

The object of research is an element of equipment in a vibrating screen, which is an active working body in the form of a plate.

The key element in the system of the flush purification unit is the vibrating screen. The basic design of the sieve does not allow the flush solution to be distributed evenly over the entire working surface. The concentration of drilling fluid in the central part of the equipment leads to premature abrasive wear and failure of the working element. Therefore, the structure of the vibrating screen needs to be improved in order to extend its service life. This task has been solved through the introduction of an active element to the structure for the redistribution of the solution through additional transverse vibrations.

The use of such an active element is an important step for improving the quality and efficiency of the purification system, which could optimize production processes and reduce costs in industry.

Taking into account transverse vibrations and calculating frequency parameters could help improve the design and use the vibrating screen more productively.

An analysis of the frequencies of oscillations of the active element-plate for cleaning the flush liquid with a vibrating screen was performed and a comparison of the analytically obtained results with the simulation data using the finite element method in the COMSOL Multiphysics software was carried out. The results are the basis for designing vibrating screens, conducting experimental and industrial research, and testing the screens. Computer studies have confirmed the possibility of using the improved design of the vibrating screen. By comparing the results of the calculation and computer simulation, the error was determined to be within 5 %.

The identified patterns could make it possible to select the plate oscillation frequencies depending on the known initial parameters, which would be useful for solving similar tasks

Keywords: computer simulation, mathematical examination, purification unit, graphical images, vibrating screen, frequency

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MATHEMATICAL AND COMPUTER IDENTIFICATION OF THE CHARACTERISTICS OF OSCILLATION FREQUENCY AND DEFORMATIONS OF THE EQUIPMENT ELEMENT IN THE FLUSH PURIFICATION UNIT

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1. Introduction

In order to increase the resource of cleaning equipment, namely vibrating screens, it is necessary to conduct research

that could help in solving this task. For production, this means increasing the productivity of the drilling fluid cleaning process with vibrating screens, determining rational sizes and shapes, as well as materials that enable maximum

cleaning efficiency. This, in turn, would reduce operating costs and increase the reliability of drilling equipment.

In the context of analysis and practical measurements related to vibrating screens for washing fluid cleaning [1], there is a need to take into account the dynamics of the action of vibrating screen elements on the drilling mix. During vibratory cleaning, both the work element and the medium follow their own laws of motion, with the energy released by the work element being distributed within the machine, at the interface with the medium, and directly within the medium. In order to optimize the economy and efficiency of cleaning, the active element of the vibrating screen was analyzed, and the most suitable option was chosen [2].

During calculations of vibrating equipment for filtering flush liquid, various modeling methodologies are used, which were described in [3, 4].

In the modern production process, special attention is paid not only to improving the quality and reducing the costs of manufactured products [5, 6] but also to minimizing the costs of the equipment used. Important aspects are the simplicity of the design, reliability, and ease of maintenance of the equipment [7]; it is precisely such production challenges that the practical side of these studies must satisfy. This emphasizes the importance of continuing research and development to further improve the efficiency of production processes.

2. Literature review and problem statement

Work [7] reports the results of research on the influence of parameters such as the supply of flush solution, drive power, and the behavior of vibrating frames for different operating conditions of vibrating screens in drilling rigs. It was shown that these parameters significantly affect the efficiency and stability of the system. But the questions related to the exact determination of the optimal parameters for various operating conditions remained unresolved. It is noted in the work that the largest part of the flush solution on the vibrating screen is cleaned precisely in its central zone (approximately 60 % of the total volume). This leads to the deflection of the screen cloth downwards. The reason for such a configuration may be objective difficulties associated with a large number of variable factors, the fundamental impossibility of achieving universal parameters for all situations, the costly part of conducting experiments under real conditions, which makes relevant research impractical. Irregularity leads to a decrease in the efficiency of the sieving process and to accelerated wear of the sieve cloths, especially in places of intensive contact with the flush liquid.

In works [8, 9], the results of research on the effect of oscillatory movements of the mesh on the efficiency of the vibrating screen are reported. It was shown that the use of different trajectories of motion of the vibrator, such as linear, circular, and elliptical, affects the quality of screening, the speed of transportation of drilled rock and the rate of wear of the screen cloth. But the issues related to the optimization of vibrating screen operating modes remained unresolved. The authors of the cited works point to the lack of a clearly defined methodology for choosing the optimal operating modes of vibrating screens, which would take into account structural and mode parameters and enable control over their changes during operation. The reason for this may be objective difficulties, which the authors associate with the lack of systematized professional recommendations on the practical use of appro-

prate equipment, the fundamental impossibility of devising a universal methodology. An option to overcome the relevant difficulties may be the development of a method of mathematical and computer detection of the peculiarities of the vibration frequencies, which would take into account all these factors and allow optimizing the parameters of vibrating screen operation. This is the approach used in the work, in which the influence of the frequency of oscillations of the plate on the efficiency of the vibrating screen was investigated. All this gives reason to assert that it is expedient to conduct a study aimed at devising systematized recommendations for the selection of vibrating screen operating modes, which would increase the efficiency and cost-effectiveness of the well drilling process.

In paper [10], the results of studies on the natural frequencies of thin rectangular plates clamped along the contour using the finite element method (FEM) are reported. It was shown that the presence of any changes in the plates significantly affects their dynamic characteristics, such as natural frequencies and forms of oscillations. But the questions related to the optimization of the model for different types of deformations and operating conditions remained unresolved. The reason for this may be the objective difficulties associated with the complexity of modeling real conditions and the variety of possible damages, the fundamental impossibility of constructing a universal model, as well as the costly part of conducting large-scale experiments, which makes relevant research impractical for our case.

In [11], the issue of the dynamics of thin and thick plates in a wide range of frequencies is considered, which is important for designing structures with good vibration indicators. Research results are reported that demonstrate that traditional methods such as the Finite Element Method (FEM) can calculate several low frequencies. However, issues related to the choice of basis functions, which must satisfy the essential boundary conditions in the Ritz method, remained unresolved. The reason for this may be the objective difficulties associated with the need for many interpolations to determine the smallest degree of the complete polynomial basis function in each direction, which makes relevant studies impractical in many cases.

An option to overcome the difficulties is the use of computer modeling and simulation of processes to reduce costs and increase the accuracy of results. All this gives reason to assert that it is expedient to conduct a study on the further optimization of the parameters of vibrating screens using advanced methods of modeling and analysis. Based on the above review, the introduction of additional horizontal movements to the operation of the vibrating screen could be a potential solution to improve the distribution of the flush liquid, which, in turn, would increase productivity and reduce equipment wear.

The construction of computational mathematical models that would take into account these unique aspects could significantly improve the design of new vibrating screens, provide effective control over their operation, and help in the calculation of dynamic parameters that arise during operation. This approach could not only increase productivity and reduce equipment wear but would also open up new opportunities to achieve better performance and reliability.

3. The aim and objectives of the study

The purpose of our study is to identify the features of vibration frequencies of the active elements of the vibrating

screen for cleaning the drilling mud, which are implemented in the form of rectangular metal plates. This could make it possible to increase the efficiency of drilling mud cleaning in the process of washing wells.

To achieve the goal, the following tasks were solved:

- to solve the differential equation of oscillations;
- to perform computer studies of plate vibration frequencies under different modes and graphically simulate the deformation of the plate position at the specified frequencies.

4. The study materials and methods

The object of our research is an element of equipment (active working body in the form of a plate) of a vibrating screen, which is included in the flush liquid purification unit.

A plate is a thin two-dimensional body of a prismatic or cylindrical shape, in which one of the dimensions is much smaller than the others: $h \ll a$, $h \ll b$, that is, the thickness is many times smaller than the other two dimensions, and the middle surface is a plane. The plane equidistant from the upper and lower sides of the plate is called the median plane. After bending, the median plane turns into a median surface. The middle surface is the surface that divides the thickness of the plate in half.

Plates are distinguished by configuration (rectangular, triangular, round, etc.), material property (homogeneous, heterogeneous, isotropic, anisotropic, etc.), thickness (thick, thin, very thin), as well as by deformability (hard, flexible, very flexible).

The theory of plates, built on a number of assumptions accepted without evidence, has been tested by experience for many years, by conducting experimental studies. It was found that the formulas obtained for the deflection of the plates give satisfactory results in the case of their sufficient rigidity, when the ratio of the maximum deflection to the thickness of the plate does not exceed 0.2. With large distortions, significant deviations from the linear relationship between the loads and the corresponding deflections are observed. In this case, the deflections grow faster than the load, and the plate becomes stiffer than is taken into account in the analytical dependences.

At $f \ll \frac{a}{5}$, a thin plate is called rigid, at $\frac{h}{5} \leq f \leq 5h$ – flexi-

ble, and at $f > 5h$, a thin and very thin plate is called absolutely flexible (membranes). Here h is the thickness, a is the smaller size of the middle plane, f is the arrow of the deflection (the largest deflection). In a rigid plate, stresses in the middle surface are neglected. In a flexible plate, along with bending stresses, stresses in the middle surface (membrane stresses) are taken into account. In the membrane (absolutely flexible plate), only the stress in the middle surface is taken into account.

The transverse oscillations of the plates occur at lower frequencies than the oscillations of the plates in their plane. Therefore, more attention is paid to the former than to the latter. However, in some cases, when there is a time-varying plane stress or a plane deformed state, it is necessary to consider the oscillations of the plates in their plane.

Usually, in the practice of designing plates for metal structures, the recommended interval for the ratio of the sides of the plate in the plan is [0.1; 10] according to the EBPlate software from the Industrial Technical Center of Metal Structures (France). In our study, it is proposed to

distinguish short, long, very long plates depending on the aspect ratio in the plan: short – 1:2–2:1; long – more than 2:1, less than 10:1; very long – more than 10:1.

One of the main tasks in calculating the vibration characteristics of structural elements, especially rectangular plates at the design stage, is to determine the frequencies of free oscillations, which are obtained from the solution to the partial differential equation of oscillations under given boundary conditions. The mentioned biharmonic differential equation, which requires additional analysis, in the absence of chain and volume forces, is the basic equation of the theory of plate bending.

In our work, the peculiarities of the vibration frequencies of individual elements of the vibrating screen design, which affect the efficiency and quality of drilling mud cleaning, were checked and revealed. The COMSOL Multiphysics (Sweden) software package was used for computer simulation to identify the features of the oscillation frequencies, which should confirm the results of the analytical studies and allow us to determine the errors in the studies.

The structural scheme of the vibrating screen used in the purification unit of the flush liquid with an active working body is shown in Fig. 1. The vibrating screen consists of a metal form divided into individual cells by means of active elements in the form of plates (item 25). This form is mounted on elastic supports and is driven into oscillating motion by means of a mounted vibration exciter. The plates, located perpendicular to the direction of propagation of vibrations, are driven into oscillating motion, transferring the energy of the solution. The plate contributes to the cleaning of the drilling fluid, acting as an active partition.

The vibrating screen of the flush liquid purification unit with an active element consists of frame 1, on which fixed and movable supports 14 and 11 are installed, to which the movable part on which receiving hopper 2 is, is attached. The structure also contains a receiving nozzle with flange 17, a vibrating exciter with drive 6, the upper grid 20 and the lower grid 23 on which the active elements (plates) 25 are installed and the fastening nodes 26 of the active element are attached to side walls 5.

The operation of the vibrating screen in the flush liquid purification unit is carried out as follows. The drilling fluid enters the receiving pipe with flange 17, after which it falls on upper grid 20, where it is partially cleaned. Then the solution passes to lower grid 23, where owing to the active element (plates) 25, located at a close distance from each other, a better cleaning is carried out to the required parameters at the output.

As part of the study of vibration processes in vibratory cleaning installations with vertically oriented oscillations, attention is focused on the mechanism of transmission of vibration action to the drilling mix. In particular, it was identified that the main influence is carried out through the side walls and partitions of the active element, as well as through the grid. An important feature of the process is that the vibrations generated in the grid drilling mix are supported mainly by tangential stresses, while the normal stresses arise from the vibrations of the sidewalls and baffles. This phenomenon is of key importance for understanding the processes of vibration energy transfer in the system.

Actively pulling the plates of the active element into the oscillation process is an effective means of increasing the intensity of drilling fluid dispersion. This approach makes it possible to optimize the cleaning process, avoid unwanted

suction in the contact zone, which can significantly affect the effectiveness of vibration cleaning. However, it should be noted that there is a risk of insufficient cleaning of the drilling fluid in the case of a significant distance from the partitions of the active element, which is due to the damping of oscillations with distance.

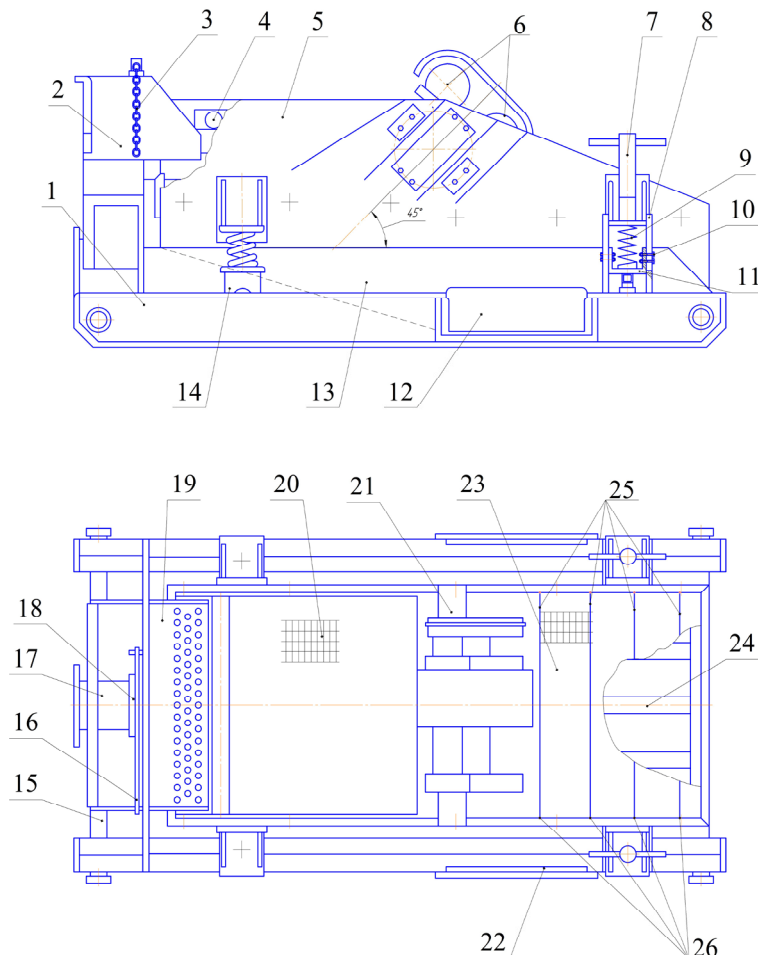


Fig. 1. The designed vibrating screen for the flush liquid purification unit with active elements: 1 – vibrating screen frame; 2 – receiving hopper; 3 – lever locking chain; 4 – spacer; 5 – side panel; 6 – vibration exciter with drive; 7 – lifting screw; 8 – guide of the movable support; 9 – spring; 10 – bolt of fixation of the movable support; 11 – movable support; 12 – damper; 13 – pallet enclosure; 14 – fixed support; 15 – pipe; 16 – weight; 17 – intake pipe with a flange; 18 – sliding door; 19 – shelf; 20 – upper grid; 21 – vibration exciter support; 22 – hole for draining drilling fluid; 23 – lower grid; 24 – stiffener of the vibrogram; 25 – active element (plate); 26 – fastening unit

For an analytical description of the oscillations of the active element, the following prerequisites are accepted:

- the active element is modeled as a uniform plate with the same thickness, which is limited by a simple geometric shape (rectangle);
- elastic deformations of the plate during oscillations are considered minimal and correspond to the laws of linear elasticity according to Hooke's principle;
- the existence of a so-called neutral layer in the structure of the plate is assumed, where the distances between points remain constant with its slight bends. In the case of a uniform plate, this layer corresponds to its median plane, which divides the thickness of the plate equally.

5. The result of investigating the active element (plate) for improving the efficiency of drilling mud cleaning

5.1. Solutions to the differential equation of oscillations

After finding out the boundary conditions on the contour of a rectangular plate, in the case of constant values of cylindrical stiffness and plate thickness, the Sophie Germain equation was represented in the form:

$$D\Delta\Delta\omega + \rho h\omega'' = q(x, y, t), \quad (1)$$

where D – cylindrical stiffness of the plate;

$$\Delta = \nabla^2 = \partial^2 / (\partial x^2) + \partial^2 / (\partial y^2),$$

$$\Delta\Delta = \nabla^4 = \partial^4 / (\partial x^4) + 2\partial^4 / (\partial x^2\partial y^2) + \partial^4 / (\partial y^4)$$

– partial differentiation operators.

Equation (1) is valid as long as the deformation wavelength is significantly greater than the thickness of the plate. In the absence of load q , the following equation of free transverse vibrations of a thin rigid plate was derived [5, 12, 13]:

$$D \left(\frac{\partial^4 \omega}{\partial x^4} + \frac{\partial^4 \omega}{\partial y^4} + \frac{\partial^4 \omega}{\partial x^2\partial y^2} \right) + \rho h \frac{\partial^2 \omega}{\partial t^2} = 0. \quad (2)$$

The solution of this equation:

$$\omega(x, y, t) = W(x, y) A \cos(\omega t - \varphi). \quad (3)$$

Gives equation for eigenforms of oscillations:

$$D\Delta\Delta W - \rho h\omega^2 W = 0. \quad (4)$$

The general solution to the equation with eight constants of integration is unknown, but all the properties of the solutions, including orthogonality conditions, have been proved. Approximate solutions of such equations can be found using the Rayleigh-Ritz method. In this case, the calculations are very complicated, so for simplification, approximate methods can be used. It was easiest to solve the problem of plates with all sides simply supported for a rectangular plate. An exact solution to the problem of free oscillations of a rectangular plate is possible under Navier boundary conditions:

$$W = (\partial^2 W) / (\partial x^2) = (\partial^2 W) / (\partial y^2) = 0, \quad (5)$$

where $x=0$ and $x=a$, $y=0$ and $y=b$.

When using the equations, it can be seen that the boundary conditions of Navier were met by the following solution:

$$W_{mn}(x, y) = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \quad (6)$$

where A_{mn} is the amplitude coefficient determined from the initial conditions of the problem, m and n are integers.

After substituting (6) into (4), the formula for determining the characteristic numbers took the form:

$$k^4 = \frac{\rho h}{D} \omega^2 = \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2. \tag{7}$$

According to (7), the natural oscillation frequency of a continuous plate freely supported along the entire contour will be determined from the following formula [14]:

$$\omega_{mn} = \pi^2 \left[(m/a)^2 + (n/b)^2 \right] \sqrt{(D/\rho h)}, \tag{8}$$

where h – plate thickness;
 ρ – plate material density;
 a – plate size along the y axis;
 b – dimensions of the plate along the x axis;
 m and n are the number of half-waves along the x and y axes, respectively.

Levy boundary conditions (two parallel edges are hinged).

Conditions for hinged fixing $W = \frac{\partial^2 W}{\partial x^2} = 0$ of the edges $x=0$ and $x=a$.

There are a total of 21 combinations of simple boundary conditions (i.e., any clamped, simply supported, or free) for rectangular plates. The frequency parameters are expressed by $\omega a^2 \sqrt{\frac{\rho}{D}}$, where a is the length dimension and does not depend on Poisson's ratio, unless at least one of the edges of the plate is free. However, since it contains D , the frequencies themselves depend on it in all cases.

The deflection function can be represented as a product of beam functions, as was done in the complex solution for rectangular plates [13]:

$$W(x,y) = X(x)Y(y), \tag{9}$$

where $X(x)$ and $Y(y)$ are chosen as the principal mode shapes of beams having plate boundary conditions. Then this choice of functions exactly satisfied all boundary conditions for the plate, except for the case of the free edge, in which the shear condition is approximately met. Six possible different sets of boundary conditions along the edges $x=0$ and $x=a$ are satisfied by the following mode shapes:

a) simply supported at $x=0$ and $x=a$:

$$X(x) = \sin \frac{(m-1)\pi x}{a}, \quad (m=2, 3, 4...); \tag{10}$$

b) fixed at $x=0$ and $x=a$:

$$X(x) = \cos \gamma_1 \left(\frac{x}{a} - \frac{1}{2} \right) + \frac{\sin(\gamma_1/2)}{\sinh(\gamma_1/2)} \cosh \gamma_1 \left(\frac{x}{a} - \frac{1}{2} \right), \tag{11}$$

($m=2, 4, 6...$),

where γ_1 values are obtained as roots from:

$$\tan(\gamma_1/2) + \tanh(\gamma_1/2) = 0. \tag{12}$$

And:

$$X(x) = \sin \gamma_2 \left(\frac{x}{a} - \frac{1}{2} \right) + \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)} \cosh \gamma_2 \left(\frac{x}{a} - \frac{1}{2} \right), \tag{13}$$

($m=3, 5, 7...$),

where γ_2 values are obtained as roots from:

$$\tan(\gamma_2/2) + \tanh(\gamma_2/2) = 0; \tag{14}$$

c) free at $x=0$ and $x=a$:

$$X(x) = 1, \quad (m=0), \tag{15}$$

$$X(x) = 1 - \frac{2x}{a}, \quad (m=1), \tag{16}$$

$$X(x) = \cos \gamma_1 \left(\frac{x}{a} - \frac{1}{2} \right) + \frac{\sin(\gamma_1/2)}{\sinh(\gamma_1/2)} \cosh \gamma_1 \left(\frac{x}{a} - \frac{1}{2} \right), \tag{17}$$

($m=2, 4, 6 ...$).

And:

$$X(x) = \sin \gamma_2 \left(\frac{x}{a} - \frac{1}{2} \right) + \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)} \cosh \gamma_2 \left(\frac{x}{a} - \frac{1}{2} \right), \tag{18}$$

($m=3, 5, 7 ...$).

With γ_1 and γ_2 , as determined in equations (12) and (14);
 d) fixed at $x=0$ and free at $x=a$:

$$X(x) = \cos \frac{\gamma_3 x}{a} - \cosh \frac{\gamma_3 x}{a} + \left(\frac{\sin \gamma_3 - \sinh \gamma_3}{\cos \gamma_3 - \cosh \gamma_3} \right) \left(\sin \frac{\gamma_3 x}{a} - \sinh \frac{\gamma_3 x}{a} \right), \tag{19}$$

($m=2, 4, 6 ...$),

where:

$$\cos \gamma_3 - \cosh \gamma_3 = -1; \tag{20}$$

e) fixed at $x=0$ and simply supported at $x=a$:

$$X(x) = \sin \gamma_2 \left(\frac{x}{2a} - \frac{1}{2} \right) - \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)} \sinh \gamma_2 \left(\frac{x}{2a} - \frac{1}{2} \right), \quad (m=2,3,4, ...). \tag{21}$$

With γ_2 , as determined in equation (14);

f) free at $x=0$ and simply supported at $x=a$:

$$X(x) = 1 - \frac{x}{a}, \quad (m=1), \tag{22}$$

$$X(x) = \sin \gamma_2 \left(\frac{x}{2a} - \frac{1}{2} \right) + \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)} \sinh \gamma_2 \left(\frac{x}{2a} - \frac{1}{2} \right), \quad (m=2, 3, 4 ...), \tag{23}$$

where γ_2 is determined according to equation (14).

The functions $Y(y)$ are chosen in a similar way under the condition that $y=0$ $y=a$ by replacing x with y , a with b , m with n in equations (10) to (23). Indicators n and m are treated as the number of nodal lines lying in the x - and

y-directions, respectively, including the border in the form of nodal lines, except when the border is free.

The frequency ω is equal to:

$$\omega^2 = \frac{\pi^4 D}{a^4 \rho} \times \left[G_y^4 \left(\frac{a}{b} \right)^4 + 2 \left(\frac{a}{b} \right)^2 \left[\nu H_x H_y + (1-\nu) J_x J_y \right] \right], \quad (24)$$

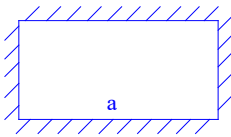
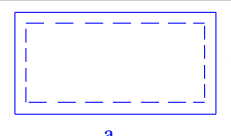
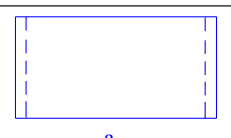
where G_x, H_x and J_x are the functions determined from Table 1 according to the conditions at $x=0$ and $x=a$. The values of G_y, H_y and J_y are determined by replacing x with y and m with n .

If another exhaustive set of solutions, which is represented by formula (25), is used to identify frequency features, the results showed almost identical convergence with the data obtained from formula (24). The components of the indicated formula are contained in Table 1. The specified comparison testifies to the reliability of the results since the difference between them is not significant. Therefore, subsequently, it was decided to compare the results of computer simulation only with the frequency determined from equation (24). The fundamental frequencies were obtained for 18 combinations of boundary conditions. To obtain the calculation formulas given in Table 1, the Rayleigh method was also used, and simple trigonometric functions that satisfied only geometric boundary conditions. The forms of modes that were used are given in Table 1 for $\nu=0.25$:

$$\omega^2 = \frac{\pi^4 D K}{a^4 \rho N}, \quad (25)$$

K and N are given in Table 1.

Frequency coefficients for the equation and various mode shapes; $\nu=0,25$

| No. | Boundary conditions | Deflection function or mode shape | N | K |
|-----|---|---|--------|--|
| 1 |  | $\left(\cos \frac{2\pi x}{a} - 1 \right) \left(\cos \frac{2\pi y}{b} - 1 \right)$ | 0.0514 | $12 + 8 \left(\frac{a}{b} \right)^2 + 12 \left(\frac{a}{b} \right)^4$ |
| 2 |  | $\sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$ | 0.25 | $0.25 + 0.50 \left(\frac{a}{b} \right)^4 + 0.25 \left(\frac{a}{b} \right)^4$ |
| 3 |  | $\sin \frac{\pi x}{a}$ | 0.50 | 0.50 |

Since equations (24) and (25) are based on the Rayleigh method, they give an upper limit to the frequency values. However, it should be noted that both sets of results have limitations in terms of accuracy. Force-type boundary conditions, as well as geometric ones, are usually satisfied, which improves the accuracy of the solution, but sometimes degrades it. The results determined from Table 1 will reduce the accuracy for higher mode shapes (increasing the values of m and n).

To calculate the frequency of oscillations for the active element, it is necessary to determine the initial data for the material and its geometric parameters from Table 2.

Table 2

Initial data for calculating oscillation frequencies

| No. | m_x | m_y | $E, \text{N/m}^2$ | ν | h, m | $D, \text{N}\cdot\text{m}$ | b, m | a, m | $g, \text{m/s}^2$ | $\rho, \text{kg/m}^3$ |
|-----|-------|-------|-------------------|-------|---------------|----------------------------|---------------|---------------|-------------------|-----------------------|
| 1 | 2 | 0 | | | | | | | | |
| 2 | 3 | 0 | | | | | | | | |
| 3 | 4 | 0 | | | | | | | | |
| 4 | - | - | | | | | | | | |
| 5 | 5 | 0 | | | | | | | | |
| 6 | 6 | 0 | | | | | | | | |
| 7 | 7 | 0 | | | | | | | | |
| 8 | - | - | | | | | | | | |
| 9 | 8 | 0 | | | | | | | | |
| 10 | 9 | 0 | 2.1E+11 | 0.3 | 0.002 | 153.8462 | 0.015 | 1.3 | 9.81 | 7,850 |
| 11 | 10 | 0 | | | | | | | | |
| 12 | - | - | | | | | | | | |
| 13 | - | - | | | | | | | | |
| 14 | 11 | 0 | | | | | | | | |
| 15 | 12 | 0 | | | | | | | | |
| 16 | 13 | 0 | | | | | | | | |
| 17 | - | - | | | | | | | | |
| 18 | - | - | | | | | | | | |
| 19 | 14 | 0 | | | | | | | | |

The initial data for determining vibration frequencies correspond to the operating conditions and geometric parameters of the vibrating screen for the most common steel grade. The selected parameters of boundary conditions and initial data satisfy the conditions of the given solution.

The solutions to the defining differential equation presented in the chapter are classical but they do not make it possible to establish new forms of oscillations, characteristic of the behavior of the working element of the vibrating screen when the frequency approaches the operational parameters. The specified solutions will make it possible to specify the operating frequency, in order to avoid unnecessary fluctuations in future computer studies.

5. 2. Computer studies of the position of the plate at different frequencies of oscillation and comparison with analytical studies

Computer studies of vibration frequencies and deformation of the plate were carried out in the COMSOL Multiphysics software. The study of the frequency of oscillations of the plate was carried out under the initial conditions indicated in Table 2.

The results of the designed active element (in the form of a plate) for the vibrating screen in the flush liquid purification unit, its deformation in the initial position, Fig. 2, at the frequency: 2.7762 Hz, 25.043 Hz are shown in Fig. 3, 4.

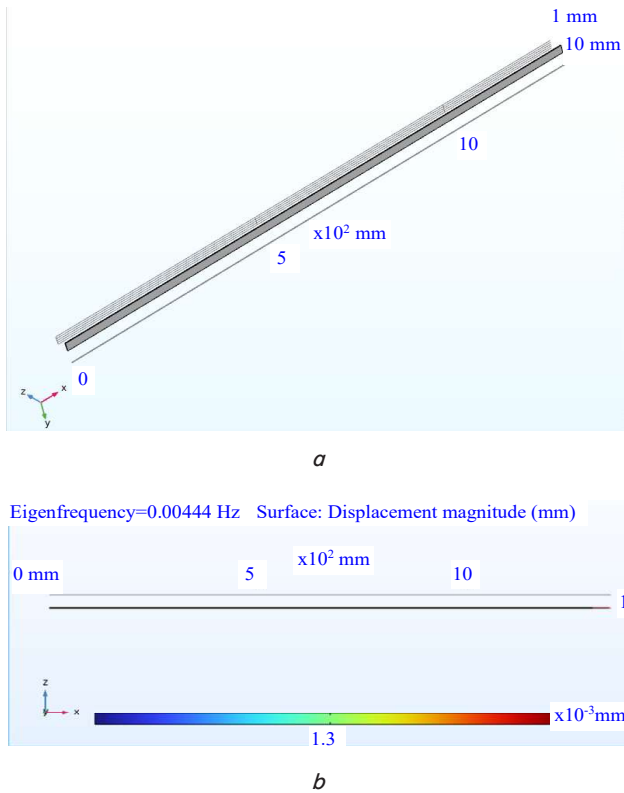


Fig. 2. The designed active element (in the form of a plate) for the vibrating screen in the flush liquid purification unit in the initial position, its deformation at zero frequency: *a* – axonometry in the plan; *b* – view along the *y* axis

The results of calculation of oscillation frequencies for the active element (in the form of a plate) for the vibrating screen in the flush liquid purification unit with

comparisons of analytical results with simulation data are given in Table 3.

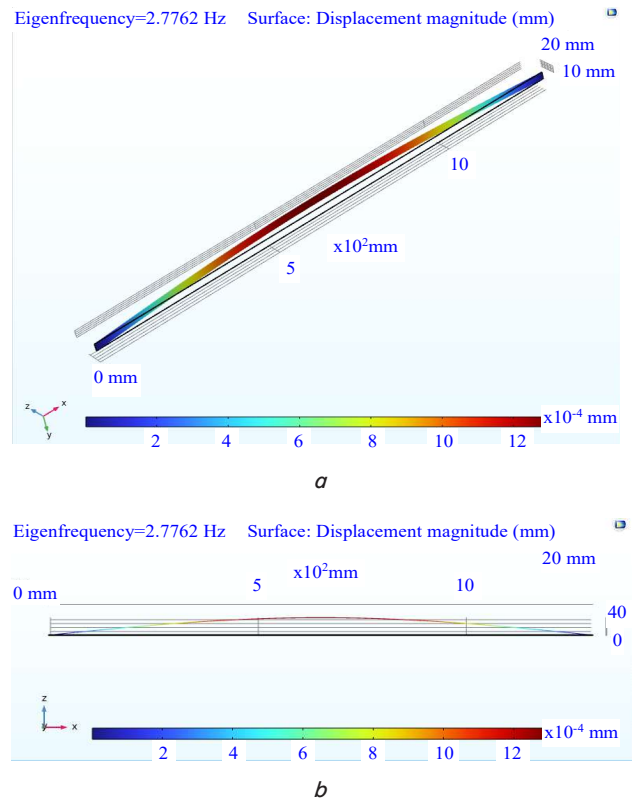


Fig. 3. The designed active element (in the form of a plate) for the vibrating screen in the flush liquid purification unit, with a purity of 2.7762 and its deformation: *a* – axonometry in the plan; *b* – view along the *y* axis

Table 3

Calculation of oscillation frequencies for the active element (in the form of a plate) for the vibrating screen in the flush purification unit

| No. | G_x | G_y | H_x | H_y | J_x | J_y | ω, s^{-1}, Hz | f, s^{-1}, Hz^* | Mode No. | f, s^{-1}, Hz^{**} | Error, % |
|-----|-------|-------|-------|-------|-------|-------|----------------------|-------------------|----------|----------------------|----------------------|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 18.3 | 2.910 | 1 | 2.776 | 4.691 |
| 2 | 2 | 0 | 4 | 0 | 4 | 0 | 73.1 | 11.64 | 2 | 11.11 | 4.617 |
| 3 | 3 | 0 | 9 | 0 | 9 | 0 | 164.5 | 26.19 | 3 | 25.04 | 4.462 |
| 4 | – | – | – | – | – | – | – | – | 4 | 27.75 | 1 sine in the plane |
| 5 | 4 | 0 | 16 | 0 | 16 | 0 | 292.5 | 46.553 | 5 | 44.58 | 4.325 |
| 6 | 5 | 0 | 25 | 0 | 25 | 0 | 457.0 | 72.74 | 6 | 69.79 | 4.128 |
| 7 | 6 | 0 | 36 | 0 | 36 | 0 | 658.1 | 104.7 | 7 | 100.84 | 3.798 |
| 8 | – | – | – | – | – | – | – | – | 8 | 110.9 | 2 sines in a plane |
| 9 | 7 | 0 | 49 | 0 | 49 | 0 | 895.8 | 142.6 | 9 | 137.6 | 3.568 |
| 10 | 8 | 0 | 64 | 0 | 64 | 0 | 1170.0 | 186.2 | 10 | 180.3 | 3.231 |
| 11 | 9 | 0 | 81 | 0 | 81 | 0 | 1480.8 | 235.7 | 11 | 229.2 | 2.785 |
| 12 | – | – | – | – | – | – | – | – | 12 | 243.5 | 1 sine in thickness |
| 13 | – | – | – | – | – | – | – | – | 13 | 249.1 | 3 sines in a plane |
| 14 | 10 | 0 | 100 | 0 | 100 | 0 | 1828.1 | 290.9 | 14 | 284.3 | 2.317 |
| 15 | 11 | 0 | 121 | 0 | 121 | 0 | 2212.0 | 352.1 | 15 | 346.01 | 1.732 |
| 16 | 12 | 0 | 144 | 0 | 144 | 0 | 2632.5 | 418.9 | 16 | 413.4 | 1.335 |
| 17 | – | – | – | – | – | – | – | – | 17 | 441.7 | 4 sines in a plane |
| 18 | – | – | – | – | – | – | – | – | 18 | 487.5 | 2 sines in thickness |
| 19 | 13 | 0 | 169 | 0 | 169 | 0 | 3089.5 | 491.7 | 19 | 489 | 0.554 |

Note: * – analytical frequency calculations according to formula (24); ** – frequency obtained as a result of finite element simulation.

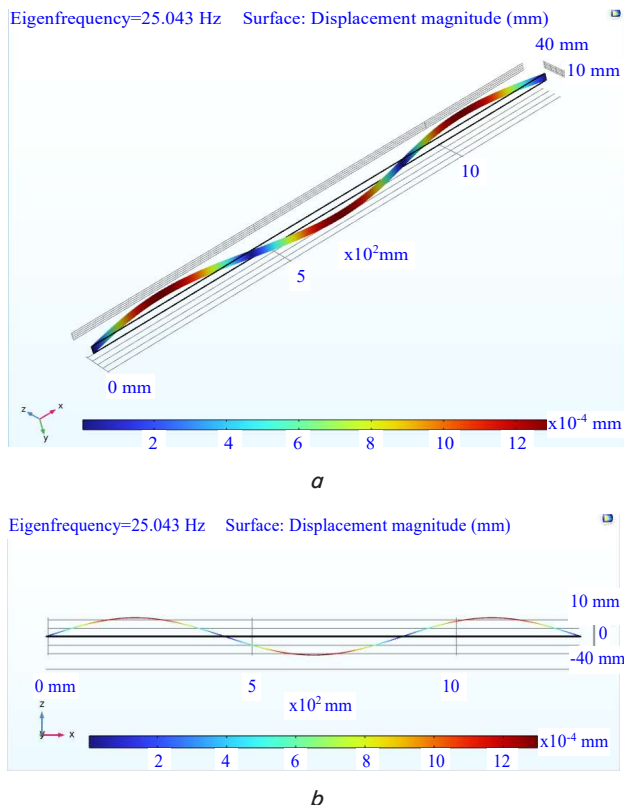


Fig. 4. The designed active element (in the form of a plate) for the vibrating screen in the flush liquid purification unit, with a purity of 25.043 and its deformation: *a* – axonometry in the plan; *b* – view along the *y* axis

The study of graphic models (Fig. 2–4) made it possible to observe different types of deformations at different frequencies of plate oscillations, namely, at some frequencies, sinuses (half-wave sinusoids) in the thickness and sinuses in the plane are detected. This has made it possible to understand the behavior of an active working body at different oscillation frequencies.

6. Discussion of results of investigating the improved vibrating screen for drilling wells with active elements

In this work, the concept of an active element represented in the form of a rectangular metal plate was introduced, which has the following boundary conditions that impose limitations on the research. The two shorter opposite sides of the plate are assumed hinged, the two longer sides are free. The plate is fixed over the mesh of the vibrating screen and interacts with the flush liquid. The adopted fixings of the plate correspond to the real operating conditions of the vibrating screen.

With the specified fixings, the convergence of the results is satisfactory, but when the boundary conditions are changed, a discrepancy can be observed, which can be investigated using software modeling according to the methodology given in the paper.

In the work, the differential equation was solved by classical methods, which would make it possible to clarify the operating frequency, in order to avoid unnecessary fluctuations during computer studies. We have calculated frequencies using formula (25) and checked with data obtained

according to formula (24), the results of which were almost identical and listed in Table 3. But they do not allow us to establish new forms of oscillations, characteristic of the behavior of the working element in the vibrating screen when the frequency approaches the operating parameters.

In the process of conducting computer studies of the active element (in the form of a plate) for the vibrating screen in the flush liquid purification unit at different frequencies of its operation, graphic models were acquired. The data of the position of the plate at different frequencies of oscillations are shown in Fig. 2–4. Also, during the study of the oscillations of the plate at different frequencies, frequency manifestations were observed in different directions (namely: sinuses in the plane and sinuses along the thickness of the plate), which are listed in Table 3. The error was calculated from the comparison of the results of the analytical calculation of Table 3, column 9, and the results of computer simulation, Table 3 column 11; the error in general does not exceed 5 %.

The research was carried out by analogy with the performed experiment [2], in which elastic plates were also placed, located across the spread of vibrations of the vibration exciter on the vibrating screen.

The built graphic models in Fig. 2–4 make it possible to state that when conducting similar studies, it will be possible to predict the frequency of oscillations of the plate and the position of its points in the coordinate axes when applying these types of mathematical calculations and similar computer simulation. All of this will enable future researchers to obtain correct results when changing the parameters of the researched object, namely very long plates with an aspect ratio of more than 10:1 as there was no procedure for calculating oscillation frequencies for this type of plate.

In the process of using the modernized vibrating screen of the flush liquid purification unit with active elements, it is important to take into account the real limits and conditions of use, namely taking into account its technical characteristics (Table 2).

Further research should focus on the construction of graphic relationships between the ratio of the sides of the plates to the frequencies of oscillations that occur in it. These graphic dependences could make it possible to select plate oscillation frequencies depending on the known parameters of the plates. The results will be the basis for conducting experimental and industrial research, as well as testing plates with different aspect ratios, in particular for adaptation to different structures of vibrating screens. Consideration of the frequency of natural oscillations of the investigated vibrating screen plate is an important scientific task for optimization of the work process, cleaning of drilling fluid.

7. Conclusions

1. Classical solutions to differential equations were derived for the working element in the vibrating screen, which allowed us to identify the operating frequencies characteristic of this system when the frequency approaches the operating parameters under the specified boundary conditions. The peculiarity of the behavior of the working element revealed during the solution of the equations emphasizes the need to specify the working frequency. This is an important step in preventing unnecessary oscillations in future computational studies and optimizing the parameters of the vibrating screen to improve its efficiency and reliability. Peculiarities and dis-

tinguishing features of the result are associated with accurate determination of optimal parameters for various operating conditions. Using the differential equations solved in this work, more accurate data were obtained. A larger number of boundary conditions and initial data affecting the reliability of the obtained results were also taken into account.

Thus, the theoretical results of the derived solutions to differential equations contribute to more accurate modeling and improvement of operational characteristics of the vibrating screen, which is essential for increasing productivity and reducing equipment wear.

2. Our computer studies of plate vibrations and deformations have confirmed the results of analytical studies. An error within 5% was found. Analysis of the results revealed that at different vibration frequencies (27.746 Hz, 110.87 Hz, 243.49 Hz, 249.05 Hz, 441.7 Hz, 487.46 Hz) of the plate, various types of deformations are observed, including sinusoidal deformations in thickness and in the plane. These data allow a better understanding of the behavior of an active working body when the frequency of oscillations changes, which is important for optimizing its work and increasing efficiency.

Our results of computer studies of plate oscillation frequencies confirm the correctness of the performed mathematical calculations and computer simulation.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

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