as that of helical knives, and secondly, if the blade is straight, it has a variable gap with the counter-cutting plate during operation. To eliminate this drawback, the blade must be outlined along the arc of the ellipse. In order to bring the drum with flat knives closer to the work with helical ones, the flat knives are made short and placed on the drum in such a way as to balance the load on it as much as possible. Thus, in terms of the quality of grinding and the uniformity of the load, screw-shaped knives are superior to flat ones, but they are more difficult to manufacture.

Thus, the issue of simplifying the process of manufacturing helical knives is relevant that can be resolved by designing them from an unfolding surface. In this case, it is possible to find its exact sweep, which, when bent into the finished article, would undergo minimal plastic deformations.

2. Literature review and problem statement

In agricultural machines, the shape of the surface of the working bodies significantly affects the technological process. These surfaces are in contact with the process material. In the process of their interaction, certain requirements must be met. If the technological material is grain, then it is necessary to prevent its injury during interaction. In paper [2], the impact of the bucket working body of the grain mixture transporter on its injury is investigated. In [3], a similar study concerns cultivator paws. But the issues related to the possible variability of the forms of working bodies remained unresolved. After all, the shape of the working surface in tillage tools is of great importance.

In paper [4], a methodology for modeling soil cultivation is proposed, which allows analyzing the power characteristics of working bodies and the quality of the soil at the stage of designing soil cultivation machines. The resulting characteristics make it possible to optimize the structural and technological parameters of the working bodies of the machines. But the authors did not focus on the issues of manufacturing working bodies of appropriate forms.

The experience of Great Britain regarding the issues of classical tillage, working bodies of tools and tools on its condition after performing the relevant operations is outlined in [5]. The authors took into account the variability of the forms of working bodies, but the issue of their manufacture remains unresolved.

The process of manufacturing the working bodies of tillage tools by bionic methods was studied in [6]. But the proposed approach cannot be used for the design of helical working bodies. The authors of study [7] propose the design of working bodies of machines and tools by means of computer modeling. This makes it possible to simulate the interaction of the proposed structures with the soil environment at the same time as to study the feasibility of the proposed solutions. However, the issue of manufacturing working bodies of appropriate forms remains unsolved.

The first step to the production of working bodies is the construction of their sweep. Work [8] substantiates the geometric model and parameters of the surface of the cultivator paw, taking into account its subsequent manufacture. The method of designing the surface of the cultivator paw has been developed, which includes the formation of the guide curve, the construction of the surface frame, the determination of horizontal, frontal, profile sections, and the construction of the sweep. However, the proposed technique is relevant only for unfolding surfaces. For folded surfaces, only an approximate sweep can be obtained.

Screw surfaces have become widespread in agricultural machines. Helical surfaces can be folded and non-folded. The design of a harrow with screw working bodies is considered in [9] without analysis of the type of surface, and this is critically important for the issue of its manufacture. Study [10] considers the issue of generating structures of energy-saving transport and technological systems with helical working bodies by the synthesis of hierarchical groups using morphological analysis. However, in the study, the question of obtaining a flat workpiece of the working body for its further manufacture remained unsolved.

Works [11, 12] consider the construction of a spiral descent, in which the working surface is an folded (oblique) helicoid. The disadvantage of these studies is that it is possible to obtain only an approximate sweep for the proposed surface. Instead, in [13] a screw tillage body is considered, the working surface of which is made in the form of an unfolding helicoid. The authors emphasize that the unfolded surface is characterized by the manufacturability of the production of the working body as it makes it possible to calculate the geometric dimensions of the flat workpiece, which is formed into the finished product by simple bending with the minimization of plastic deformations. The proposed approach is limited to the design of auger soil tillage bodies, but it can be adapted to solving a wide range of problems, in particular helical knives.

In the shredding drums of forage harvesters, the working bodies are knives, which can be flat or helical, that is, have a helical shape. The first ones are easy to manufacture and must be installed at an angle to the counter-cutting plate, but at the same time, the constancy of the cutting angles is not ensured [1]. To reduce this disadvantage, they must be made as short as possible, which complicates the construction of the drum due to the large number of knives, each of which is attached in a certain order, that is, the drum is made sectional. The helical blade can have the form of unfolded or folded helicoid. To facilitate the manufacture of the knife, it is suggested to give it the shape of an unfolding helicoid (unfolding helical surface). In this case, one can accurately calculate the contours of the flat workpiece. When it is formed into a finished product, plastic deformations will be minimal, which reduces the energy consumption of manufacturing the final product. Works [14, 15] consider the strengthening of surfaces and preventing their wear. In these works, the issue of increasing the wear resistance of parts already after their manufacture by applying a suitable coating is considered. However, this issue can be partially resolved already at the stage of designing a flat blank for manufacturing a working body.

Paper [16] considers the modeling of a helical knife using SolidWorks software, and paper [17] presents a new machine for cold drawing of a spiral blade. This technology of making a knife implies a high labor intensity of the process due to significant plastic deformations of the workpiece. Meanwhile, this process can be improved by analyzing the surface of the knife with the involvement of the differential geometry apparatus. These questions remained outside the attention of researchers due to the simple, in their opinion, shape of the knife. A knife with a simpler shape, whose cross-section is a straight line, is more difficult to manufacture than a knife whose cross-section is a curved line – the involute of a circle. However, it was not possible to find an analytical description of the shape of the helical knife and its flat blank in the available literature.

3. The aim and objectives of the study

The purpose of our study is to develop an analytical description of the shape of the helical knife for the shredding drum and its flat workpiece based on the condition that the surface of the knife is unfolded. This will make it possible to minimize plastic deformations when bending a flat blank into a finished product.

To achieve the goal, the following tasks were set:

– according to the given structural parameters of the knife, derive the parametric equations of the unfolding screw surface and analytically outline the contours of the knife on it;

– to find the contours of a flat workpiece for making a knife, indicating the required dimensions.

4. The study materials and methods

The object of our research is the helical blade for the shredding drum from the unfolded surface for the purpose of finding the exact contours of its flat workpiece.

The main hypothesis of the study assumes that the replacement of the folded surface of the helical knife with an unfolding one could simplify the knife manufacturing technology. In the work, a simplification is adopted, according to which the thickness of the material from which the knife is made is not taken into account. The assumption is that this will not affect the accuracy of the construction of the knife sweep in the form of a flat workpiece.

Research materials are based on the provisions of the internal geometry of the surfaces. According to this theory, a piece of the unfolded surface can be bent into a flat compartment and vice versa, while the correspondence between the points of the surface and the points of the flat compartment is established, which makes it possible to find the contours of the flat workpiece based on the known contours of the piece of surface. Correspondence is determined by the first quadratic form of surface equations in space and on the scan, which is a confirmation of the reliability of mathematical statements.

5. Results of investigating the unfolding surface, for which the guiding spiral line is the blade of the knife

5. 1. Cutting a piece from the helical surface that corresponds to the contours of the helical knife

Fig. 1, *a, b* shows a schematic representation of a drum with a helical knife in projections. At any point, the cross section of the knife is a segment of a straight line that makes an angle φ with the radial direction (Fig. 1, *b*). Its other structural parameters are the radius of the drum *R*, its length *L*, the angle τ, which forms the outer helical line of the knife (blade) with the axis of rotation or counter-cutting plate.

The helical surface for cutting a knife from it for convenience is designed with a vertical axis of rotation. The guide curve is taken as a knife blade, i.e. a helical line with a rise angle of 90°–τ. A folded helical surface (straight open helicoid) will be formed by the helical movement of the horizontal segment so that it intersects the knife blade and forms a constant angle φ with the radial direction, as shown in Fig. 1, *b*. The surface formed in this way is shown in Fig. 1, in which the contours of the knife are indicated by a thick line.

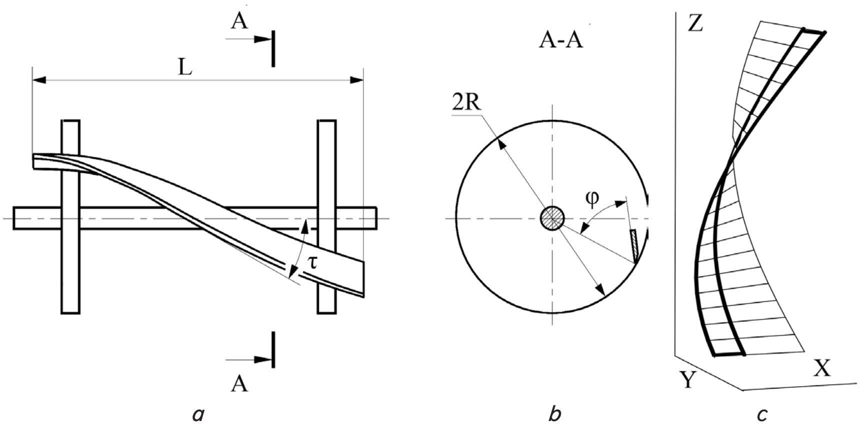

Fig. 1. Graphical illustrations of the shape of a helical knife from a folded surface: *a* – diagram of the location of the knife on a drum with a horizontal axis of rotation; *b* – side view, on which the knife blade is projected by an arc of a circle; *c* – a spatial image of a knife in a vertical position, as a section of a helical surface

The blade from the surface of the unfolded helicoid is formed by a set of rectilinear generatrices tangent to the helical line, which for it is called the turning edge. The parametric equations of the torso-helicoid are written:

$$X = a\cos\alpha + u\cos\beta\sin\alpha; \tag{1}$$

$$Y = a\sin\alpha - u\cos\beta\cos\alpha;$$

$$Z = a\alpha\,\mathrm{tg}\beta - u\sin\beta,$$

where α and *u* are independent variable surfaces, and α is the angle of rotation of the point around the axis when it moves along the turning edge, *u* is the length of the rectilinear generatrix, the count of which starts from the point on the turning edge; *a* is the radius of the cylinder on which the turning edge is located (Fig. 2, *a*); β is the angle of elevation of the turning edge.

When changing the sign before *u* in equations (1), the linear generatrix will be directed upwards. Fig. 2 shows the constructed projections of the unfolded helicoid section, which with the proper parameters will be a helical knife with the simplification that its thickness is zero. Also, the proportions of the surface section do not correspond to the proportions of the knife, but at the same time it is convenient to mark the necessary parameters to find their connection with the structural parameters of the knife.

The cross-section (perpendicular to the axis of the surface) of the folded helicoid will be a straight line since all its generatrices are parallel to the horizontal plane (Fig. 1, *c*). For an unfolded helicoid, this will be a well-known curve – the involute of a circle. To find its equation, you need to set the constant value of the coordinate $Z=Z_h$ in the last equation (1). From here, the dependence $u=u(\alpha)$ can be derived:

$$u = \frac{a\alpha\,\mathrm{tg}\beta - Z_h}{\sin\beta}, \tag{2}$$

when $Z_h=0$ and substituting (2) into the parametric equation (1), an involute arc will be described, which limits the length of the knife below, and when $Z_h=L$ – above. Fig. 2, *b* shows the constructed torso-helicoid, in which the rectilinear generatrices are tangents to the turning edge 1 and when meeting the horizontal plane (that is, at $Z_h=0$) in the intersection with it give involute 2. The part of the involute that concerns the torso strip (helix-shaped knife) is indicated by a thick line. In all the given dependences (1) to (3), the radius *a* and the angle β refer to the turning edge. They need to be expressed through the parameters of the knife.

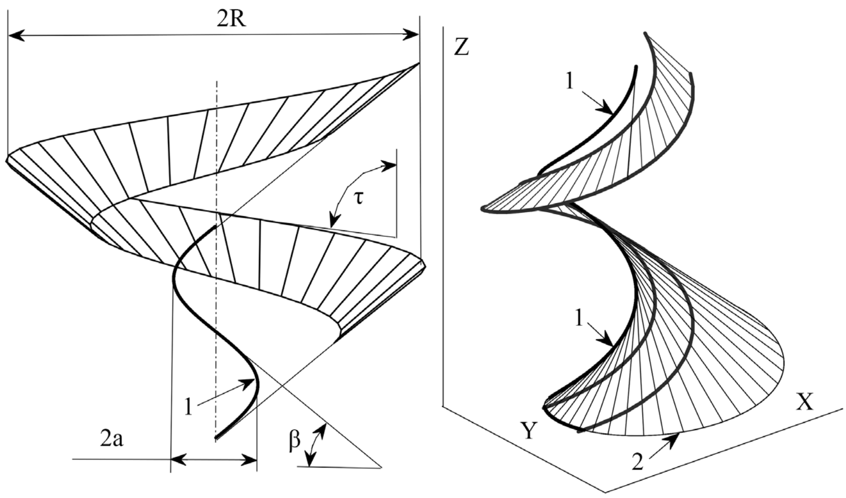

Fig. 2. Projections of the unfolded helicoid strip with the turning edge, which is helical line 1: *a* — frontal projection (generatrices of the unfolded helicoid are directed upwards); *b* — axonometric projection (generatrices of the torso-helicoid are directed downwards)

Fig. 3, *a* shows the top view of the unfolded helicoid depicted in Fig. 2, *b*.

The result of its cross-section is involute 2, which is indicated by a thick line within the section between the radii *R* and *r*. The angle φ between the radial direction and the tangent to the involute at the point of the blade is shown. Fig. 3, *b* introduces other notations, in particular, an additional angle γ was introduced to show that the angle φ is present in the right triangle *OAB*. From it, the expression for the radius *a* can be found through the parameters of the knife:

$$a = R\cos\varphi. \tag{3}$$

It is appropriate to find the expression of the angle β through the parameters of the knife. For this, the length of the rectilinear generatrix *u*, which corresponds to the helical line of radius ρ on the surface, must be determined. The distance ρ from the point of this helical line to the axis of the helicoid is determined as follows:

$$\rho^2 = X^2 + Y^2 = a^2 + u^2\cos^2\beta \Rightarrow$$

$$\Rightarrow u = \frac{\sqrt{\rho^2 - a^2}}{\cos\beta}. \tag{4}$$

When $\rho=R$, from (4) the expression *u* for the helical line – the knife blade – will be derived.

By substituting the value of *a* from (3) and the value of *u* from (4) taking into account $\rho=R$ into the parametric equations (1) of the surface, the equations of the helical line, the knife blade, can be obtained:

$$x = R\cos\varphi\cos\alpha + R\sin\varphi\sin\alpha; \tag{5}$$

$$y = R\cos\varphi\sin\alpha - R\sin\varphi\cos\alpha;$$

$$z = R\alpha\cos\varphi\,\mathrm{tg}\beta - R\sin\varphi\,\mathrm{tg}\beta.$$

The tangent to the helical line (5) forms an angle τ with the surface axis (that is, with the *Z* axis). It can be written as the angle between the tangent vector to the helical line (5) and the *Z* axis. The expression for this angle can be derived after finding the derivatives in equations (5):

$$\cos\tau = \frac{\cos\varphi\,\mathrm{tg}\beta}{\sqrt{1+\cos^2\varphi\,\mathrm{tg}^2\beta}} \Rightarrow \beta = \mathrm{Arctg}\frac{\cos\tau}{\cos\varphi\sin\tau}. \tag{6}$$

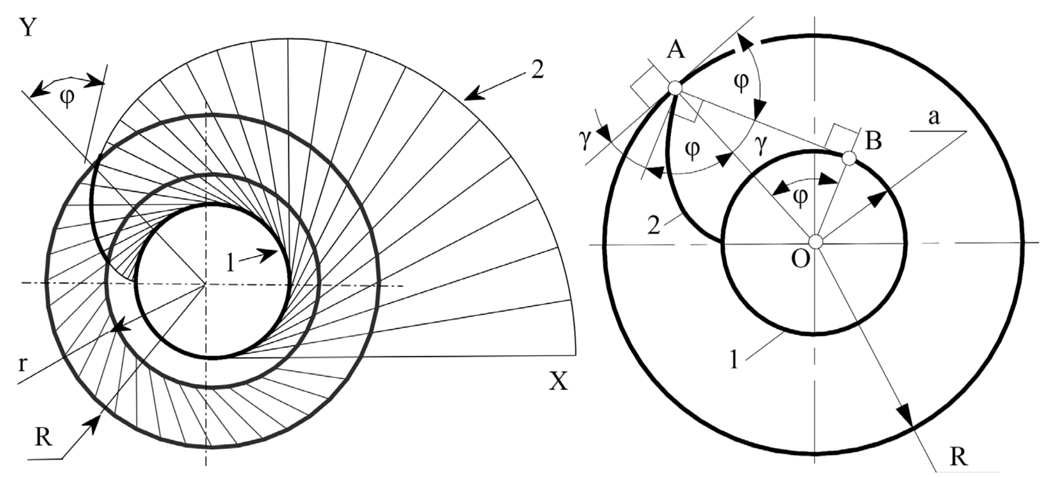

Fig. 3. The top view of the helicoid torso shown in Fig. 2, *b*: 1 — turning edge, 2 — circle involute; *a* — the selected strip of the unfolded helicoid and its cross-section, which is indicated within the strip of the involute circle; *b* — designation of the necessary parameters

In the equations of the unfolded helicoid (1), constants α and β are expressed through the structural parameters of the knife according to (3) and (6). The interval in which the variables α and u acquire values was considered on a specific example.

Substitution of the expression a from (3) and β from (6) into the second formula (4) and the transformation give the result:

$$u=\frac{\sqrt{\left(\rho^2-R^2\cos^2\varphi\right)\left(\cos^2\varphi\sin^2\tau+\cos^2\tau\right)}}{\cos\varphi\sin\tau}. \quad (7)$$

Let the knife parameters be as follows: R=0.25 m, τ=20°, φ=65°. Substitution of the given data into formula (7) makes it possible to obtain the value of u for a given value of the radius ρ. For example, at ρ=R, which corresponds to the blade of a knife, the value of u is u=1.49 m. At u=0, equation (1) will describe another spiral line – the turning edge. These values will correspond to the maximum possible width of the knife in the form of the length of the arc of the circle involute (cross-section of the knife), which is located between the corresponding circles (Fig. 3, *b*). The width of the knife was limited by an internal spiral line, which corresponds to a radius of ρ=0.2 m. Substituting this value into formula (7) gives the result: u=1.12 m. Therefore, the u parameter will vary within u=1.12...1.49. The resulting section of the unfolded helicoid will be located between uniaxial cylinders of 0.2 m and 0.25 m radii.

The limits of change in the angle α depend on the length of the knife, which in turn depends on their number on the drum. The load on the drum should be balanced, and this will be on the condition that when cutting with one knife is finished, the next one immediately begins work. From the last equation (1) at u=0 we write: Z=a·α·tg β. If the number of knives is n, then for one knife its rotation angle should be α=2π/n. Therefore, Z=2a·π·tg β/n. The value of Z will be the length of the knife, i.e. L according to the designations in Fig. 1, *a*. When n=6, we shall obtain: Z=L=0.72 m.

When constructing a section of the surface (1), which corresponds to the contours of the knife, the change in parameters u and α can be taken within larger limits and the desired section can be cut from the obtained surface. Fig. 4 shows the constructed surface of the unfolded helicoid with lines that cut out the desired section on it. Numbers 1 and 2 indicate the involutes that limit the length of the knife. They are constructed by substituting expression (2) into the surface equation (1).

By analogy with the section of an open helicoid (Fig. 1, *c*), which is formed by the helical movement of a horizontal rectilinear generatrix, the section of an unfolded helicoid can also be formed in this way. At the same time, instead of a rectilinear generatrix, there should be an arc of an involute circle, the plane of which during helical motion remains parallel to the horizontal plane of the projections. The strip (section of the surface) with this formation technique is built in Fig. 4, *c*. At the same time, the rotational movement of the involute arc around the axis of the helicoid and its translational movement along the axis must be aligned.

5. 2. Finding the contours of a flat workpiece for making a screw-shaped knife from it

For unfolded surfaces, one can find their parametric equations on the sweep, that is, after bending them onto a plane. They are found by means of differential geometry on the basis that the lengths of the lines on the surface do not change after bending. Without going into the subtleties of the transition from the equations of the unfolding surface in space to its equations on the plane, the result takes the following form:

$$X_0=\frac{a}{\cos^2\beta}\cos(\alpha\cos\beta)+u\sin(\alpha\cos\beta); \quad (8)$$

$$Y_0=\frac{a}{\cos^2\beta}\sin(\alpha\cos\beta)-u\cos(\alpha\cos\beta).$$

It is possible to make sure that the parametric equations (1) and (8) describe the same helical surface of the unfolded helicoid in space and on the plane by finding the first quadratic form of these equations. A pair of numerical values of coordinates α and u defines a point on the surface in space according to equations (1) and the corresponding point on the sweep according to equations (8). The dependence u=u(α), which is the internal equation of the line, when substituted into parametric equation (1), describes the line on the surface in space, and when substituted into equation (8), it describes the corresponding line on the sweep.

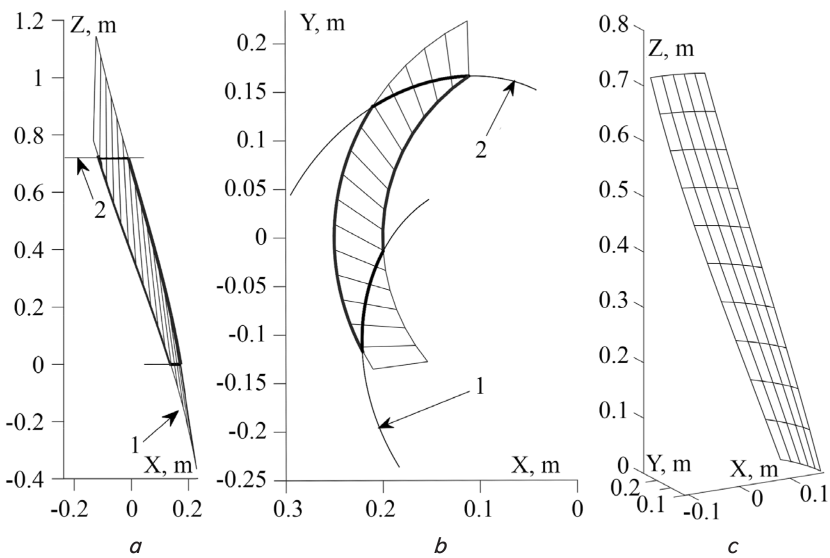

Fig. 4. Projections of the section of the unfolded helicoid (1 – arc of the involute of the circle at Z=0, 2 – at Z=L): *a* – frontal projection; *b* – horizontal projection; *c* – the axonometry of the section, the generating curve of which is the arc of the involute of a circle

For example, when substituting (2) into (8) when $Z_h=L$, involute 1 was obtained (Fig. 5, *a*), and when $Z_h=0$ – involute 2. Accordingly, when substituting expression (7) (in this case, it is a constant value) in (8), the curve of knife blade 3 at $\rho=R=0.25$ m and the opposite curve (4) at $\rho=0.2$ m were obtained.

The knife sweep can be built more simply. Its blade on the sweep and the opposite curve turn into arcs of circles. The radii of these circles can be found as follows. The distance from the origin of the coordinates to an arbitrary point on the sweep (8) is determined from the formula:

$$\rho_0=\sqrt{X_0^2+Y_0^2}=\frac{\sqrt{a^2+u^2\cos^4\beta}}{\cos^2\beta}. \quad (9)$$

All values included in expression (9) are constant, therefore the distance ρ_o is constant, that is, the corresponding curve will be a circle. Substitution into formula (9) of the expressions included in it from (3), (6), (7) makes it possible to obtain an expression for ρ_o through the structural parameters of the knife:

$$\rho_0=\frac{1}{\cos\varphi}\sqrt{\left(\rho^2+R^2\text{ctg}^2\tau\right)\left(\cos^2\varphi+\text{ctg}^2\tau\right)}. \quad (10)$$

At $\rho=R=0.25$ m, formula (10) gives the radius R_0 of the arc of the circle that outlines the blade on the sweep, and at $\rho=0.2$ m – the radius r_o (Fig. 5, *a*). With the given parameters of the knife: $R_0=4.8$ m, $\rho_o=4.7$ m. Two concentric circles can be built according to the obtained dimensions (Fig. 5, *b*).

To find the boundaries of the section between concentric circles, one can proceed as follows. By substituting the value $\rho=a$ in (10), the radius a_o of the circle, into which the turning edge on the sweep is transformed, is determined. Its length *s*, within the length of the knife *L*, is the same in space and on the sweep. In space, it is located through the elevation angle β: $s=L/\sin\beta$. On the sweep, it corresponds to the arc *AB* (Fig. 5, *b*) and is found through the angle α_o: $s=a_o\cdot\alpha_o$. Hence, the central angle α_o for the arc *AB* can be determined: $\alpha_o=L/(a_o\cdot\sin\beta)$.

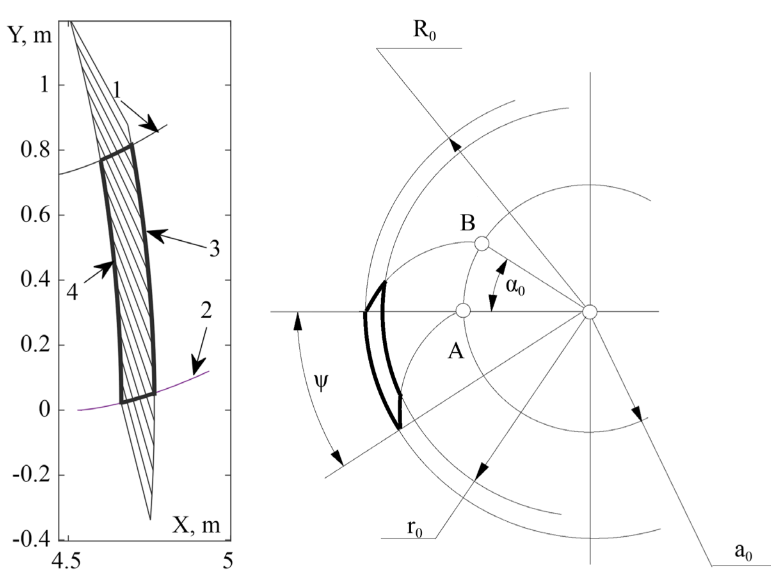

Fig. 5. The sweep of the knife:
a — built according to equations (8) by substituting the internal equations of the contour lines into them; *b* — constructed according to equation (10), as a section between two concentric circles

Switching to the structural parameters of the knife makes it possible to obtain the following expression:

$$\alpha_0=\frac{L\cos\varphi\sin^2\tau}{R\cos\tau\sqrt{\cos^2\varphi\sin^2\tau+\cos^2\tau}}. \quad (11)$$

Based on the known value of the angle α_o according to (11), the arc *AB* can be placed on a circle of radius a_o. From points *A* and *B*, involutes are drawn to this circle, which, at the intersection with circles of radii r_o and R_o, cut the blade of the knife from the ring. Analogously to the angle α_o, the angle ψ can be found: $\psi=L/(R_o\cdot\cos\tau)$. When moving to the structural parameters of the knife, the result is a formula that exactly coincides with formula (11), i.e. $\psi=\alpha_o$. This means that the knife blade rests on the same central angle as the arc *AB*. For the given structural parameters of the knife, this angle is $\psi=\alpha_o=0.765$ rad.

6. Discussion of results regarding the design of a helical knife and the construction of its sweep

Design and manufacture of helical knives are not given due attention in the scientific literature due to the supposedly simple shape of this part. The designation of the knife indicates that its surface is helical. However, a helical surface can be linear or non-linear, and non-linear surfaces are all non-expandable, and a linear surface can be both non-expandable and expandable. The simplest helical knife based on its surface structure is a knife whose cross-section is a rectilinear segment (Fig. 1). However, its surface is folded, and it is impossible to accurately find the contours of a flat workpiece. Accordingly, when molding the approximate workpiece into the finished product, plastic deformations occur, in which cracks and even metal rupture may occur if the plasticity threshold is exceeded.

Our results are explained by the fact that it is proposed to adopt the surface of the helical knife in the form of an unfolded helicoid. A feature of the unfolded helicoid is that the cross-section of its surface is an involute arc, the parameters of which are strictly coordinated with the parameters of the knife. The process of formation of folded and unfolded helicoids is similar: while in the first case the surface is formed by the helical movement of a straight-line segment (Fig. 1, *c*), in the second – by the helical movement of an involute arc (Fig. 4, *c*). However, in the second case, it is possible to accurately build the sweep of the knife, that is, the contours of the flat workpiece. When it is bent into the finished article, plastic deformations caused by the thickness of the workpiece will also occur, but they will be minimal. It is these features of the proposed solutions that provide advantages. In contrast to [17], in which the manufacture of a knife by cold drawing of a workpiece is considered, in this work a flat workpiece with precise curvilinear contours is proposed. This became possible owing to the analysis of the knife surface from the point of view of the differential geometry of surfaces.

Our solutions resolve the problem specified in the second chapter. It involves devising a methodology for the accurate calculation of a flat workpiece for a helical knife from the unfolded surface, while in works [6, 16, 17] this was not paid attention to, and the production of knives was carried out from approximate blanks without taking into account the type of surface.

There are limitations on the length of the knife due to the technology of its manufacture. For example, one can theoretically imagine a knife in which the blade has one full turn, that is, the angle of its twist is 360° (for example, Fig. 2, *a*). However, it is technologically difficult to manufacture, so it is necessary to limit the twisting angle to 90°.

The disadvantage of our study is that the thickness of the knife is assumed to be zero. However, this does not detract from the practical value of the results since such an assumption is quite acceptable for parts of small thickness, and the error in the calculations decreases with a decrease in the thickness of the part.

Further development of the study may focus on an unambiguous correspondence between points and lines on the surface and on the sweep for other unfolded surfaces. This would make it possible to precisely build the contours of flat workpieces for the manufacture of work surfaces that are expandable.

7. Conclusions

1. Analysis of the surface of the helical knife has revealed that it can be a section of both folded and unfolded helicoids. Although the surface of the unfolded helicoid is more complex in its structure, in this case it is possible to accurately construct the sweep of the knife in the form of a contour of a flat workpiece. This is an advantage in the manufacture of a knife due to the reduction of plastic deformations. In this regard, the parametric equations of the unfolded helicoid were constructed, to which the structural parameters of the knife were entered. The internal equations of the curved lines that outline the contours of the knife on the surface of the unfolded helicoid were also derived.

2. On the basis of the theory of differential geometry, the parametric equations of the unfolded helicoid were built after its alignment with the plane. These equations established a one-to-one correspondence between the points on the helicoid sweep and the points on the surface in space. This correspondence has made it possible to derive the internal equations of the contours of the knife. Substituting them into the parametric equations of the helicoid gives the contours of the knife on the surface, and when substituting them into the parametric equations of the sweep – the contours of the flat workpiece. Since the helical line – the blade of the knife – turns into an arc of a circle on the sweep, a simplified technique for constructing a flat workpiece was devised. It implies finding the concentric circles that outline the flat workpiece and the central angle on which the knife blade rests. Appropriate calculations were performed for the specified structural parameters of the knife.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Tian, F., Xia, K., Wang, J., Song, Z., Yan, Y., Li, F., Wang, F. (2021). Design and experiment of self-propelled straw forage crop harvester. Advances in Mechanical Engineering, 13 (7), 168781402110244. https://doi.org/10.1177/16878140211024455
2. Derevyanko, D. A., Polishchuk, V. M., Chichilyuk, S. B., Derevyanko, A. D. (2020). Substantiation of effect of ladle conveyors on injury and quality of seeds of cereals. Machinery & Energetics, 11 (1), 163–169. Available at: https://technicalscience.com.ua/web/uploads/pdf/Machinery%20&%20Energetics_Vol.%2011,%20No.%201_163_169.pdf
3. Buslaiev, D. O., Vasylenko, M. O. (2020). Traction resistance of cultivator paws with surface hardening during operation of tillage machines. Machinery & Energetics, 11 (1), 177–182. Available at: https://technicalscience.com.ua/uk/journals/t-11-1-2020/tyagoviy-opir-kultivatornikh-lap-z-povyerkhnyevim-zmitsnyennyam-pri-yekspluatatsiyi-gruntoobrobnikh-mashin
4. Mudarisov, S. G., Gabitov, I. I., Lobachevsky, Y. P., Mazitov, N. K., Rakhimov, R. S., Khamaletdinov, R. R. et al. (2019). Modeling the technological process of tillage. Soil and Tillage Research, 190, 70–77. https://doi.org/10.1016/j.still.2018.12.004
5. Morris, N. L., Miller, P. C. H., Orson, J. H., Froud-Williams, R. J. (2010). The adoption of non-inversion tillage systems in the United Kingdom and the agronomic impact on soil, crops and the environment – A review. Soil and Tillage Research, 108 (1-2), 1–15. https://doi.org/10.1016/j.still.2010.03.004
6. Tesliuk, H., Volik, B., Sokol, S., Ponomarenko, N. (2019). Design of working bodies for tillage tools using the methods of bionics. Eastern-European Journal of Enterprise Technologies, 3 (1 (99)), 49–54. https://doi.org/10.15587/1729-4061.2019.169156

7. Kobets, A., Aliiev, E., Tesliuk, H., Aliieva, O. (2023). Simulation of the process of interaction of the working bodies of tillage machines with the soil in Simcenter STAR-CCM+. Machinery & Energetics, 14 (1). https://doi.org/10.31548/machinery/1.2023.09
8. Kobets, A. S., Pugach, A. M., Kharytonov, M. M. (2018). Justification of the cultivator sweep and strengthening elements on the working surface. INMATEH – Agricultural Engineering, 54 (1), 161–170. Available at: https://dspace.dsau.dp.ua/bitstream/123456789/726/1/1.pdf
9. Klendii, M., Klendii, M. I., Shatrov, R. V. (2020). Research of influence of constructive and technological parameters of developed variants of screws with screw working bodies rabbit. Machinery & Energetics, 11 (4), 65–71. https://doi.org/10.31548/machenergy2020.04.065
10. Chvartatskiy, I., Flonts, I., Grabar, A., Shatrov, R. (2021). Synthesis of energy-saving transport-technological systems with screw working bodies. Machinery & Energetics, 12 (4), 77–84. https://doi.org/10.31548/machenergy2021.04.077
11. Kresan, T. A. (2020). Calculation of gravitation descent formed by surface of skew closed helicoid. Machinery & Energetics, 11 (2), 49–57. Available at: https://technicalscience.com.ua/web/uploads/pdf/Machinery%20&%20Energetics_Vol.%2011,%20No.%202_49-57.pdf
12. Kresan, T. (2021). Movement of soil particles on surface of developable helicoid with horizontal axis of rotation with given angle of attack. Machinery & Energetics, 12 (2), 67–75. https://doi.org/10.31548/machenergy2021.02.067
13. Pylypaka, S., Klendii, M., Kresan, T. (2022). Study of the movement of soil particles on the surface of a screw tillage working body. Machinery & Energetics, 13 (2), 62–72. https://doi.org/10.31548/machenergy.13(2).2022.62-72
14. Paladiychuk, Yu. B., Melnik, Ju. M. (2021). Restoration resource of working bodies of soil treatment machines by applying wear-resistant coatings. Machinery & Energetics, 12 (4), 43–52. https://doi.org/10.31548/machenergy2021.04.043
15. Borak, K. V. (2020). Influence of degree of abrasive particle clinging to wear mechanism of soil-cultivating machines operational mechanisms. Machinery & Energetics, 11 (2), 35–40. Available at: https://technicalscience.com.ua/uk/journals/t-11-2-2020/vpliv-stupyenya-zakriplyennya-abrazivnoyi-chastinki-na-myekhanizm-znoshuvannya-robochikh-organiv-gruntoobrobnikh-mashin
16. Tan, C. M., Lin, G. Y. (2016). An Innovative Compression Mold Design for Manufacture of Reel Mower Helical Blades. Applied Mechanics and Materials, 851, 255–258. https://doi.org/10.4028/www.scientific.net/amm.851.255
17. Li, Z. F., Jiang, L. Q. (2013). Design of Combined Helical Blade Manufacturing Device. Advanced Materials Research, 753-755, 1386–1390. https://doi.org/10.4028/www.scientific.net/amr.753-755.1386