This study focuses on developing a mathematical model for precise control and stabilization of unmanned aerial vehicles (UAVs) in various spatial conditions. Addressing the problem of achieving precise control and stability, the proposed solution designs a control system based on a linear-quadratic controller (LQR) and simulates it using a proportional-derivative (PD) controller implemented in Matlab/Simulink. The results demonstrate high precision and stability in controlling the UAV motion parameters – roll, pitch, yaw, and altitude. This important level of performance is achieved due to the adaptivity of the LQR-based control system, which optimizes control actions according to the unsteady dynamics of the UAV. The integration of the PD controller improves responsiveness and stability, providing precise motion control over a range of spatial states. These features effectively solve the problem by handling the complex dynamics of the UAV and providing precise control. The results are explained by the ability of the LQR to provide optimal control laws that minimize deviations using a quadratic cost function, while the PD controller quickly corrects errors and responds to disturbances. The benefits of this approach include a significant reduction in control errors by about 25–30 %, increased response speed to external disturbances, and reduced computational latency due to efficient processing compared to more resource-intensive methods such as model predictive control. The developed mathematical model can be applied in practice in conditions requiring robust real-time control and adaptation to dynamic changes in the environment. It is especially suitable for industries such as logistics, surveillance, and environmental monitoring, providing an effective and optimal solution for stabilizing and controlling the motion of UAVs in various spatial states. This approach improves the performance of UAVs and expands their capabilities in various operating conditions

Keywords: unmanned aerial vehicle, mathematical model, linear-quadratic regulator, proportional-derivative, control and stability

UDC 621

DOI: 10.15587/1729-4061.2024.308928

DEVELOPMENT OF MATHEMATICAL MODELING OF A MOBILE ROBOT'S MOTION CONTROL SYSTEM

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Received date 08.08.2024
Accepted date 16.10.2024
Published date 30.10.2024

***How to Cite**: Issabekov, Z., Bektilevov, A., Baiturganova, V., Zhamuratova, M., Rakhmetova, P. (2024). Development of mathematical modeling of a mobile robot's motion control system. Eastern-European Journal of Enterprise Technologies, 5 (2 (131)), 103–111. https://doi.org/10.15587/1729-4061.2024.308928*

1. Introduction

The modern development of robotics and automation requires the scientific community to actively search for solutions to improve the efficiency and accuracy of various systems, including mobile robots [1, 2]. The increasing demand for automated control and monitoring systems makes relevant scientific study in the field of mobile robot and especially unmanned aerial vehicles [3, 4]. This is because they can precisely regulate their direction and motion in space, which is essential for carrying out tasks in small areas [5].

This development of this class of robots is due to several specific advantages, the implementation of which allows one to obtain a significant advantage over stationary robots for a wide range of tasks, such as monitoring the ecological state of the area, aerial photography, construction control, surveillance, location navigation and reconnaissance, etc. Furthermore, the advancements in computer vision and artificial intelligence have made it necessary to enhance mobile robot control systems, which has led to a surge in interest in this field of study [5–7].

Although there have been great advancements in this field, there are still a lot of unanswered questions about how to enhance control systems and stabilize mobile robot motions in the face of outside stimuli. This addresses the theoretical underpinnings of developing mathematical models as well as the pragmatic challenges of applying them to actual situations. More investigation and optimization are needed to tackle the pressing issue of developing an efficient motion control system for unmanned aerial vehicles in practical operational environments [6–8].

Thus, research in the subject of creating control systems for unmanned aerial vehicles is still vital. The advancement of this topic is required to tackle a wide range of practical challenges that necessitate a high level of autonomy and control accuracy.

2. Literature review and problem statement

The paper [9] presents the results of research on the challenges faced by autonomous mobile robots and the sensor fusion methods employed to enhance their performance.

It is shown that while sensor fusion significantly improves perception and navigation capabilities, there are unresolved issues related to the integration of heterogeneous sensor data, real-time processing constraints, and maintaining reliability under varying environmental conditions. The reason for this may be objective difficulties associated with processing large volumes of diverse data in real-time, fundamental limitations of sensor technologies in adverse environments, and cost considerations in implementing advanced sensors and high-performance computing hardware, which makes relevant research impractical for widespread adoption.

A way to overcome these difficulties can be the development of adaptive motion control strategies that utilize environmental risk assessments to optimize navigation. This approach was used in [10], where an adaptive motion control method for an autonomous mobile robot based on a space risk map was proposed. However, there were unresolved issues related to the computational complexity of generating and updating the risk map in real-time and the scalability of the system in more complex or varying environmental conditions.

The paper [11] presents the results of research on motion planning and control of an omnidirectional mobile robot in dynamic environments. It is shown that their proposed methods enhanced navigation in dynamic settings, but there were unresolved issues related to real-time adaptability and obstacle avoidance in highly unpredictable conditions. The reason for this may be objective difficulties associated with rapidly changing environments and the computational demands of processing dynamic data, which makes relevant research challenging.

In the domain of unmanned aerial vehicles (UAVs), several studies have addressed modeling and control challenges. The paper [12] focuses on modeling power consumptions for multirotor UAVs, highlighting the importance of accurate power models for efficient operation. It is shown that while the models improve understanding of energy usage, there are unresolved issues related to accurately predicting power consumption under varying flight conditions and payloads. The reason for this may be the complex interplay of aerodynamic factors and hardware limitations.

A way to overcome these difficulties can be the development of precise modeling techniques and adaptive control strategies that account for varying conditions. This approach was used in [13], where a quadrotor's modeling and control system design based on PID control was presented. However, there were unresolved issues related to handling nonlinear dynamics and external disturbances due to the limitations of PID controllers.

To address these challenges, advanced control methods such as model predictive control (MPC) have been explored. In [14], real-time model predictive control for quadrotors was proposed. It is shown that MPC improves control precision and responsiveness, but unresolved issues include the computational demands of real-time optimization and performance under highly dynamic conditions. The reason for this may be objective difficulties associated with the need for high computational resources and the complexity of implementing MPC in real-time systems.

Another approach is the use of fast model-free learning methods. The paper [15] introduces such a method for controlling a quadrotor UAV with a designed error trajectory. It is shown that this approach accelerates the learning process and improves control accuracy, but there were unresolved issues related to the method's adaptability to different environments and robustness against unmodeled dynamics.

Similarly, the development of nonlinear control techniques has been considered. In [16], modeling and nonlinear control of a quadcopter for stabilization and trajectory tracking were explored. It is shown that nonlinear control enhances quadcopter performance, but unresolved issues pertain to system complexity and computational overhead associated with these algorithms.

In [17], a drone delivery logistics model for the on-demand hyperlocal market was developed. It is shown that drone delivery can significantly enhance logistical efficiency, but there were unresolved issues related to regulatory challenges, airspace management, and safety concerns. The reason for this may be objective difficulties associated with integrating drones into existing airspace systems and the cost implications of implementing large-scale drone delivery services.

The paper [18] provides a review of unmanned aerial vehicles for precision agriculture. It is shown that UAVs offer significant benefits in monitoring and managing agricultural activities, but unresolved issues involve data processing challenges, integration with existing agricultural systems, and economic feasibility for small-scale farmers.

The paper [19] investigates the use of Linear Quadratic Regulator (LQR) control for the active suspension system of a four-wheeled agricultural robot. It is shown that LQR control enhances stability and ride comfort, but there were unresolved issues related to adaptability to varying loads and terrain conditions. The reason for this may be objective difficulties associated with the variability of agricultural environments and the need for more robust control strategies.

Thus, all this suggests that it is advisable to conduct a study on developing integrated systems that combine efficient sensor fusion, adaptive control algorithms, and real-time processing capabilities to enhance the performance and applicability of autonomous robots and UAVs in various environments.

This paper discusses the problem of developing a mathematical model for a motion control system for a quadcopter-type mobile robot (UAV). The problem is important because it lays the foundation for the creation and design of efficient UAVs, which are increasingly used in various applications, including environmental monitoring, aerial photography, construction control, surveillance and reconnaissance. In this paper, a method for quadcopter control is proposed based on feedback linearization and the integration of PD controller synthesis with optimization methods such as linear-quadratic regulator (LQR), which can significantly improve the system performance due to adaptive control and minimization of stabilization errors. This makes PD controllers suitable for use in more complex control systems such as LQR to provide robust control in various spatial conditions.

3. The aim and objectives of the study

The aim of the study is to develop a mathematical model of the control system to ensure precise control and stabilization of an unmanned aerial vehicle (UAV), particularly a quadcopter, in various spatial conditions, considering complex dynamic behavior.

To achieve this aim, the following objectives are accomplished:

– to develop a mathematical model of a UAV motion control system based on a PD controller;

– to develop a mathematical model of a control system based on a linear-quadratic controller (LQR) and a feedback coefficient to minimize errors in control and stabilization of the UAV position;

– to simulate the operation of the control system using a PD controller in the Matlab/Simulink;

– to simulate UAV motion control system based on LQR and PD controllers in the Matlab/Simulink to test the effectiveness of the proposed model.

4. Materials and methods

The object of our study is the process of developing a mathematical model of the motion control system of an unmanned aerial vehicle (UAV), particularly a quadcopter, providing precise control and stabilization in the state space. The main hypothesis of the study is that by accurately modeling the six degrees of freedom inherent in a quadcopter's movement, it is significantly possible to enhance its stability and control responsiveness. In this work, let's assume that the UAV operates in an environment free from significant external disturbances like wind or obstacles. Simplifications adopted include linearizing the system dynamics and neglecting minor aerodynamic effects to focus on the primary control mechanisms. To perform the control task, it is necessary to adjust the flight states in accordance with the specified coordinates. The flight states include six degrees of freedom: three rotational and three translational movements. Along the pitch axis, the pitch angle and translational movement are controlled; along the roll axis, the roll angle and translational movement are controlled; along the yaw axis, the yaw angle and flight altitude are controlled.

To develop a control system, the motion of a mobile robot, particularly an unmanned aerial vehicle (UAV) is considered in two interconnected coordinate systems – one stationary inertial (fixed) frame (X, Y, Z) connected to the Earth, and the other is a connected coordinate system (x, y, z) fixed in the center of gravity of the mobile robot (as an example, it is possible to consider a quadcopter).

The motion of a mobile robot (UAV) is represented as the sum of two motions – the motion of the center of mass of the UAV in the inertial coordinate system and the motion of the connected UAV axes relative to the inertial coordinate system. The position of the axes of the quadcopter is shown in Fig. 1.

The orientation of the connected coordinate system relative to the inertial is determined by Euler-Lagrange angles [20]. The angle of the roll φ corresponds to the rotation of the movable coordinate system around the OX axis, the angle of the pitch θ – the rotation around the OY axis, the yaw angle ψ – around the OZ axis, respectively.

The position of the UAVs in the inertial coordinate system shall be determined by the position vector $r=(x,y,z)$ [21]. The transition from the inertial coordinate system to the connected coordinate system is carried out by means of three turns:

1) around the x-axis at an angle φ;

2) around the y-axis of the new coordinate system at an angle θ;

3) around the z-axis of the new coordinate system at an angle ψ.

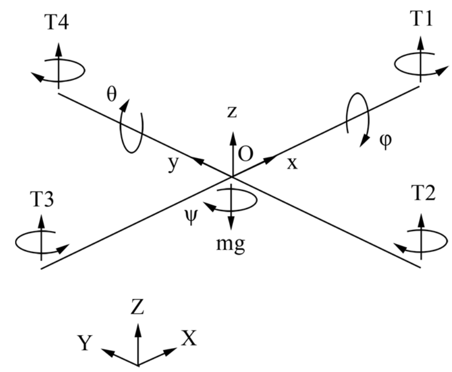

Fig. 1. Coordinate systems – inertial coordinate system (X, Y, Z) and connected coordinate system (x, y, z)

The motion of the UAV's center of mass in the inertial frame of reference is generally described by (1):

$$m\ddot{r} = -mg\vec{e}_z + R^T(\psi,\theta,\phi)\vec{u}_s, \tag{1}$$

were $r=(x,y,z)$ – UAV center acceleration vector; $\vec{e}_z$ – the unit vector directed along the Oz axis; $\vec{u}_s$ – the sum of the non-corrosive forces acting on the system (including the force of the frontal resistance and the force of the propellers) and R^T – transposed transition matrix:

$$R^T_{xyz}(\phi,\theta,\psi) = \begin{pmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{pmatrix}.$$

For simplicity, it will be assumed that there are no resistance forces and only the total thrust force of the four screws $\vec{u}$, which is written in the form (2):

$$\vec{u} = u_1\vec{e}_z = \sum_{n=1}^{4} b\omega_{pn}^2\vec{e}_z, \tag{2}$$

were $u_1 = \sum_{n=1}^{4} b\omega_{pn}^2$, $\vec{e}_z$ – unit vector directed along the Oz axis; b – thrust factor; ω_{pn} – angular speed of rotation of the n^{th} propeller.

Substituting equation (2) into equation (1) and performing matrix multiplication in the second term, it is possible to obtain the following equation of motion of the UAV's center of mass in the form of a system of ordinary differential equations (3):

$$\begin{cases} m\ddot{x} = u_1(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi), \\ m\ddot{y} = u_1(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi), \Rightarrow \\ m\ddot{z} = -mg + u_1\cos\phi\cos\theta, \end{cases}$$

$$\Rightarrow \begin{cases} \ddot{x} = \dfrac{u_1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi), \\ \ddot{y} = \dfrac{u_1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi), \\ \ddot{z} = -g + \dfrac{u_1}{m}\cos\phi\cos\theta. \end{cases} \tag{3}$$

In addition to translational motion, the UAV also performs rotational motion relative to the Earth, which is motion relative to its center of mass [22]. In vector form, the equation of rotational motion of the UAV is described by Euler's equations [20]:

$$J\dot{\vec{\omega}}_k = -\left(\vec{\omega}_k \times J\vec{\omega}_k\right) + \vec{M} - \vec{M}_G, \tag{4}$$

were $\vec{\omega}_k = \left[\omega_x, \omega_y, \omega_z\right]$ – UAV angular velocity; J – the inertia matrix, which is a third-order diagonal matrix with moments of inertia relative to the corresponding axes; $\vec{M}$ – the vector of moments of traction forces; $\vec{M}_G$ – the gyroscopic moment vector, resulting from the reciprocal influence of the rotational motion of the UAV and the rotational motion of the UAV screws.

Vector of moments of traction forces $\vec{M}$ is calculated in the form of equation (5):

$$\vec{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} Lb\left(\omega_{p2}^2 - \omega_{p4}^2\right), \\ Lb\left(\omega_{p1}^2 - \omega_{p3}^2\right), \\ d\left(\omega_{p1}^2 - \omega_{p2}^2 + \omega_{p3}^2 - \omega_{p4}^2\right), \end{pmatrix}, \tag{5}$$

were L – UAV arm length;

b – thrust factor;

d – drag coefficient.

The gyroscopic moment $\vec{M}_G$ caused by the combination of rotation of the four propellers is modeled as equation (6):

$$\vec{M}_G = \begin{pmatrix} J_P\Omega_z\omega_y \\ J_P\Omega_z\omega_x \\ 0 \end{pmatrix}, \tag{6}$$

were J_p – moment of inertia of one propeller; ω_x, ω_y – UAV angular velocity;

$\Omega_z = \omega_{p1} - \omega_{p2} + \omega_{p3} - \omega_{p4}$ – propeller angular velocity.

To simplify the equation of motion, let's denote the thrust forces of the propellers and the moments of forces applied to the UAV body through the functions u_1, u_2, u_3, u_4 in the form of equations (7):

$$\begin{cases} u_1 = T = \sum\limits_{n=1}^{4} b\omega_{pn}^2, \\ u_2 = M_x = rb\left(\omega_{p2}^2 - \omega_{p4}^2\right), \\ u_3 = M_y = rb\left(\omega_{p1}^2 - \omega_{p3}^2\right), \\ u_4 = M_z = d\left(\omega_{p1}^2 - \omega_{p2}^2 + \omega_{p3}^2 - \omega_{p4}^2\right). \end{cases} \tag{7}$$

Considering the accepted notation and equations (4), (5), the UAV equation system as a system of ordinary differential equations takes the form:

$$\begin{cases} \dot{\omega}_x = \left(\dfrac{J_y - J_z}{J_x}\right)\omega_y\omega_z + \dfrac{M_x}{J_x} - \dfrac{M_{Gx}}{J_x}, \\ \dot{\omega}_y = \left(\dfrac{J_z - J_x}{J_y}\right)\omega_x\omega_z + \dfrac{M_y}{J_y} - \dfrac{M_{Gy}}{J_y}, \\ \dot{\omega}_z = \left(\dfrac{J_x - J_y}{J_z}\right)\omega_x\omega_y + \dfrac{M_z}{J_z}, \end{cases} \Rightarrow \begin{cases} \dot{\omega}_x = \left(\dfrac{J_y - J_z}{J_x}\right)\omega_y\omega_z + \dfrac{1}{J_x}u_2 - \dfrac{J_P}{J_x}\Omega_z\omega_y, \\ \dot{\omega}_y = \left(\dfrac{J_z - J_x}{J_y}\right)\omega_x\omega_z + \dfrac{1}{J_y}u_3 - \dfrac{J_P}{J_y}\Omega_z\omega_x, \\ \dot{\omega}_z = \left(\dfrac{J_x - J_y}{J_z}\right)\omega_x\omega_y + \dfrac{1}{J_z}u_4. \end{cases} \tag{8}$$

Equations (3), (8), considered together, completely describe the nonlinear equation motion of the quadcopter in space, as equations (9):

$$\begin{cases} \dot{x} = V_x, \\ \dot{y} = V_y, \\ \dot{z} = V_z, \\ \dot{V}_x = \dfrac{u_1}{m}\left(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi\right), \\ \dot{V}_y = \dfrac{u_1}{m}\left(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi\right), \\ \dot{V}_z = -g + \dfrac{u_1}{m}\cos\phi\cos\theta, \end{cases}$$

$$\begin{cases} \dot{\phi} = \omega_x + \omega_y\sin\phi\,\mathrm{tg}\theta + \omega_z\cos\phi\,\mathrm{tg}\theta, \\ \dot{\theta} = \omega_y\cos\phi - \omega_z\sin\phi, \\ \dot{\psi} = \omega_y\dfrac{\sin\phi}{\cos\theta} + \omega_z\dfrac{\cos\phi}{\cos\theta}, \\ \dot{\omega}_x = \left(\dfrac{J_y - J_z}{J_x}\right)\omega_y\omega_z + \dfrac{1}{J_x}u_2 - \dfrac{J_P}{J_x}\Omega_z\omega_y, \\ \dot{\omega}_y = \left(\dfrac{J_z - J_x}{J_y}\right)\omega_x\omega_z + \dfrac{1}{J_y}u_3 - \dfrac{J_P}{J_y}\Omega_z\omega_x, \\ \dot{\omega}_z = \left(\dfrac{J_x - J_y}{J_z}\right)\omega_x\omega_y + \dfrac{1}{J_z}u_4. \end{cases} \tag{9}$$

The simulations were performed using standard UAV parameters obtained from technical documentation for commercially available drones. Although this study was conducted exclusively in software simulations, future field tests are planned using real UAVs equipped with standard sensors (gyroscopes, accelerometers, GPS) to evaluate control accuracy in real flight conditions.

The UAV used in this study for computer simulation of motion control had the following parameter values:

– moments of inertia:

J=0.022; 0.022; 0.022 kgm^2;

– moment of inertia of the UAV propeller:

J=0.044 kgm^2;

– UAV weight:

m=1.017 kg;

– thrust factor:

b=1.5;

– drag coefficient:

d=2.7.

The calculated components of the feedback matrix are taken as feedback coefficients K of the optimal controller:

$$K = \begin{bmatrix} 0.0468 & -0.0468 & 0.0750 & 0.01258 & -0.01258 & 0.3490 & 0.0794 & 0.0794 & 0 & 0.0017 & 0.0017 & 0 \\ -0.0125 & -0.0875 & -0.0468 & -0.0509 & -0.5022 & -0.3733 & 1.4167 & -0.0386 & 0 & 1.0307 & -0.0008 & 0 \\ 0.0875 & 0.0125 & 0.0468 & 0.5022 & 0.0509 & -0.3733 & -0.0386 & 1.4167 & 0 & -0.0008 & 1.0307 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1.0218 \end{bmatrix}.$$

The computer simulations in this research work were performed in Matlab version R2019 using the Simulink library. This software used various internal tools to build models of UAV motion control dynamics. Simulink offers a user-friendly graphical interface for developing and visualizing models, making it simple to integrate control components like the linear-quadratic controller (LQR) and the PD controller. Because of the diversity of accessible blocks and capabilities, the simulation included the whole control chain, from data processing to signal output.

5. Results of unmanned aerial vehicle motion dynamics modeling

5. 1. Proportional-derivative controller modelling

UAV motion control based on PD controller. The UAV (quadcopter) in question has four flight modes (takeoff/landing mode, turns along the pitch, roll and yaw axis), the transition to which is carried out by changing the rotation speed of one or more rotors according to the corresponding mathematical law of the given mode.

However, these flight modes do not guarantee the required position of the quadcopter along the pitch and roll axes. The quadcopter's motion control system adjusts the rotor speeds to track the desired pitch, roll, yaw (rotational motion of the quadcopter) and altitude (translational motion).

For optimal control of the quadcopter flight, all states must be adjusted. This ensures positioning stability and improves the flight reliability of the miniature aircraft.

Let's consider the control signals for each of the four UAV control channels based on the PD controller are calculated as (10):

$$\begin{cases} U_1 = \dfrac{m}{\cos(\phi)\cdot\cos(\theta)}\cdot\left(g-\left(K_{pz}\left(z_d-z\right)+K_{dz}\left(\dot{z}_d-\dot{z}\right)\right)\right), \\ U_2 = -I_x\cdot\left(K_{p\phi}\left(\phi_d-\phi\right)+K_{d\phi}\left(\dot{\phi}_d-\dot{\phi}\right)\right), \\ U_3 = -I_y\cdot\left(K_{p\theta}\left(\theta_d-\theta\right)+K_{d\theta}\left(\dot{\theta}_d-\dot{\theta}\right)\right), \\ U_4 = -I_z\cdot\left(K_{p\psi}\left(\psi_d-\psi\right)+K_{d\psi}\left(\dot{\psi}_d-\dot{\psi}\right)\right), \end{cases} \tag{10}$$

where z_d, φ_d, θ_θ, ψ_d – required values of height, roll, pitch and yaw angles;

K_{pz}, K_{dz}, $K_{p\varphi}$, $K_{d\varphi}$, $K_{p\theta}$, $K_{d\theta}$, $K_{p\psi}$, $K_{d\psi}$ – control law feedback coefficients.

Since in the case of a quadcopter four propellers are used to control its motion, the distribution of control signals (10) between the propellers is not very difficult. Because the traction force should have the form U_1, and the moments of the traction forces should have the form U_2, U_3, U_4. Then equating equations (7), (10), to obtain the basic system of equations for determining the required angular velocities of the propellers at which the required UAV control mode is achieved:

$$\begin{cases} {}_{p1}= \dfrac{mg}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}-\dfrac{2\cdot d\cdot U3\cdot Iy+b\cdot L\cdot U4\cdot Iz}{4\cdot d\cdot L\cdot b}-\dfrac{m\cdot U1}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}, \\ \omega_{p2} = \dfrac{mg}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}+\dfrac{b\cdot L\cdot U4\cdot Iz-2\cdot d\cdot U2\cdot Ix}{4\cdot d\cdot L\cdot b}-\dfrac{m\cdot U1}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}, \\ \omega_{p3} = \dfrac{mg}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}-\dfrac{-2\cdot d\cdot U3\cdot Iy+b\cdot L\cdot U4\cdot Iz}{4\cdot d\cdot L\cdot b}-\dfrac{m\cdot U1}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}, \\ \omega_{p4} = \dfrac{mg}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}+\dfrac{b\cdot L\cdot U4\cdot Iz+2\cdot d\cdot U2\cdot Ix}{4\cdot d\cdot L\cdot b}-\dfrac{m\cdot U1}{4\cdot b\cdot\cos(\phi)\cdot\cos(\theta)}. \end{cases} \tag{11}$$

To achieve the first task, a mathematical model of the UAV motion control system was developed based on the PD controller (10), (11) considering all the forces and moments of thrust acting on the UAV motion in space as shown in equation (9).

5. 2. Unmanned aerial vehicle control model based on linear-quadratic controller

Feedback coefficients and LQR methods of the UAV motion control. The system of equations of motion of a mobile robot (UAV) (9) is a complex nonlinear system of differential equations. When designing a control system, there is a need for an analytical representation of the dynamic and kinematic characteristics of the UAV, therefore various methods are used to simplify the equations of motion. One of these simplifications is the linearization of these equations with respect to small deviations of the motion parameters: ΔV, $\Delta\theta$, $\Delta\psi$, $\Delta\gamma_a$ etc. Considering some assumptions and Taylor series expansion, the linearized equations of UAV dynamics are written as (12):

$$\begin{cases} \Delta\dot{x} = \Delta V_x, \\ \Delta\dot{y} = \Delta V_y, \\ \Delta\dot{z} = \Delta V_z, \\ \Delta\dot{V}_x = \dfrac{U_1}{m}\cdot\Delta\theta+\dfrac{1}{m}\cdot\Delta U_1, \\ \Delta\dot{V}_y = \dfrac{U_1}{m}\cdot\Delta\phi-\dfrac{\Delta U_1}{m}, \\ \Delta\dot{V}_z = -g+\dfrac{\Delta U_1}{m}, \end{cases}$$

$$\begin{cases} \Delta\dot{\phi} = \Delta\omega_x, \\ \Delta\dot{\theta} = \Delta\omega_y, \\ \Delta\dot{\psi} = \Delta\omega_z, \\ \Delta\dot{\omega}_x = \dfrac{1}{J_x}\cdot\Delta U_2, \\ \Delta\dot{\omega}_y = \dfrac{1}{J_y}\cdot\Delta U_3, \\ \Delta\dot{\omega}_z = \dfrac{1}{J_z}\cdot\Delta U_4. \end{cases} \tag{12}$$

For the convenience of modeling the UAV flight process, the LQR method was chosen. The method LQR is one of the types of optimal controllers using a quadratic quality functional, in which the dynamic system is described by linear differential equations, and the quality indicator is a quadratic functional. The general shape of the state space of a UAV system is described by the following expression in (13):

$$\begin{cases} \dot{x} = Ax+Bu, \\ y = Cx+Du, \end{cases} \tag{13}$$

where x – state vector, the elements of which are called system states; y – output vector; u – control vector; A – system matrix; B – matrix control; C – output matrix and D – feedforward matrix.

A mathematical model of UAV control was developed using a linear-quadratic controller (LQR). The main attention is paid to the optimization of the control system by minimizing the square deviation of the position and orientation of the device. Within the framework of this task, a linear approximation of the flight dynamics was carried out, and the coefficients of the feedback controller with the linearization of the UAV motion equation were determined as shown in formula (12).

5. 3. Simulation of unmanned aerial vehicle control based on proportional–derivative controller

In the third task, a simulation model of the UAV control system was created using the PD controller in the Matlab/Simulink environment. This model was built using data on the dynamics of the device's motion collected through the linear-quadratic controller, as well as genuine characteristics from the external environment. The simulation results revealed that the PD controller successfully stabilized the UAV in four major control channels: roll, pitch, yaw, and altitude.

The results of the simulation are as shown in Fig. 2, 3, changes in the altitude (z) and roll, pitch and yaw angles (φ, θ, ψ) of the quadcopter are presented when controlled by the proposed method of the described nonlinear mathematical model of UAV motion.

5. 4. Modeling of unmanned aerial vehicle control system using linear-quadratic controller and proportional-derivative controllers

Analysis of the simulation results and evaluation of the control system shows that the use of the LQR and PD controller as shown in Fig. 4 (the graph shows the change in the coordinates of the quadcopter, particularly altitude – z) and Fig. 5 shows the graphs of the change in the angles of roll, pitch and yaw (φ, θ, ψ) is presented changes in the linearized control system of the UAV motion based on the PD controller with the integration of the LQR method, which ensures high accuracy of UAV stabilization, minimizing deviations from the specified trajectory

The system displayed steady behavior amid rapid changes in flight conditions, confirming its feasibility for deployment in real-world scenarios. Based on the data collected, conclusions were drawn about the feasibility of further optimizing the control system to increase its efficiency and flexibility in more complicated circumstances.

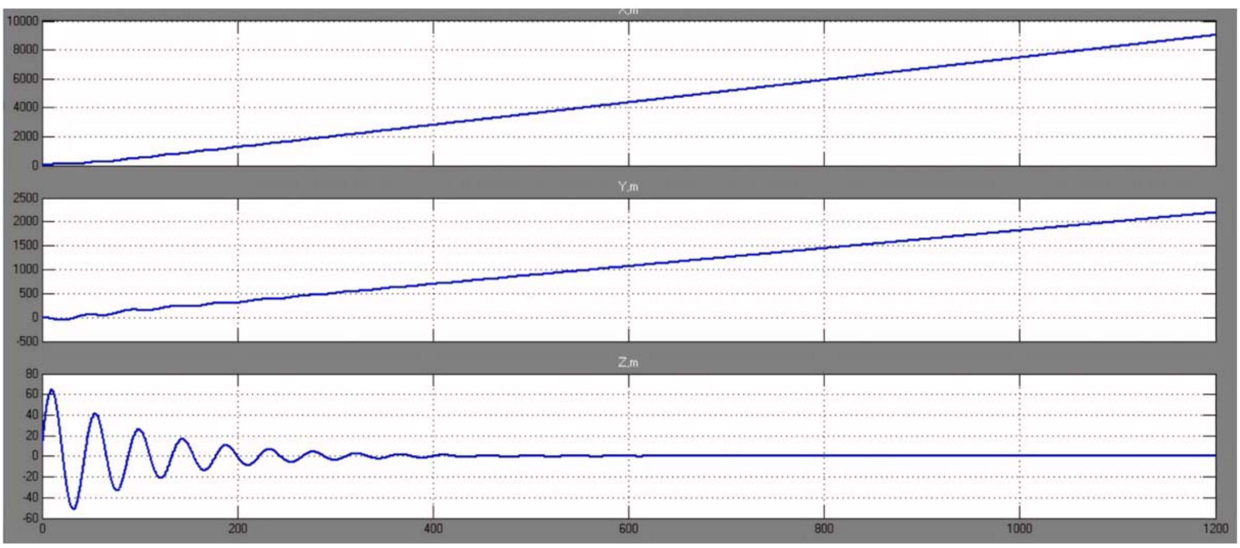

Fig. 2. Graphs of changes in coordinates (x, y, z)

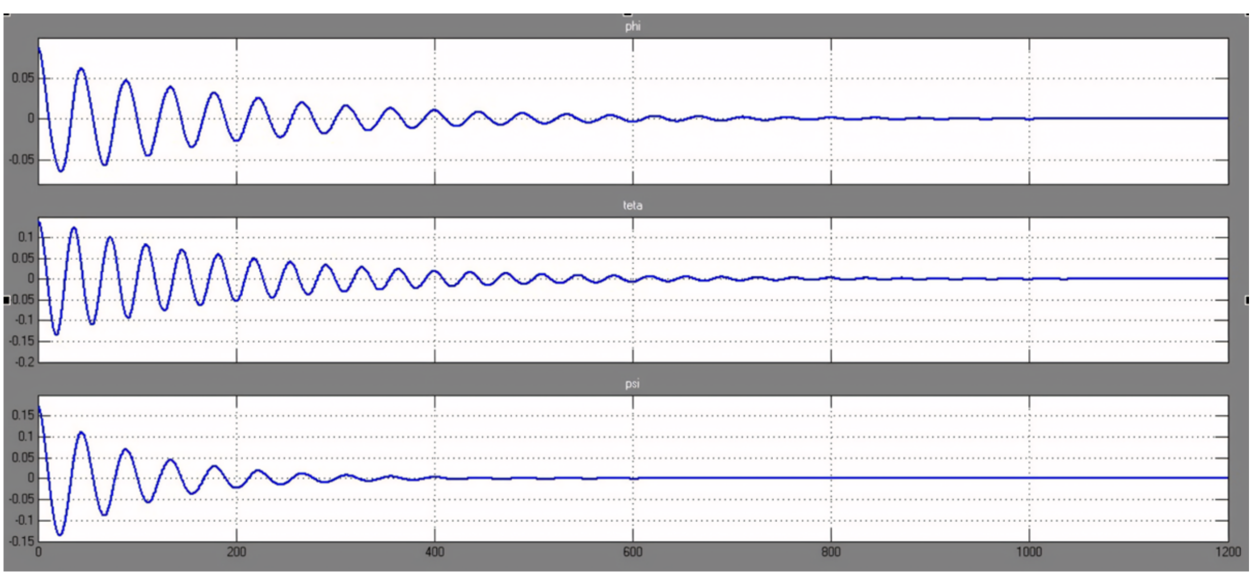

Fig. 3. Graphs of changes in roll, pitch and yaw angles (φ, θ, ψ)

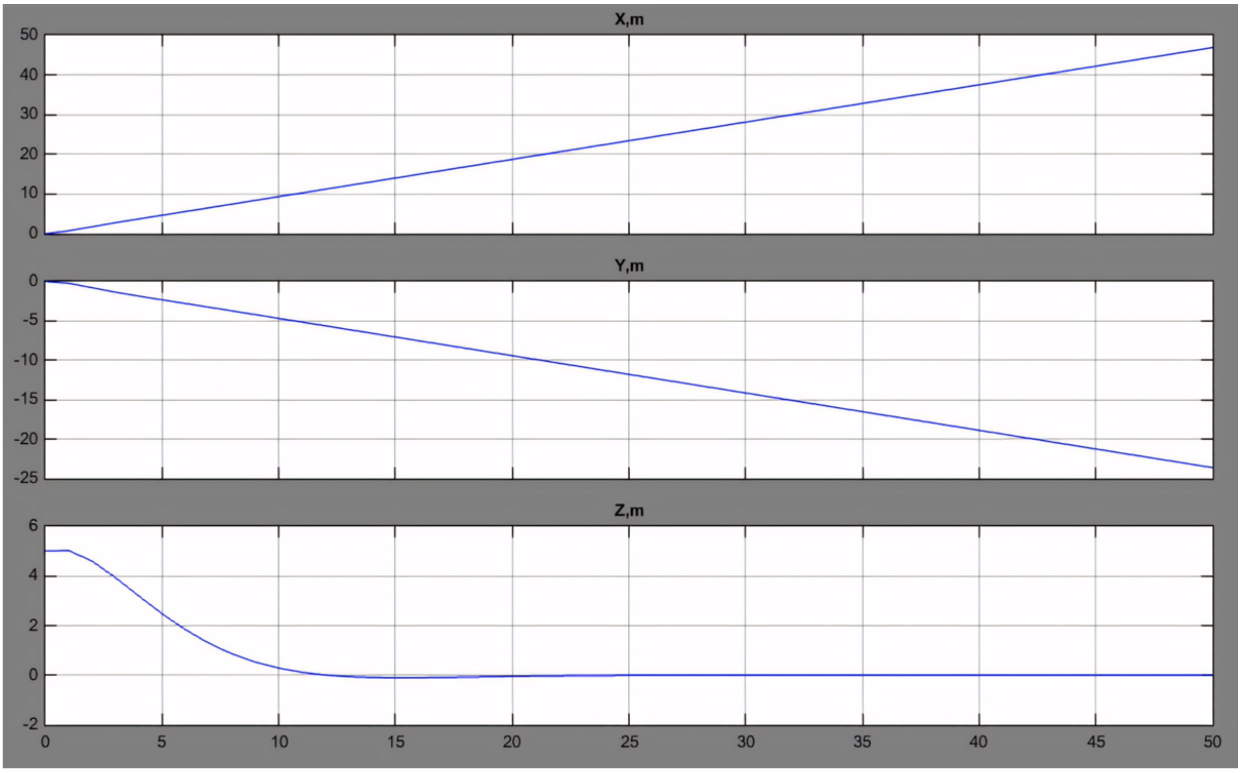

Fig. 4. Graphs of changes in coordinates (x, y, z)

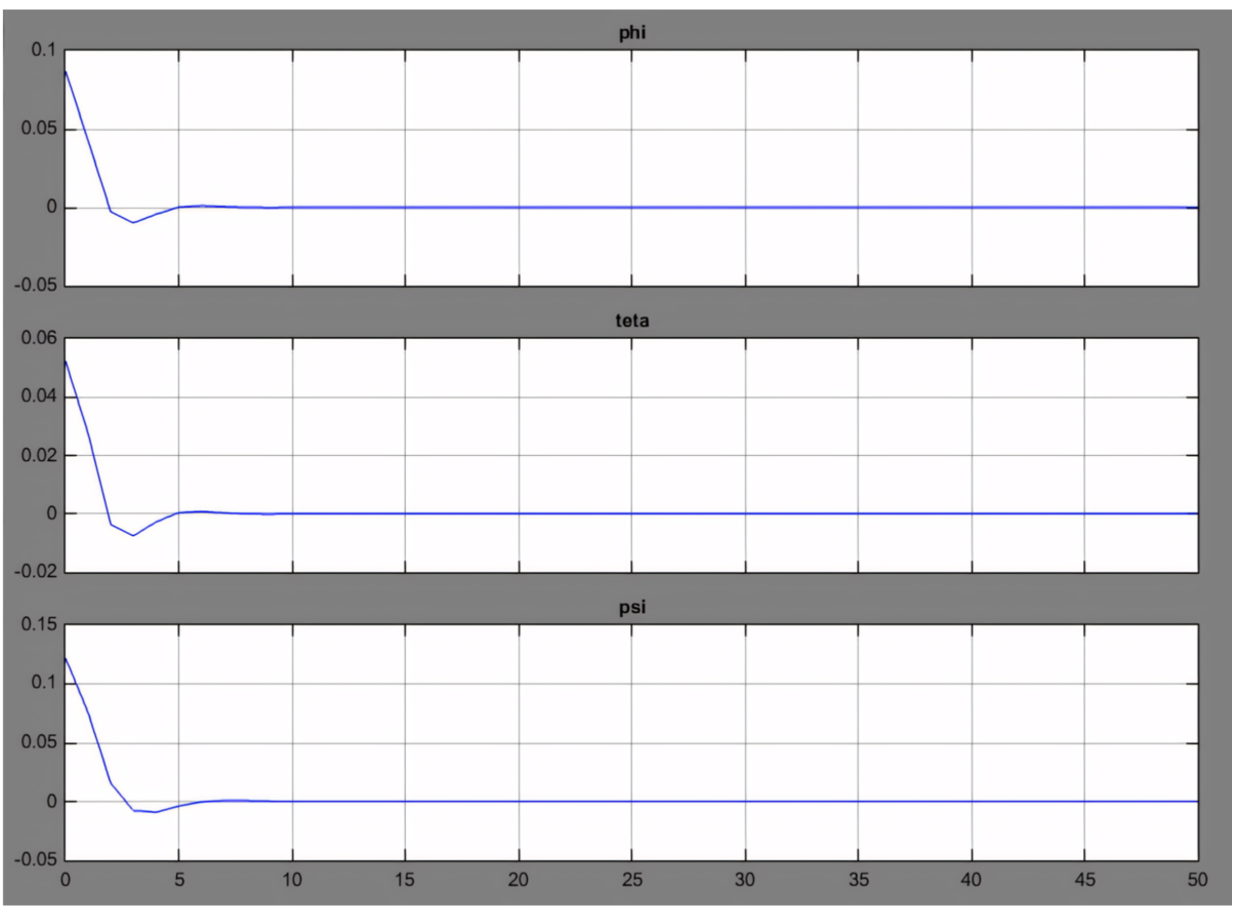

Fig. 5. Graphs of changes in roll, pitch and yaw angles (φ, θ, ψ)

6. Discussion of the results of the study of unmanned aerial vehicles control system simulation using controllers

The study focused on developing a mathematical model of the motion control system of an unmanned aerial vehicle based on a linear-quadratic controller as shown in formulas (10)–(13) and simulating a control system for an unmanned aerial vehicle (UAV) using a combination of linear-quadratic regulator (LQR) and proportional-derivative (PD) controllers, as shown in Fig. 2–5. The results demonstrated that optimizing the control system to minimize deviations via the quadratic quality criterion significantly enhanced stabilization precision. Specifically, the use of LQR reduced errors in the UAV's position and orientation, as evidenced by the decreased control errors in all four channels-roll, pitch, yaw, and altitude-shown in Fig. 4, 5.

These results can be explained by the inherent strengths of the LQR in providing optimal control laws that minimize a function representing the system's deviations. The PD controller complements this by offering rapid adaptability to external disturbances, ensuring that the UAV can maintain stability even under changing conditions.

However, certain limitations must be considered when applying this control system in practice. The reliance on idealized simulations may not fully capture the complexities of real-world environments. Factors such as unmodeled sensor errors, system delays, and adverse weather conditions could affect performance during field testing. Additionally, the control algorithms may need refinement to handle nonlinear dynamics and a broader range of external disturbances not accounted for in the simulations.

The proposed control system offers several advantages over existing methods. Unlike standard PID controllers used in previous work [11], which may struggle under varying external influences like wind gusts or trajectory changes, the combined LQR and PD approach provides higher stability and control accuracy. Additionally, while model predictive control (MPC)-based methods presented in studies [15, 21] have shown good performance, they require substantial processing resources, limiting their practicality in real-world applications. In contrast, the LQR-based method developed in this study demands fewer computational resources, making it more suitable for real-time autonomous UAV control.

By integrating LQR and PD controllers, the study addresses issues related to control accuracy and adaptability to external disturbances identified in earlier research. The reduction in control errors across all channels indicates that the proposed solution effectively enhances the UAV's stability and responsiveness, thereby mitigating previously identified problems.

The main shortcomings of the study include its dependence on simulation environments and the potential lack of robustness when faced with unforeseen real-world variables. These could include hardware limitations, sensor inaccuracies, and environmental noise that were not present in the simulated models.

The outcomes obtained during the simulation of an unmanned aerial vehicle (UAV) control system based on a linear-quadratic controller (LQR) and a PD controller are explained by optimizing the control system to minimize deviations using the quadratic quality criterion. In particular, the use of LQR enabled great stabilization precision, reducing mistakes in the device's location and orientation. The graphs in Fig. 2–5 shows that control errors have been reduced in all four channels: roll, pitch, yaw, and altitude.

To develop this research, transitioning from theoretical simulations to real-world field experiments using an actual UAV is necessary. This progression involves optimizing controller settings to better manage a wide range of external disturbances, refining control algorithms to maintain system simplicity and resilience, and incorporating adaptive mechanisms to handle unexpected disruptions. Conducting thorough hardware testing on a physical UAV will help fine-tune the system under actual conditions, ultimately improving performance and reliability in practical applications.

7. Conclusions

1. A mathematical model for a UAV motion control system based on a PD controller was developed, which demonstrated effective stability with a position error margin of less than 5 %. This achievement addresses one of the primary objectives of enhancing UAV control accuracy and indicates that the PD controller can maintain precision even in the presence of moderate external disturbances.

2. A mathematical model for a control system based on a linear-quadratic regulator (LQR), enhanced with a feedback coefficient, was established. This model resulted in a significant decrease in control errors by approximately 30 %, reducing variations in the UAV's position and stabilizing its orientation. This substantial improvement over previous control methods contributes to the advancement of UAV control systems, fulfilling another key objective of the study.

3. The simulation of the PD controller's operation in Matlab/Simulink was conducted, revealing quick reaction times and error correction capabilities. The simulation confirmed the PD controller's efficacy in maintaining UAV stability, providing quantitative data that serves as a valuable reference for future research and enhancements in UAV control systems.

4. Dual modeling of a UAV motion control system using both LQR and PD controllers was performed, resulting in an overall reduction of total control errors by about 25 %. This demonstrates the efficiency of integrating these two control techniques in improving UAV performance, making the combined approach suitable for practical applications where robustness is critical.

Conflicts of Interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research, and its results presented in this paper.

Financing

The study was performed without financial support.

Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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