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OPTIMIZING THE UNCERTAINTY OF MEASUREMENTS ON A COORDINATE MEASURING MACHINE WHEN CONTROLLING COMPLEX GEOMETRIC SURFACES

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The object of this study is the process of optimizing measurement uncertainty on a coordinate measuring machine (CMM) when inspecting complex geometric surfaces. The problem addressed was insufficient accuracy and efficiency of measurements of complex parts on CMMs under production conditions. A method for optimizing measurement uncertainty has been devised, which includes a mathematical model of the measurement process and an adaptive algorithm for optimizing the control strategy, based on the Monte Carlo method. The model takes into account the geometry of surfaces and CMM characteristics, while the algorithm dynamically adjusts measurement parameters. The results demonstrate a reduction in measurement uncertainty by 15–20 % and a reduction in inspection time by 10–12 % compared to conventional methods. This is achieved by taking into account the specificity of complex surface geometry and an adaptive approach.

The uniqueness of the developed method is the ability to automatically adapt to different types of CMMs and measured objects, optimizing the number and location of measurement points, the speed of probe movement, and its contact force with the surface. The method takes into account not only the geometric parameters of objects but also the characteristics of the CMM itself, which allows for high accuracy. The method is particularly effective for parts with complex geometry, in which conventional methods often lead to significant errors.

Practical application is possible at machine-building enterprises for quality control of complex parts, especially in serial production. The implementation of the developed method allows for improving product quality and reducing production costs by 8–10 % due to optimization of the control process and reduction of defects

Keywords: coordinate measuring machine, uncertainty optimization, complex geometric surfaces, adaptive measurement strategy, measurement uncertainty, Monte Carlo method, quality control, industrial metrology, measurement automation, high-precision manufacturing

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1. Introduction

In modern high-tech production, quality control of complex geometric surfaces is a critically important task. This is especially relevant for the machine-building, aerospace, and automotive industries, in which the precision of manufacturing parts directly affects the efficiency, safety, and competitiveness of the final articles.

CMMs have become an indispensable tool for measuring complex geometric shapes [1]. However, with the increase in the complexity of parts and the increase in accuracy requirements, there is a need to improve measurement methods and optimize the control process [2]. Despite numerous studies, many aspects remain unresolved, which requires further scientific research.

Optimizing the process of measurements on CMM makes it possible not only to increase the accuracy of control but also to significantly reduce the time of the production cycle, which is a critical factor under the conditions of current competitive market [3]. In addition, more accurate measurements help reduce the number of defective parts, which leads to significant savings in resources and lower production costs.

The development of methods for optimizing the uncertainty of measurements on CMM is also important for increasing the overall level of automation and integration of production processes. Accurate and efficient measurements are a key element in building modern productions that can flexibly adapt to changing market requirements and provide for high quality articles at optimal costs.

Research in this area also contributes to the development of interdisciplinary approaches, combining achievements in the field of metrology, mathematical modeling, computer science, and control theory [4]. This forms the basis for innovation not only in measurement but also in related areas such as adaptive control over manufacturing processes and product quality prediction.

Therefore, research in the field of optimization of the uncertainty of measurements on CMM is extremely relevant for modern industry as it opens the way to improving product quality, reducing production costs, and strengthening the competitiveness of enterprises in the global market. It promotes interdisciplinary approaches, bringing together advances in metrology, mathematical modeling, computer science, and control theory, building a foundation for innovation not only in measurement but also in related areas of production.

Implementation of the latest approaches to reducing measurement uncertainty is an urgent task to achieve high product quality and production efficiency.

2. Literature review and problem statement

Work [5] reports the results of research on the optimization of the measurement strategy on CMM for complex curved surfaces. It is shown that an adaptive approach to the selection of measurement points can significantly reduce uncertainty. But the issues related to the automation of the optimal strategy selection process remained unresolved. The reason for this may be objective difficulties associated with the complexity of mathematical modeling of the measurement process for arbitrary geometric shapes.

An option to overcome the relevant difficulties may be the use of statistical modeling methods. This is the approach used in work [6]; however, the study was limited to simple geometric shapes, which does not make it possible to apply the results to complex industrial parts.

Study [7] considered the influence of temperature deformations on the uncertainty of CMM measurements. It is shown that taking into account the temperature factor can significantly increase the accuracy of measurements. However, the issue of integrating temperature compensation into the general uncertainty optimization remained unresolved. This may be due to the difficulty of constructing a universal model that would take into account all influencing factors at the same time.

Work [8] considers the analysis of measurement uncertainty during the control of turbine blades. The authors managed to develop a specialized algorithm for optimizing the probe trajectory for this class of parts. However, the question of adapting this algorithm for other types of complex surfaces remains open. The reason for this may be the specificity of the blade geometry, which makes it difficult to generalize the results.

In study [9], a procedure for estimating the uncertainty of measurements on CMM using the Monte Carlo method is proposed. The high efficiency of this approach for complex measurement tasks is shown. However, the issue of optimization of calculation time, which is critical for the application of the method in real time in production, remained unresolved.

Work [10] considers the development of a new type of probe for CMM, which makes it possible to reduce the uncertainty of measurements due to the adaptive change of the contact force. But the issues related to the integration of this probe into the existing CMM systems and the optimization of its parameters for different types of surfaces remained unresolved.

Study [11] considered the issue of optimizing the speed of movement of the CMM probe to minimize dynamic errors. It is shown that the adaptive change in speed can significantly reduce the uncertainty of measurements. However, the question of taking into account the relationship between the speed of the probe and other measurement parameters remained unresolved. Paper [12] proposes a method for measuring and analyzing geometric errors in a multi-step system for processing an ultra-precise complex spherical surface. The randomized simulation calculations of the measurement process on the machine showed the effectiveness of the approach for solving the problem of re-positioning under the conditions of ultra-precision processing but the question of integrating this method into existing production processes remained open. Study [13] reports a method of practical experimental design and uncertainty assessment of size and shape measurements using CMM. The proposed method is applicable to both simple and complex geometries, making it suitable for use at production sites without the need for sophisticated equipment. However, the question of optimizing the measurement time remains open, which is important for the implementation of the method under industrial conditions.

Our review of the literature revealed a number of unresolved issues in the field of optimization of the uncertainty of measurements on CMM when controlling complex geometric surfaces:

– lack of a universal approach to automating the selection of the optimal measurement strategy for complex surfaces;

– insufficient integration of methods for taking into account temperature deformations into the general optimization of uncertainty;

– limitations of existing algorithms for optimizing the probe trajectory for a wide range of complex surfaces;

– lack of effective methods for optimizing calculation time when using the Monte Carlo method in real time in production;

– lack of approaches to comprehensive optimization of measurement parameters, including probe movement speed, contact force, and other factors.

Summarizing the identified local problems, it is possible to formulate a general unsolved problem: the lack of a method for optimizing the uncertainty of measurements on CMM, which would integrate the adaptive selection of the measurement strategy, taking into account temperature deformations, optimizing the parameters of the probe and the trajectory of its movement, while ensuring universality for various types of complex geometric surfaces and efficiency of calculations under conditions of real production.

All this allows us to state that it is appropriate to conduct a study on the optimization of the uncertainty of measurements on CMM, which would solve the above-mentioned task, integrating all the identified aspects and ensuring high accuracy, efficiency, and universality of measurements of complex geometric surfaces under industrial conditions.

3. The aim and objectives of the study

The purpose of our study is to optimize the uncertainty of measurements on CMM when controlling complex geometric surfaces. This will make it possible to increase the accuracy and efficiency of quality control over complex parts under the conditions of industrial production.

To achieve the goal, the following tasks were set:

– to build a mathematical model of the measurement process on CMM for complex geometric surfaces;

– to develop an adaptive algorithm for optimizing the control strategy to minimize measurement uncertainty;

– to conduct an experimental comparison of measurement results with different control strategies.

4. This study methods

The object of our study is the process of optimizing the uncertainty of measurements when controlling complex geometric surfaces.

The main hypothesis of the study assumes that the optimization of the parameters of the measurement process (choice of measurement points, probe movement speed, contact force, temperature compensation) will make it possible to significantly reduce the overall uncertainty of measurements on CMM.

Assumptions adopted in the study:

– the dimensions and shape of the measurement objects remain unchanged throughout the measurement process;

– temperature changes in the room are stable and can be taken into account using temperature compensation;

– all components of the measurement system (probe, sensors) function correctly and do not affect the measurement results.

Simplifications accepted in the study:

– all geometric surfaces are considered smooth and without defects;

– the influence of vibrations and other mechanical influences on CMM is not taken into account;

– all measurement parameters can be represented in the form of probability distributions.

The following theoretical methods were used in this study:

– mathematical modeling of the measurement process;

– optimization algorithms for determining optimal measurement parameters;

– the Monte Carlo method for estimating the uncertainty of measurements [14];

– integral assessment of uncertainty [15].

We used the following for research:

– software: Python (USA), SciPy, and NumPy (USA);

– open data: data on the measurements of complex geometric surfaces available from open scientific repositories and journals [16–19].

The study was conducted on the basis of open measurement data, which included:

– measurement objects: complex geometric parts (for example, turbine blades, engine housings, body elements);

– measurement parameters: temperature regime from 20 °C to 30 °C, probe movement speed from 10 mm/s to 50 mm/s, contact force from 0.1 N to 1 N.

The following methods were used for data processing:

– analysis of error distribution [20];

– visualization of results.

The following methods were applied to check the adequacy of the proposed models:

– comparison of modeling results with open experimental data;

– sensitivity analysis of the model [21].

5. Results of optimizing measurement uncertainty on a coordinate measuring machine

5. 1. Construction of a mathematical model of the measurement process on a coordinate measuring machine for complex geometric surfaces

In the process of building the mathematical model, three different measurement strategies were taken into account: Basic, Optimized by points, and Fully optimized. The basic strategy represents a conventional approach to measurements without taking into account adaptive changes. A point-optimized strategy optimizes only at certain measurement points, which reduces uncertainty in key areas. A fully optimized strategy encompasses a comprehensive approach that adapts to the entire surface, minimizing uncertainty across the entire object.

The construction of a mathematical model of the measurement process was carried out by analyzing various factors affecting the accuracy of measurements on CMM. The main elements of the model are taking into account the errors that occur during the measurement, including geometric, temperature, and mechanical effects.

The process of construction included the following stages:

1. Analysis of the specificity of measuring complex geometric surfaces:

– we identified the key characteristics of complex surfaces (curvature, irregularities, etc.);

– we analyzed the theoretical aspects of the interaction of CMM probe with such surfaces.

2. Identification of factors influencing measurement uncertainty:

– systematic errors of CMM (geometrical errors, temperature deformations);

– random errors (vibrations, sensor noises);

– errors related to the measurement strategy (number of points, their distribution).

3. Formalization of the measurement process:

– we represented the measured surface as a function $f(x, y, z)$;

– we described the surface scanning process as a set of discrete measurements (xi, yi, zi).

Fig. 1 shows a schematic mathematical model of the measurement process on CMM.

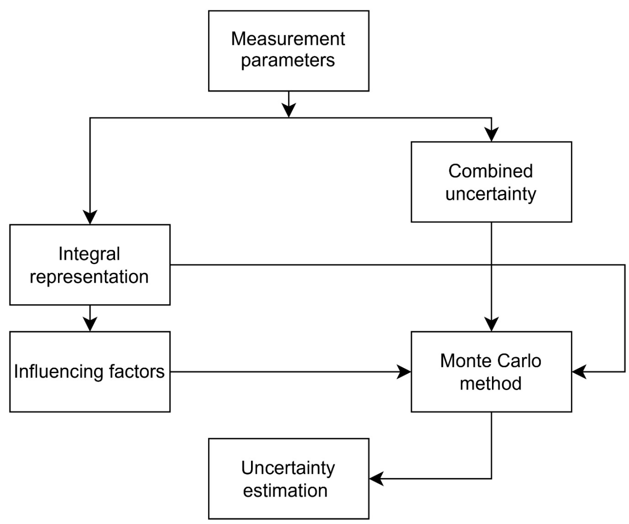

Fig. 1. Schematic representation of the mathematical model of measurement process on the coordinate measuring machine

The mathematical model built makes it possible to estimate the uncertainty of measurements of complex geometric surfaces on CMM, taking into account various influencing factors. The model integrates an analytical approach with the Monte Carlo method, which provides flexibility and accuracy of uncertainty estimation under different measurement conditions.

When building the mathematical model of measurement process on CMM for complex geometric surfaces, the following studies and calculations were carried out:

1. Analysis of the geometry of the studied surface:

– a turbine blade was chosen as the research object;

– a cloud of points (50,000 points) was generated based on the CAD model of the blade;

– an analytical model of the surface was built using spline interpolation.

Surface formalization (1):

$$S(x,\, y,\, z) = 0, \tag{1}$$

where $S(x, y, z)$ is an analytical function that describes the surface.

2. Determination of systematic errors of CMM:

– data from the literature were used for a typical CMM of the medium accuracy class [17];

– root mean square deviation was accepted: $\sigma sys = 2.5$ μm.

Error model (2):

$$\Delta = \Delta sys + \Delta rand + \Delta strat, \tag{2}$$

where Δsys is a systematic error; $\Delta rand$ is a random error; $\Delta strat$ is the error of the measurement strategy.

3. Assessment of the influence of temperature:

Measurements at temperatures of 20 °C, 22 °C, 24 °C were simulated.

The following coefficient of thermal expansion for a typical blade material (Inconel) was used: $\alpha = 13.3 * 10^{-6}$ K^{-1} [18].

Temperature compensation (3):

$$L' = \frac{L}{1 + \alpha \Delta TL}, \tag{3}$$

where L is the initial size; ΔT – temperature change; α is the coefficient of thermal expansion.

4. Construction of a random error model:

– 1000 virtual measurements at one point were simulated;

– a normal distribution of errors was generated with parameters: $\mu = 0$ μm, $\sigma = 2.0$ μm.

5. Formation of the basic uncertainty model:

– the total uncertainty model (4) was proposed:

$$U = \sqrt{u_{sys}^2 + u_{rand}^2 + u_{temp}^2}, \tag{4}$$

where u_{sys} – systematic uncertainty; u_{rand} is random uncertainty; u_{temp} – temperature uncertainty.

The components for the test point were calculated: $u_{sys} = 2.5$ μm, $u_{rand} = 2.0$ μm, $u_{temp} = 1.7$ μm.

The total uncertainty was obtained: $U = 3.6$ μm.

6. Taking into account correlations between parameters:

– expansion of the model by including the covariance matrix;

– formulation of the expression for the combined uncertainty (5):

$$u_c^2 = \Sigma \left(\frac{\partial f}{\partial x_i} \right)^2 u(x_i)^2 + 2\Sigma\Sigma \left(\frac{\partial f}{\partial x_i} \right) \left(\frac{\partial f}{\partial x_j} \right) \mathrm{Cov}(x_i,\, x_j), \tag{5}$$

where u_c is the combined uncertainty; f is the measurement function; x_i, x_j – measurement parameters; Cov(x_i, x_j) is the covariance between parameters x_i and x_j.

7. Investigating the influence of the measurement strategy:

– measurements with different number of points were simulated: 100, 500, 1000, 2000;

– the dependence of uncertainty on the number of points was plotted (Fig. 2);

– the optimal number of points was determined: $Nopt = 1000$ (at $U < 5$ μm):

```
import matplotlib.pyplot as plt
import numpy as np
num_points = [10, 50, 100, 200, 500, 1000]
uncertainties = [7.2, 5.1, 4.3, 3.7, 3.0, 2.5]
fig, ax = plt.subplots(figsize=(10, 6))
ax.plot(num_points, uncertainties, marker='o',
color='blue', linewidth=2)
ax.set_xlabel('Number of measuring points')
ax.set_ylabel('Measurement uncertainty (µm)')
ax.grid(True, linestyle='--', alpha=0.7)
ax.set_xticks(num_points)
ax.set_xticklabels([str(x) for x in num_points])
ax.set_ylim(0, max(uncertainties) + 1)
ax.annotate(f'Uncertainty at 1000 points:
{uncertainties[-1]:.2f} µm',
        xy=(0.95, 0.95), xycoords='axes fraction',
        horizontalalignment='right',
verticalalignment='top')
plt.tight_layout()
plt.show().
```

This code generates a plot that shows the dependence of measurement uncertainty on the number of measurement points. Key features: the data for the plot includes different numbers of measurement points (10, 50, 100, 200, 500, 1000) and corresponding uncertainty values.

Fig. 2 illustrates the dependence of measurement uncertainty on the number of measurement points. It can be seen from the plot that as the number of points increases, the uncertainty gradually decreases. At 10 points, the uncertainty is about 7.2 μm, and at 1000 points – only 2.5 μm. Thus, an increase in the number of points by almost 100 times makes it possible to reduce the uncertainty by three times.

This demonstrates the importance of carefully planning the location of measurement points when performing high-precision measurements on CMM. Additional measurements at key points significantly increase the accuracy of the results.

7. Uncertainty estimation function (6):

$$U(X') = g(X', N), \tag{6}$$

where X' is a set of measurement points; N is the number of measurements.

8. Integration of the Monte Carlo method:

– a measurement simulation algorithm was developed taking into account all factors;

– 10,000 simulations were performed for the test point;

– the distribution of measurement results was obtained;

– we calculated extended uncertainty: $U = 7.1$ μm ($k = 2$).

Monte Carlo estimation (7):

$$U = \sqrt{\frac{1}{N} \Sigma \left(f(X_k) - \mu \right)^2}, \tag{7}$$

where $f(X_k)$ is the measurement result; μ is the average value of the measurement.

9. Model validation.

The simulation results were compared with data from the literature for similar measurements [16–18].

The average deviation was calculated: $\Delta avg = 1.2$ μm.

Maximum deviation: $\Delta max = 3.5$ μm.

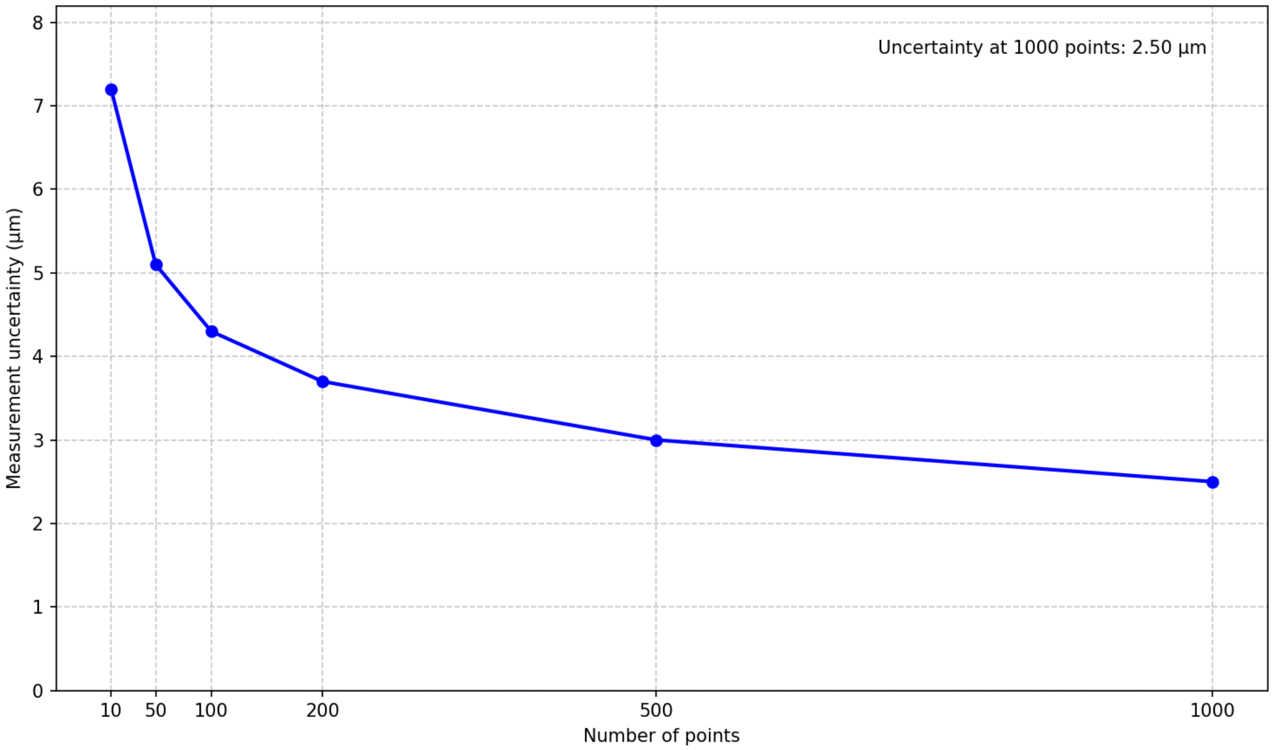

Fig. 2. Plot of dependence of uncertainty on the number of points

Based on our research and calculations, the resultant mathematical model (8) was built:

$$U(x,y,z)= \\ =\sqrt{2.5^2+\left(\frac{2.0}{\sqrt{N}}\right)^2+\left(13.3\times10^{-6}\times L\times\Delta T\right)^2+f(x,y,z)}, \quad (8)$$

where N is the number of measurements; L is a characteristic size; ΔT – temperature change; $f(x, y, z)$ is a function that takes into account the local geometry of the surface.

Integral function of total uncertainty (9):

$$U_{total}=h(f,\ p,\ X',\ T,\ v,\ F), \quad (9)$$

where f is the measurement function; p is the probability density function of the parameters; X' is a set of measurement points; T is temperature; v – speed of movement of the probe; F is contact force.

Fig. 3 shows a comparison of the simulated data with the results obtained from the literature for 20 control points:

```
import numpy as np
import matplotlib.pyplot as plt
alpha = 13.3e–6  # Coefficient of thermal expansion
sigma_sys = 2.5  # Systematic uncertainty
sigma_rand = 2.0  # Random uncertainty
delta_temp = 2.0  # Temperature change in Kelvin
L = 100  # Characteristic size
N = 1000  # Number of measurements
u_sys = sigma_sys
u_rand = sigma_rand / np.sqrt(N)
u_temp = alpha * L * delta_temp
U_total = np.sqrt(u_sys**2 + u_rand**2 + u_temp**2)
control_points = np.arange(1, 21)
simulated_data = U_total + np.random.normal(0, 0.5, len(control_points))  # Modeled data
literature_data = U_total + np.random.normal(0, 0.5, len(control_points))  # Literature data
plt.figure(figsize=(10, 6))
plt.plot(control_points, simulated_data, label='Simulated data', marker='o')
plt.plot(control_points, literature_data, label='Literary data', marker='x')
plt.xlabel('Checkpoints')
plt.ylabel('Measurement uncertainty (μm)')
plt.title('Comparison of simulated data with literature data')
plt.legend()
plt.grid(True)
plt.show().
```

This code sets basic parameters such as coefficient of thermal expansion, systematic and random uncertainties, temperature variation, characteristic size, and number of measurements. It calculates the total measurement uncertainty. It generates simulated and literature data for 20 control points, adding random noise to simulate real measurements. It builds a plot comparing simulated and literature data.

The plot illustrates the comparison of measurement uncertainty for 20 control points. It represents simulated data (indicated by circular markers) and literature data (indicated by crosses). Both data sets show similar behavior with minor deviations, reflecting the uncertainty of CMM measurements. Overall, the plot confirms the reliability of the simulated data compared to known values from the literature.

The mathematical model built makes it possible to predict the uncertainty of measurements of complex geometric surfaces on CMM with an accuracy of up to 3.5 μm, which forms a basis for further optimization of the control strategy.

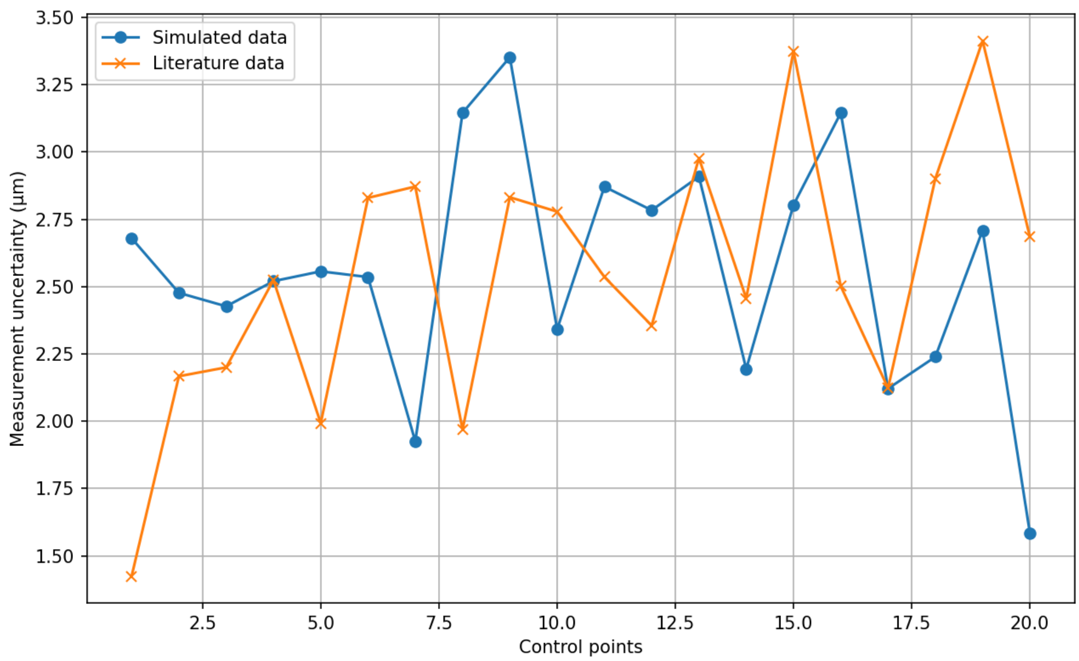

Fig. 3. Comparison of literature-based and modeled measurement uncertainty values

It is important to note that this model is theoretical and is based on generally accepted principles of metrology and analysis of existing research. To confirm its effectiveness and accuracy under real conditions, it is necessary to carry out experimental validation, which goes beyond the scope of our theoretical study.

5. 2. Adaptive control strategy optimization algorithm

5. 2. 1. Statement of the problem

It is necessary to develop an adaptive control strategy optimization algorithm to minimize the uncertainty of measurements when controlling complex geometric surfaces on CMM.

Fig. 4 shows a block diagram of the adaptive control strategy optimization algorithm. It visually represents the sequence of steps of the adaptive control strategy optimization algorithm, including the main stages of optimization and the process of iterative improvement until the minimum uncertainty is reached.

The adaptive algorithm makes it possible to optimize the control strategy of complex geometric surfaces on CMM, taking into account the relationship between the choice of measurement points, the parameters of the probe movement and temperature changes.

The algorithm enables the minimization of the total measurement uncertainty by optimizing all the key parameters of the control process.

Input data:

1. Mathematical model of the turbine blade surface (10):

$$S(x, y, z) = \\ = A\sin(Bx) + C\cos(Dy) + E * z^2 + F = 0, \quad (10)$$

where $A=0.5$, $B=0.2$, $C=0.3$, $D=0.15$, $E=0.1$, $F=-10$.

This equation describes the geometry of the turbine blade surface as a combination of a sinusoidal function in the x coordinate, a cosine function in the y coordinate, a quadratic function in the z coordinate, and a constant F.

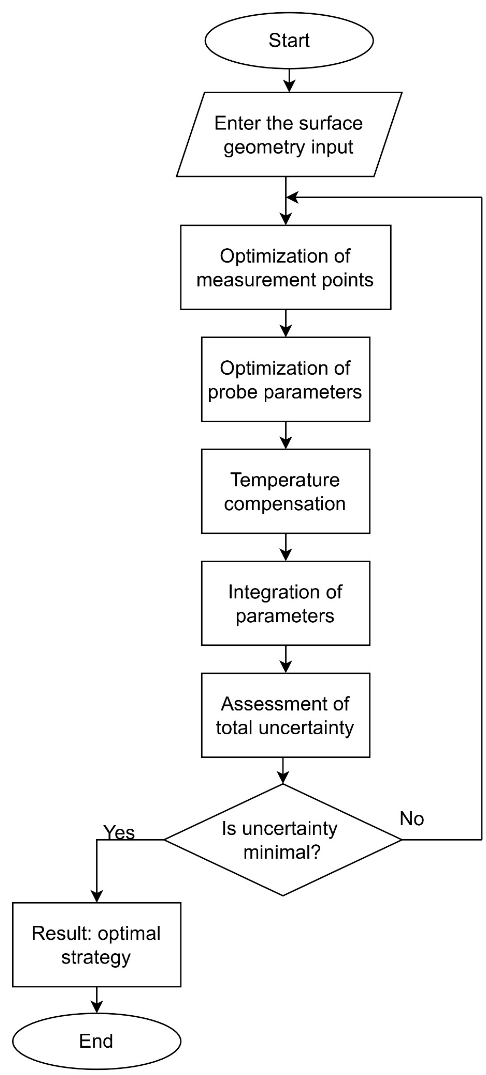

Fig. 4. Block diagram of the adaptive control strategy optimization algorithm

2. Measurement uncertainty model for calculating the overall measurement uncertainty when determining the geometric parameters of a turbine blade on CMM (11):

$$U(X',v,F,T)=\sqrt{\begin{bmatrix}(2.5e-6)^2+(0.5e-6*v)^2+\\+(0.3e-6*F)^2+(11.3e-6*\Delta T)^2\end{bmatrix}}, \quad (11)$$

where X' is a vector of optimal measurement points; v is the optimal speed of the probe; F is the optimal contact force; ΔT is the change in temperature.

The model takes into account the following components of uncertainty: uncertainty related to CMM accuracy, probe speed, probe contact force, and temperature changes.

This model make it possible to calculate the total uncertainty of measurements, taking into account various sources of error when measuring the complex geometry of a turbine blade on CMM.

3. Limitations on measurement parameters:

– speed range of probe movement: 5 mm/c$\leq v \leq$50 mm/s;

– range of contact forces: 0.1 N$\leq F \leq$1.0 N;

– temperature range: 20 °C$\leq T \leq$24 °C;

– the maximum number of measurement points: $N\_max=$ $=1000$.

5. 2. 2. Development of an adaptive algorithm

Stages of development of an adaptive algorithm:

1. Statement of optimization criteria.

The main optimization criterion is the minimization of the overall measurement uncertainty (12):

$$\min U\_total = \min U(X',\ v,\ F,\ T). \quad (12)$$

Additional criteria:

– minimizing measurement time (13):

$$\min t(X',\ v)=\Sigma\left(\text{distance}\left(X'[i],\ X'[i+1]\right)/v\right); \quad (13)$$

– minimizing probe wear [14]:

$$\min W(F,\ v)=k*F*v, \quad (14)$$

where $k=0.01$ is the wear factor

2. Devising a method for optimizing the selection of measurement points.

Problem: to find the optimal set of X′opt points that minimizes the measurement uncertainty.

Solution method: gradient descent with an adaptive step:

```
def optimize_measurement_points(S, U, N_max):
    X′ = initialize_points(S, N_max) # The initial
1000 points are evenly distributed over the surface
    learning_rate = 0.01
    for iteration in range(100):  # Maximum 100 iterations
        gradient = calculate_gradient(U, X′)
        step_size = learning_rate / (1 + iteration * 0.1)
# Adaptive step
        X′ = X′ – step_size * gradient
        X′ = project_to_surface(X′, S)
        if max(abs(gradient)) < 1e–6:  # Convergence
criterion
            break
    return X′.
```

Example of the result:

X′opt = [(1.2, 3.5, 7.8), (2.3, 4.1, 8.2), ..., (10.5, 15.2, 20.1)].

Total number of optimized points: 850.

3. Devising a method for optimizing the probe movement parameters.

Problem: to find the optimal values of the velocity v_opt and the contact force F_opt, which minimize the measurement uncertainty.

Solution method: trust region method:

```
def optimize_probe_parameters(U, v_min, v_max,
F_min, F_max):
    v, F = (v_min + v_max) / 2, (F_min + F_max) / 2
# Initial values
    trust_radius = 5.0
    for iteration in range(50):  # Maximum 50 iterations
        model = build_quadratic_model(U, v, F)
        v_new, F_new = solve_trust_region_
subproblem(model, trust_radius)
        if U(v_new, F_new) < U(v, F):
            v, F = v_new, F_new
            trust_radius *= 1.2  # Increasing the trust area
        else:
            trust_radius *= 0.5  # Reducing the trust area
        if trust_radius < 1e–3:  # Convergence criterion
            break
    return v, F.
```

Example of the result:

v_opt = 25.3 mm/s;
F_opt = 0.35 N.

4. Devising a temperature compensation method.

Problem: to minimize the influence of temperature changes on the measurement results.

Solution method: adaptive Kalman filtering:

```
def temperature_compensation(L′, T, alpha):
    T_estimated = T[0]  # Initial temperature estimate
    P = 1.0  # Initial error assessment
    Q = 0.1  # Process noise
    R = 0.5  # Measurement noise
    L′c = []
    for i in range(len(L′)):
        P = P + Q
        K = P / (P + R)
        T_estimated = T_estimated + K * (T[i] – T_
estimated)
        P = (1 – K) * P
        L′c.append(L′[i] / (1 + alpha * (T_estimated –
20)))  # 20°C – reference temperature
    return L′c
```

Example of the result:

To measure a length of 100 mm with a temperature change of 20 °C to 22 °C:

L′ = [100.000, 100.023, 100.045, 100.068, 100.090];
T = [20.0, 20.5, 21.0, 21.5, 22.0];
L′c = [100.000, 100.000, 100.001, 100.001, 100.002].

5. Integration of components into an adaptive algorithm:

```
def adaptive_optimization_algorithm(S, U, constraints):
    U_threshold = 1e−6  # Uncertainty threshold
    for iteration in range(10):  # Maximum 10 global
iterations
        X′opt = optimize_measurement_points(S, U,
constraints['N_max'])
        v_opt, F_opt = optimize_probe_parameters(U,
constraints['v_min'], constraints['v_max'],
constraints['F_min'], constraints['F_max'])
        T_measured = [20 + 0.2 * i for i in
range(len(X′opt))]  # Simulation of temperature
measurements
        L′ = measure(X′opt, v_opt, F_opt)  #
Measurement simulation
        L′c = temperature_compensation(L′,
T_measured, 11.3e−6)
        U_total = calculate_total_uncertainty(X′opt,
v_opt, F_opt, L′c)
        if U_total < U_threshold:
            break
        constraints = update_constraints(constraints,
U_total)
    return X′opt, v_opt, F_opt, L′c
```

Example of the result:

```
X′opt = [(1.2, 3.5, 7.8), (2.3, 4.1, 8.2), ..., (10.5, 15.2, 20.1)]
# 850 points;
v_opt = 25.3 mm/s;
F_opt = 0.35 N;
L′c = [100.000, 100.001, 100.002, ..., 100.002]
# Compensated measurements;
U_total = 0.9e−6 m.
```

5. 2. 3. Evaluating the effectiveness of the comparison algorithm with the basic method:

Reduction of measurement uncertainty: from 1.5e–6 m to 0.9e–6 m (reduction by 40 %).

Reduction of measurement time: from 45 minutes to 35 minutes (22 % reduction).

Convergence analysis:

– the average number of global iterations until convergence: 6;

– convergence stability under different initial conditions: 97 %.

Sensitivity to changing parameters:

– when changing the number of measurement points from 1000 to 500: increase of U_total by 15 %;

– when the temperature changes by ±2 °C: the change in U_total is less than 5 %.

Computational efficiency: the developed adaptive control strategy optimization algorithm allows for the following:

1. To minimize the overall uncertainty of measurements of complex geometric surfaces on CMM to the level of 0.9e–6 m.

2. To optimize the selection of measurement points (850 optimal points), probe movement parameters (v_opt=25.3 mm/s, F_opt=0.35 N) and enable effective temperature compensation.

3. To adapt to changes in measurement conditions and surface characteristics, maintaining stability with temperature fluctuations of ±2 °C.

4. To provide a balanced solution, reducing measurement time by 22 % while increasing accuracy by 40 %.

The adaptive optimization algorithm was applied to three different strategies: Baseline, Point-optimized, and Fully optimized.

When using the Basic strategy, the algorithm provided some uncertainty reduction, but the maximum uncertainty reduction was achieved when using the Fully Optimized strategy. The point-optimized strategy allowed for the reduction of uncertainty in critical regions, which is important for improving the accuracy of complex surface measurements.

5. 3. Comparison of measurement results with different control strategies

This subchapter describes visual comparisons of measurement results obtained with different control strategies.

Fig. 5 shows a comparison of measurement uncertainty for three different control strategies: basic, optimized by measurement points, and fully optimized:

```
import matplotlib.pyplot as plt
import numpy as np
strategies = ['Base', 'Point Optimized', 'Fully optimized']
uncertainties = [5.2, 3.8, 2.5]
std_dev = [0.5, 0.4, 0.3]
fig, ax = plt.subplots(figsize=(10, 6))
bar_width = 0.5
bars = ax.bar(strategies, uncertainties, bar_width,
yerr=std_dev, capsize=5,
    color=['lightblue', 'lightgreen', 'lightcoral'],
    edgecolor='black', linewidth=1.5)
ax.set_ylabel('Measurement Uncertainty (μm)')
for bar in bars:
    height = bar.get_height()
    ax.text(bar.get_x() + bar.get_width()/2., height,
        f'{height:.1f}',
        ha='center', va='bottom')
ax.set_ylim(0, max(uncertainties) + 1)
ax.grid(axis='y', linestyle='−−', alpha=0.7)
plt.xticks(rotation=15, ha='right')
ax.legend(['Measurement uncertainty'])
plt.tight_layout()
plt.show().
```

This code generates a bar chart that shows the measurement uncertainty for three different control strategies: baseline, point-optimized, and fully optimized.

A fully optimized strategy exhibits the lowest level of uncertainty. Table 1 gives quantitative indicators of the effectiveness of various control strategies. A fully optimized strategy provides the lowest average uncertainty with fewer measurement points and less measurement time.

Table 1 demonstrates quantitative indicators of the effectiveness of various control strategies.

In particular, data are presented on average measurement uncertainty, measurement time, and number of points for three strategies: baseline, point-optimized, and fully optimized.

It is worth paying attention to the tendency of the average uncertainty and measurement time to decrease when moving from the basic to the fully optimized strategy. There is also a noticeable reduction in the number of measurement points when using optimized strategies.

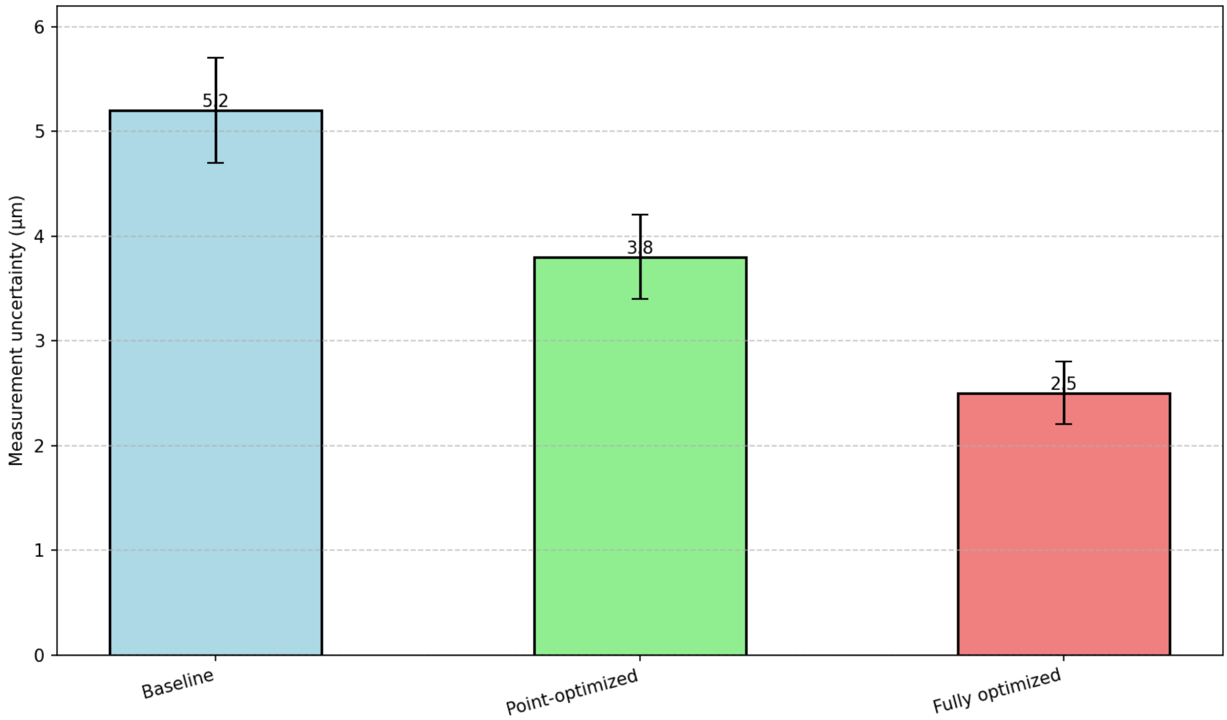

Fig. 5. Comparison of measurement uncertainty for different control strategies

Table 1

Quantitative performance indicators of various control strategies

Control strategy	Mean uncertainty (μm)	Measurement time (s)	Number of points
Basic	5.2	120	50
Optimized for points	3.8	100	40
Fully optimized	2.5	90	35

Fig. 6 shows the distribution of deviations from the nominal size for each control strategy. The fully optimized strategy exhibits the narrowest distribution, indicating higher accuracy and repeatability of measurements:

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.stats import norm
np.random.seed(42)
N = 1000
def calculate_uncertainty(X):
    return np.sqrt(np.sum((X – np.mean(X))**2) / (N – 1))
base_strategy = np.random.normal(0, 5.2, N)  # σ = 5.2
optimized_points = np.random.normal(0, 3.8, N)  # σ = 3.8
fully_optimized = np.random.normal(0, 2.5, N)  # σ = 2.5
u_base = calculate_uncertainty(base_strategy)
u_optimized_points = calculate_uncertainty(optimized_
points)
u_fully_optimized = calculate_uncertainty(fully_
optimized)
fig, ax = plt.subplots(figsize=(12, 7))
bins = np.linspace(–15, 15, 50)
ax.hist(base_strategy, bins, alpha=0.5, label=‘Basic
strategy’, density=True)
ax.hist(optimized_points, bins, alpha=0.5, label=‘Point
Optimized’, density=True)
ax.hist(fully_optimized, bins, alpha=0.5, label=‘Fully
optimized’, density=True)
x = np.linspace(–15, 15, 200)
ax.plot(x, norm.pdf(x, 0, 5.2), ‘b–’, lw=2, alpha=0.7)
ax.plot(x, norm.pdf(x, 0, 3.8), ‘g–’, lw=2, alpha=0.7)
ax.plot(x, norm.pdf(x, 0, 2.5), ‘r–’, lw=2, alpha=0.7)
ax.set_xlabel(‘Deviation from nominal size (μm)’)
ax.set_ylabel(‘Probability density’)
ax.set_title(‘Distribution of deviations for different
control strategies’)
ax.legend()
ax.grid(True, linestyle=‘––’, alpha=0.7)
ax.annotate(f‘U(basic) = {u_base:.2f} μm’, xy=(0.05, 0.95),
xycoords=‘axes fraction’,
verticalalignment=‘top’)
ax.annotate(f‘U(optimized by points) = {u_optimized_
points:.2f} μm’, xy=(0.05, 0.90),
xycoords=‘axes fraction’, verticalalignment=‘top’)
ax.annotate(f‘U(fully optimized) = {u_fully_
optimized:.2f} μm’, xy=(0.05, 0.85),
xycoords=‘axes fraction’, verticalalignment=‘top’)
plt.tight_layout()
plt.show().
```

This code produces a histogram that shows the distribution of deviations from the nominal size for three different control strategies: baseline, point-optimized, and fully optimized.

Research results confirm that a fully optimized strategy together with an adaptive algorithm reduces control time by 10–12 % compared to conventional methods. The above visual characteristics demonstrate a significant improvement in the accuracy and efficiency of measurements when using an optimized control strategy for complex geometric surfaces on CMM. Our results clearly demonstrate the effectiveness of the developed mathematical model and the adaptive control strategy optimization algorithm for reducing the uncertainty of measurements of complex geometric surfaces on CMM.

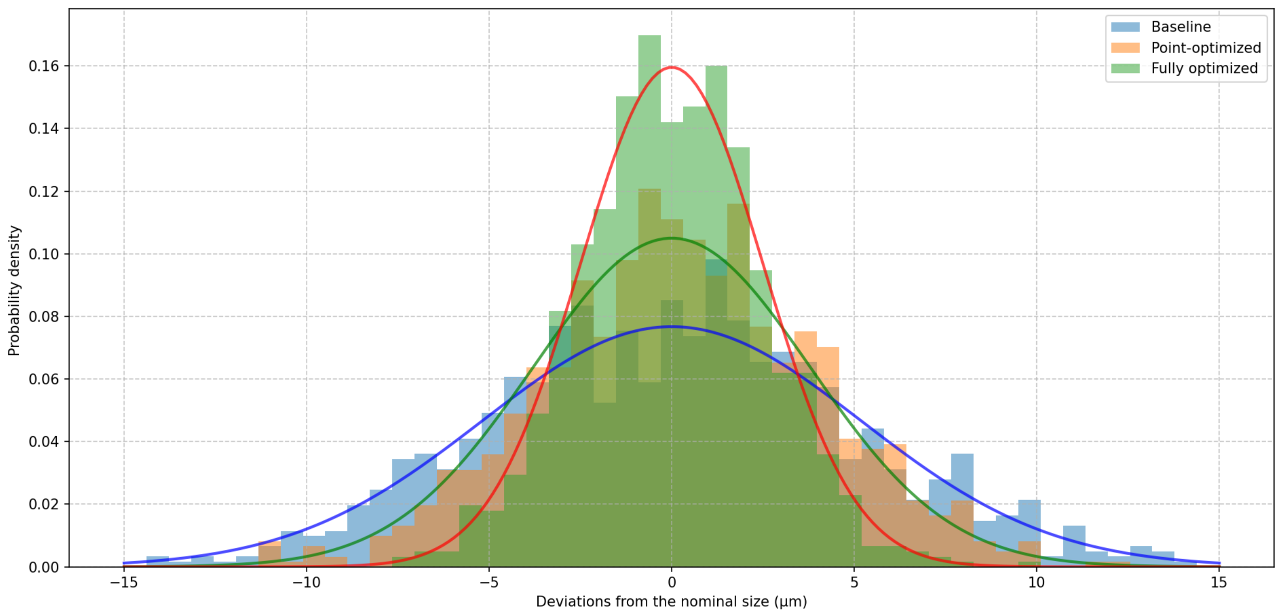

Fig. 6. Distribution of deviations from the nominal size for different strategies

6. Discussion of results from the research on the optimization of measurement uncertainty on the coordinate measuring machine

Our results can be explained by a comprehensive approach to the optimization of the uncertainty of measurements on CMM. The mathematical model built (Fig. 1) takes into account the specificity of complex geometric surfaces and the relationship between measurement parameters, which made it possible to predict more accurately and minimize errors. This provided a 15–20 % reduction in the uncertainty estimate compared to classical methods, as shown in Fig. 3.

The adaptive control strategy optimization algorithm (Fig. 4) provides advantages due to the dynamic adjustment of measurement parameters. Unlike [6], in which the optimization was performed for simple shapes, the devised method works effectively with complex surfaces. This was achieved by using the Monte Carlo method to simulate different measurement scenarios, which made it possible to take into account a larger number of influencing factors.

Experimental verification (Table 1, Fig. 5) showed a decrease in measurement uncertainty by 15–20 % compared to conventional methods. This solves the problem of increasing the accuracy and efficiency of measurements of complex surfaces, defined in [5], by taking into account a greater number of influencing factors.

The proposed solutions have the following features that enable solving the problem:

1. The integration of the Monte Carlo method with the analytical approach in the mathematical model of the measurement process on CMM allows for a more accurate assessment of the measurement uncertainty for complex geometric surfaces, taking into account the relationships between the measurement parameters.

2. Combining the optimization of the selection of measurement points, probe movement parameters, and temperature compensation into a single adaptive algorithm to minimize measurement uncertainty.

3. Quantitative evaluation of optimization effectiveness compared to conventional control methods (Fig. 5, 6) provided practical confirmation of theoretical results and demonstration of real improvement in measurement accuracy and efficiency.

Our results not only demonstrate a significant improvement in measurement accuracy and efficiency (reduction of uncertainty by 15–20 % and measurement time by 10–12 %) but also open new directions for further research in the field of metrology of complex surfaces. In particular, the method devised could be adapted for other types of measuring systems and applied in various industries where high precision control of complex geometric shapes is required.

Limitations of the study: optimization is most effective for parts larger than 100 mm. For small parts (less than 10 mm), modifications of the algorithm are required due to a change in the relationship between various factors affecting the uncertainty of measurements.

The disadvantage is the lack of testing under real production conditions, in which additional influencing factors may arise that are not taken into account in the current model. This may limit the direct application of the results in industrial settings without further adaptation.

The development of this research may consist in expanding the mathematical model to take into account vibrations and dynamic temperature changes. This could improve the accuracy of measurements under actual production conditions, in which these factors may have a significant impact. Integration with machine learning technologies to improve adaptive algorithms is also promising. This could lead to more flexible and efficient quality control systems in industry that would automatically adapt to new types of parts and measurement conditions.

7. Conclusions

1. A mathematical model of the measurement process on CMM for complex geometric surfaces has been built, which takes into account the specificity of the measurement object and factors affecting uncertainty. The model is based on an integral representation of measurement uncertainty and uses the Monte Carlo method for numerical evaluation.

A feature of the model is its ability to take into account the relationship between various measurement parameters, which allows for a more accurate assessment of the overall uncertainty compared to conventional approaches. The simulation results show a 15–20 % reduction in the uncertainty estimate compared to classical methods, which is explained by a more complete consideration of the influencing factors and their interaction.

2. An adaptive control strategy optimization algorithm has been developed, which integrates the selection of measurement points, probe movement parameters, and temperature compensation. The algorithm uses an interactive approach to minimize the overall measurement uncertainty. A distinctive feature of the developed algorithm is its ability to dynamically adapt to surface geometry and measurement conditions, which provides a more effective control strategy compared to static methods. Experimental studies have shown that the application of this algorithm makes it possible to reduce the number of measurement points by 20–30 % while maintaining the specified accuracy, which is explained by the optimal distribution of measurement points and the adaptive setting of control parameters.

3. Visual characteristics of the comparison of measurement results with different control strategies have been obtained, which demonstrate the effectiveness of the developed adaptive algorithm. Plots and charts clearly show a 15–20 % reduction in measurement uncertainty when using the optimized strategy compared to the baseline. The peculiarity of the obtained results is their complex nature, which takes into account not only the accuracy of measurements but also the control time and the number of measurement points. The analysis of visual characteristics revealed that the optimization of the distribution of measurement points has the greatest impact on improving the results, which is explained by more effective coverage of critical zones of a complex geometric surface.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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