

*The object of the research is to identify the influence of the pitch of the hammers with cutting edges on the determination of the average length of feed, crushed from by-product stem raw materials in hammer crushers. However, a problem was identified such as the lack of an analytical expression for determining the average length of chopped stem feed by a hammer grinder depending on the distance between the edges of adjacent hammers. With a simultaneous change in the angles of the location of the stem with the maximum size in the layer and, using the methods of probability theory for determining the mathematical descriptions of the function of two arguments, in this study an analytical expression was obtained for determining the average length of particles crushed by hammer crushers depending on the distance between the faces of adjacent hammers. Hammers with cutting edges are used to crush feed from by-product stem raw materials. The end working part of the hammer is milled at an angle of  $90^\circ$ , i.e. two cutting edges are processed at the end of the hammer. As a result of the practical calculations, the average length of feed crushed from by-product stem raw materials was determined depending on the parameters of the hammer crusher. From the resulting graph of the influence of the distance between the faces of adjacent hammers on the average length of crushed particles, it is evident that when the distance between the faces of adjacent hammers changes, the average length of crushed particles varies within the range of 36.19...53.39 mm. To verify the reliability of the obtained expression, experiments were carried out on a crusher with a distance between the faces of adjacent hammers of 20 mm. In this case, the average length of the crushed particles was equal to 38.75 mm, and the theoretical value was 40.16 mm. The difference between them was 3.64 %, which proves the reliability of the obtained analytical expression. The conducted theoretical studies with the receipt of an analytical expression, ensuring the determination of the average length of feed, chopped from by-product stem raw materials, is the solution to an important problem in the development of theoretical foundations of chopping machines, significantly affecting the efficiency of mechanization and operation of feed choppers in practice*

*Keywords: grinder, length of grinding, edges of adjacent hammers, angle of stem arrangement*

# IDENTIFYING THE INFLUENCE OF THE PITCH OF HAMMERS WITH CUTTING EDGES ON THE AVERAGE LENGTH OF FEED CRUSHED IN HAMMER GRINDERS

**Tokhtar Abilzhanuly**

Doctor of Technical Sciences, Professor  
Laboratory of Innovative Equipment for Animal Husbandry\*

**Ruslan Iskakov**

Corresponding author

PhD, Associate Professor

Department of Technological Machines and Equipment\*\*

E-mail: rus.iskakov79@mail.ru

**Daniyar Abilzhanov**

PhD, Leading Researcher\*

**Alexandr Gulyarenko**

PhD, Associate Professor

Department of Standardization, Metrology and

Certification\*\*

**Valery Khan**

PhD in Engineering Science, Senior Lecturer

Department of Technological Machines and Equipment\*\*

\*Scientific Production center

of Agricultural Engineering, LTD

Raiymbek ave., 312, Almaty,

Republic of Kazakhstan, 050005

\*\*S. Seifullin Kazakh Agro Technical Research University

Zhenis ave., 62, Astana, Republic of Kazakhstan, 010011

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## 1. Introduction

For the preparation of feed, it is possible to use stem raw materials, and especially by-product stem raw materials, often obtained as a result of the preparation of medicines. The use of waste raw materials in the preparation of feed has a beneficial effect on the environment and contributes to the sustainable state of the agro-industrial complex [1]. In addition, the desire for rational and comprehensive processing of agricultural waste is an economically advantageous direction

for obtaining additional income [2]. Processing of waste feed raw materials implies the use of technological equipment [3]. Hammer mills are especially widely used, having a number of advantages over others in terms of grinding efficiency, ease of maintenance and high technical and economic characteristics [4]. Great importance in hammer mills is given to the design and operation of hammers, a model for predicting the percentage of crushed particles [5]. As is known, the quality of grinding of stem feed, including from by-product raw materials, is estimated by the average length of the crushed feed.

When analyzing the results of the experiment, mathematical dependencies are obtained that describe the change in the average weighted length of the chopped raw material in the form of linear functions [6]. This is due to the fact that the possibility of complete consumption and assimilation of chopped feed is interconnected with the justified length of the chopped stem feed. For example, for sheep, the length of the chopped stem feed should be within 20...30 mm, and for cattle 30...50 mm.

It is important to use optimal methods in designing, assembling [7], calculating, varying and justifying the working bodies of machines. Attention should be paid to the study of the designs of the working bodies of crushers and the derivation of analytical expressions for predicting the quality of crushed feed. In research, it is necessary to study the design parameters of the working bodies of the crusher, affecting the quality of crushing and the crushing force of feed raw materials [8]. Determining the parameters of crushing machines depending on the size of the crushed particles is important in the development of feed crushers, working bodies of a hammer crusher, forecasting and optimizing straw crushers [9]. In addition, conducting theoretical studies to obtain analytical expressions that ensure the determination of the average length of feed crushed from by-product stem raw materials is a solution to an important problem in the development of the theoretical foundations of crushing machines.

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## 2. Literature review and problem statement

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Using the methods of probability theory, in [10] an analytical expression was obtained for determining the mass fraction of crushed particles depending on the pitch of the hammers and counter-hammers. However, the determination of the average length of crushed feed in hammer crushers has not been carried out.

For high-quality grinding of grain feed in sieveless grinders, a grinding method of «beveling – cutting» between two cutting edges of hammers and sharp edges of the deck faces has been proposed [11]. However, not enough attention has been paid to predicting and determining the average sizes of the ground particles, which is the most important characteristic when assessing the quality of grinding.

In the work [12] the scheme of arrangement of pre-crushed particles in horizontal and vertical planes during their metered feeding is considered. An analytical expression is obtained for determination of uniformity of feeding depending on the length of feed particles, their mass, density, humidity and geometry of the cross-section of the feed flow. However, an analytical expression is not derived for determination of the average length of crushed particles depending on the distance between the faces of adjacent hammers, the length of the loaded part and the height of the feed layer.

As a result of theoretical studies, an analytical expression was obtained for determining the average size of crushed particles in crushers with hammers with cutting edges. Here, the process of arranging stems in a layer in horizontal and vertical planes is considered separately [13]. However, the change in the arrangement of stems in the layer of supplied feed is not considered simultaneously in two planes, which occurs when crushing in a hammer mill.

The paper [14] provides a rationale for the parameters of a mini-crusher-grinder for processing coarse forage stalks in farms with a small livestock population, which has a rotary crusher that combines crushing and grinding operations.

Experiments have shown that using a flat-ground hammer with a lower sharpening and a rotation angle of 60° or more ensures high-quality crushing of stalks. However, the issue of determining the average length of the crushed forage has not been resolved.

The work [15] points out the banana pseudostem, which is a by-product of the planting process in tropical zones of China. The paper discusses a horizontal return chipper suitable for the banana pseudostem, which was bulky, fragile and had a high moisture content. The motion and force of the chopping blade were analyzed. The average length of the chopped stem was 57 mm, which met the agronomic requirements of straw recovery. However, the paper did not study the direction of obtaining analytical expressions for determining the average length of the chopped pseudostem, which would have a positive effect on predicting the process of chopping banana pseudostems.

In [16], the study showed that to improve the efficiency of hammer crushers, it is important to substantiate the parameters of the hammer rotor and the method of feeding raw materials into the crusher working chamber. The experiment used a stand that allows determining the strength of stems during destruction. The average diameter of clover samples was 1.64 mm, lupine 3.3 mm, respectively, the length of the samples was 50 and 100 mm. The results of experimental studies at different values of hammer speed and a crusher grinding chamber width of 180 mm showed that, regardless of the feeding method and type of processed raw materials, the yield of crushed products was distributed very unevenly across the width of the working chamber. At the same time, the work does not derive analytical expressions that help determine the average length of crushed stems, depending on the location and action of the hammers of the hammer crusher.

In [17], a study was conducted of the movement of particles of crushed stems interacting with hammers inside the crusher. An analytical expression was obtained to determine the speed of movement of particles of crushed stems interacting with hammers. It was found that intensive crushing of stems of coarse feed by rotor hammers is ensured at their peripheral speed of 30–35 m/s. The speed of particles from the input to the output of the working chamber gradually increases and is 15–16 m/s. The productivity of the crusher depends on the angular velocity and radius of the rotor, as well as on the density and feed rate of the processed material. However, the issue of determining the average length of crushed feed is not fully taken into account.

In [18], a need was identified for a more efficient and safe method of adjusting the distance between the swing of the crusher hammers and the fixed lattice bars. The current method of adjusting the gap is inaccurate, time-consuming and labor-intensive, and requires theoretical justification with the derivation of analytical expressions to determine the average size of the crushed particles, which optimizes the performance of the crusher hammers.

A review of previous studies shows that researchers are working to determine the quality of chopped stem feed depending on their physical and mechanical properties and machine parameters.

In previous studies, the process of stem arrangement was considered in two planes separately. However, the change in the arrangement of stems in the layer of feed supplied to the grinding chamber will be carried out simultaneously in two planes. Therefore, determining the average length of chopped feed from by-product stem raw materials in hammer grinders,

using more modern methods of probability theory, is a solution to an urgent problem in developing the theoretical foundations of feed grinders.

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### 3. The aim and objectives of the study

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The aim of the study is to obtain an analytical expression that ensures the determination of the average length of feeds crushed from by-product stem raw materials depending on the pitch of the arrangement of hammers with cutting edges in hammer crushers. This will make it possible to determine the values of rational parameters and kinematic modes of feed grinders, as well as solve an important problem in developing the theoretical foundations of grinding machines.

To achieve this aim, the following objectives are accomplished:

- determine the maximum length of crushed particles when stems are hit by adjacent hammers;
- determine the average length of feed crushed from secondary stem raw materials by a hammer working element;
- conduct experiments and calculations to determine the average length of chopped feed.

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### Materials and methods of the study

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The object of the study is to identify the influence of the pitch of the hammers with cutting edges on the determination of the average length of feed, crushed from by-product stem raw materials in hammer crushers. The main hypothesis of the study is that, depending on the feed rate into the grinding chamber, the peripheral speed of the hammers and the number of hammer rows, each row of hammers separates a portion of stem feed of a certain length from the feed layer and this length should be determined theoretically by calculation based on obtaining an analytical expression. Therefore, the need to obtain an analytical expression for determining the average length of the crushed parts is necessary. At the same time, the assumption made in this study was that the forage stem mass falling under the blows of adjacent hammers will be crushed depending on the location of the stems in a layer of a certain length, width and height. When entering the grinding chamber, the layer of forage stem raw material has the shape of a parallelepiped. In this case, the maximum length of the stem, depending on the arrangement of the stems, can have a certain length. All these data of the figure under consideration have the same size of the angle of arrangement, and inside the parallelepiped they are also the same. Therefore, to determine the average length of crushed stems, the location of a stem of a certain length with angles of location in the horizontal plane and vertical plane was considered. With known values of the parameters of the feed layer, the maximum length of the stem, the values of the angles of arrangement in the horizontal plane and vertical plane were determined by special formulas. The simplification adopted in this work was that the determination of the maximum length of crushed particles when the stems fall under the blows of adjacent hammers is rationally made on the basis of the developed scheme of the arrangement of the stems in the layer enclosed by the walls of adjacent hammers, the length of the loaded part of the hammers and the height of the feed layer, taking into account the diagonal arrangement of the stem in the parallelepiped.

When preparing feed, it is necessary to grind the particles to the required size. In the course of theoretical and experi-

mental studies, a special hammer crusher for stalk feed was used. The design and technological scheme of the crusher for feed from by-product stalk raw materials includes a belt-feed conveyor, a pressing drum, a hammer rotor, and a deflector. The crushing of stalk feed was carried out by the working parts of the hammer crusher, i.e. hammers with cutting edges. In this case, the distance between the edges of adjacent hammers in the hammer crusher was 20 mm. The power of the electric motor of the hammer crusher was 15 kW. To obtain the same productivity, 1.0 kg of alfalfa was spread out on 1 meter of the conveyor. This corresponds to the productivity of the hammer crusher of 5.4 t/h. During the experiments, the power was not measured. At the same time, it is known from previously conducted studies that for a crusher productivity of 5.4 t/h, the value of the power consumption when crushing alfalfa hay was 13.6 kW.

During the experiments, samples were taken to determine the average size of the crushed particles. Due to the complexity of conducting experiments at other levels of hammer spacing (10, 30, 40 mm), experiments were not conducted.

To determine the maximum length of crushed feed from by-product stem raw materials, the layout of stems in the feed layer into the grinding chamber was considered.

The theoretical determination of the average length of crushed particles ensures the determination of the optimal pitch of the hammers in the rows, depending on the required size of the crushed particles. In reality, the arrangement of the stems in the layer simultaneously changes in the vertical and horizontal planes. This explains the reliability of the chosen method for solving problems. The peculiarity of this method for determining the average length of crushed particles is the simultaneous change in the angles  $\theta$  and  $\varphi$ . At the same time, this method is used in probability theory to determine mathematical descriptions of functions of several arguments. Therefore, the obtained analytical expression is of primary importance when choosing the pitch of adjacent hammers in one row for crushing stem feed. The influence of the distance between the faces of adjacent hammers on the values of the average length of crushed particles was determined by calculation.

The length of the layer when the feed mass enters the grinding chamber, depending on the kinematic parameters of grinding and the number of encounters of the hammers with the feed mass, was determined using the derived formulas. The height of the feed mass layer was selected based on the wear of the hammers along the length during the operation of the hammer grinders, the entry of the feed mass into the grinding chamber and the movement of the feed mass in the grinding chamber.

The average size of the crushed particles was determined using the method of determining the numerical characteristics of the random variable function of probability theory. The influence of the distance between the faces of adjacent hammers on the average length of crushed particles was determined by calculation. The analytical expression for determining the average length of chopped feed when simultaneously hit by adjacent hammers was obtained using methods of probability theory and mathematical analysis. It is clear from this that the obtained analytical expression determines the step of placement between the faces of adjacent hammers, i.e. the main parameter of the technological process of grinding stem feed raw materials.

To determine the experimental values of the average length of the crushed particles, the method of conducting a single-factor study was used. Hammers with cutting edges were used to crush feed from by-product stem raw materials.

In this case, the end working part of the hammer was milled at an angle of 90°, i.e. two cutting edges were processed at the end of the hammer. In experiments on crushing stalked fodder, the feed conveyor belt speed was 1.5 m/s, the peripheral speed of the hammers was 60 m/s, and the number of hammer rows was 4 pcs. The crusher had hammers 10 mm thick. The experiments were conducted on waste from freshly mown alfalfa with a moisture content of 63.36 %. The average length of crushed particles was determined based on the work of three samples. The results of analyzing samples of crushed stem raw materials are given in a separate table. The standard deviation was determined using a special formula and experimental data. Experimental determination of the sizes of crushed particles of stem feed raw materials was carried out by analyzing samples using measuring instruments (a ruler with a measurement of up to 500 mm with a maximum deviation of no more than 3 microns, a measuring tape 2 m long with a measurement error of ±0.3 mm). The use of laboratory electronic scales MW-300T made it possible to output the result of weighing the feed material as a percentage of the norm. Communication with an external device via an RS-232C interface connector. Scale accuracy class according to MP 76 MOZM. The permissible error limit is 0.01 g. When analyzing three samples, the average length of the crushed particles was 38.75 mm, the theoretical value of the average length of the crushed particles was 40.16 mm. Moreover, the difference between the theoretical and actual values of the average length of crushed particles was only 3.64 % error, which proves the reliability of the theoretical studies. The reliability of the studies was carried out by comparing the theoretically calculated and experimentally determined average lengths of crushed particles. When determining the average length of crushed particles, well-known statistical methods for processing empirical data were used [19].

When designing a hammer crusher, the KOMPAS-3D v23 program was used to create three-dimensional models and technical drawings. Typical calculations were performed using built-in tools in the Kompas 3D application library program. Mathcad Prime 8 was used to perform mathematical calculations and data analysis, facilitating the process of calculating formulas and numerical data. To determine the effect of the distance between the faces of adjacent hammers, calculations were performed for different values of the distance between the faces of adjacent hammers. The calculation results are presented in the form of the average length of crushed particles, depending on the distance between the faces of adjacent hammers, i.e. a graphical dependence of the effect of the distance between the faces of adjacent hammers on the average length of crushed particles was obtained by calculation. Microsoft Excel was used to process and analyze the data, which allows to effectively build diagrams and graphs. To create more complex visualizations and diagrams, specialized graphic editors CorelDRAW Graphics Suite 2024, Adobe Photoshop and Illustrator were used, which provide extensive opportunities for working with vector graphics and allow to create detailed and professional images.

**5. Results of research on determining the average length of crushed particles during operation of hammer crushers**

**5.1. Determination of the maximum length of crushed particles when stems are hit by adjacent hammers**

For grinding feed from by-product stem, grain [20] and bone raw materials, grinders equipped with hammer working

elements are widely used. In this case, a hopper with flaps is used to feed grain and bone feed into the grinding chamber, and in many cases a belt conveyor with a pressing drum is used to feed stem feed. In this case, the design and technological scheme (3D) of grinders has the following appearance (Fig. 1).

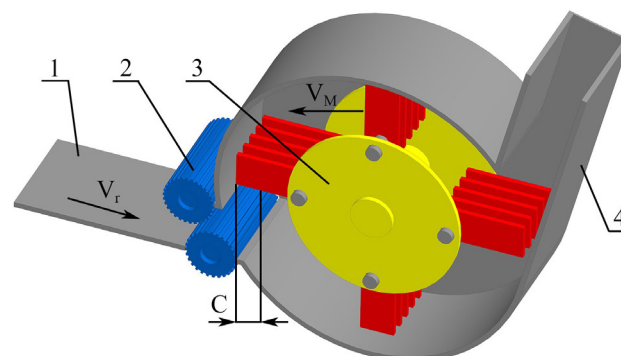


Fig. 1. Design and technological scheme (3D) of a feed grinder from by-product stem raw materials: 1 – belt feed conveyor; 2 – pre-pressing drum; 3 – hammer rotor; 4 – deflector

Let’s consider the determination of the maximum length of crushed particles when stems are hit by hammer blows. Depending on the feed rate into the grinding chamber  $v_T$ , the peripheral speed of the hammers  $v_M$  and the number of rows of hammers  $K_M$ , each row of hammers separates a portion of length  $C$  from the fed layer and this length is determined by the formula:

$$C = v_T \cdot t_K, \tag{1}$$

where  $t_K$  – the time spent on turning the rotor between adjacent rows of hammers, which is determined by the formula:

$$t_K = \frac{60}{n_p \cdot K_M}, \tag{2}$$

where  $n_p$  – hammer rotor speed,  $\text{min}^{-1}$ .

In this case, the value of  $C$  is determined by the formula:

$$C = \frac{v_T \cdot 60}{n_p \cdot K_M} = \frac{v_T \cdot 2\pi \cdot R_p}{v_M \cdot K_M}, \tag{3}$$

where  $R_p$  – rotor radius at the ends of the hammers.

In hammer crushers, the peripheral speed of the hammers is set at about 60 m/s, i.e. this speed is destructive for feed from by-product stem raw materials, regardless of their moisture content. Therefore, it is possible to assume that the mass falling under the blows of adjacent hammers will be crushed depending on their location in a layer of length  $a$ , width  $C$  and height  $h_C$  (Fig. 2).

At the entrance to the grinding chamber, the layer has the shape of a parallelepiped  $ABCDMNLK$ . In this case, the maximum length of the stem, depending on the arrangement of the stems, can have a length of  $CM, DN, AL, BK$ . All these data of the figure under consideration have the same size of the angle of arrangement, and inside the parallelepiped they are also the same. Therefore, to determine the average length of the crushed stems, it is necessary to consider in this study the location of the stem of length  $CM$  with angles of arrangement in the horizontal plane  $\theta$  and vertical plane  $\varphi$ . With known

values of the layer parameters, the maximum length of the stem  $l_m$  is determined by the formula:

$$l_m = \sqrt{a^2 + c^2 + h_c^2}. \quad (4)$$

In this case, the values of the angles can also be determined using the formulas:

$$\begin{aligned} \theta &= \arccos \frac{a}{\sqrt{a^2 + c^2}}, \\ \varphi &= \arccos \frac{\sqrt{a^2 + c^2}}{l_m}. \end{aligned} \quad (5)$$

Thus, it is rational to determine the maximum length of crushed particles when stems fall under the blows of adjacent hammers on the basis of the developed scheme of the arrangement of stems in a layer enclosed by the walls of adjacent hammers, the length of the loaded part of the hammers and the height of the fed layer, taking into account the diagonal arrangement of the stem in the parallelepiped.

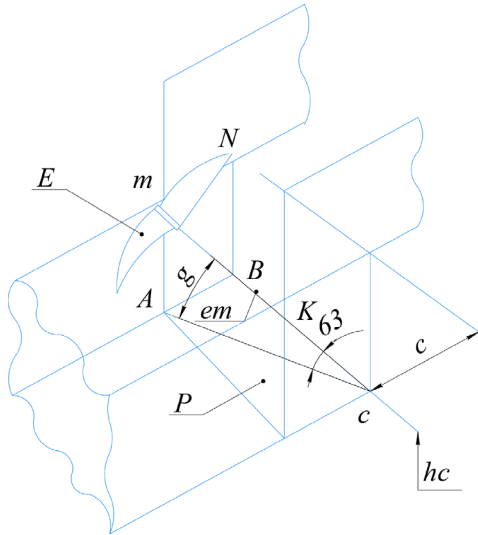


Fig. 2. Schematic diagram of the arrangement of stems of length  $l_m$  at the entrance of the layer into the grinding chamber

### 5.2. Determination of the average length of feeds crushed from by-product stem raw materials using a hammer working element

In case of determining the average length of feed, crushed from by-product stem raw materials by a hammer working element, the location of the stem length the location of the stem with length  $l_m$  depends on the angles  $\theta$  and  $\varphi$  and the orientation of the stem within these angles is equally probable, i.e. depends on two random variables, and the law of distribution of arguments is known.

In this case, the mathematical expectation of the function  $\varphi(x, y)$  from two random variables  $x$  and  $y$  for continuous random variables [21]:

$$M[\varphi(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varphi(x, y) f(x, y) dx dy, \quad (6)$$

where  $\varphi(x, y)$  – argument distribution function;  $f(x, y)$  – argument distribution density.

The direction of the stem  $l_m$  will be characterized by a unit vector  $l_m$ . The direction of the vector  $l_m$  in the spherical coordinate system associated with the plane  $P$  onto which the projection is made is determined by two angles: the angle  $\theta$  lying in the plane  $P$ , and the angle  $\varphi$  lying in the plane perpendicular to  $KP$ . When changing the specified angles, the vector  $l_m$  describes a spherical surface  $E$  and the entire direction  $l_m$  has the same probability density, therefore the probability element:

$$f(\theta, \varphi) d\theta d\varphi, \quad (7)$$

where  $f(\theta, \varphi)$  – angle distribution density  $\theta, \varphi$ .

When changed, these angles form an elementary area  $dS$  on the sphere  $E$ , which is expressed by the formula:

$$dS = d\theta d\varphi \cos \varphi. \quad (8)$$

Where the probability element is expressed by the formula:

$$\begin{aligned} f(\theta, \varphi) d\theta d\varphi &= A \cos \varphi d\theta d\varphi, \\ f(\theta, \varphi) &= A \cos \varphi, \end{aligned} \quad (9)$$

where  $A$  – proportionality coefficient.

Here the proportionality coefficient is determined by the formula:

$$\begin{aligned} &\int_0^{\arccos \frac{a}{\sqrt{a^2 + c^2}}} d\theta \cdot \int_0^{\arccos \frac{\sqrt{a^2 + c^2}}{l_m}} A \cos \varphi d\varphi = \\ &= A \arccos \frac{a}{\sqrt{a^2 + c^2}} \sin \left( \arccos \frac{\sqrt{a^2 + c^2}}{l_m} \right) = 1. \end{aligned} \quad (10)$$

From formula (10):

$$A = \frac{1}{\arccos \frac{a}{\sqrt{a^2 + c^2}} \sin \left( \arccos \frac{\sqrt{a^2 + c^2}}{l_m} \right)}. \quad (11)$$

The distribution density of a function is expressed by the formula:

$$f(\theta, \varphi) = \frac{1}{\arccos \frac{a}{\sqrt{a^2 + c^2}} \sin \left( \arccos \frac{\sqrt{a^2 + c^2}}{l_m} \right)} \cos \varphi. \quad (12)$$

Here the function for determining the current value of the crushed particles is determined by projecting the maximum length onto the plane  $P$ :

$$l_{mT} = l_m \cdot \cos \varphi. \quad (13)$$

Considering  $l_{mT}$  as a function of two arguments  $\theta$  and  $\varphi$ , and applying formula (6), as a result, in this study the mathematical expectation of the function or the average length of crushed particles:

$$\begin{aligned}
 l_{m_c} &= \frac{l_m}{\arccos \frac{a}{\sqrt{a^2+c^2}} \sin \left( \arccos \frac{\sqrt{a^2+c^2}}{l_m} \right)} \times \\
 &\times \int_0^{\arccos \frac{a}{\sqrt{a^2+c^2}}} d\theta \cdot \int_0^{\arccos \frac{\sqrt{a^2+c^2}}{l_m}} \cos^2 \varphi d\varphi = \\
 &= \frac{l_m}{\sin \left( \arccos \frac{\sqrt{a^2+c^2}}{l_m} \right)} \cdot \frac{1}{2} \cdot \int_0^{\arccos \frac{\sqrt{a^2+c^2}}{l_m}} (1+\cos 2\varphi) d\varphi = \\
 &= \frac{l_m}{\sin \left( \arccos \frac{\sqrt{a^2+c^2}}{l_m} \right)} \cdot \frac{1}{2} \times \\
 &\times \left( \int_0^{\arccos \frac{\sqrt{a^2+c^2}}{l_m}} d\varphi + \int_0^{\arccos \frac{\sqrt{a^2+c^2}}{l_m}} \cos 2\varphi d\varphi \right) = \\
 &= \frac{l_m}{\sin \left( \arccos \frac{\sqrt{a^2+c^2}}{l_m} \right)} \times \\
 &\times \left( \frac{\arccos \frac{\sqrt{a^2+c^2}}{l_m}}{2} + \frac{\sin \left( 2 \arccos \frac{\sqrt{a^2+c^2}}{l_m} \right)}{4} \right). \tag{14}
 \end{aligned}$$

Thus, an analytical expression was obtained for determining the average length of crushed feed when simultaneously hit by adjacent hammers.

**5.3. Conducting experiments and calculations to determine the average length of crushed feed**

Analysis of formula (14) shows that in order to carry out a calculation to determine the average length of crushed particles, it is necessary to clarify the value of adjacent hammers and the height of the layer of the supplied mass.

Hammers with cutting edges are used to crush feed from by-product stem raw materials. In this case, the end working part of the hammer is milled at an angle of 90°, i.e. two cutting edges are processed at the end of the hammer. Therefore, when simultaneously hit by hammer blows, the stems are cut at the level between the edges. In addition, if the hammers did not have cutting edges, all the same, when simultaneously hit by hammer blows, the length of the crushed particles would be equal to the distance between the edges of adjacent hammers.

Here, the length of the layer when the mass exits the grinding chamber, depending on the kinematic parameters of grinding, can be determined using formula (3).

When the crusher is operating, the mass entering the crushing chamber is struck by hammers several times. The number of times the hammers meet the feed mass can be determined using the formula [10]:

$$K_B = \frac{\varphi}{2\pi} = \frac{\varphi v_M K_M}{2\pi v_c} \tag{15}$$

where  $\varphi$  – deck coverage angle, rad.;  $v_M$  – peripheral speed at the ends of the hammers, m/s;  $K_M$  – number of rows of hammers, pcs.;  $v_c$  – velocity of the circulating layer in the chamber,  $v_c=0.35v_M$ .

When operating hammer feed grinders, the wear of the hammers occurs over a length of 20...40 mm. Therefore, to calculate the layer height when the feed mass enters the grinding chamber and when the initial feed mass moves in the grinding chamber, the layer height in this study was taken to be 30 mm. When the feed mass entering the grinding chamber moves at a speed of 1.5 m/s, the peripheral speed at the ends of the hammers is 60 m/s and when the hammers are arranged in two rows,  $C=25$  mm.

In our designs, the distance between the edges of adjacent hammers is 20 mm.

To determine the influence of the value of the distance between the faces of adjacent hammers, calculations were carried out for different values of the distance between the faces of adjacent hammers. The calculation results are presented in the form of the value of the average length of crushed particles, depending on the distance between the faces of adjacent hammers (Fig. 3).

From the graph (Fig. 3) it is evident that when the distance between the faces of adjacent hammers changes, the average length of the crushed particles varies within the range of 36.19...53.39 mm. To determine the reliability of the obtained analytical expression, an experiment was conducted on a special stalk feed crusher (Fig. 4).

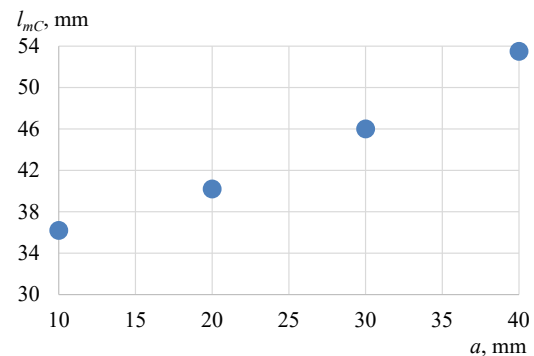


Fig. 3. Effect of the distance between the faces of adjacent hammers on the average length of crushed particles



Fig. 4. General view of the stem feed grinder

During the experiments, the feed conveyor belt speed was 1.5 m/s, the peripheral speed of the hammers was 60 m/s, the number of hammer rows was 4 pcs., and the distance between the faces of adjacent hammers was 20 mm. The crusher had 10 mm thick hammers with cutting edges.

The experiments were carried out on waste from freshly mown alfalfa with a moisture content of 63.36 %. In this case, the wet feed mass is crushed without grinding and is crushed

mainly when it is hit by hammer blows, i.e. the size of the crushed particles depends on the distance between the edges of adjacent hammers.

To determine the average size of the crushed particles, 3 samples were analyzed. The results of the sample analysis are presented in Table 1.

**Table 1**  
Results of sample analysis to determine the average size of crushed particles

Sample No.	Average size of chopped feed $l_{ci}$ , mm	$(l_{ci}-l_c)$	$(l_{ci}-l_c)^2$
1	38.02	0.73	0.5329
2	38.68	0.07	0.0049
3	39.55	0.8	0.64
–	$l_c=38.75$	–	$\sum 1.1778$

It is possible to determine the standard deviation:

$$\sigma = \sqrt{\frac{\sum_{c=1}^n (l_{ci} - l_c)^2}{n-1}} \quad (16)$$

The results show that the difference between replicates is only 0.767 mm. In addition, the sample spread is very small, i.e., it is within 38.02...39.55 mm.

When analyzing three samples, the average length of crushed particles was 38.75 mm, the theoretical value of the average length of crushed particles was 40.16 mm. At the same time, the difference between the theoretical and actual values of the average length of crushed particles is only 3.64 %, which proves the reliability of theoretical studies.

Analysis of the obtained analytical expression shows that using the obtained expressions, it is possible to determine the values of rational parameters and kinematic modes of feed grinders.

## 6. Discussion of the results of studies on determining the average length of particles in hammer mills

In this study, a new scheme of arrangement of stems when they fall under hammer blows is developed. With simultaneous change of angles  $\theta$  and  $\varphi$  an analytical expression for determination of average length of crushed particles is obtained. As a result of calculation, the influence of distance between edges of adjacent hammers on average length of crushed particles is received, which provides determination of optimal step of arrangement of hammers in rows, depending on required size of crushed particles. The average length of crushed stalk feed was determined for hammer crushers with a belt-feed conveyor according to the design and technological scheme of the crusher, presented in Fig. 1. The maximum length of the crushed particles when the stems fall under the hammer blows was determined depending on the feed rate into the crushing chamber  $v_T$ , the peripheral speed of the hammers  $v_M$  and the number of hammer rows  $K_M$ , each row of hammers separates a portion of length  $C$  from the fed layer, calculated using (1). This study found that the crushing of stems by hammer blows occurs depending on their location in a layer of length  $a$ , width  $C$  and height  $h_C$ , which is evident from the arrangement scheme of stems of length  $l_M$  when the layer enters the crushing chamber. Therefore, it is rational to determine the maximum

length of crushed particles when stems fall under the blows of adjacent hammers on the basis of the developed diagram of the arrangement of stems in a layer (Fig. 2), enclosed by the walls of adjacent hammers, the length of the loaded part of the hammers and the height of the fed layer, taking into account the diagonal arrangement of the stem in the parallelepiped.

When crushing stem feeds with hammer crushers, the stems are located in the feed layer depending on two angles  $\theta$  and  $\varphi$ , characterized by the position of the stems in the horizontal and vertical planes. In the main case, to determine the average size of the crushed particles depending on the pitch of the hammers, it is necessary to know the laws of angle distribution, i.e. angle distribution density. Here, using the basic position of probability theory, an analytical expression (14) was obtained for the first time to determine the average size of crushed stem feeds. The derived analytical expression is used to determine the average length of crushed particles when using hammers installed in four rows. Here, compared to the possible two-row arrangement of hammers on the disk of a hammer crusher with a hinged mounting of hammers in two rows [22], a clear increase in productivity to 5.4 t/h of stem feed is observed, including due to the use of four rows of hinged hammers. In this case, it is possible to apply the research results for crushing feed raw materials using six-row hammer crushers. The effectiveness of the obtained analytical expression, confirmed experimentally, can be successfully used to determine the pitch of the hammers in the rows on the axes of the hammer crushers based on the required size of the crushed particles of feed materials. Therefore, it can be especially emphasized that the derived analytical expression gives reliable results within the zootechnical requirements for feed materials, and also ensures the selection of the main design parameter of the hammer crusher – the pitch of the hammers in the rows. It is worth noting the practical solution of this theoretical problem is accompanied by Fig. 3, which shows a certain theoretical value of the average size of crushed particles depending on the pitch of the hammers. The solution of the theoretical problem ensures the determination of the optimal value of the main parameter of hammer crushers. The required pitch of the hammers is set (20 mm). A special experiment was conducted when crushing stem feed using a special hammer crusher for stem feed (Fig. 4). The actual value of the average size of crushed particles corresponds to the theoretical value of the average size of crushed feed, which proves the reliability of theoretical studies. It is clear from this that the obtained results of theoretical studies ensure the determination of the main parameter of the hammer crusher.

In contrast to the analytical expression derived in this work for determining the average length of crushed particles of stem feed raw materials, in a previously published work [10] obtained an analytical expression for determining the mass fraction of the required fractions in units of a unit (or in percent). In previous studies [13], considering changes in angles separately, i.e. the position of the stems in the horizontal and vertical planes separately, analytical expressions were obtained for determining the average size of the crushed particles. However, in reality, changes in these two angles occur simultaneously. In this case, this article considers the simultaneous change of two angles  $\theta$  and  $\varphi$  obtaining an analytical expression for determining the average size of the crushed particles. Thus, an expression was obtained when considering the actual process and this is the main feature of theoretical studies. In the work [12], taking into account the location of crushed feed particles in the horizontal and vertical planes of

the working area, a mathematical description of the process of dosed unloading of crushed feed particles was obtained. In this case, the probability of finding particles with a certain size in the feed layer is determined, however, in the presented studies, analytical expressions for determining the average length of crushed feed raw materials are not presented.

It is known that by applying probability theory, it is possible to determine the numerical characteristics of the function of random variables. Here, an important issue is the definition of mathematical expectations of functions of several arguments. Therefore, the application of probability theory in many cases gives positive results. In our case, the reliability of the obtained expression is proven by experimental studies. Therefore, the theoretical definition of the average length of crushed particles proposed in this study for the first time is reliable and this is proven by the result of the experiment, which ensures the determination of a reliable value of the average length of crushed particles. The solution to this problem with one temporary grinding of two arguments in previously conducted studies of other authors was not found. Therefore, the solution to such a problem using the methods of probability theory should be of some interest to any researchers.

As a result of the conducted research, it was established that in reality the arrangement of stems in the layer changes simultaneously in the vertical and horizontal planes, which explains the reliability of the chosen method for solving problems simultaneously in two planes. It can be considered that this method of solving the problem does not provide for a parallel and slightly angled arrangement of the stems to the side surface of the hammer. This is a certain limitation of this method. It is known that in this case the stems do not fall under the blows of the hammers, given that the stems fall under the blows of neighboring hammers several times before exiting the grinding chamber. In addition, the exit of the stems from the grinding chamber with such a parallel arrangement to the side walls of the hammers is unlikely. Here, the method used does not consider the arrangement of stems with large values of angles  $\theta$  and  $\varphi$ . However, here the arrangement of stems is limited by the volume of the layer when stems are simultaneously hit by adjacent hammers, so these unaccounted for positions of stems are justified by their arrangement in a limited layer. In the work of agricultural machines, such processes are often encountered. Therefore, the proposed method for solving the problem can have experience of application in the processes of crushing, mixing and picking up hay, agricultural crops, waste of plant origin from the surface of the earth.

The peculiarity of this method of determining the average length of crushed particles is the simultaneous change of angles  $\theta$  and  $\varphi$ . At the same time, this method is used in probability theory to determine mathematical descriptions of functions of several arguments. Therefore, it should be noted that as a result of theoretical research, the necessary analytical expression for determining the average length of crushed particles was obtained and its reliability was confirmed by the results of experiments, which is a significant advantage of the study.

The disadvantages of the conducted and presented study include incomplete experiments with some levels of variation of the pitches of the arrangement of adjacent hammers in a hammer crusher. To develop the results of this study, further experimental studies should be conducted with the installation of a different range of pitches of adjacent hammers, with an increase in the number of hammer rows, using new reinforced hammer designs. However, further research can

significantly extend the effectiveness and applicability of the obtained analytical expression for determining the average length of crushed particles for different types of hammer mills.

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## 7. Conclusions

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1. A scheme of arrangement of stems in a layer enclosed by walls of adjacent hammers, the length of the loaded part of the hammers and the height of the supplied layer has been developed. Taking into account the diagonal arrangement of the stem in the parallelepiped, the maximum length of the stem when simultaneously hit by hammers has been determined. The developed design and technological scheme of the chopper of feed from by-product stem raw materials allowed to consider the determination of the maximum length of crushed particles when the stems fall under the blows of hammers. The features and distinctive features of determining the maximum length of crushed particles when the stems fall under the blows of adjacent hammers should be noted rational arrangement of stems in the layer of feed supplied to the chopping chamber simultaneously in 2 (two) planes. This arrangement of stems is not uncommon compared to the arrangement of stems in the layer in horizontal and vertical planes separately, i.e. in 1 (one) plane. It has been established that the mass falling under the blows of adjacent hammers will be crushed depending on their arrangement in a layer of length  $a$ , width  $C$  and height  $h_C$  according to the constructed scheme of arrangement of stems of length  $l_M$  at the entrance of the layer to the crushing chamber. At the entrance to the crushing chamber the layer has the shape of a parallelepiped ABCDMNLK and therefore the maximum length of the stem depending on the arrangement of the stems can have a length of CM, DN, AL, BK. All these data of the figure under consideration have the same size of the arrangement angle, and inside the parallelepiped they are also the same. Therefore, to determine the average length of crushed stems, the arrangement of a stem of length CM with arrangement angles in the horizontal plane  $\theta$  and vertical plane  $\varphi$  is considered. Thus, it is rational to determine the maximum length of crushed particles when stems fall under the blows of adjacent hammers based on the developed scheme of the arrangement of stems in the layer enclosed by the walls of adjacent hammers, the length of the loaded part of the hammers and the height of the fed layer, taking into account the diagonal arrangement of the stem in the parallelepiped. All this contributes to achieving the set goal, i.e. obtaining an analytical expression that ensures the determination of the average length of feed crushed from secondary stem raw materials depending on the pitch of the arrangement of hammers with cutting edges in hammer crushers.

2. It has been established that when determining the average length of feed crushed from secondary stem raw materials by a hammer working element, the location of a stem with a length  $l_m$  depends on angles  $\theta$  and  $\varphi$  and the orientation of the stem within these angles is equally probable, i.e. depends on two random variables  $x$  and  $y$ . The direction of the stem is characterized by a unit vector  $l_m$ . The direction of the vector  $l_m$  in a spherical coordinate system associated with the plane  $P$  onto which the projection is made is determined by two angles: angle  $\theta$ , lying in the plane  $P$ , and angle  $\varphi$ , lying in the plane perpendicular to  $KP$ . When changing these angles, the vector  $l_m$  describes a spherical surface  $E$  and the entire direction  $l_m$  has the same probability density. When



changing, these angles form an elementary area  $dS$  on the sphere  $E$ . Considering  $l_{mT}$  as a function of two arguments  $\theta$  and  $\varphi$ , the mathematical expectation of the function or the value of the average length of crushed particles is obtained. By simultaneously crushing the angles  $\theta$  and  $\varphi$  of the stem location with the maximum size in the layer and applying the methods of probability theory to determine the mathematical descriptions of the function of two arguments, an analytical expression was obtained for determining the average length of crushed particles by a hammer crusher depending on the distance between the faces of adjacent hammers.

3. As a result of the practical calculations, the average length of chopped feed from by-product stem raw materials was determined depending on the parameters of the hammer mill. To verify the reliability of the obtained expression, experiments were conducted on a mill with a distance between the faces of adjacent hammers of 20 mm. In this case, the average length of the crushed particles was equal to 38.75 mm, and the theoretical value was 40.16 mm. The difference between them was 3.64 %, which proves the reliability of the obtained analytical expression.

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#### Conflict of interest

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The authors declare that they have no conflict of interest in relation to this research, whether financial, personal,

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#### Data availability

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#### Use of artificial intelligence

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