This paper considers a heat conduction process for isotropic layered media with internal thermal heating. As a result of the heterogeneity of environments, significant temperature gradients arise as a result of the thermal load. In order to establish the temperature regimes for the effective operation of electronic devices, linear and non-linear mathematical models for determining the temperature field have been constructed, which would make it possible to further analyze the temperature regimes in these heat-active environments. The coefficient of thermal conductivity for the above structures was represented as a whole using asymmetric unit functions. As a result, the conditions of ideal thermal contact were automatically fulfilled on the surfaces of the conjugation of the layers. This leads to solving one heat conduction equation with discontinuous and singular coefficients and boundary conditions at the boundary surfaces of the medium. For linearization of nonlinear boundary value problems, linearizing functions were introduced. Analytical solutions to both linear and nonlinear boundary value problems were derived in a closed form. For heat-sensitive environments, as an example, the linear dependence of the coefficient of thermal conductivity of structural materials on temperature, which is often observed when solving many practical problems, was chosen. As a result, analytical relations for determining the temperature distribution in these environments were obtained. Based on this, a numerical experiment was performed, and it was geometrically represented depending on the spatial coordinates. This proves that the constructed linear and nonlinear mathematical models testify to their adequacy to the real physical process. They make it possible to analyze heat-active media regarding their thermal resistance. As a result, it becomes possible to increase it and protect it from overheating, which can cause the destruction of not only individual nodes and their elements but also the entire structure

***Keywords:** temperature field, thermal conductivity of material, thermal resistance of structures, thermosensitive material, thermally active surface*

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CONSTRUCTION OF MATHEMATICAL MODELS OF THERMAL CONDUCTIVITY FOR MODERN ELECTRONIC DEVICES WITH ELEMENTS OF A LAYERED STRUCTURE

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1. Introduction

Modern society is characterized by a high level of use of electronic devices of modern technology for various needs. They are operated under certain temperature regimes, which require ensuring reliable operation, reducing mass and dimensions (miniaturization), and increasing the service life. The requirement for the compactness of devices and their elements predetermines the concentration of high power of heat generation. At the same time, the constant operation of microelectronic devices is affected by both internal and external heat flows. Therefore, the task of determining temperature fields and analyzing temperature regimes in individual elements and nodes of these devices remains relevant since temperature is an important factor that significantly affects their effective operation.

Individual elements and nodes of microelectronic devices function in a wide range of temperatures due to the action of high thermal loads. In the process of their design and operation, a number of complex engineering problems arise, for the solution of which it is necessary to have reliable information about their thermal state and temperature regimes. Since experimental studies are often impossible to conduct due to high temperatures and hermetic properties of heat removal systems, such information can only be obtained by calculation. This method, at the same time, requires the solution to complex boundary value problems of thermal conductivity obtained on the basis of mathematical models, which maximally reflect the most important features of thermophysical processes manifested in the above structures.

Composite materials, the development of which is one of the leading problems of modern materials science, are of particular importance for the production of modern technology devices. The emergence of new composite materials with improved operational physical and mechanical properties contributes to the creation of new technologies in aviation, space, shipbuilding, energy, electronic industry, mechanical engineering, and transport. Layered structures occupy an important place among composite materials. They are widely used in the construction of microelectronic devices, in particular, in built-in sensors for temperature and humidity control, light-emitting elements for dynamic LED illumination, in smartphones, etc.

In particular, smartphone manufacturers are focusing on increasing processor speed and battery life, creating wireless charging sources, improving monitor performance, and equipping them with various functions. Large companies are introducing wireless charging of batteries, abandoning cables, and offering charging using various stations. In the future, frameless smartphones may appear, owing to which it is possible to increase the size of the screen and the completeness of the representation of information. There is a trend to transform gadgets that currently serve as a camera, payment device, etc., into a source of medical information acquisition.

The design and development of such devices, the individual elements and nodes of which have a layered and homogeneous structure, consists not only in expanding the possibilities and improving their parameters. These devices often function under conditions of intense heating or cooling, which requires ensuring their stable operation, high reliability, and heat resistance. It also enables high reliability and heat resistance during operation. Increasing the power of such devices and their integration into the system greatly complicates the problem of thermal resistance to thermal loads of their structures, which partially or completely fail due to thermal overloads.

Such structures work in a wide range of temperatures. Their high operating parameters require the consideration and solution of nonlinear boundary value problems. This is due to the dependence of the thermophysical parameters of the materials on the temperature and conditions of heat exchange on the surfaces. Calculations of temperature fields based on linear heat conduction models do not always give satisfactory results. Therefore, in order to build adequate mathematical models for a real physical process, it is necessary to take into account the dependence of thermophysical parameters on temperature, the density of surface flows, and the intensity of internal heat sources, as well as changes in the shape of the environment. In addition, phase and structural transformations of media are important.

Fig. 1 shows a diagram of the influence of various external factors on the reliability of microelectronic devices [1].

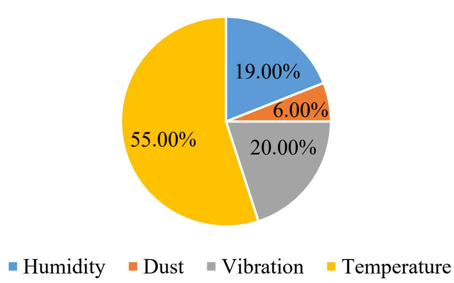

Fig. 1. Diagram of the influence of various external factors on the reliability of microelectronic devices: 1 – humidity – 19.00 %; 2 – dust – 6.00 %; 3 – vibration – 20.00 %; 4 – temperature – 55.00 %

As a result, construction of mathematical models of the heat conduction process is an urgent task since as a result of the operation of modern electronic devices, they heat up. The heterogeneity of environments and the intensity of heating leads to the emergence of significant temperature gradients, which contribute to the appearance of defects, which leads to destruction. To prevent this, it is necessary to establish acceptable temperature regimes for the effective operation of the devices. Without conducting expensive experiments for media with a layered structure, our research results make it possible to achieve this goal.

2. Literature review and problem statement

Work [2] reports a methodology for effective determination and research of the thermal stress state of bodies with thin multilayer coatings based on the procedure of modeling these coatings with shells with the appropriate geometric, thermal, and thermomechanical properties of the coating. In this approach, the effect of coatings on the thermal stress state of the body-coating system is described by special generalized boundary conditions. The effectiveness of this approach is demonstrated by the analysis of test tasks. Solutions to non-classical linear and nonlinear boundary-value problems of thermoelasticity for bodies with layered thin coatings subjected to thermal load are presented. The coatings are thin and, as a result, their geometric parameters are not taken into account, which leads to an error in the results.

Study [3] proposed a method for determining the temperature fields arising in the plate, taking into account thermal radiation, temperature dependence of thermal parameters, densities of surface and volume heat sources with uneven distribution of the initial temperature. The Kirchhoff transformation, Green's function, generalized functions and linear splines were used, and heat conduction problems were reduced to solving a recurrent nonlinear algebraic equation to determine the values of the Kirchhoff variable at the spline nodes on the corresponding boundary surface. The results of the numerical analysis are given. The study was performed for a homogeneous plate, which made it possible to apply the Kirchhoff transformation to linearize the nonlinear boundary value problem. The given approach does not make it possible to conduct research for environments with a layered structure.

In paper [4], partial boundary elements are considered as a variant of the indirect method of boundary elements. Using the example of two-dimensional problems of the potential theory, the accuracy and efficiency of their use were investigated. For objects of canonical form (circle, square, rectangle, ellipse) and arbitrary polygons, the effectiveness of partial boundary elements is shown. The use of this method ensures the accuracy of the solution commensurate with the accuracy of the boundary element method. It is also an order of magnitude higher than when using the boundary element method. At the same time, the calculation time is reduced by 2–2.5 times, compared to when applying the boundary element method. The software implementation of the proposed approach was carried out using the Python programming language. The given approach has been tested for the problems of electro profiling and vertical electrical probing in a half-plane with polygonal inclusion. Recommendations for the use of partial boundary elements in geophysical practical tests are given. The study was performed for homogeneous media of the canonical form, which is a fairly simple geometric form.

An analytical-numerical approach for determining physical scalar quantities (temperature, potential, pressure) or vector quantities (components of the electromagnetic field) is given. This approach is applied in a piecewise homogeneous region of arbitrary shape with mixed boundary conditions. It also takes into account the conditions of ideal contact at the media interface [5]. Using the method of indirect boundary elements and the scheme of the time sequence of initial conditions, software was developed, computational experiments were performed to estimate the errors of discretization of boundary regions and approximation of the mathematical model. The effect of piezoelectric coefficients on the pressure distribution in composite formations was investigated. The given method is approximate and with its use there is a significant accumulation of errors in the results.

Improving the accuracy and efficiency of temperature field prediction is an important requirement for iterative calculation and online monitoring of the temperature field, especially for combined structures. The movement of the heat flow is hindered by the incomplete contact of the interface, which leads to a temperature jump. To solve this problem, paper [6] gives an equivalent thin layer temperature field model (ETTM) instead of the interface thermal contact conductivity (TCC) model. First, based on fractal geometry, a fractal model of the TCC bolted connection is obtained. Subsequently, the contact surface of the circular bolted joint is considered to be a parallel connection of several bolted thermal resistances. The TCC of the equivalent thin layer is obtained. Based on this, the ETTM of the built-in rotor structure was developed and the temperature field model of the bolted rotor was obtained. The reliability of the models was checked by comparing the given model with the results of previous studies and the results from ANSYS analysis. Numerical simulation results show that TCC is affected by several factors: contact pressure, nominal contact area on the surfaces of the fractal domain, fractal dimensionality, fractal roughness, and root mean square (RMS) height. In addition, the influence of the thickness of the thin layer model and the thermal conductivity of the material on heat transfer was investigated. Ultimately, the results indicate that the heat transfer at the interface is significantly hindered by the thermal contact resistance due to the different temperature boundary conditions. Notably, lower contact pressure increases this impedance. There is an accumulation of errors in research results. It is difficult to predict the behavior of the temperature field according to the predictive model.

Paper [7] presents the basic equations and data set of the thermal model for predicting temperature fields and heating rates when applying localized laser treatments to the Fe-C-Ni alloy. The model takes into account the transient properties of the material and the relationship between temperature and microstructure with an emphasis on the phase dependence of thermal parameters and hysteresis in the phase change. The model provides temperature fields that are consistent with experimental microstructures in the zones of laser exposure. The given model can be applied to other materials that exhibit solid-state transformations during laser processing. Thermophysical parameters are averaged, which leads to errors in the reported results.

The temperature is important for controlling the shape of the plate due to its hardening by the roller. The shape of the plate has an important effect on the efficiency of using the steel plate. Paper [8] provides a model of the temperature field that can be used to control the shape of the plate. First, the cooling mechanism during quenching was analyzed and the heat transfer coefficients of each surface were obtained. Secondly, the model of the temperature field was determined by the heat conduction equation and the uniformity of cooling in the directions of width and thickness was investigated. Third, based on the model of temperature field and uniformity of cooling, a typical plate shape control structure was developed. Finally, the temperature field model was tested; the results show that it can simulate the temperature of the steel plate. An experimental model of the temperature field is given, and for a homogeneous medium. The research results contain significant errors.

Work [9] considered a one-dimensional non-stationary thermal state in a medium with a temperature dependence of thermal conductivity. For this case, the corresponding heat conduction equation becomes nonlinear, and a weakening boundary condition is applied to the surfaces. This condition is time-dependent, which approaches a certain time-independent condition as time passes. This behavior at the surfaces of the medium occurs naturally in some physical systems. As an example, a simple model system that generates a Dirichlet condition or a convective relaxing boundary condition is proposed. Due to the dependence of the thermal conductivity coefficient on temperature, the convective state is non-linear. For the solution of the boundary value problem, a numerical approach is given, which makes it possible to discretize the heat conduction equation in time, as a result of which a sequence of two-point boundary value problems (TPBVP) is obtained. The authors used implicit discretization of time, which ensures unconditional stability of the method. If the initial condition is given, then for this case it is possible to successively solve the TPBVP and obtain approximate values of the temperature at different time levels. The finite difference method was used to solve the TPBVP. The resulting systems of nonlinear algebraic equations are solved by Newton's method. A number of simple model problems are presented that confirm the effectiveness of this approach.

Thermal modeling of electronic devices is one of the most important tools for assessing their reliability under various operating modes. In [10], a thermal model of electronic devices is presented, which is based on experimental temperature measurement data obtained by an infrared camera. These data are input to the constructed mathematical model, which is based on the method of finite differences and some known physical dependences. The model built was verified by comparing simulation data with experimental data. It can be used to study the thermal behavior of the device under various operating conditions. The temperature distribution is determined experimentally, which introduces an error into the constructed mathematical model based on the finite difference method. As a result, the results contain significant errors.

In most portable electronic devices, in addition to the temperature of several heat sources, i.e., the junction temperature, the body temperature, i.e., the skin temperature, must also be monitored to protect the user. Thus, creating a compact device-level thermal model to predict skin temperature will not only improve the efficiency of thermal design at an early stage but also help devise a model-based temperature control strategy. In paper [11] dynamic compact thermal models of two portable electronic devices, including a smartphone and a laptop, were built based on the convolution method. Under the assumption of linear time-invariant systems, the skin temperature for the two test devices can be quickly determined after the step response of each heat source is obtained. The model built is experimental and does not make it possible to determine temperature regimes for more than two portable electronic devices.

The increase in the specific power of electronic devices, due to high performance and miniaturization requirements, has prompted researchers to search for new and alternative methods of temperature control. Most electronic devices are frequently subjected to high frequency power cycles. Cooling systems must be able to manage transient thermal profiles to delay the temperature response and reduce temperature gradients within the device that can lead to thermal stresses. In the long run, this can lead to the failure of the electronic device. The integration of phase change materials (PCM) in heatsinks for electronic devices represents an interesting technical system to increase the thermal inertia of the cooling system while providing a more stable operating temperature in the electronic components. Paper [12] discusses recent research trends in this field, with special emphasis on electric batteries, power electronics, and portable device applications. In the studies, the value of the working temperature was determined experimentally. The errors contained in these values significantly affect the efficient operation of the components of electronic devices.

Much of the effort in electronics thermal management has focused on devising cooling solutions that provide steady-state operation. However, electronic devices are increasingly used in applications with time-varying workloads. These include microprocessors (especially those used in portable devices), power electronic devices, and arrays of powerful semiconductor laser diodes. Transient solutions for temperature management are becoming essential to ensure the performance and reliability of such devices. New requirements for temperature control in transient processes are defined in [13]; cooling recommendations described in the literature for such applications are given, focused on the time scales of the thermal response. Control over temperature regimes is executed experimentally, which significantly limits the establishment of optimal values of the temperature field for the effective functioning of electronic devices.

Work [14] analyzed the features of the temperature field distribution and the reaction of the heat-conducting material as a function of its grinding parameters. Grinding of rails is widely used as a technique for re-profiling the surfaces of rails in the case of wear and tear, as well as for eliminating missing damage. However, grinding can burn the surface and form a white etching layer. Taking into account the position of the rail surface, the result of the study was the construction of an analytical thermal model based on an unevenly distributed heat source for predicting the temperature field during the grinding process. The temperature as a result of the rail grinding experiment was measured using special thermocouples. At the same time, the reaction of the rail material from the point of view of surface heating and the white etching layer was analyzed in detail. The results show that at a grinding temperature of about 400 °C, WEL starts to appear on the rail surface. Remains of austenite were found on the polished surfaces of the rails, which indicates the existence of martensite as a result of the effect of a combination of thermal and mechanical interactions. In order to describe the relationship between grinding temperature, surface burnout and WEL, suitable diagrams have been constructed for use in real production. To obtain a high-quality surface of the rails during grinding, it would be advisable to build a mathematical model of the heat conduction process, which would significantly increase the accuracy of determining temperature gradients.

An analytical solution to the three-dimensional problem of thermal conductivity together with the field of temperature and heat flow is one of the important tasks that are not solved in the mechanics of solid bodies. Taking into account the temperature dependence of the thermomechanical parameters of the material complicates the task. In paper [15], the authors first reduce the nonlinear three-dimensional problem of heat conduction to the solution of the three-dimensional Laplace equation by introducing an intermediate function. Then a generalized ternary function is proposed, and a general solution to the three-dimensional Laplace equation is given. Finally, analytical solutions to three specific problems were obtained and corresponding temperature-heat flow fields were analyzed. The results show that the heat flow field of the nonlinear three-dimensional problem coincides with the results obtained by the classical method for the linear problem, and the temperature field differs. The heat flow at the flat boundary of the defect has a singularity of $r-1/2$, and its intensity is proportional to the root of the fourth power of the width of the defect. On the other hand, when it is blocked by a planar defect, its distribution is rearranged so that it flows at the same speed from all sides of the planar defect.

A functional defect leads to high temperature, as well as significant thermal stress in thermoelectric materials, which is one of the main mechanisms of reducing the reliability of thermoelectric devices. In this perspective, thermoelectric-elastic fields around an elliptical functional defect in a two-dimensional thermoelectric plate are analyzed based on the complex variable method, and the field distribution is obtained in closed form. The results of work [16] show that the temperature at the top of the defect increases with the size of the defect and can exceed the temperature at the hot end and even exceed the melting temperature of the material. In addition, both the von Mises stresses within the matrix and the defect can exceed the yield strength of many materials. These results can be used for fracture studies and reliability analysis of thermoelectric materials.

Existing methods have been improved and new approaches have been devised to construct mathematical models that make it possible to analyze heat exchange in piecewise homogeneous media [17, 18]. Planar and spatial models of heat exchange are given, in which the differential equations contain coefficients that depend on the thermophysical properties of the phases and the geometric structure. Approaches for determining analytical and analytical-numerical solutions to boundary value problems of thermal conductivity are presented in [19]. Heat exchange processes occurring in homogeneous and layered structures with foreign inclusions of canonical form were analyzed in [20, 21]. In the cited works [17–21], models that take into account local heating, the heterogeneity of environments and the thermal sensitivity of their structural materials have remained little studied. The use of classical analytical and numerical methods does not make it possible to effectively take into account the given factors for individual elements and nodes of electronic device structures. Therefore, a technique for building mathematical models of thermal conductivity, in which these factors are taken into account, is given.

Our review of the literature reveals that there is no strict, logical, theoretically grounded technique for building linear and nonlinear mathematical models of thermal conductivity for media with a layered structure. Since the components of modern digital devices are small in size and have concentrated significant thermal power, it is important to take into account the heterogeneity of these elements, the thermal sensitivity of structural materials, and the locality of thermal heating.

3. The aim and objectives of the study

The purpose of our study is to construct linear and nonlinear mathematical models of thermal conductivity for isotropic layered media that are subject to internal heating. As a result, there is an opportunity to increase the accuracy of determining temperature fields, which will further affect the effectiveness of the design methods of modern electronic devices.

To achieve this goal, the following tasks must be solved:

– to build a linear mathematical model for determining the temperature field in a layered environment with internal heating;

– to build a non-linear mathematical model for determining the temperature field in a thermosensitive (thermophysical parameters of the material depend on temperature) layered environment with internal heating;

– to build a linear mathematical model for determining the temperature field in a two-layer environment with internal heating on the surface of the conjugation of layers;

– to build a non-linear mathematical model for determining the temperature field in a thermosensitive two-layer environment with internal heating on the surface of the conjugation of layers.

4. The study materials and methods

The object of research is the heat conduction process for isotropic layered media heated by internal heat sources.

Our research hypothesis assumes that adequate mathematical models of the heat conduction process in layered media can be derived when it is possible to obtain solutions to the corresponding linear and nonlinear boundary value problems by an analytical method.

Accepted assumptions and simplifications in the research process: the media are not anisotropic, that is, the values of thermophysical parameters are constant in spatial directions. The heat conduction process is stationary since the change in the temperature field is determined only by the spatial coordinate.

The theory of generalized functions was used to construct linear and nonlinear mathematical models for determining the temperature field and analyzing temperature regimes in layered environments with internal thermal heating. This approach led to an effective representation of thermophysical parameters of materials for layered environments. This led to the solution of boundary value problems of heat conduction, which contain one differential equation with a singular right-hand side and boundary conditions on the boundary surfaces of the medium. Linearizing functions were introduced to linearize nonlinear mathematical models of thermal conductivity due to the thermal sensitivity of materials in a layered environment.

An isotropic layered plate of thickness 2δ and width $2l$ is given, with heat-insulated front surfaces $|z|=\delta$, $|x|=l$ and uniformly distributed internal heat sources with power q_0=const, which consists of n layers. The given construction is attributed to the Cartesian rectangular coordinate system (x, y, z), on the surfaces $K_0=\{(x,0,z):|x|\le l,|z|\le\delta\}$, $K_n=\{(x,y_n,z):|x|\le l,|z|\le\delta\}$ of which boundary conditions of the first kind are set. On the surfaces of the layers $K_i=\{(x,y_i,z):|x|\le l,|z|\le\delta\}$ $(i=\overline{1,n-1})$ there is an ideal thermal contact $t_i=t_{i+1}$, $\lambda_i\dfrac{dt_i}{dy}=\lambda_{i+1}\dfrac{dt_{i+1}}{dy}$ (Fig. 2). In the given structure, it is necessary to determine the temperature distribution $t(y)$ along the spatial coordinate y, which is obtained by solving the heat conduction equation:

$$\frac{d}{dy}\left[\lambda(y)\frac{dt}{dy}\right]=-q_0, \tag{1}$$

with boundary conditions:

$$t(0)=t(y_n)=t_0, \tag{2}$$

where t_0 is the specified value of the temperature on the boundary surfaces of the plate; $\lambda(y)$ is the thermal conductivity coefficient of the laminated plate.

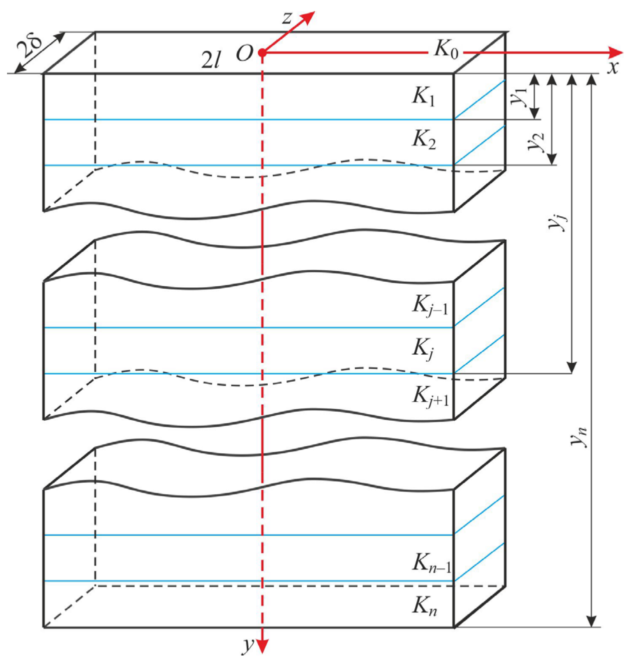

Fig. 2. Isotropic multilayer plate under the action of internal heat sources

An isotropic thermosensitive layered plate is given, in which the thermophysical parameters of the materials depend on temperature. The plate has a thickness of 2δ and a width of $2l$ with thermally insulated front surfaces $|z|=\delta$ and $|x|=l$. It has uniformly distributed internal heat sources with power q_0=const and consists of n layers. Due to thermal sensitivity, the conditions of ideal thermal contact on the surfaces of the layers K_i $(i=\overline{1,n-1})$ are set in the form $t_i=t_{i+1}$, $\lambda_i(t)\dfrac{\partial t_i}{\partial y}=\lambda_{i+1}(t)\dfrac{\partial t_{i+1}}{\partial y}$ (Fig. 1). In the given structure, it is necessary to determine the temperature distribution $t(y)$ along the spatial coordinate y, which is obtained by solving the nonlinear heat conduction equation:

$$\frac{d}{dy}\left[\lambda(y,t)\frac{dt}{dy}\right]=-q_0, \tag{3}$$

with boundary condition (2). Here, $\lambda(y, t)$ is the coefficient of thermal conductivity of the thermosensitive layered plate.

An isotropic two-layer plate with a thickness of 2δ and a width of $2l$ with heat-insulated front surfaces $|z|=\delta$, $|x|=l$ is given. It is located in the Cartesian rectangular coordinate system (x, y, z). The plate is heated by uniformly distributed heat sources on the surface of the conjugation of layers $K_0=\{(x,0,z):|x|<\infty,|z|\le\delta\}$ with power q_0=const. On this surface, there is an ideal thermal contact $t_1=t_2$, $\lambda_1\frac{dt_1(y)}{dy}=\lambda_2\frac{dt_2(y)}{dy}$ for $y=0$ (1, 2 – for the first and second layers of the plate, respectively). Boundary conditions of the first kind are set on the boundary surfaces $K_-=\{(x,-y_1,z):|x|\le l,|z|\le\delta\}$ and $K_+=\{(x,y_2,z):|x|\le l,|z|\le\delta\}$ of the plate (Fig. 3).

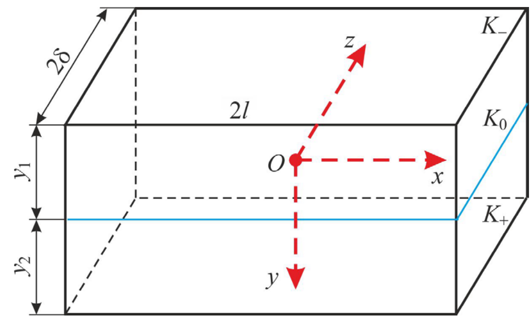

Fig. 3. Double-layer plate under the influence of internal heating

For the given structure, the temperature distribution $t(y)$ was determined by solving the thermal conductivity equation:

$$\frac{d}{dy}\left[\lambda(y)\frac{dt}{dy}\right]=-q_0\delta(y), \tag{4}$$

with boundary conditions:

$$t\big|_{y=-y_1}=t_1, t\big|_{y=-y_2}=t_2, \tag{5}$$

where $\delta(\zeta)$ is the Dirac delta function.

An isotropic thermosensitive two-layer plate is given (Fig. 2). Taking into account thermal sensitivity, the conditions of ideal thermal contact are written as $t_1(y)=t_2(y)$, $\lambda_1(t)\frac{dt_1(y)}{dy}=\lambda_2(t)\frac{dt_2(y)}{dy}$ for $y=0$ (1, 2 – for the first and second layers of the plate, respectively).

For the given structure, the temperature distribution $t(y)$ was determined by solving the nonlinear heat conduction equation:

$$\frac{d}{dy}\left[\lambda(t,y)\frac{dt}{dy}\right]=-q_0\delta(y), \tag{6}$$

with boundary conditions (5).

5. Research results related to the process of constructing mathematical models of thermal conductivity for isotropic layered media

5. 1. Linear mathematical model for determining the temperature field in a layered plate with internal heating

The coefficient of thermal conductivity of a layered plate is represented in the form:

$$\lambda(y)=\lambda_1+\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)S_+(y-y_i), \tag{7}$$

where λ_i is the coefficient of thermal conductivity of the material of the i-th layer $(i=\overline{1,n})$ of the plate; $S_\pm(\zeta)$ are asymmetric unit functions:

$$S_\pm(\zeta)=\begin{cases}1, & \zeta>0,\\ 0.5\mp0.5, & \zeta=0,\\ 0, & \zeta<0.\end{cases}$$

The following function has been introduced:

$$T(y)=\lambda(y)t(y), \tag{8}$$

and it was differentiated by the variable y taking into account expression (7) for the thermal conductivity coefficient $\lambda(y)$. As a result, the ratio was obtained:

$$\lambda(y)\frac{dt}{dy}=\frac{dT}{dy}-\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)t(y_i)\delta_+(y-y_i),$$

which made it possible to transform the original equation (1) into the following form:

$$\frac{d^2T}{dy^2}-\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)t(y_i)\delta'_+(y-y_i)=-q_0.$$

Here $\delta_+(\zeta)=dS_+(\zeta)/d\zeta$ is Dirac's asymmetric delta function, and $\delta'_+(\zeta)$ is its derivative.

The general solution to this equation was determined from the following relation:

$$T(y)=\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)t(y_i)S_+(y-y_i)-\frac{q_0}{2}y^2+c_1y+c_2, \tag{9}$$

and the temperature value on the surfaces of the conjugation of the medium layers $t(x_i)$ $(i=\overline{1,n-1})$ using this expression was found in the form:

$$t(y_1)=\frac{1}{\lambda_1}\left(c_1y_1+c_2-\frac{q_0}{2}y_1^2\right),$$

$$t(y_i)=c_1\left[\sum_{j=1}^{i-1}\left(\frac{1}{\lambda_j}-\frac{1}{\lambda_{j+1}}\right)y_j+\frac{y_i}{\lambda_i}\right]+$$

$$+\frac{c_2}{\lambda_1}-\frac{q_0}{2}\left[\sum_{j=1}^{i-1}\left(\frac{1}{\lambda_j}-\frac{1}{\lambda_{j+1}}\right)y_j^2+\frac{y_i^2}{\lambda_i}\right].$$

Taking into account boundary conditions (2) and relation (9), the constants of integration c_1 and c_2 were determined:

$$c_1=\frac{q_0}{2}\frac{\lambda_n\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_i}-\frac{1}{\lambda_{i+1}}\right)y_i^2+y_n^2}{\lambda_n\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_i}-\frac{1}{\lambda_{i+1}}\right)y_i+y_n};\ c_2=\lambda_1t_0.$$

As a result, the temperature field in a layered plate with uniformly distributed internal heat sources is fully determined by expression (9).

Analysis of numerical results. Fig. 4 shows the behavior of temperature field in the structure of a five-layer assembly of a lithium-ion battery, in which the material of the first, third, and fifth layers is aluminum ($\lambda_1=\lambda_3=\lambda_5=282$ W/(degree·m) at a temperature of 627 °C) , and the second and fourth – lithium ($\lambda_2=\lambda_4=52.9$ W/(degree· m) at a temperature of 627 °C), for the following values of the spatial coordinate y: $y_1=0.05$ m; $y_2=0.25$ m; $y_3=0.3$ m; $y_4=0.5$ m; $y_5=0.55$ m.

As can be seen from Fig. 4, the temperature reaches its highest value in the middle aluminum layer and monotonically decreases as a function of the spatial coordinate y to the value $t_0=627$ °C, given in the boundary conditions (2).

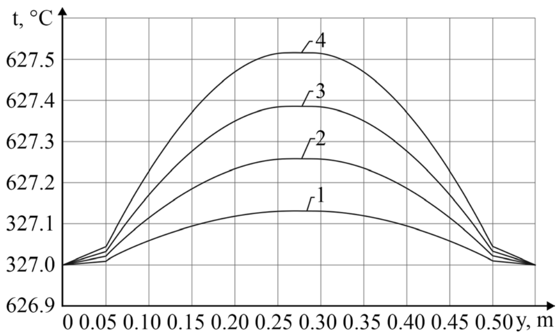

Fig. 4. Temperature distribution in the structure of the lithium-ion battery unit for different values of power q_0 of heating sources: 1 – q_0=250 W/m^3; 2 – q_0=500 W/m^3; 3 – q_0=750 W/m^3; 4 – q_0=1000 W/m^3

The presence of corner points on the curves in the area of the inner surfaces of the first and fifth aluminum layers of the assembly of lithium-ion batteries indicates a change in the temperature field. This phenomenon occurs during the transition from a solid phase state (aluminum) to liquid (lithium).

5. 2. Nonlinear mathematical model for determining the temperature field in a layered plate with internal heating

The coefficient of thermal conductivity for a thermosensitive laminated plate is given in the form:

$$\lambda(y,t)=\lambda_1(t)+\sum_{i=1}^{n-1}\left(\lambda_{i+1}(t)-\lambda_i(t)\right)S_+(y-y_i), \tag{10}$$

where $\lambda_i(t)$ is the coefficient of thermal conductivity of the i-th layer material $(i=\overline{1,n})$ of the thermosensitive plate.

A linearizing function has been introduced:

$$\vartheta(y)=\int_0^{t(y)}\lambda_1(\zeta)\mathrm{d}\zeta+$$
$$+\sum_{i=1}^{n-1}S_+(y-y_i)\int_{t(y_i)}^{t(y)}\left[\lambda_{i+1}(\zeta)-\lambda_i(\zeta)\right]\mathrm{d}\zeta, \tag{11}$$

it was differentiated by the variable y and the ratio was obtained:

$$\lambda(y,t)\frac{\partial t}{\partial y}=\frac{\partial\vartheta}{\partial y},$$

taking into account which the original nonlinear heat conduction equation (3) is transformed into an ordinary differential equation of the second order with constant coefficients relative to the function $\vartheta(y)$:

$$\frac{d^2\vartheta}{dy^2}=-q_0, \tag{12}$$

the general solution to which is defined as:

$$\vartheta(y)=-\frac{q_0}{2}y^2+c_1y+c_2.$$

The linearizing function (11) made it possible to transform the boundary conditions (2) for determining the integration constants c_1, c_2 into the following form:

$$\vartheta\big|_{y=0}=\int_0^{t_0}\lambda_1(t)\mathrm{d}t,$$
$$\vartheta\big|_{y=y_n}=\int_0^{t_0}\lambda_1(t)\mathrm{d}t+\sum_{i=1}^{n-1}\int_{t(y_i)}^{t_0}\left[\lambda_{i+1}(t)-\lambda_i(t)\right]\mathrm{d}t. \tag{13}$$

As a result, the solution to the boundary value problem (12), (13) is obtained:

$$\vartheta(y)=y\left\{\begin{aligned}&\frac{q_0}{2}(y_n-y)+\\&+\frac{1}{y_n}\sum_{i=1}^{n-1}\int_{t(y_i)}^{t_0}\left[\lambda_{i+1}(\zeta)-\lambda_i(\zeta)\mathrm{d}\zeta\right]\mathrm{d}\zeta+\\&+\int_0^{t_0}\lambda_1(\zeta)\mathrm{d}\zeta\end{aligned}\right\}. \tag{14}$$

As an example, a two-layer thermosensitive plate is given. To solve many practical tasks, the dependence of the coefficient of thermal conductivity of structural materials on temperature is used in the form:

$$\lambda=\lambda_m^0(1-k_mt), \tag{15}$$

where λ_m^0, k_m is the reference and temperature coefficients of thermal conductivity of materials for the first (m=1) and second (m=2) layers of the plate. The material of the first layer of the plate is silicon, and germanium is chosen for the second layer. By interpolation, the temperature dependence of the thermal conductivity coefficient in the temperature range [0 °C; 1127 °C] for the given materials, as partial case (15), is determined from the following ratios:

– for silicon:

$$\lambda(t)=67.9\frac{\mathrm{W}}{\mathrm{degree\cdot m}}\left(1-0.0005\frac{1}{\mathrm{degree}}t\right);$$

– for germanium:

$$\lambda(t)=60.3\frac{\mathrm{W}}{\mathrm{degree\cdot m}}\left(1-0.0008\frac{1}{\mathrm{degree}}t\right).$$

Taking into account these ratios and expressions (11), (14) to determine the temperature distribution $t(y)$ in the given structure, quadratic equations were obtained for the first $\{(x,y,z)$: $|x|<\infty$, $0\le y\le y_1$, $|z|\le\delta\}$:

$$\lambda_1^0k_1t^2-2\lambda_1^0t+\lambda_1^0t_0(2-k_1t_0)+\vartheta(y)=0, \tag{16}$$

and the second $\{(x,y,z)$: $|x|<\infty$, $y_1<y\le y_2$, $|z|\le\delta\}$:

$$\lambda_2^0k_2t^2-2\lambda_2^0t+\lambda_1^0t_0(2-k_1t_0)+$$
$$+t(y_1)\left[\lambda_2^0(2-k_2t(y_1))-\lambda_1^0(2-k_1t(y_1))\right]+\vartheta(y)=0, \tag{17}$$

layers of the plate and on the surface of their conjugation $K_1=\{(x,y_1,z)$: $|x|<\infty$, $|z|\le\delta\}$:

$$\left[\lambda_1^0k_1+\frac{y_1}{y_2}\left(\lambda_2^0k_2-\lambda_1^0k_1\right)\right]t^2(y_1)-$$
$$-2\left[\lambda_1^0+\frac{y_1}{y_2}(\lambda_2^0-\lambda_1^0)\right]t(y_1)+\vartheta(y_1)+\lambda_1^0t_0(2-k_1t_0)=0, \tag{18}$$

where

$$\vartheta(y) = y\left\{q_0(y_2 - y) + \frac{1}{y_2}\left[t_0\Lambda_1 - ((y_1))_2\right]\right\},$$

$$\vartheta(y_1) = y_1\left[q_0(y_2 - y_1) + \frac{t_0}{y_2}\Lambda_1\right];$$

$$\Lambda_1 = \lambda_2^0(2 - k_2 t_0) - \lambda_1^0(2 - k_1 t_0);$$

$$\Lambda_2 = \lambda_2^0(2 - k_2 t(y_1)) - \lambda_1^0(2 - k_1 t(y_1)).$$

Based on numerical calculations, the behavior of the temperature field $t(y)$ in a two-layer plate is depicted in Fig. 5. For constant values of the coefficient of thermal conductivity for the structural materials of the plate: $\lambda_1=67.9$ W/(degree·m), $\lambda_2=60.3$ (W/(degree·m) at a temperature of 27 °C, curve 1 is shown. For a linearly variable coefficient of thermal conductivity from the temperature, curve 2 is shown. The values of the power of the internal heat sources q_0 and the temperature t_0 on the boundary surfaces of the plate are equal to 200 W/m^3 and 100 °C. The values of the thickness of the plate layers are chosen as $y_1=0.2$ m and $y_2=0.4$ m. As can be seen from Fig. 4, the curves describing the temperature $t(y)$ as a function of the spatial coordinate y are continuous without corner points on the surface of the conjugation of layers, where the conditions of ideal thermal contact are specified.

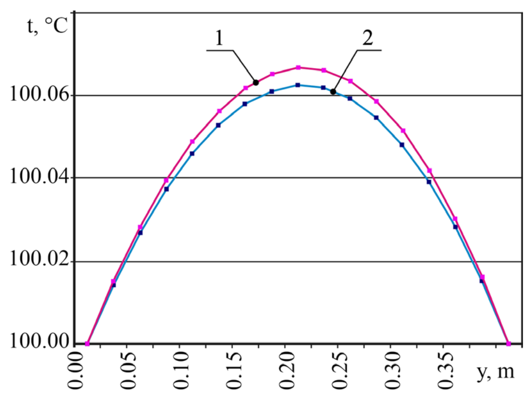

Fig. 5. Temperature distribution $t(y)$ in an isotropic two-layer plate for linear (curve 1) and nonlinear (curve 2) models

This testifies to the correctness of mathematical models, both linear and non-linear. The numerical experiment confirms that taking into account the thermal sensitivity leads to a decrease in the temperature values $t(y)$ for the given materials of the plate layers.

5. 3. Linear mathematical model for determining the temperature field in a two-layer plate with internal heating

The coefficient of thermal conductivity for a two-layer plate is represented in the form:

$$\lambda(y) = \lambda_1 + (\lambda_2 - \lambda_1)\, S_-(y), \tag{19}$$

where λ_1, λ_2 are thermal conductivity coefficients of the 1st and 2nd layers of the plate, respectively.

The function (8) was introduced, it was differentiated with respect to the variable y taking into account expression (19) for the thermal conductivity coefficient $\lambda(y)$. As a result, the ratio is obtained:

$$\lambda(y)\frac{dt}{dy} = \frac{dT}{dy} - (\lambda_2 - \lambda_1)t(0)\delta_-(y),$$

taking into account which the original equation (4) is transformed into the following form:

$$\frac{d^2T}{dy^2} = (\lambda_2 - \lambda_1)t(0)\delta'_-(y) - q_0\delta(y),$$

which is integrated and its general solution is obtained as a result:

$$T(y) = (\lambda_2 - \lambda_1)t(0)S_-(y) + y(c_1 - q_0 S(y)) + c_2.$$

The use of boundary conditions (5) made it possible to obtain a solution to problem (4), (5) in the form:

$$T(y) = \frac{(y + y_1)\left[\lambda_2 t_2 + y_2 q_0 - (\lambda_2 - \lambda_1)t(0)\right] + (y_2 - y)\lambda_1 t_1}{y_1 + y_2} -$$
$$- y q_0 S(y) + (\lambda_2 - \lambda_1)t(0)S_-(y).$$

Using this solution, the relationship for determining the temperature $t(y)$ in the first layer of the plate ($-y_1 \le y < 0$) is obtained:

$$t(y) = \frac{(y + y_1)\left[\lambda_2 t_2 + y_2 q_0 - (\lambda_2 - \lambda_1)t(0)\right] + (y_2 - y)\lambda_1 t_1}{\lambda_1(y_1 + y_2)}, \tag{20}$$

on the conjugation surface of the plate layers ($y=0$):

$$t(0) = \frac{y_1(\lambda_2 t_2 + y_2 q_0) + y_2\lambda_1 t_1}{\lambda_1 y_2 + \lambda_2 y_1}, \tag{21}$$

and in the second layer of the plate ($0 \le y \le y_2$):

$$t(y) =$$
$$= \frac{(y + y_1)\left[\lambda_2 t_2 + y_2 q_0 - (\lambda_2 - \lambda_1)t(0)\right] + (y_2 - y)\lambda_1 t_1}{\lambda_2(y_1 + y_2)} +$$
$$+ \frac{(\lambda_2 - \lambda_1)t(0) - y q_0}{\lambda_2}. \tag{22}$$

The resulting expressions (20) to (22) describe the temperature field in an isotropic two-layer plate, which is heated by internal heat sources concentrated on the surface of the layer junction.

5. 4. Nonlinear mathematical model for determining the temperature field in a two-layer plate with internal heating

The coefficient of thermal conductivity for a thermosensitive two-layer plate is given in the form:

$$\lambda(y,t) = \lambda_1(t) + \left[\lambda_2(t) - \lambda_1(t)\right]S_-(y),$$

where $\lambda_1(t)$, $\lambda_2(t)$ are thermal conductivity coefficients of the 1st and 2nd layers of the thermosensitive plate, respectively.

A linearizing function has been introduced:

$$\vartheta(y) = \int_{t_1}^{t}\lambda_1(\zeta)d\zeta + S_-(y)\int_{t_0}^{t}\left[\lambda_2(\zeta) - \lambda_1(\zeta)\right]d\zeta, \tag{23}$$

it was differentiated by the variable y, and as a result the ratio was obtained:

$$\lambda(t,y)\frac{dt(y)}{dy}=\frac{d\vartheta(y)}{dy},$$

using which equation (6) is reduced to an ordinary linear differential equation of the second order with constant coefficients and a singular right-hand side relative to the linearizing function $\vartheta(y)$:

$$\frac{d^2\vartheta}{dy^2}=-q_0\delta(y), \qquad (24)$$

with boundary conditions:

$$\vartheta\big|_{z=-z_1}=0,\ \vartheta\big|_{z=-z_2}=\int_{t_1}^{t_2}\lambda_1(t)\mathrm{d}t+\int_{t_0}^{t_2}\left[\lambda_2(t)-\lambda_1(t)\right]\mathrm{d}t. \qquad (25)$$

Equation (24) was integrated, and the general solution is obtained in the following form:

$$\vartheta(y)=\frac{y+y_1}{y_1+y_2}\left\{\begin{matrix} y_2q_0+\int_{t_1}^{t}\lambda_1(\zeta)\mathrm{d}\zeta+ \\ +S_-(z)\int_{t_0}^{t}\left[\lambda_2(\zeta)-\lambda_1(\zeta)\right]\mathrm{d}\zeta \end{matrix}\right\}-yq_0S(y), \qquad (26)$$

where $t_0=t(0)$.

The material of the first and second layer of the plate is U12 and 08 steel, respectively. In the temperature range [0 °C; 700 °C] the temperature dependence of the thermal conductivity coefficient on temperature for these materials is determined by interpolation of the relations:

$$\lambda_1(t)=47.5\frac{\mathrm{W}}{\mathrm{degree}\cdot\mathrm{m}}\left(1-0.00037\frac{1}{\mathrm{degree}}t\right),$$

$$\lambda_2(t)=64.5\frac{\mathrm{W}}{\mathrm{degree}\cdot\mathrm{m}}\left(1-0.00049\frac{1}{\mathrm{degree}}t\right). \qquad (27)$$

Taking into account expressions (22), (25), and (26) to determine the temperature distribution $t(y)$ in the given structure, quadratic equations were obtained for the first:

$$\lambda_1^0k_1t^2-2\lambda_1^0t+\Lambda_2+$$
$$+\frac{y+y_1}{y_1+y_2}\left\{2y_2q_0-\Lambda_2+\Lambda_3+t_0\Lambda_1\right\}=0, \qquad (28)$$

and the second:

$$\lambda_2^0k_2t^2-2\lambda_2^0t+\Lambda_2-t_0\Lambda_1+$$
$$+\frac{y+y_1}{y_1+y_2}\left\{2y_2q_0-\Lambda_2+\Lambda_3+t_0\Lambda_1\right\}-2yq_0=0, \qquad (29)$$

layers of the plate and on the surface of the conjugation $K_0=\left\{(x,0,z):|x|<\infty,|z|\leq\delta\right\}$:

$$\left[\lambda_1^0k_1+y_p\left(\lambda_2^0k_2-\lambda_1^0k_1\right)\right]t_0^2-2\left[\lambda_1^0+y_p\left(\lambda_2^0-\lambda_1^0\right)\right]t_0+$$
$$+\Lambda_2+y_p\left[2y_2q_0-\Lambda_2+\Lambda_3\right]=0, \qquad (30)$$

where

$$y_p=\frac{y_1}{y_1+y_2};\ \Lambda_1=\lambda_1^0\left(2-k_1t_0\right)-\lambda_2^0\left(2-k_2t_0\right),$$

$$\Lambda_2=\lambda_1^0t_1\left(2-k_1t_1\right);\ \Lambda_3=\lambda_2^0t_2\left(2-k_2t_2\right).$$

Numerical calculations of the temperature field $t(y)$ were performed for the linear model (constant coefficient of thermal conductivity of the materials of the plate layers; λ_1=38.7 W/(degree·m), λ_2=48.7 W/(degree·m)) according to the formulas (20) to (22) (Fig. 6, curve 1).

The temperature distribution for the nonlinear model is determined by solving quadratic equations (28) to (30). In this model, the coefficient of thermal conductivity of the materials of the plate layers, which varies linearly with temperature, is expressed by relations (27). The results are shown in Fig. 6 (curve 2); in this case, $y_1=y_2=1$ m.

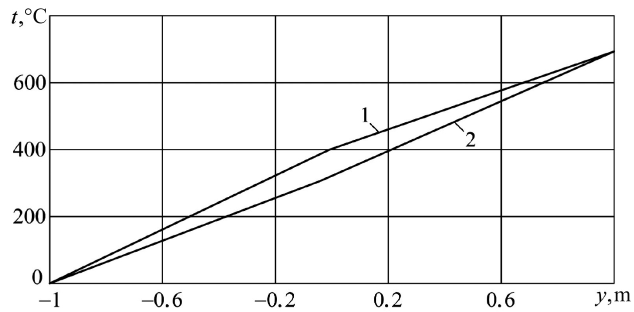

Fig. 6. Dependence of temperature t on coordinate y for constant (curve 1) and linearly variable (curve 2) coefficient of thermal conductivity of the materials of plate layers

The behavior of the curves indicates the correspondence of the mathematical model to the real physical process, since on the surfaces of the plate layer conjugation ($y=0$) we observe the fulfillment of the conditions for ideal thermal contact (there is no temperature jump). The results obtained for the selected materials based on the linear dependence of the coefficient of thermal conductivity on temperature differ from the results obtained for a constant coefficient of thermal conductivity by 15 %.

6. Discussion of research results related to the constructed mathematical models for determining temperature fields in layered environments

The boundary value problems of thermal conductivity are stated in accordance with the real physical process, which is given in the considered environments. As a result, differential equations of heat conduction and boundary conditions rigorously describe mathematical models of the stationary process of heat conduction, which correspond to the corresponding physical models. The shape of the curves in Fig. 4–6, which are constructed on the basis of determined numerical values of temperature as a function of the spatial coordinate, obtained using analytical solutions to boundary value problems, testifies to the correspondence of the results to the physical process. This is confirmed by the smoothness of the temperature function along the spatial coordinate, the presence of corner points due to the phase transition in the layered structure, and the fulfillment of the given boundary conditions on the boundary surfaces of the medium.

In our studies, the theory of generalized functions was used, which made it possible to effectively describe the stratification of environments and local temperature disturbances, as a result of which the obtained differential equations contain singular right-hand parts. For the linearization of nonlinear boundary value problems (2), (3), and (5), (6), linearizing functions (11) and (23) were introduced, respectively, which made it possible to effectively obtain the corresponding linear boundary value problems (12), (13), and (24), (25). Subsequently, this made it possible to derive quadratic equations (16) to (18) and (28) to (30), the solutions to which determine the temperature distribution, which is geometrically represented in Fig. 5, 6.

It should be noted that the reviewed works did not consider an approach for linearization of boundary value problems of thermal conductivity for thermosensitive media by an analytical method. In contrast to [2], in which a homogeneous environment is considered, and the use of the Kirchhoff transformation made it possible to linearize the boundary value problem. In our studies, new linearizing functions were introduced; by applying them to nonlinear problems, their analytical solutions were effectively obtained. And this, in turn, leads to a minimal error in the obtained results, in contrast to the use of numerical methods, which was not achieved in [3, 4]. The use of generalized functions makes it possible to effectively describe thermophysical parameters for layered media, which leads to the solution of one differential equation of heat conduction with a singular right-hand side, which was not done in [1].

The current studies concern only the stationary process of heat conduction, and these studies were performed for layered media. In the future, such studies can be continued for layered media with foreign inclusions, for non-stationary heat conduction processes, as well as for anisotropic layered media.

Since the architecture of modern electronic devices includes separate heat-active nodes and their elements in the form of structures with foreign inclusions, there is a need to construct mathematical models of the heat conduction process. These models can be linear or nonlinear for isotropic layered media containing foreign heat-active inclusions. As a result, the given mathematical models of heat conduction are simplified, but they make it possible to build more complex mathematical models of the heat conduction process for composite media based on them.

Based on the obtained analytical solutions to both linear and nonlinear boundary value problems of heat transfer, it is proposed to develop computational algorithms and software tools for their numerical implementation. This will make it possible to carry out research for a number of materials used in the process of designing digital electronic devices, regarding the effect of their thermal sensitivity on the temperature distribution.

It is proposed to take into account the thermal sensitivity of structural materials, which significantly complicates the process of solving the corresponding nonlinear boundary value problems of thermal conductivity. The sought-after solutions to these problems describe the temperature behavior as a function of spatial coordinates somewhat more adequately to the real physical process.

Our study was carried out for a stationary heat conduction process, as a result of which the constructed models are limited, as they allow determining the temperature change only by spatial coordinate. Heat conduction problems contain boundary conditions of the first kind at the boundary surfaces of media, which is a disadvantage, although this does not reduce the generality of research.

Further research may involve the construction of mathematical models for determining temperature fields in layered media for the non-stationary process of heat conduction and for more complex boundary conditions.

7. Conclusions

1. A linear mathematical model for determining the temperature field, and subsequently for the analysis of thermal regimes in layered structures of electronic devices due to internal heating, has been built. An analytical solution to the boundary value problem was obtained and on its basis the behavior of the temperature as a function of the spatial coordinate for a five-layer structure was determined graphically.

2. A non-linear mathematical model for determining the temperature field, and subsequently for the analysis of thermal regimes in thermosensitive layered structures of electronic devices due to internal heating, has been built. A linearizing function was introduced, using which a nonlinear boundary value problem is linearized. On this basis, quadratic equations for determining the temperature were derived for the linear dependence of the thermal conductivity coefficient of the layer materials. They determine the temperature in a two-layer structure both inside the layers and on their conjugation surface. The temperature behavior as a function of the spatial coordinate is graphically displayed both for constant values of the thermal conductivity coefficient and for a linearly variable one.

3. A linear mathematical model for determining the temperature field, and subsequently for the analysis of thermal regimes in two-layer structures of electronic devices due to their internal heating on the surface of the conjugation of layers, has been built. Analytical solutions to the linear boundary value problem were obtained, which determine the temperature in the layers of the structure and on their surfaces of conjugation.

4. A nonlinear mathematical model has been constructed for determining the temperature field, and subsequently for the analysis of thermal regimes in two-layer thermosensitive structures of electronic devices due to their internal heating on the surface of the layer junction. A linearizing function was introduced, using which a nonlinear boundary value problem is linearized. On this basis, for the linear dependence of the coefficient of thermal conductivity of the materials of the layers, quadratic equations were derived for determining the temperature in a two-layer structure both inside the layers and on their mating surface. The behavior of the temperature as a function of the spatial coordinate is graphically displayed both for constant values of the thermal conductivity coefficient and for a linearly variable one. The results obtained for the selected materials based on the linear dependence of the coefficient of thermal conductivity on temperature differ from the results obtained for a constant coefficient of thermal conductivity by 15 %.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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