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DEVISING AN APPROACH TO THE CONSTRUCTION OF AN ADAPTED MODEL OF THE RECONNAISSANCE-FIRE SYSTEM FUNCTIONING

Oleksandr Maistrenko
Corresponding author
Doctor of Military Sciences, Professor
Department of Special Operations Forces*
E-mail: maj_Alex@ukr.net

Andrii Saveliev
PhD, Associate Professor, Deputy Head of the Institute*

Oleksandr Pechorin
PhD, Associate Professor, Head of Department
Department of Special Operations Forces*

Oleksandr Karavanov
PhD, Professor
Department of Artillery Reconnaissance Systems and Devices**

Stanislav Stetsiv
PhD, Associate Professor
Department of Missile Forces**

Mykola Shvets
PhD, Senior Researcher
The Scientific and Methodological Center of Scientific, Scientific and Technical Activities Organization
Povitrianykh Syl ave., 28, Kyiv, Ukraine, 03049

Oleksandr Lykholot
PhD
Department of Missile Troops and Artillery
Command and Staff Institute of Troops (Forces) Employment
Povitrianykh Syl ave., 28, Kyiv, Ukraine, 03049

Serhii Voitenko
PhD, Associate Professor
Research Laboratory
Ivan Kozhedub Kharkiv National Air Force University
Sumska str., 77/79, Kharkiv, Ukraine, 61023

Oleksandr Khimchenko
PhD***

Yulii Kondratenko
PhD***

*National Defence University of Ukraine
Povitrianykh Syl ave., 28, Kyiv, Ukraine, 03049
**Hetman Petro Sahaidachnyi National Army Academy
Heroiv Maidanu str., 32, Lviv, Ukraine, 79026
***Centre for Military and Strategic Studies
Povitrianykh Syl ave., 28, Kyiv, Ukraine, 03049

The object of this study is the operational model of a reconnaissance-fire system.

The problem that was solved is the lack of an approach to building a model of the functioning of a combat system, in particular a reconnaissance-fire system, which would take into account the influence of all subsystems and include the necessary number of system states.

An improved procedure for building an adapted operational model of the reconnaissance-fire system has been proposed. The essence of the improved methodology is the formalization of processes through the definition of system states and intensities of transitions from state to state. The improved procedure is based on the Kolmogorov-Chapman equations and the goal tree construction method.

A feature of the improved methodology is the breakdown of the states of the reconnaissance-fire system by hierarchy levels, which allows taking into account more necessary states of the system.

The field of practical use of the improved methodology is planning and management processes during the development of action algorithms during combat operations.

An adapted operational model of the reconnaissance-fire system has been built. The essence of the model is to determine the probability of the reconnaissance-fire system being in a certain state based on the Chapman-Kolmogorov equations, taking into account the necessary level of detail in the process of its operation.

Special feature of the proposed model is that it makes it possible to model by taking into account 39 states of the system with the necessary accuracy both for the system as a whole and separately for subsystems. This is explained by the fact that the test of the adequacy of the model showed that the discrepancy of the results is within the statistical error from 2 to 9 %.

The field of application of the adapted operational model of the reconnaissance-fire system is the processes of making a decision on the application of the operation of the intelligence-fire system during hostilities and their management during combat operations

Keywords: reconnaissance-fire system, modeling, Kolmogorov-Chapman equation, combat operations, military control

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1. Introduction

The results of the analysis of recent military conflicts [1, 2], and especially the Russian-Ukrainian war [3–5], testify to the transformation of management processes in the military domain from general (centralized) to decentralized. This is due to the peculiarities of modern military conflicts, in particular, the «blurring» of the battle line, high speed of processes, rapid changes in the situation, significant autonomy of units. The specified features require the search for ways to prevent the negative impact of these features on the process of functioning of the military group in the process of carrying out combat tasks. One of the ways is the introduction of new approaches to management processes.

One of the most effective approaches is the principle of «mission command», which is essentially a concept that focuses on giving subordinates the authority to make decisions within the overall mission goal. This gives commanders the flexibility to act and quickly respond to changing situations, while maintaining control over the group [6]. One of the manifestations of the application of «mission command» in practice is the unification of individual functional elements of a military group into reconnaissance and fire circuits (systems).

Reconnaissance and fire systems (RFS) make it possible to significantly speed up management processes and, accordingly, increase the effectiveness of combat missions. However, the use of RFS requires improvement of approaches to their use and, accordingly, integration into the overall structure of the group's functioning processes in a combat environment.

One of the main challenges associated with the use of anti-tank missiles is the need to determine the possibility of performing a combat mission depending on the situation. It is clear that the existing approaches to assessing the predicted effectiveness of the use of combat systems can only give a general assessment, without taking into account the specificity of RFS. This is explained by the fact that existing approaches are based on models that take into account centralized control, which leads to the neglect of certain processes that are inherent in decentralized management. In particular, this is due to the increase in the number of states in which the system can be, which requires the reconstruction of models and methodological apparatus in general.

In addition, it is necessary to note the widespread use of decision-making support systems in the military domain, which are based on formalized models of the functioning of both individual functional elements and systems as a whole. This leads to an increase in the requirements for the selection of these models because the accuracy of model selection in the specified decision support systems directly affects the accuracy of the proposed action options.

Thus, on the one hand, the need to use models of the functioning of complex systems for military purposes, in particular, RFS, and on the other hand, the lack of approaches to building such adapted models, predetermine the relevance of our research.

2. Literature review and problem statement

Paper [7] reports the results of research on the «network-centric» concept of conducting hostilities. It is shown that its application influences the development of RBS, and based on this, the management cycle was decomposed and the role and place of each element of the cycle was determined. But the issues related to taking into account all possible states of RFS, in which it can be, remained unresolved. The reason for this may be objective difficulties associated with the impossibility of taking into account all the necessary states of RFS due to the difficulty of aggregating them at one level, which makes relevant studies impractical. The option to overcome the difficulties can be the selection of the appropriate scientific and methodological apparatus. This is the approach used in work [8]; however, it is necessary to take into account the peculiarities of the process of the operation of RFS itself. All this gives reason to assert that it is expedient to carry out a study on the development of approaches to building a model of the functioning of RFS with the required number of states.

Paper [8] reports the results of research on scientific and methodological approaches to modeling the process of functioning of military systems, in particular, models based on the Kolmogorov equations, Lanchester equations, and the Lotka-Volterra equation. It is shown that the models based on the Kolmogorov equations make it possible to determine the probability of the system being in a certain state, however, in the case when there are no more than 10 states. However, issues related to modeling when the required number of states exceeds 10 remain unresolved. The reason for this is objective difficulties associated with the impossibility of compiling the appropriate number of equations at one level of the hierarchy according to the existing approach, which makes relevant research impractical. A variant of overcoming the relevant difficulties can be the distribution of modeling by subsystems. This is the approach used in [9]; however, when modeling by individual subsystems, the integrity of the modeling may be lost. All this gives reason to assert that it is expedient to conduct a study on the development of modeling approaches for the system as a whole, taking into account the required number of states.

Paper [9] reports the results of research on the issue of modeling the intelligence process under complex urban conditions using unmanned aerial vehicles (UAVs). It is shown that modeling based on a metaheuristic algorithm allows optimal route planning for UAVs. But the questions related to the modeling of the process of functioning of this RFS subsystem as part of a larger system remained unresolved. The reason for this may be the fundamental impossibility of combining multifaceted processes, in particular intelligence, control, and execution (fire influence using the existing approach, which makes relevant studies impractical. An option to overcome the relevant difficulties may be the introduction of the interconnection (interaction) functions of various subsystems into the model. Exactly such an approach is used in work [10]; however, the proposed approach considers the same type of processes. All this gives reasons to assert that it is appropriate to conduct a study on the development of modeling approaches for the system as a whole, taking into account the required number of states and various functional processes.

Work [10] reports the results of research on the simulation of reconnaissance by a swarm of cooperative UAVs. It is shown that the model, which is based on stochastic algorithms, allows one to plan intelligence processes in such a way as to perform tasks as effectively as possible when jointly applied. But the questions related to taking into account the entire set of processes that take place in the combat environment remained unresolved. The reason for this is the objective difficulties associated with modeling the combat environment with the necessary set of processes, which makes relevant studies impractical. An option to overcome the difficulties can be the development of a simulated combat environment. This is the approach used in work [11]; however, this approach requires the formalization of the entire set of heterogeneous states of RFS. All this gives reason to assert that it is expedient to carry out a study on the formalization of processes related to the functioning of RFS, which are different in terms of functions.

In [11], the results of studies into the mechanisms and structure of simulation software for combat simulation are presented. It is shown that AFSIM (Advanced framework for simulation, integration, and modeling) and E-CARGO (Environments – classes, agents, roles, groups, and objects) are the most suitable tools for combat simulation. But the questions related to the modeling of the process of the operation of RFS, taking into account the peculiarities of the processes of management, reconnaissance, and execution (fire influence) remained unresolved. The reason for this is the objective difficulties associated with the complexity of modeling various processes with a large number of states, which makes relevant research impractical. An option to overcome the difficulties can be the modeling of different processes separately using the appropriate methodical apparatus. This is the approach used in [12]; however, the combination of simulation results requires the development of an additional methodological apparatus. All this gives reason to assert that it is expedient to conduct a study on the development of a methodical apparatus for modeling the processes of functioning of various subsystems of RFS with the required number of states.

Work [12] reports the results of research on the quantitative analysis of combat operations on the basis of various models, in particular probabilistic, Markov, optimization models; Monte Carlo. It is shown that the proposed models make it possible to solve various modeling problems. But issues related to modeling processes with a relatively large number of system states remained unresolved. The reason for this may be objective difficulties associated with the ability of the mathematical apparatus to describe the appropriate number of states, which makes relevant research impractical. A variant of overcoming the difficulties may be the aggregation of states according to the relevant characteristics. This is the approach used in work [13]; but there are no defined signs by which aggregation can be carried out. All this gives reason to assert that it is expedient to conduct a study on determining the necessary feature of the aggregation of the states of the functioning of RFS.

Work [13] reports the results of research into the method of modeling combat operations by aggregating forces while simulating the battle process using Lanchester equations. It is shown that the proposed model makes it possible to obtain the parameters of the conflict result with numerical indicators of success, resource consumption, etc. But the issues related to not taking into account different states of the system depending on the set of situations (conditions) remained unresolved. The reason for this is the fundamental impossibility of taking into account the entire set of states in accordance with various conditions, which makes relevant research impractical. An option for overcoming the relevant difficulties may be the formalization of the necessary set of events. This is the approach used in [14]; however, the formalization of the set of events requires the formalization of system states according to each type of process. All this gives reason to assert that it is expedient to carry out a study on the development of approaches to building a model of the functioning of RFS, taking into account the necessary number of states determined by situations.

Paper [14] reports the results of modeling research based on the formalization of the specification of a discrete system of events, which includes the use of several combat objects to hit several targets. It is shown that the proposed model makes it possible to obtain experimental results regarding combat situations with the use of weapons at the tactical level. But the issues related to taking into account different types of processes during modeling remained unresolved. The reason for this may be the objective difficulties associated with the mathematical apparatus, which makes relevant research impractical. A combination of several mathematical methods can be an option for overcoming these difficulties. This is the approach used in work [15]; however, the methods proposed there do not solve the issue of modeling different types of processes in particular, RFS. All this gives reason to assert that it is expedient to carry out a study on the development of approaches to building a model of the functioning of RFS, taking into account the modeling of different types of processes.

Work [15] reports the results of research into the simulation of a combat collision, which is based on Markov chains and Monte Carlo methods. It is shown that the proposed approach makes it possible to improve the interpretation of data on the results of a combat encounter by taking into account the accuracy of shooting. But the issues related to the failure to take into account the processes of functioning of the intelligence and management subsystems remained unresolved. The reason for this is the objective difficulties associated with the lack of distribution of system states by hierarchy levels, which makes relevant studies impractical. An option to overcome the difficulties may be to take into account the correlation between various properties of the process of functioning of RFS. This is the approach used in work [16]; but there only the correlation between management and efficiency is taken into account, which focuses attention on the management subsystem. All this gives reason to assert that it is expedient to carry out a study on the development of approaches to building a model of the functioning of RFS, taking into account the correlation between various properties of the process of its functioning.

Paper [16] reports the results of research on the combat model and its additions. It is shown that the specified model is based on the model of network-centric operations, which makes it possible to demonstrate the correlation between management and efficiency. But the issues related to taking into account the properties of other subsystems of RFS remained unresolved. The reason for this is the objective difficulties associated with the impossibility of distributing system states by hierarchy level according to existing approaches. An option to overcome the difficulties may be the development of approaches to the construction of an adapted model of the functioning of RFS.

Therefore, our review of studies on modeling the functioning of combat systems shows that existing studies focus on modeling only a certain subsystem of these systems, which does not allow taking into account the entire set of their states. Also, it should be noted that existing studies consider only a limited number of combat system states due to the complexity of their formalization. Thus, an unsolved theoretical problem is the lack of an approach to building a model of the functioning of a combat system, in particular RFS, which would take into account the influence of all subsystems and include the necessary number of states of RFS.

3. The aim and objectives of the study

The purpose of our study is to devise approaches to building an adapted model of the functioning of RFS, which would take into account the influence of all subsystems and include the required number of states of RFS. This will make it possible to increase the accuracy of forecasting the results of the combat use of RFS and, accordingly, the effectiveness of their use.

To achieve the goal, the following tasks were set:

– to improve the methodology of building an adapted model of the functioning of RFS;

– to build an adapted model of RFS functioning.

4. The study materials and methods

The object of our study is the model of RFS functioning.

The main hypothesis assumes that the distribution of simulations by hierarchy levels will make it possible to increase the number of states that can be taken into account and to take into account the influence of all subsystems of RFS. This will make it possible to increase the accuracy of the forecast and, accordingly, the efficiency of the application of RFS.

The main assumption adopted in the research: the process of RFS functioning, which is characterized by a flow of events (transitions from state to state), is stationary, ordinary, and without aftereffects. Stationarity is determined by the fact that the probability of hitting a certain number of transitions in a certain time interval depends only on the length of the interval. Also, the number of events does not depend on where this interval is located in time. Ordinariness is determined by the fact that the probability of two or more transitions in an elementary time interval is infinitesimally small. A flow of events without an aftereffect is due to the fact that, for any time intervals, the number of transitions falling on one of them does not depend on the number of events falling on others.

The main simplification accepted in the work is that the flow of transitions from state to state of the system is the simplest (Poisson). Accordingly, it is subject to restrictions on aftereffect, ordinariness, and stationarity. It is also a simplification that the list of states is constant and requires the formalization of all processes under these states.

The work uses approaches from the theory of Markov stochastic processes in the theory of probabilities, in particular the Chapman-Kolmogorov equation [17]. These equations are an identity that connects the joint probability distributions of different sets of coordinates on a stochastic process. The essence of modeling using these equations is to determine the probability of system states ($P(t)$) at any instant of time of a system with discrete states in which a Markov random process with continuous time takes place. To determine these probabilities, it is necessary to know: the number of possible states of the system (n), initial states of the system (S), matrix of probability density (intensities of transition from state to state (λ)) [17, 18].

The general view of the marked state graph of a certain system is shown in Fig. 1.

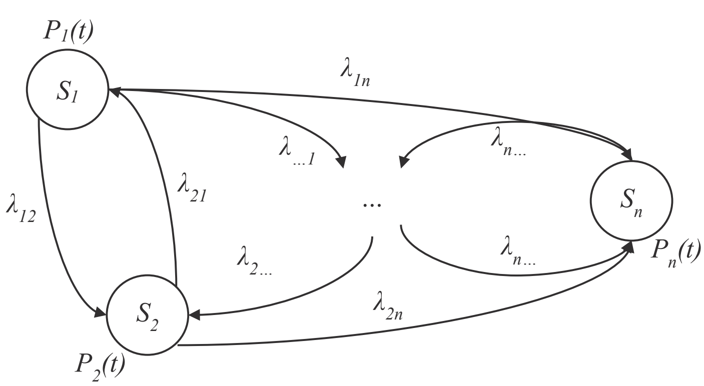

Fig. 1. General view of the marked state graph of a certain system

To determine the probabilities of the states of a continuous Markov chain, a system of equations is built, each of which describes the boundary probability of the system being in a certain state at any moment in time. The general form of the equations is shown in dependence (1) [18]:

$$\begin{cases} \dfrac{dP_1(t)}{dt} = \sum\limits_{i=1}^{k_1} (\pm\lambda_{1i}) P_i(t), \\ \dfrac{dP_2(t)}{dt} = \sum\limits_{i=1}^{k_2} (\pm\lambda_{2i}) P_i(t), \\ \ldots \\ \dfrac{dP_n(t)}{dt} = \sum\limits_{i=1}^{k_n} (\pm\lambda_{ni}) P_i(t), \\ \sum\limits_{i=1}^{n} P_i(t) = 1. \end{cases} \quad (1)$$

Note that the sign of intensity is chosen depending on the direction of the transition; if the transition occurs from this state to another, then «–», if on the contrary, then «+».

Subsequently, the equations can be solved under the condition of stationary process (P_i=const), that is, the left-hand side of the equations can be equated to 0. Accordingly, (1) is transformed into a system of linear algebraic equations (2):

$$\begin{cases} 0 = \sum\limits_{i=1}^{k_1} (\pm\lambda_{1i}) P_i, \\ 0 = \sum\limits_{i=1}^{k_2} (\pm\lambda_{2i}) P_i, \\ \ldots \\ 0 = \sum\limits_{i=1}^{k_n} (\pm\lambda_{ni}) P_i, \\ \sum\limits_{i=1}^{n} P_i = 1. \end{cases} \quad (2)$$

The method of building a goal tree was used in the work. Here, the goal tree should be understood as a hierarchical tree-like structure that is obtained by dividing the general goal (function, etc.) into sub-goals.

The advantage of this method is that it enables the division of a complex goal (function, task, etc.), which is difficult to formalize, into a set of simpler goals. The division of goals continues until simple goals are obtained that can be solved by known methods [19].

The initial data for checking the model for adequacy are the composition of RFS (Table 1); set of events $\{N_i\}$ (Table 2). For verification, an operational-level RFS with a closed circuit of information exchange and three channels of intelligence data was used. The state under investigation is movement along fire position (S_{421}); given probability of being in a certain state P_{set}=0.01; modeling period – 24 hours.

In general, the adopted composition of RFS (Table 2) makes it possible to determine the number of events according to the states that can occur in a certain period of time and the intensity of these events, according to the level of the hierarchy (Table 3).

In the study, the Microsoft Excel 2010 software environment (USA) was used for calculations. We directly applied Microsoft Excel spreadsheets for computation using formulas, data representation, structuring. Also, when working with Microsoft Excel, the capabilities of this application for working with standard formulas, including functions, were used.

Table 1

The composition of RFS for the conditions of the example

Subsystem	Element type	Quantity
Intelligence	Optical-electronic complex «Sych 5K10» [20]	1
	Radar station Kh1-M «Oko» [21]	1
	Leleka-100 unmanned aircraft complex	1
Control	Control point based on the KROPYVA software complex [22]	1
Damage	Self-propelled howitzer M109 [23]	6

Table 2

The set of the number of certain events for a certain time related to RFS functioning $\{Ni\}$ according to formula (3)

N_i		λ_i	N_i		λ_i	N_i		λ_i	N_i		λ_i	N_i		λ_i
$N_{1\to2}$	4	0.25	$N_{22\to23}$	2	0.50	$N_{43\to41}$	2	0.50	$N_{231\to232}$	4	0.25	$N_{332\to331}$	4	0.25
$N_{2\to3}$	2	0.50	$N_{23\to21}$	3	0.33	$N_{111\to112}$	4	0.25	$N_{232\to231}$	3	0.33	$N_{411\to412}$	3	0.33
$N_{3\to1}$	3	0.33	$N_{31\to32}$	5	0.20	$N_{112\to111}$	5	0.20	$N_{311\to312}$	7	0.14	$N_{411\to413}$	2	0.50
$N_{3\to4}$	2	0.50	$N_{32\to31}$	6	0.17	$N_{121\to122}$	3	0.33	$N_{312\to311}$	2	0.50	$N_{412\to413}$	5	0.20
$N_{4\to3}$	3	0.33	$N_{32\to33}$	4	0.25	$N_{122\to121}$	5	0.20	$N_{321\to322}$	6	0.17	$N_{413\to411}$	4	0.25
$N_{4\to1}$	2	0.50	$N_{33\to31}$	2	0.50	$N_{211\to212}$	6	0.17	$N_{322\to321}$	7	0.14	$N_{421\to422}$	4	0.25
$N_{11\to12}$	4	0.25	$N_{41\to42}$	3	0.33	$N_{212\to211}$	4	0.25	$N_{322\to323}$	3	0.33	$N_{422\to421}$	3	0.33
$N_{12\to11}$	2	0.50	$N_{42\to41}$	2	0.50	$N_{221\to222}$	2	0.50	$N_{323\to321}$	5	0.20	$N_{431\to432}$	2	0.50
$N_{21\to22}$	5	0.20	$N_{42\to43}$	3	0.33	$N_{222\to221}$	5	0.20	$N_{331\to332}$	6	0.17	$N_{432\to431}$	4	0.25

The Matrix calculator online application [24] was also applied to solve the system of linear equations.

5. Results of the development of approaches to the construction of an adapted operational model of the reconnaissance-fire system

5. 1. Improving the procedure of building an adapted operational model of the reconnaissance-fire system

The results of analysis of approaches to the modeling of management processes in the military domain show that it is an important and necessary element of both military control and research in the military domain.

Analyzing the modeling process in the military sector, it is necessary to note the complexity of this process due to the need to take into account the uncertainty of the results of the interaction of two opposing sides. This leads to a higher level of detail in modeling and the need to take into account a larger number of system parameters and states. However, such detailing leads to a significant complication of the model due to the need to take into account all possible variants of the development of events. In other words, to obtain more accurate results, it is necessary to significantly complicate the system's functioning model.

On the other hand, the complexity of the model can lead to both objective and subjective errors. Objective errors are related to the ranges of possible values of the simulated indicators, the greater the number of these indicators, the greater the range of values of the highest level indicator. Subjective errors are associated with the possibility of operator error, as well as with the process of rounding of indicator values. Accordingly, the greater the number of calculations (indicators), the greater the probability of an error by the operator and the greater the error of accumulating an unaccounted value of the indicator because of rounding.

In the military field, during simulation, there are two main approaches to prevent the influence of these errors on simulation results [25]. First, it is a simplification of the model itself to a level that will provide relatively accurate results. However, this approach gives only a general idea of the simulated processes and does not allow for detailed simulation.

The second approach is based on reducing the range of values of input values on the basis of their more accurate definition [26]. This allows for a more accurate forecast, but it requires significant effort to determine accurate input data.

In view of the above, it is proposed to apply the approach of distribution by hierarchy level of modeling processes and coordination of these models among themselves. The general idea of improving the methodology of building an adapted model of the functioning of RFS consists in the distribution of system states according to the level of process detail and step-by-step modeling according to the level of the hierarchy.

At the first stage of the proposed methodology, it is necessary to obtain input data, which include a set number of certain events for a certain time related to RFS functioning $\{N_i\}$. Also, the input data is the state, the probability of being in which, it is necessary to obtain, S_{set}; given probability of being in a certain state, P_{set}, as well as the period to be simulated.

The second stage involves determining the necessary degree of detailing of the process of RFS functioning (the number of hierarchy levels) depending on the purpose of modeling. That is, at this stage, it is necessary to formulate the state of the system that is interesting for research. Such a formulation is proposed to be carried out with the use of cluster analysis, which will make it possible to determine the level of the hierarchy of system states to which the state under investigation corresponds. The schematic diagram of determining the required level of detail of the system state is shown in Fig. 2.

At the third stage, determination of the number of RFS states (m) is provided, according to the level of the hierarchy. Such determination is proposed to be carried out using the «goal tree» method. Application of this method will allow gradation and separation of system states while preserving their functional and fundamental (essential) integrity. The schematic diagram of determining the number of states of RFS, according to the level of the hierarchy using the goal tree, is shown in Fig. 3.

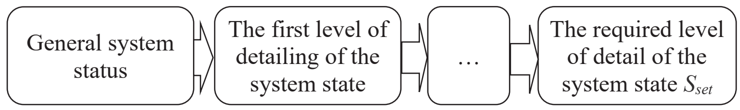

Fig. 2. Schematic diagram of determining the required level of detail of the system state

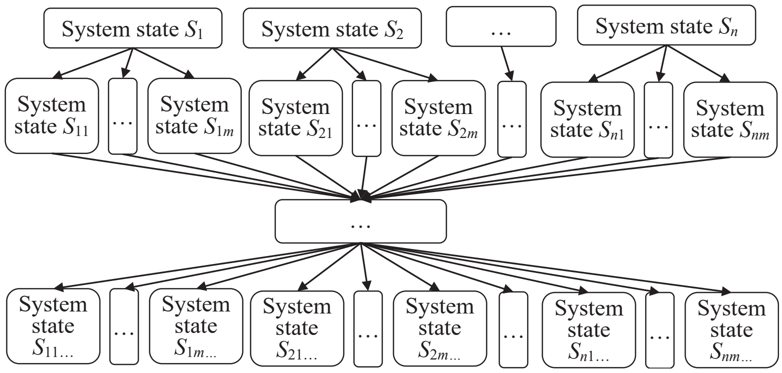

Fig. 3. Schematic diagram of determining the number of states of RFS, according to the level of the hierarchy using a goal tree

The fourth stage of the methodology involves several steps, in particular, the construction of a marked graph of the states of RFS according to the level of the hierarchy. This will make it easier to understand the process of RFS functioning in accordance with the conditions of the operation. Moreover, it is necessary to take into account that the number of states at each level of the hierarchy should not exceed 10, which will ensure ease of working with this model. The general view of the marked state graph of a certain system is shown in Fig. 1.

At the fifth stage, determination of the intensities of transitions from the state to the state of RFS is provided in accordance with the accepted conditions $\{\lambda_i\}$. In general, the intensity of transition from state to state is defined as the number of these transitions per unit of time. Accordingly, taking into account for this the set of the number of certain events for a certain time associated with RFS functioning $\{N_i\}$, the intensities for each transition are determined according to formula (3):

$$\lambda_i = \frac{1}{N_i}, \tag{3}$$

where 1 is a unit of time taken according to the dimensionally of the period to be modeled T.

At the sixth stage, several steps are also provided, in particular, regarding the compilation of systems of Chapman-Kolmogorov equations according to the level of hierarchy and input data. This will allow us to move on to determining the probabilities of staying in these states of RFS. The general approach to constructing equations is demonstrated in (1) and (2).

At the seventh stage, it is envisaged to determine the probabilities of the presence of RFS in a certain state.

This is done by transforming the equations compiled in the previous steps. For this purpose, all the probabilities of staying in RFS in certain states are determined by the known intensities.

At the eighth stage of the methodology, there is provision for checking the sensitivity of the obtained model by changing the intensities of transitions from state to state, and further checking the compliance of the obtained probability values with the established limits (4):

$$P_i \geq P_{set} \text{ or } P_i \leq P_{set}, \tag{4}$$

In the event that the obtained values of probabilities are lower (higher) than the established norms, it is necessary to change the input data.

Thus, the study of several levels of the hierarchy of states of RFS makes it possible to adapt this model directly to the conditions and goals of the study. Moreover, this approach makes it possible to overcome the shortcomings of existing approaches, in particular, low modeling accuracy (first approach) and high complexity of model formalization (second approach).

The general view of the improved methodology for building an adapted model of the functioning of RFS is shown in Fig. 4.

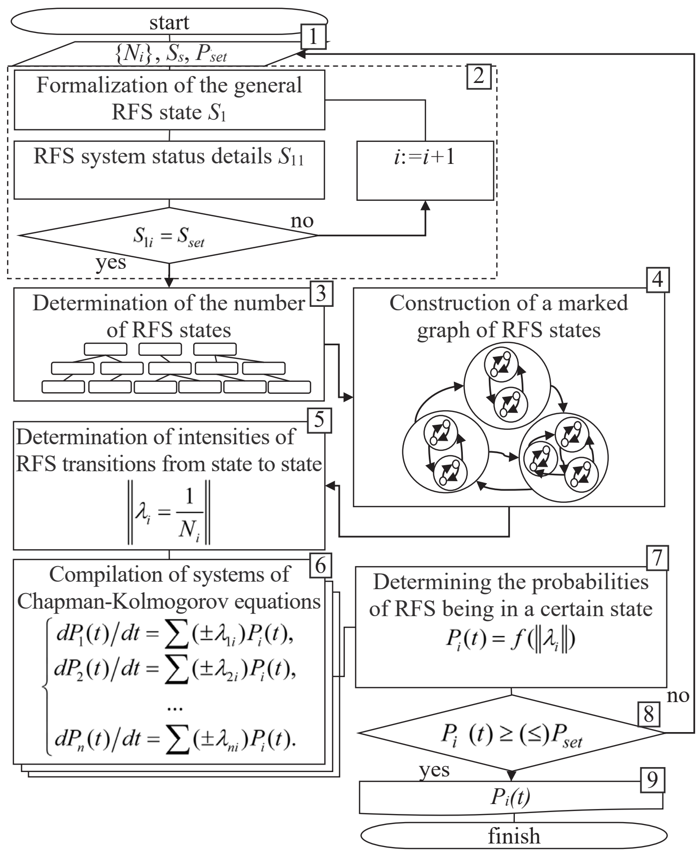

Fig. 4. General view of the improved methodology for building an adapted operational model of the reconnaissance-fire system

Analysis of the improved methodology for building an adapted model of RFS functioning (Fig. 4) reveals that its application could significantly simplify the modeling of the RFS functioning process. In general, the construction of a model of RFS operation according to the proposed method will make it possible to increase the accuracy of forecasting while maintaining relative simplicity. This is explained by the fact that the distribution of states by hierarchy levels allows simulation with a relatively small number of states, taking into account their relationship with other levels of the hierarchy.

The limitations of the application of this procedure include the fact that the process of RFS functioning, which is characterized by a flow of events (transitions from state to state), must be stationary, ordinary, and without aftereffects. The stationarity is explained by the fact that the probability of hitting one or another number of transitions in a certain time interval depends only on the length of the interval. Also, the number of events does not depend on where this interval is located in time. Ordinariness is explained by the fact that the probability of two or more transitions in an elementary time interval is infinitesimally small. A flow of events without an aftereffect is explained by the fact that, for any time intervals, the number of transitions falling on one of them does not depend on the number of events falling on others. These restrictions will make it possible to significantly simplify the calculations, given that these conditions are fulfilled even without artificial restrictions during relatively short intervals of time during the process of RFS operation.

The shortcomings of this procedure include the dependence of the formalization of conditions on the level of experience of the researcher. However, it should be noted that the use of expert evaluation methods could significantly reduce the level of subjectivity in the formalization of the states of RFS.

5. 2. Construction of an adapted operational model of a reconnaissance-fire system

In accordance with the approaches proposed while improving the methodology for building an adapted model of RFS functioning, the model is built from the purpose of the study. However, taking into account the urgent need to use forecasted data when planning operations, it is advisable to build an adapted model RFS operation, taking into account the necessary level of detail.

Therefore, based on the improved methodology for building an adapted model of RFS functioning, the initial step is to determine the necessary degree of detailing of RFS functioning process (the number of hierarchy levels) depending on the purpose of modeling. Taking into account that the main purpose of RFS operation is to identify and destroy the target while preserving the ability to function, it is suggested we start the analysis (clustering with this purpose in mind).

The following processes in terms of the hierarchy are the processes of obtaining intelligence information (S_1) [27, 28], making a decision to perform a task (S_2) [29], defeating a target (S_3) [30], ensuring the preservation of a combat-ready state of anti-aircraft missiles (S_4) [31].

The next level of the process hierarchy is detailing by homogeneous processes. Regarding the receipt of intelligence information, sub-processes are directly intelligence (S_{11}) and processing of received information (S_{12}) [32]. Regarding the decision to perform the task, the sub-processes are the analysis of the received information (S_{21}), target distribution (S_{22}) and giving the combat order (S_{23}) [33]. With regard to target engagement, the sub-processes are aiming (aiming) (S_{31}), conducting fire (S_{32}), adjusting fire (S_{33}) [34]. With regard to maintaining the operational condition of the anti-aircraft missile system, this is a maneuver to fire positions (FP) (S_{41}), occupation of FP (S_{42}), abandonment of FP (S_{43}).

The next level of distribution of processes is carried out in accordance with the identified clusters. Intelligence includes the processes of determining the intelligence area (S_{111}), object detection (S_{112}) [35]. Processing of received information includes generalization of intelligence information (S_{121}) and identification of intelligence objects (S_{122}).

Analysis of the received information includes summarizing the information (S_{211}) and ranking the goals by importance (S_{212}). Target allocation includes determining the possible effect of hitting a certain target with a certain fire means (S_{221}) and assigning a certain fire means to hit a certain target (S_{222}). Giving a combat order (order) includes defining the main elements of the order, such as terms, routes, signals, tasks, etc. (S_{231}) and sending an order to the executor (S_{232}) [36].

Targeting (aiming) includes determination of the mutual location of the target and the means of destruction (S_{311}), determination of settings for firing (launch) (S_{312}). Firing includes loading (S_{321}), firing (launching) (S_{322}), discharging (S_{323}). Fire correction includes determination of deviation of the explosion from the target (S_{331}), calculation of corrections (S_{332}). The maneuver to the firing (starting) position (FP (SP)) includes the formation of a column (S_{411}), movement (S_{412}), dispersal (S_{413}). Occupying the FP (SP) includes moving along the FP (SP) to the designated location (S_{421}), bringing the equipment to a combat state (S_{422}). Leaving the FP (SP) includes collapsing the equipment (S_{431}), moving from the FP (SP) (S_{432}) [37].

According to the third point of the improved methodology, it is necessary to determine the number of states of RFS (m), in accordance with the level of the hierarchy using the «goal tree» method. The general diagram of the number of states of RFS, according to the level of the hierarchy defined by the tree of goals, is shown in Fig. 5.

The next step of the methodology involves the construction of a marked graph of the state of RFS according to the level of the hierarchy (Fig. 5). The general view of the marked state graph of RFS is shown in Fig. 6.

The next step of the methodology involves determining the intensities of the transitions from the state to the state of RFS according to the accepted conditions {λ_i}. Usually, determination of intensities of transitions from state to state is carried out according to (3); however, taking into account the need to ensure the accuracy of modeling, such determination is proposed to be carried out according to the conditions. There are three ways to search for the set number of certain events over a certain time related to RFS functioning {N_i}. The first technique is the analysis of open sources regarding the performance of tasks of RFS, for example [28, 38, 39]. The second is the use of analytical and statistical data [40, 41]. The third technique is combined – a combination of research with analytical data.

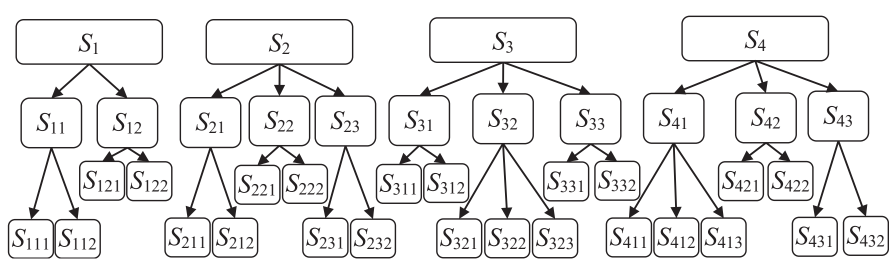

Fig. 5. The general scheme of the number of states of RFS, according to the level of the hierarchy defined by the goal tree

The sixth step in the development of an adapted model of RFS functioning involves the construction of systems of Chapman-Kolmogorov equations according to the level of the hierarchy and input data, based on (1) and (2).

To simplify the mathematical interpretation of the formulas, it is proposed to establish letter designations for the intensities of the RFS transition from state to state (Table 3).

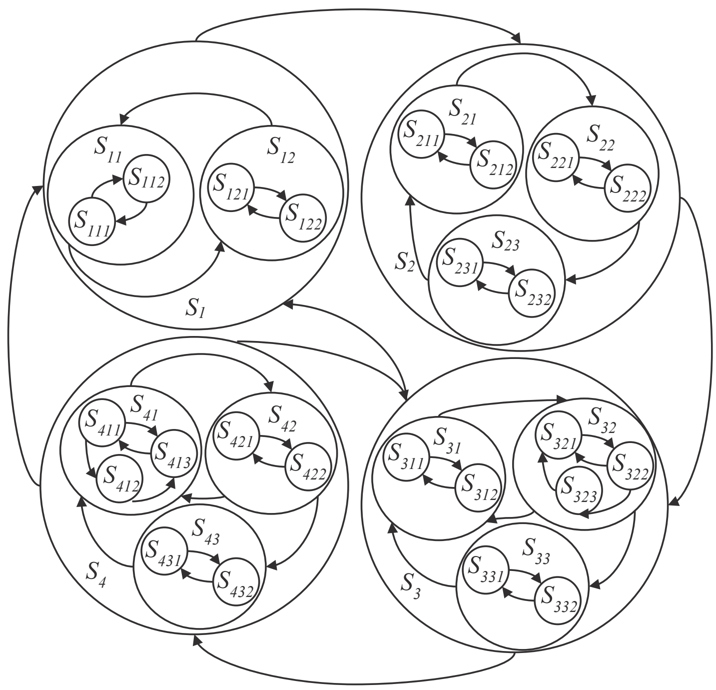

Fig. 6. General view of the marked graph of the states of a reconnaissance-fire system

Table 3

Letter notation for the intensities of the RFS transition from state to state

Intensity	Notation	Intensity	Notation	Intensity	Notation	Intensity	Notation	Intensity	Notation
$\lambda_{1\to2}$	Q	$\lambda_{22\to23}$	A	$\lambda_{43\to41}$	Z	$\lambda_{231\to232}$	o	$\lambda_{332\to331}$	l
$\lambda_{2\to3}$	W	$\lambda_{23\to21}$	S	$\lambda_{111\to112}$	q	$\lambda_{232\to231}$	a	$\lambda_{411\to412}$	z
$\lambda_{3\to1}$	E	$\lambda_{31\to32}$	D	$\lambda_{112\to111}$	w	$\lambda_{311\to312}$	s	$\lambda_{411\to413}$	x
$\lambda_{3\to4}$	R	$\lambda_{32\to31}$	F	$\lambda_{121\to122}$	e	$\lambda_{312\to311}$	d	$\lambda_{412\to413}$	c
$\lambda_{4\to3}$	T	$\lambda_{32\to33}$	G	$\lambda_{122\to121}$	r	$\lambda_{321\to322}$	f	$\lambda_{413\to411}$	v
$\lambda_{4\to1}$	Y	$\lambda_{33\to31}$	H	$\lambda_{211\to212}$	t	$\lambda_{322\to321}$	g	$\lambda_{421\to422}$	b
$\lambda_{11\to12}$	U	$\lambda_{41\to42}$	J	$\lambda_{212\to211}$	y	$\lambda_{322\to323}$	h	$\lambda_{422\to421}$	n
$\lambda_{12\to11}$	I	$\lambda_{42\to41}$	K	$\lambda_{221\to222}$	u	$\lambda_{323\to321}$	j	$\lambda_{431\to432}$	m
$\lambda_{21\to22}$	O	$\lambda_{42\to43}$	L	$\lambda_{222\to221}$	i	$\lambda_{331\to332}$	k	$\lambda_{432\to431}$	p

In accordance with the notation of the intensities of RFS transition from state to state (Table 3) and equations (1) and (2), the equations for the states S_{111}, S_{112} (5) and S_{121}, S_{122} (6) were derived:

$$\begin{cases} 0 = wP_{112} - qP_{111}, \\ 0 = qP_{111} - wP_{112}, \\ P_{11} = P_{111} + P_{112}, \end{cases} \quad (5)$$

$$\begin{cases} 0 = eP_{121} - rP_{122}, \\ 0 = rP_{122} - eP_{121}, \\ P_{12} = P_{121} + P_{122}. \end{cases} \quad (6)$$

Equations for states S_{211}, S_{212} (7), S_{221}, S_{222} (8) and S_{231}, S_{232} (9):

$$\begin{cases} 0 = tP_{211} - yP_{212}, \\ 0 = yP_{212} - tP_{211}, \\ P_{21} = P_{211} + P_{212}, \end{cases} \quad (7)$$

$$\begin{cases} 0 = uP_{221} - iP_{222}, \\ 0 = iP_{222} - uP_{221}, \\ P_{22} = P_{221} + P_{222}, \end{cases} \quad (8)$$

$$\begin{cases} 0 = oP_{231} - aP_{232}, \\ 0 = aP_{232} - oP_{231}, \\ P_{23} = P_{231} + P_{232}. \end{cases} \quad (9)$$

Equations for states S_{311}, S_{312} (10), S_{321}, S_{322}, S_{323} (11) and S_{331}, S_{332} (12):

$$\begin{cases} 0 = sP_{311} - dP_{312}, \\ 0 = dP_{312} - sP_{311}, \\ P_{31} = P_{311} + P_{312}, \end{cases} \quad (10)$$

$$\begin{cases} 0 = jP_{323} + gP_{322} - fP_{321}, \\ 0 = fP_{321} - gP_{322} - hP_{322}, \\ 0 = hP_{322} - jP_{323}, \\ P_{32} = P_{321} + P_{322} + P_{323}, \end{cases} \quad (11)$$

$$\begin{cases} 0 = kP_{331} - lP_{332}, \\ 0 = lP_{332} - kP_{331}, \\ P_{33} = P_{331} + P_{332}. \end{cases} \quad (12)$$

Equations for states S_{411}, S_{412}, S_{413}, (13), S_{421}, S_{422} (14) and S_{431}, S_{432} (15):

$$\begin{cases} 0 = -xP_{411} - zP_{411} + vP_{413}, \\ 0 = zP_{411} - cP_{412}, \\ 0 = xP_{411} + cP_{412} - vP_{413}, \\ P_{41} = P_{411} + P_{412} + P_{413}, \end{cases} \quad (13)$$

$$\begin{cases} 0 = nP_{422} - bP_{421}, \\ 0 = bP_{421} - nP_{422}, \\ P_{42} = P_{421} + P_{422}, \end{cases} \quad (14)$$

$$\begin{cases} 0 = pP_{432} - mP_{431}, \\ 0 = mP_{431} - pP_{432}, \\ P_{43} = P_{431} + P_{432}. \end{cases} \quad (15)$$

Equations for states S_{11}, S_{12}, (16), S_{21}, S_{22}, S_{23} (17), S_{31}, S_{32}, S_{33} (18) and S_{41}, S_{42}, S_{43} (19):

$$\begin{cases} 0 = UP_{11} - IP_{12}, \\ 0 = IP_{12} - UP_{11}, \\ P_1 = P_{11} + P_{12}, \end{cases} \quad (16)$$

$$\begin{cases} 0 = SP_{23} - OP_{21}, \\ 0 = OP_{21} - AP_{22}, \\ 0 = AP_{22} - SP_{23}, \\ P_2 = P_{21} + P_{22} + P_{23}, \end{cases} \quad (17)$$

$$\begin{cases} 0 = HP_{33} + FP_{32} - DP_{31}, \\ 0 = DP_{31} - FP_{32} - GP_{32}, \\ 0 = GP_{32} - HP_{33}, \\ P_3 = P_{31} + P_{32} + P_{33}, \end{cases} \quad (18)$$

$$\begin{cases} 0 = ZP_{43} + KP_{42} - JP_{41}, \\ 0 = JP_{41} - KP_{42} - LP_{42}, \\ 0 = LP_{42} - ZP_{43}, \\ P_4 = P_{41} + P_{42} + P_{43}. \end{cases} \quad (19)$$

Equations for states S_1, S_2, S_3 and S_4 (20):

$$\begin{cases} 0 = YP_4 + EP_3 - QP_1, \\ 0 = QP_1 - WP_2, \\ 0 = WP_2 + TP_4 - EP_3 - RP_3, \\ 0 = RP_3 - YP_4 - TP_4, \\ 1 = P_1 + P_2 + P_3 + P_4. \end{cases} \quad (20)$$

The next, seventh, and eighth steps involve determining the probabilities of being in a certain state and checking the sensitivity of the resulting model (4). However, this is done in the case when the goal of the simulation is determined, that is, the determination of the state (states) that are directly investigated. Considering that this model carries a general interpretation of the process of RFS functioning, it is proposed to consider these stages on a specific example.

Further use of the received intensities (Table 2) allows us to transform equations (5) to (20) to determine the probabilities of the occurrence of certain events.

Accordingly, equations for the probabilities P_{111}, P_{112} (21) and P_{121}, P_{122} (22):

$$\begin{cases} P_{111} = \dfrac{w}{w+q}, \\ P_{112} = \dfrac{q}{q+w}, \end{cases} \quad (21)$$

$$\begin{cases} P_{121} = \dfrac{r}{r+e}, \\ P_{122} = \dfrac{e}{e+r}. \end{cases} \quad (22)$$

Equations for probabilities P_{211}, P_{212} (23), P_{221}, P_{222} (24) and P_{231}, P_{232} (25):

$$\begin{cases} P_{211} = \dfrac{y}{y+t}, \\ P_{212} = \dfrac{t}{t+y}, \end{cases} \quad (23)$$

$$\begin{cases} P_{221} = \dfrac{i}{i+u}, \\ P_{222} = \dfrac{u}{u+i}, \end{cases} \quad (24)$$

$$\begin{cases} P_{231} = \dfrac{a}{a+o}, \\ P_{232} = \dfrac{o}{o+a}. \end{cases} \quad (25)$$

Equations for probabilities P_{311}, P_{312} (26), P_{321}, P_{322}, P_{323} (27) and P_{331}, P_{332} (28):

$$\begin{cases} P_{311} = \dfrac{d}{d+s}, \\ P_{312} = \dfrac{s}{s+d}, \end{cases} \quad (26)$$

$$\begin{cases} P_{321} = \dfrac{gj+hj}{fh+fj+gj+hj}, \\ P_{322} = \dfrac{fj}{fh+fj+gj+hj}, \\ P_{323} = \dfrac{fh}{fh+fj+gj+hj}, \end{cases} \quad (27)$$

$$\begin{cases} P_{331} = \dfrac{l}{l+k}, \\ P_{332} = \dfrac{k}{l+k}. \end{cases} \quad (28)$$

Equations for probabilities P_{411}, P_{412}, P_{413}, (29), P_{421}, P_{422} (30) and P_{431}, P_{432} (31):

$$\begin{cases} P_{411} = \dfrac{cv}{cv+cx+cz+vz}, \\ P_{412} = \dfrac{vz}{cv+cx+cz+vz}, \\ P_{413} = \dfrac{cx+cz}{cv+cx+cz+vz}, \end{cases} \quad (29)$$

$$\begin{cases} P_{421} = \dfrac{n}{n+b}, \\ P_{422} = \dfrac{b}{n+b}, \end{cases} \quad (30)$$

$$\begin{cases} P_{431} = \dfrac{p}{p+m}, \\ P_{432} = \dfrac{m}{p+m}. \end{cases} \quad (31)$$

Equations for probabilities P_{11}, P_{12}, (32), P_{21}, P_{22}, P_{23} (33), P_{31}, P_{32}, P_{33} (34) and P_{41}, P_{42}, P_{43} (35):

$$\begin{cases} P_{11} = \dfrac{I}{I+U}, \\ P_{12} = \dfrac{U}{I+U}, \end{cases} \quad (32)$$

$$\begin{cases} P_{21} = \dfrac{AS}{AO+AS+OS}, \\ P_{22} = \dfrac{OS}{AO+AS+OS}, \\ P_{23} = \dfrac{AO}{AO+AS+OS}, \end{cases} \quad (33)$$

$$\begin{cases} P_{31} = \dfrac{FH+GH}{DG+DH+FH+GH}, \\ P_{32} = \dfrac{DH}{DG+DH+FH+GH}, \\ P_{33} = \dfrac{DG}{DG+DH+FH+GH}, \end{cases} \quad (34)$$

$$\begin{cases} P_{41} = \dfrac{KZ+LZ}{JL+JZ+KZ+LZ}, \\ P_{42} = \dfrac{JZ}{JL+JZ+KZ+LZ}, \\ P_{43} = \dfrac{JL}{JL+JZ+KZ+LZ}. \end{cases} \quad (35)$$

Equations for probabilities P_1, P_2, P_3 and P_4 (36):

$$\begin{cases} P_1 = \dfrac{EYW+YRW+ETW}{EYQ+YQR+EQT+EYW+YQW+YRW+QRW+ETW+QTW}, \\ P_2 = \dfrac{EYQ+YQR+EQT}{EYQ+YQR+EQT+EYW+YQW+YRW+QRW+ETW+QTW}, \\ P_3 = \dfrac{YQW+QTW}{EYQ+YQR+EQT+EYW+YQW+YRW+QRW+ETW+QTW}, \\ P_4 = \dfrac{QRW}{EYQ+YQR+EQT+EYW+YQW+YRW+QRW+ETW+QTW}, \end{cases} \quad (36)$$

Based on the derived equations (21) to (36), the numerical values of the probabilities of finding RFS in a certain state were obtained (Table 4).

Analysis of the results (Table 4) reflects a separate study on the aggregates of states of RFS. However, considering that the value of this model lies precisely in the complex study of the states of the system, it is appropriate to determine the probabilities precisely in the set of states (Table 5).

Table 4

Numerical values of the probabilities of finding RFS in a certain state when considering each set of states separately

Probability	Value	Probability	Value	Probability	Value
P_1	0.468	P_{42}	0.240	P_{312}	0.219
P_2	0.234	P_{43}	0.158	P_{321}	0.511
P_3	0.186	P_{111}	0.444	P_{322}	0.185
P_4	0.112	P_{112}	0.556	P_{323}	0.305
P_{11}	0.667	P_{121}	0.377	P_{331}	0.595
P_{12}	0.333	P_{122}	0.623	P_{332}	0.405
P_{21}	0.498	P_{211}	0.595	P_{411}	0.168
P_{22}	0.199	P_{212}	0.405	P_{412}	0.276
P_{23}	0.302	P_{221}	0.286	P_{413}	0.556
P_{31}	0.583	P_{222}	0.714	P_{421}	0.569
P_{32}	0.278	P_{231}	0.569	P_{422}	0.431
P_{33}	0.139	P_{232}	0.431	P_{431}	0.333
P_{41}	0.602	P_{311}	0.781	P_{432}	0.667

Table 5

Numerical values of the probabilities of finding RFS in a certain state when considering the entire set of states

Probability	Value	Probability	Value	Probability	Value
P_1	0.468	P_{42}	0.027	P_{312}	0.024
P_2	0.234	P_{43}	0.018	P_{321}	0.026
P_3	0.186	P_{111}	0.139	P_{322}	0.010
P_4	0.112	P_{112}	0.174	P_{323}	0.016
P_{11}	0.312	P_{121}	0.059	P_{331}	0.015
P_{12}	0.156	P_{122}	0.097	P_{332}	0.010
P_{21}	0.117	P_{211}	0.070	P_{411}	0.011
P_{22}	0.047	P_{212}	0.047	P_{412}	0.019
P_{23}	0.071	P_{221}	0.013	P_{413}	0.037
P_{31}	0.108	P_{222}	0.033	P_{421}	0.015
P_{32}	0.052	P_{231}	0.040	P_{422}	0.012
P_{33}	0.026	P_{232}	0.031	P_{431}	0.006
P_{41}	0.067	P_{311}	0.085	P_{432}	0.012

Analysis of the results given in Table 5 reveals that the specified model can be used both for the study of individual groups of RFS states (Table 4) and according to the level of the hierarchy (Table 5).

The next step, according to the conditions of the example, is to check the achievement of the set probability level, in particular, the given probability of being in a certain state $P_{set}=0.01<P_{421}=0.015$. Thus, the goal of researching RFS functioning with the help of an adapted model of RFS functioning has been achieved.

The next step of the research is to check the model for adequacy. It is proposed to carry out this check by comparing the simulation results with the results obtained on the tested models [37, 42]. For verification, the state characterized by the maneuver on FP was chosen: S_{41}, according to the proposed model, S_1 – the model proposed in paper [37], and S_4 – the model proposed in report [42].

With regard to the parameters that were used in RFS models, these are states that are directly related to ensuring the preservation of the combat-ready state of RFS (S_4) [30]. The state of the maneuver on FP (S_{41}) was directly considered. Accordingly, transitions from the state of maneuvering on FP to occupying FP ($\lambda_{41\to42}$), from occupying FP to maneuvering to another FP ($\lambda 42\to41$). Also, transitions from occupying the FP to leaving the FP ($\lambda_{42\to43}$), from leaving FP to the maneuver on FP ($\lambda_{43\to41}$).

The results of the simulation of the process of RFS operation using proven models were carried out on 10 different sets of input data (Table 6). This makes it possible to achieve a confidence probability of at least 0.82 according to rule (37) [43, 44]:

$$P_{con} \le \frac{\Omega-1}{\Omega+1}, \quad (37)$$

where Ω is the number of experiments.

Analysis of the simulation results (Table 6) reveals that the deviation of the results is in the range from 2 to 9 %, which does not exceed the limit of statistical error (10 %). For a better interpretation of the simulation results, the simulation results are shown in Fig. 7.

The results of the experimental studies were obtained by modeling the process of RFS operation, in particular the state of the maneuver on FP, using existing models No. 1 [42], No. 2 [37], and the proposed adapted model.

Analysis of the simulation results using different models (Table 6, Fig. 7) reveals that the deviation of the data is within the limits of statistical error and does not exceed 10 %. Thus, the proposed adapted model is adequate and can be used to model the processes related to RFS functioning.

The main advantages of the proposed model are its modularity, i.e., it is possible to model both the general process of RFS functioning and its elements. This is explained by the fact that the model is built according to the levels of the hierarchy of functioning processes.

Another advantage of the proposed adapted model is the detailed simulation of the process of RFS operation with 39 system states, which is too difficult to do when using existing approach. This is explained by the fact that the processes are considered as a set of separate sub-processes, which makes it possible to distribute the modeling process and simplify it, respectively.

Table 6

Results of simulating the process of functioning of RFS elements according to the adapted and existing models

No. Ω	Intensity of transitions				Simulation results					Mean Δ
	$\lambda_{41\to42}$	$\lambda_{42\to41}$	$\lambda_{42\to43}$	$\lambda_{43\to41}$	Adapted model	Model 1 [41]		Model 2 [36]		
					P_{41}	P_4	Δ	P_1	Δ	
1	0.33	0.50	0.33	0.50	0.602	0.584	0.03	0.554	0.08	0.06
2	0.50	0.33	0.25	0.33	0.400	0.396	0.01	0.372	0.07	0.04
3	0.50	0.50	0.50	0.50	0.500	0.460	0.08	0.485	0.03	0.06
4	0.25	0.25	0.33	0.50	0.583	0.554	0.05	0.577	0.01	0.03
5	0.25	0.20	0.33	0.20	0.444	0.431	0.03	0.417	0.06	0.05
6	0.20	0.20	0.17	0.50	0.579	0.544	0.06	0.538	0.07	0.07
7	0.33	0.17	0.50	0.17	0.333	0.326	0.02	0.326	0.02	0.02
8	0.25	0.33	0.17	0.25	0.545	0.512	0.06	0.501	0.08	0.07
9	0.20	0.25	0.33	0.20	0.522	0.485	0.07	0.512	0.02	0.05
10	0.50	0.50	0.50	0.17	0.333	0.310	0.07	0.303	0.09	0.08

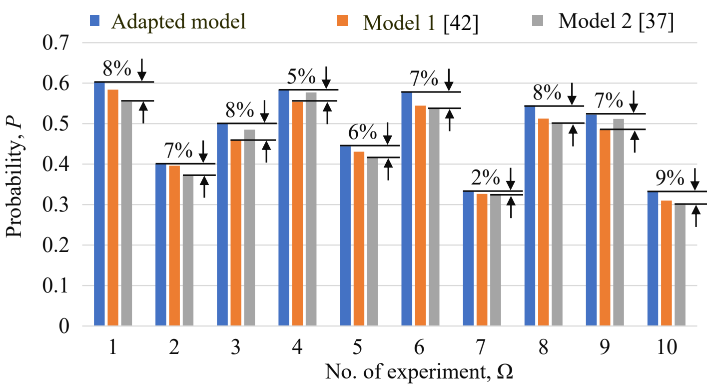

Fig. 7. Graphic representation of results of simulating the process of functioning of RFS elements according to the adapted and existing models

The main disadvantage of this model is that if it is necessary to introduce a new state into the existing model, it is necessary to review all formula dependences because they are interconnected. However, this drawback can be overcome using machine programming or a simulation environment similar to MATLAB (USA).

The main assumption regarding is that the flow of transitions from state to state of the system is the simplest (Poissonian) and is subject to restrictions on aftereffect, ordinariness, and stationarity [45].

The main simplification adopted in the work is that the list of states is constant and requires the formalization of all processes under these states. However, considering that the states are determined based on the generalization of existing studies and based on the results of the test of the adequacy of the adapted model, it can be argued that this does not significantly affect the result.

6. Discussion of results related to devising approaches to the construction of an adapted operational model of a reconnaissance-fire system

An improved procedure for building an adapted model of RFS functioning has been proposed (Fig. 4). It has been established that its application could significantly simplify the processes, significantly simplify the modeling of the process of RFS operation and, accordingly, ensure the accuracy of forecasting while maintaining relative simplicity. This is explained by the fact that the distribution of states by hierarchy levels (Fig. 3) allows simulation with a relatively small number of states, taking into account their relationship with other levels of the hierarchy.

The application of this procedure will make it possible to adapt the model of RFS functioning directly to the conditions and goals of the study (Fig. 2). Moreover, this approach makes it possible to overcome the shortcomings of existing approaches, in particular, low modeling accuracy (4) and high complexity of model formalization (Fig. 3).

The essence of the developed and improved methodology for building an adapted model of RFS functioning is the formalization of processes through the determination of system states and intensities of transitions from state to state. The improved procedure is based on the Kolmogorov-Chapman equations (1), (2), and the method of building a goal tree (Fig. 3).

A feature of the developed improved methodology for building an adapted model of RFS functioning is the division of RMS states by hierarchy levels (Fig. 3), which allows taking into account more system states.

Unlike studies [8, 10–12, 15], which consider a limited number of states of one level of the hierarchy, the proposed methodology considers the aggregation of states at different levels of the hierarchy. The specified feature of the developed improved methodology resolves the task associated with the impossibility of taking into account the required number of states according to the existing approaches to modeling the process of RFS operation. This is explained by the fact that the modeling is proposed to be carried out in accordance with the levels of the hierarchy of objectives defined using the tree method. This makes it possible to adapt existing scientific and methodical apparatus to the needs of modeling the process of RFS functioning in accordance with existing conditions.

The limitations of the developed improved methodology are that the simulation results can be applied under the condition that the functioning of the support subsystems is limited. That is, the simulation results will reflect the functioning process with a certain accuracy in the event that the support subsystems will fully and timely perform their functions. However, these limitations can be overcome by introducing additional states of RFS, in particular, taking into account the refusals to perform tasks due to non-provision. When using the approach proposed in the study, these limitations must be taken into account in order to obtain accurate data.

The disadvantage of the developed improved methodology is the dependence of the formalization of states on the level of experience of the researcher. However, it should be noted that the use of expert evaluation methods could significantly reduce the level of subjectivity in the formalization of RFS states.

The improved methodology for building an adapted model of RFS functioning can be advanced by studying the non-stationary flows of events (transitions). This would significantly expand the limits of application of the proposed improved procedure.

An adapted model of RFS operation has been built (Fig. 6, (5) to (20)), which is based on the improved methodology for building an adapted model of RFS operation (Fig. 4). The essence of the model is to determine the probabilities of RFS being in a certain state on the basis of the Chapman-Kolmogorov equations (21) to (36), taking into account the necessary degree of detailing of RFS functioning process (Fig. 5, 6). The refined model was constructed based on the generalization of existing approaches to the modeling of individual functioning processes or the modeling of general processes. In contrast to existing models, the developed adapted model makes it possible to model both the system as a whole (Table 5) and separate subsystems (Table 4).

Checking the adequacy of the model for the accepted conditions (Table 1) by comparing the simulation results with known models (Table 6, Fig. 7) proves that the developed adapted model is adequate. This is explained by the fact that the discrepancy of the results is within the statistical error from 2 to 9 %. Also, the number of conducted experiments makes it possible to ensure the condition when the confidence probability is within 0.82 (37). However, it is possible to assume that increasing the number of experiments to ensure a higher confidence probability would not significantly change the trend.

The peculiarity of the developed adapted model of RFS functioning (Fig. 6, (5) to (20)) is that it is based on the Chapman-Kolmogorov equations and the goal tree method.

Unlike studies [13–16], in which general modeling is performed, the developed model allows for both general (Table 5) and distributed (Table 4) simulation. This will significantly expand the limits of this model's application and unify the modeling processes of various subsystems (subprocesses).

The developed adapted model resolves the task regarding the complexity of the formalization of the process of RFS functioning due to the limited number of states that can be taken into account according to the existing approaches. There is also a lack of approaches to building a model of RFS functioning, which would take into account the influence of all subsystems. This is explained by the fact that the combination of the goal tree method and the Chapman-Kolmogorov equations allows one to combine the modeling of processes at different levels of the hierarchy while preserving the integrity of the overall functioning process.

This makes it possible to increase the degree of detailing of the processes of RFS operation during the planning of military operations and the study of processes related to combat operations.

The advantage of the developed model is its modularity, i.e., it is possible to simulate both the general process of RFS functioning and its elements. This is explained by the fact that the model is built according to the levels of the hierarchy of functioning processes. Also, the advantage of the model is the detailed simulation of the process of RFS operation with 39 states of the system, which is too difficult to do using existing models. This is explained by the fact that the processes are considered as a set of separate sub-processes, which makes it possible to distribute the modeling process and simplify it, respectively.

The limitations of this model are that the flow of transitions from state to state of the system is the simplest and is subject to aftereffect, ordinariness, and stationarity constraints. Also, the list of states is permanent and requires the formalization of all processes specifically for these states. However, considering that the states are determined based on the generalization of existing studies and based on the results of the test of the adequacy of the adapted model, it can be argued that this does not significantly affect the result.

The disadvantage of the model is the need to review all model elements in the case of introducing a new state into the existing model. However, this disadvantage can be overcome using machine programming or a simulation environment similar to MATLAB.

Another area of our future research may be the development of approaches to the modeling of the process of RFS operation, taking into account the support subsystems. This could significantly expand the range of application scope of this approach and create prerequisites for the construction of a comprehensive model of the functioning of an operational group of troops during activities.

7. Conclusions

1. An improved methodology for building an adapted model of RFS operation has been proposed. It has been established that its application could significantly simplify the processes, significantly simplify the modeling of the process of RFS operation and, accordingly, ensure the accuracy of forecasting while maintaining relative simplicity. A feature of the proposed methodology is the breakdown of the states of the system by hierarchy levels, which makes it possible to take into account more states of the system. In contrast to previous studies, which consider a limited number of states of one level of the hierarchy, the proposed methodology considers the aggregation of states at different levels of the hierarchy. The proposed algorithm makes it possible to resolve the task associated with the lack of an approach to the construction of a model of RFS operation, taking into account the necessary set of states. It also makes it possible to adapt existing scientific and methodological apparatus to the needs of modeling the process of RFS operation in accordance with existing conditions. The field of application of the proposed improved methodology is planning and management processes while developing action algorithms during combat operations.

2. An adapted model of RFS operation has been developed. The essence of the model is to determine the probabilities of RFS being in a certain state based on the Chapman-Kolmogorov equations, taking into account the necessary degree of detailing of the RFS functioning process. The adapted model built allows simulation both for the system as a whole and separately for subsystems. The peculiarity of the proposed model is that it is based on the Chapman-Kolmogorov equations and the goal tree method. A distinctive feature of the model is that it makes it possible to model with a relatively large number of states with the required accuracy. This is explained by the fact that the test of the adequacy

of the model showed that the discrepancy of the results is within the statistical error from 2 to 9 %. Also, the number of conducted experiments makes it possible to ensure the condition when the confidence probability is within 0.82. The field of application of the adapted model of RFS functioning is the process of making a decision on the use of RFS during hostilities and their management during military operations.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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