

The object of research is a three-phase electricity metering unit, which includes a digital meter and measuring current transformers. The reduction of non-technological energy losses is restrained due to the insufficient accuracy of the accounting of electric energy in distribution power networks under a reduced load current of the metering unit. The possibility of representing the dependence of the relative error of electricity measurement on current values by a fuzzy function at reduced load has been confirmed. The boundaries of such a function are approximated with sufficient accuracy by the sum of two exponents, which is explained by its significant non-linearity in the range of reduced current. The proposed EMRL software allows to estimate the real consumption and the most possible level of underaccounting based on the array of electricity meter readings. The accuracy of estimating by the EMRL the amount of electricity consumed with a probability of 0.7 can be estimated with a relative error not exceeding 2 %. The probability of psychophysical assessments of the accuracy of EMRL «very good» and «good» is at least 0.833. The trend of a significant decrease in the relative value of underaccounting with an increase in the level of electricity consumption was revealed. With a daily consumption of up to 10 kW·h, the amount of underaccounting can reach 18 %, and with a consumption of more than 20 kW·h, it does not exceed 6 %. The adequacy of the results of estimating the amount of consumed electricity at reduced load using the EMRL was confirmed by experimental data at a significance level of 0.05. The software capabilities allow to increase the accuracy of the accounting of electrical energy in distribution networks with a reduced load current of the metering unit. The program can be used as part of automated systems of commercial electricity metering or advanced metering infrastructure to determine the most possible underaccounting due to the operation of metering units at a reduced load

**Keywords:** current transformer, electricity meter, reduced load, measurement uncertainty, membership function

# INCREASING THE ACCURACY OF ELECTRICAL ENERGY ACCOUNTING AT REDUCED LOAD

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## 1. Introduction

The plan for the transformation of the energy system of the European Union REPowerEU [1] envisages a reduction in the consumption of energy from fossil fuels, instead – an increase in the use of electricity in industry and in transport. In order to improve the efficiency of the European energy system, it is planned to create a European energy data space based on intelligent energy infrastructure [2]. Total investments in the electric power system for the period 2020–2030 are estimated at 584 billion EUR, including about 400 billion EUR in the development of distribution grids. About 170 billion EUR of the latter amount is expected to be invested in grids digitalization. One of the important directions in this context is the introduction of intelligent electricity metering, which will allow to increase accuracy. The global smart electricity meter market was estimated at 10.4 billion USD in 2020. Its volume is expected to be 13.5 billion USD for 2024 [3]. Such meters are operated as part of metering units in distribution grids. Their current circuits are connected to the network using measuring current transformers, most often – of the electromagnetic type. The global market for such devices was estimated at 7.46 billion USD in 2021. It is expected to grow from 7.94 billion

in 2022 to 13.14 billion USD in 2030 at a compound annual growth rate of 6.5 % [4].

Electricity metering units are installed at industrial enterprises, charging stations for electric vehicles, renewable sources (solar, wind stations), accumulators, at household consumers etc. More than 186 million smart electricity meters were in use in the European Union by the end of 2023, which is 4 % more than in 2022. With an annual growth rate of 6 %, it is predicted that such measuring equipment will be equipped at the level of 78 % by 2028 [5]. The disadvantages of such measuring equipment include a decrease in the accuracy of accounting in non-standard modes, in particular – in an incomplete phase mode [6] or at a reduced load. In particular, in the latter case, accounting errors can reach – 90 % [7]. Electricity, that is not accounted due to the shortcomings of measuring equipment, is the main component of non-technological losses [8], which also include cases of theft, fraud, non-payment of bills etc. The level of non-technological losses in the power system of Albania is approaching 7 %, in Portugal – up to 9.8 % [9]. Electricity losses in most countries of Latin America and the Caribbean exceed 17 % [10]. Reducing electricity losses will expand access to modern energy sources, reduce greenhouse gas emissions, and lower tariffs for end users [11].

Thus, the need to reduce non-technological losses of electricity in distribution networks determines the relevance of the disclosed scientific issues.

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## 2. Literature review and problem statement

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To detect non-technological losses of electricity, a number of technical solutions based on the use of machine learning and neural networks have been proposed. Such methods involve estimating losses by comparing actual consumption with a typical profile that has been identified based on the analysis of large data sets. In particular, based on the monthly consumption records of 37,814 consumers during the year using the k-fold cross-validation algorithm training and testing data sets, used for model training, were obtained [12]. The latter allows to assign the consumer to the «normal» or «abnormal» categories. Disadvantages of this approach include the failure to take into account the features of the measuring equipment used by each consumer, especially the accuracy class. Also, the method is characterized by significant errors with unbalanced data sets. It is known about the use of a decision tree combined K-nearest neighbor and support vector machine to detect unaccounted electricity consumption [13]. Problem consumers are identified using a combination of supervised and unsupervised machine learning. Drawbacks of this approach include complex technical implementation and focus on peak electricity consumption, the probability of which may be negligible in the case of a long-term reduced load regime. The use of a deep convolutional neural network makes it possible to distinguish between periodic and aperiodic changes in electricity consumption, which is proposed to be used to detect unauthorized consumption [14]. However, this approach may turn out to be wrong in the event of a change in the rhythm of the enterprise's production activity, implementation of energy efficiency measures etc. Detection of non-technological losses of electricity in distribution grids with an accuracy of about 0.7 is possible on the basis of long-term consumption models known as detection contrastive predictive coding [15]. However, this approach is focused on detecting abnormally large levels of consumption compared to the base level, which makes it difficult to detect the reduced load mode.

There are also methods that involve estimating errors and calibrating individual electricity meters. Such operations can be carried out remotely based on the comparison of readings of different levels meter and taking into account the losses in the lines as part of the advanced measuring infrastructure [16]. Remote accuracy control of smart meters can be performed using a modified back-propagation neural network [17]. However, with such approaches, the output of the accounting error beyond the normalized limits due to a decrease in the current of consumers cannot be detected. To detect smart meter errors, it is proposed to use a high-frequency signal processed using a fast Fourier transform in a discrete form [18]. However, the lack of current control makes it difficult to detect a significant decrease in the load level. The meter error can be estimated based on its readings and correction factors using the method of least squares [19]. Incorrect selection of the values of the correction coefficients, especially for the reduced load mode, lowers the effectiveness of the method.

There are known methods of taking into account the errors of measuring current transformers as an integral part of the metering unit. In particular, it is proposed to improve the design of such a device by placing an additional winding [20].

Signals from such a winding should be analyzed by a self-correcting algorithm that detects errors and corrects the measurement results. There is also a well-known proposal to increase the accuracy of the current transformer by using a composite axial magnetic conductor, which is made using permalloy [21]. However, the wide distribution of electromagnetic measuring transformers of classical design complicates the implementation of such methods.

The analysis of the literature [12–21] allows to establish the existing problem: insufficient accuracy of electric energy accounting in distribution power grids with reduced load current of the metering unit. Inconsistency of the actual value of the current with the permissible range according to the accuracy class of the measuring equipment leads to an increase in power losses. This, in turn, is the cause of financial losses of energy supply companies. At the same time, such companies do not have enough evidence of unaccounted energy consumption, which makes it difficult to update the measuring current transformers in the metering unit.

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## 3. The aim and objectives of the study

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The aim of the study is to increase the accuracy of the accounting of electric energy by digital meters of the transformer connection in the distribution 0.38 kV grid by taking into account the characteristics of the measuring equipment at a reduced load current. This will make it possible to reduce the level of non-technological losses of electricity. Accordingly, to increase the profitability of energy supply companies. And also, to stimulate consumers to bring the parameters of measuring equipment (in particular, the rated current of measuring transformers) into compliance with actual operating conditions.

To achieve the aim, the following objectives must be solved:

- to identify the characteristics of the measuring equipment of a specific electricity metering unit at a reduced load;
- to estimate consumption and the most possible under-accounting based on the set of experimental data;
- to assess the accuracy and adequacy of determining the amount of consumed electricity at reduced load.

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## 4. Materials and methods

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### 4. 1. Object and research hypothesis

The object of the study is a three-phase electricity metering unit, which includes a digital meter and measuring current transformers, Fig. 1. This unit is intended for commercial metering in 380 V power distribution grids.

The subject of the study is the uncertainty of electricity measurement at reduced current through the primary windings of measuring transformers. At the same time, the root-mean-square value of such a current is smaller than twice the smallest current, at which the relative error is normalized according to [22]. In particular, for the accuracy class of 0.5 S, this smallest current is 1 %. Uncertainty is supposed to be estimated by a L-R fuzzy interval, the boundaries of which depend on the type of membership function, the accepted level of significance, and the values of the phase currents.

The main hypothesis of the study is the possibility of electricity measurement at a reduced load of the metering unit, when the currents through the measuring equipment are smaller than the lower limit of the class-standardized accuracy range.

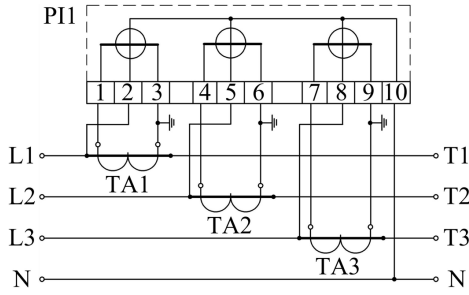


Fig. 1. Schematic diagram of a three-phase electricity metering unit: P11 – meter; TA1–TA3 – measuring current transformers; L1, L2, L3 – generator terminals; T1, T2, T3 – terminals for connecting the load; N – neutral

Research is conducted under the following assumptions:

1. The currents of each of the phases of the consumer at a reduced load are considered as implementations of a random stationary process. Such currents are uniformly discretized in time with an interval  $\Delta t$ . During each of these intervals, it is considered that the current remains unchanged and is equal to the mean of the realization of a random process. Direct current measurement may not be possible for technical or organizational reasons. If there is information about the actual amount of consumed energy  $\Delta W$ , kW-h, for the interval  $\Delta t$ , hour, the average value of the current  $\bar{I}_p$  of each phase during the specified interval can be calculated as:

$$\bar{I}_p = \frac{1,000 \cdot \Delta W}{\sqrt{3} U_r \cdot \Delta t \cdot \cos \varphi}, \quad (1)$$

where  $U_r$  – rated voltage, V;  $\cos \varphi$  – power factor, p.u.

2. The voltage  $U_\zeta(t)$  of each phase,  $\zeta = \{A, B, C\}$ , corresponds to realizations of a random stationary process. The relative values of the phase voltages with a probability of 0.95 belong to the permissible interval according to [23]:

$$P[0.9 \leq U_\zeta^*(t) \leq 1.1] \approx 0.95. \quad (2)$$

3. It is assumed that during the reduced load mode, the power factor of each phase with a probability of 0.95 lies within the interval:

$$P[0.995 \leq \cos \varphi_\zeta \leq 1] \approx 0.95. \quad (3)$$

This corresponds to the active nature of the consumer during the reduced load.

4. The positive direction of power is assumed: from the source to the consumer.

5. Changes in conductor resistance and metrological characteristics of measuring equipment due to ambient temperature fluctuations are not taken into account.

#### 4. 2. Mathematical modeling of the metering unit at reduced load

The method [24] of presenting the static characteristics of the measuring current transformer at a reduced current through the primary winding with a statistical model was used in the study:

$$\hat{I}_s^*(I^*) = \hat{\mu}' + \hat{\beta} \cdot I^*, \quad (4)$$

where  $I^*$  and  $\hat{I}_s^*$  – the primary winding current and the estimate of the secondary winding current of the measuring transformer, p.u.;  $\hat{\mu}'$ ,  $\hat{\beta}$  – estimates of linear regression parameters, p.u.

Covariance analysis methods are used to estimate the parameters values of characteristic (4).

A mathematical model of the uncertainty of electricity measurement at reduced load is also used [25]. The latter involves the use of fuzzy functions to estimate the measurement uncertainty. For the  $\zeta$  measuring channel, the boundaries of the fuzzy function, which evaluates the relative deviation of the meter readings in the metering unit from the actual value of the consumed energy, correspond to the expression:

$$\delta \bar{W}_\zeta(I_\zeta) = \left[ \delta W_{L_\zeta}(I_\zeta) \Big|_{\lambda_j^*}; \delta W_{R_\zeta}(I_\zeta) \Big|_{\lambda_j^*} \right], \quad (5)$$

where  $\lambda_j^*$  – confidence level, which is an element of the set  $\{\lambda^*\}$  at  $j = 1, \bar{\Lambda}$ ,  $\lambda_j^* > \lambda_{j+1}^*$ .

In addition, the left and right limits of (5) are obtained by approximating the experimental points with the functions  $F$ , namely:

$$\delta W_{L_\zeta}(I_\zeta) \Big|_{\lambda_j^*} = F[I_\zeta, \{L_{\zeta j}\}], \quad (6)$$

$$\delta W_{R_\zeta}(I_\zeta) \Big|_{\lambda_j^*} = F[I_\zeta, \{R_{\zeta j}\}], \quad (7)$$

where  $\{L_{\zeta j}\}$ ,  $\{R_{\zeta j}\}$  – sets of parameters of functions  $F$ .

The fuzzy function that describes the measurement uncertainty by the three-phase metering unit at the phase's currents  $I_A, I_B, I_C$ , is:

$$\delta \bar{W}(I_A, I_B, I_C) = \frac{\sum_\zeta I_\zeta \cdot \delta \bar{W}_\zeta(I_\zeta)}{\sum_\zeta I_\zeta}, \quad (8)$$

and the boundaries of such a function for the confidence level  $\lambda_j^*$  are equal to:

$$\delta W_{L(R)}(I_A, I_B, I_C) \Big|_{\lambda_j^*} = \frac{\sum_\zeta I_\zeta \cdot \delta W_{L(R)\zeta}(I_\zeta) \Big|_{\lambda_j^*}}{\sum_\zeta I_\zeta}. \quad (9)$$

Estimation of boundaries (9) for a set of confidence levels makes it possible to obtain a sample. It is provided that the sample values are approximated, it is possible to obtain the membership function:

$$\mu_{abc}(\delta W) = \begin{cases} \mu_{abc_L}(\delta W), & \text{if } \delta W \leq \delta W_v; \\ \mu_{abc_R}(\delta W), & \text{if } \delta W > \delta W_v, \end{cases} \quad (10)$$

where  $\delta W_v$  – the closest quantity to the true value of  $\delta W$ .

In order to determine, in accordance with the membership function (10), the limits of  $\delta W$  change, which can be used to calculate underaccounting, the marginal confidence level  $\lambda_b^*$  should be estimated. Such an indicator is defined for measuring equipment of a specific accuracy class and type as:

$$\lambda_b^* = m[\lambda_e^*] - 2 \cdot s[\lambda_e^*], \quad (11)$$

where  $\lambda_e^*$  – the sample values of the confidence level obtained in a series of experiments to identify the characteristics of

the metering unit;  $m$  – the sample mean;  $s$  – the sample standard deviation.

The calculation of the marginal confidence level (11) makes it possible to estimate the interval of belonging of the value  $\delta W$  at the determined values of the phase currents:

$$\begin{aligned} \widetilde{\delta W}(I_A, I_B, I_C) = & \\ = & \left[ \frac{\sum_{\zeta} I_{\zeta} \cdot \delta W_{L\zeta}(I_{\zeta})|_{\lambda_b^*}}{\sum_{\zeta} I_{\zeta}}; \frac{\sum_{\zeta} I_{\zeta} \cdot \delta W_{R\zeta}(I_{\zeta})|_{\lambda_b^*}}{\sum_{\zeta} I_{\zeta}} \right]. \end{aligned} \quad (12)$$

The obtained interval (12) characterizes the uncertainty of electricity measurement in the reduced load mode. Based on its boundaries, it is possible to estimate the lowest and highest possible levels of underaccounting.

**4. 3. General algorithm for estimating electricity consumption and the most possible underaccounting**

The general algorithm for estimating the accuracy of the electricity metering unit at reduced load (Fig. 2) involves two procedures. Metrological characteristics are identified once for specific equipment. The subsystem that implements such a procedure is marked I and includes blocks 1–11. For specific operating conditions, which are determined by the values of the consumer phases currents, the actual amount of energy consumed during the studied time is estimated based on the obtained characteristics. Such an estimate is given by a fuzzy number  $\widetilde{W} = [W_L; W_R]$ .

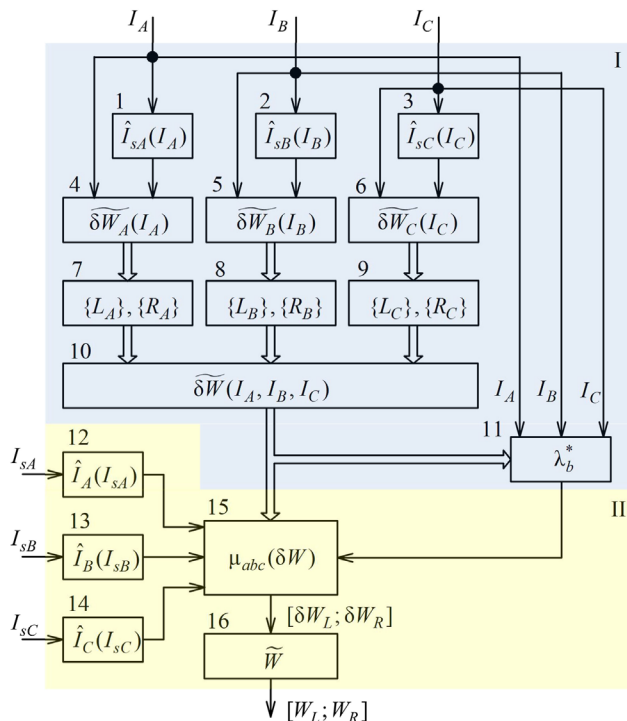


Fig. 2. Structural diagram of the general algorithm for assessing the accuracy of the electricity metering unit at reduced load

Identification of the metrological characteristics of the metering unit is based on the load currents values  $I_A, I_B, I_C$ . The parameters evaluation of the static characteristics of the measuring current transformers is carried out in accordance with dependence (4) by blocks 1, 2, 3. The evaluation of

the parameter’s values of the fuzzy functions (5) for each of the measuring channels is carried out by blocks 4–6. In blocks 7–9, a nonlinear approximation of the boundaries of such functions is carried out at given levels of significance in accordance with (6), (7). Such dependencies make it possible to establish the relationship between the boundaries (9) of the fuzzy function (8) for the metering unit with current values, block 10. The last function is used to determine, according to (11), the marginal confidence level, block 11. The output values of subsystem I are the array of parameters values of the fuzzy function (8) and the value  $\lambda_b^*$ . On the basis of such data, the uncertainty of electricity measurement is evaluated in subsystem II for specific values of the secondary currents  $I_{sA}, I_{sB}, I_{sC}$  of the measuring transformers. These values are measured by a digital meter as part of the metering unit and can be read by software. The primary currents evaluation is carried out by blocks 12–14 according to dependencies that are inverse to those defined in blocks 1–3. The key operation of the subsystem II is the determination by polynomial approximation of the membership function (10), block 15, taking into account the marginal confidence level. As a result, the boundaries of the fuzzy number are calculated:

$$\widetilde{\delta W} = [\delta W_L; \delta W_R], \quad (13)$$

which evaluates the relative deviation of meter readings as part of the metering unit from the actual amount of consumed energy during the reduced load period. The specified real value is characterized by fuzzy number:

$$\widetilde{W} = \frac{W_{PI1}}{\delta W + 1}. \quad (14)$$

The output values of subsystem II are the boundaries of the number (14), which estimate the actual consumption of electricity and are calculated in block 16 as:

$$\widetilde{W} = [W_L; W_R] = \left[ \frac{W_{PI1}}{\delta W_R + 1}; \frac{W_{PI1}}{\delta W_L + 1} \right], \quad (15)$$

where  $W_{PI1}$  – the electricity consumed during the reduced load mode according to the readings of the P11 meter in the metering unit.

For the most unfavorable conditions for the consumer, the right boundary of the interval (15) can be considered as the amount of electricity  $W_{EMRL}$  consumed during reduced load:  $W_{EMRL} = W_R$ .

**4. 4. The procedure for identifying the characteristics of the metering unit at reduced load**

To perform calculations in accordance with blocks 4–9 (Fig. 2), the GS software was developed in the MATLAB environment (The MathWorks company, USA). The program calculates the sets of parameters  $\{L\}, \{R\}$  of the  $F$  curves, approximating the fuzzy functions (6), (7), Fig. 3. The input data (block 2) are sample test values of currents  $I_{\zeta\gamma}$  and values of  $\delta W_{L\zeta\gamma}$ , and  $\gamma = 1, \overline{M}$  is the number of the test current interval. For all phases and current intervals, block 3, the average current values  $\overline{I_{\zeta\gamma}}$  of the interval are calculated, block 4. A set of confidence levels is specified, block 5. For all phases, current intervals and confidence levels, block 6, using the ‘f\_interv’ sub-program, the boundaries of the fuzzy interval  $[\delta W_{L\zeta\gamma}; \delta W_{R\zeta\gamma}]$  are calculated, block 7. The measurement result  $\delta W_{v\zeta\gamma}$  which is the closest to the true value, is also determined.



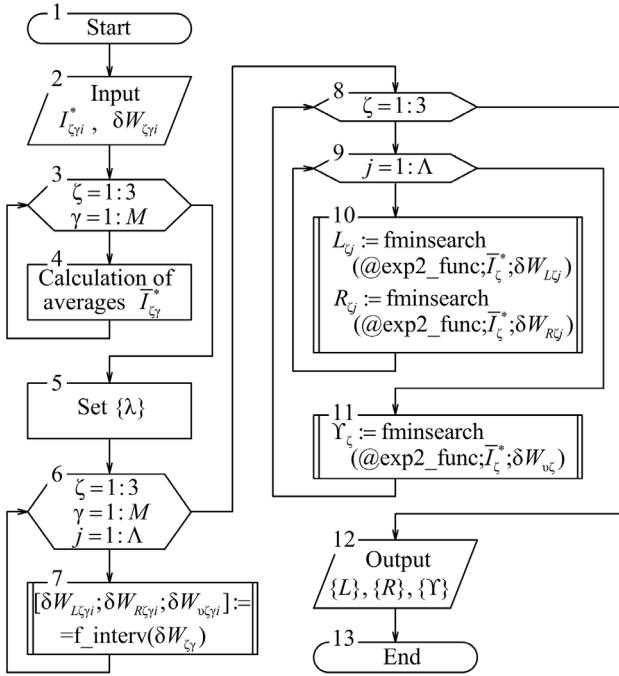


Fig. 3. Block diagram of the GS software algorithm for calculating the sets  $\{L\}, \{R\}$  of the curves parameters that approximate the fuzzy functions (6), (7)

For all phases, block 8, and confidence levels, block 9, the values of parameters  $\{L\}, \{R\}$  sets are calculated, block 10. boundaries (6) and (7) are approximated by the sum of exponents:

$$F(x, \{K\}) = K^{(1)} \cdot \exp[-x / K^{(3)}] + K^{(2)} \cdot \exp[-x / K^{(4)}] + K^{(5)}, \quad (16)$$

where  $\{K\} = \{K^{(1)}, \dots, K^{(5)}\}$  is a set of parameters.

Estimation of  $\{K\}$  is carried out by the least squares method. The minimization of the squares sum of the residuals is performed by the built-in 'fminsearch' function of the MATLAB environment, which implements the simplex Nelder-Mead method of minimization of a function with several variables. The custom function 'exp2\_func' calculates the residuals. Similarly, the value of the parameters  $\{Y\}$  of the dependence, which approximates the values of the deviations closest to the true value, is estimated, block 11.

Algorithm of the 'f\_interv' subprogram, Fig. 4, implements the method proposed in [26]. The input values are the vector of measured values  $\delta W_i$ , the degree  $L$  of polynomials approximating the branches of the membership function of the measured value, and the confidence level  $\lambda^*$ .

The measured values are sorted in ascending order, block 1. The intervals  $\Delta_k$  between adjacent sorted values and their extreme values  $\Delta_{\max}, \Delta_{\min}$ , block 2 are calculated. For all intervals, block 3, the frequency  $m_k$  of the measured values is calculated, block 4. In block 5, the number  $v$  of the measured value with the smallest interval width is determined. With the help of blocks 6, 7 and 8, 9, sample values  $\mu_1$  of the left and  $\mu_2$  of the right, respectively, branches of the membership function of the measured quantity are calculated. Moreover, the relative values of this quantity for the specified branches are  $\tau_1$  and  $\tau_2$ , respectively.

The mentioned branches are approximated by polynomials of  $L$  degree, the arrays of coefficients of which are calculated according to the criterion of the minimum of the Chebyshev norm adjustment, block 10. The components  $\xi_1$  and  $\xi_2$  of the fuzzy interval at a given confidence level are determined using the built-in MATLAB 'roots' function for finding the roots of a polynomial, block 11. This allows the width and bounds of the fuzzy interval to be calculated, block 12.

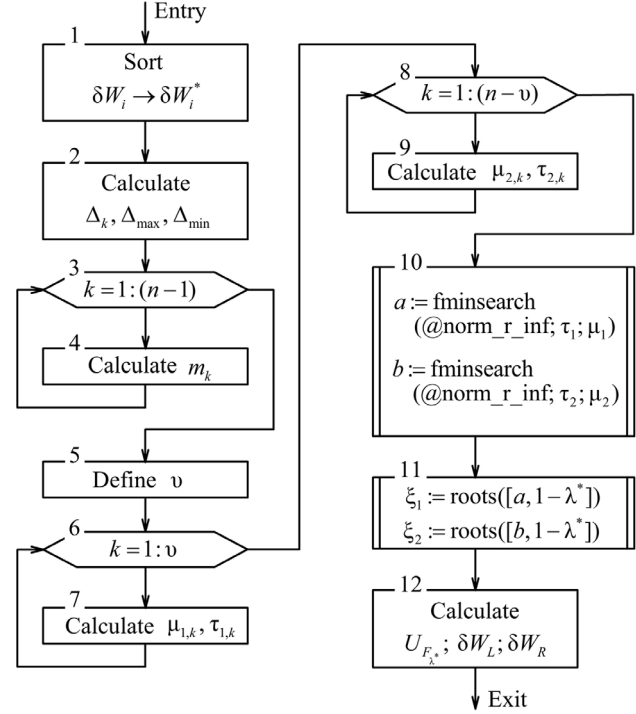


Fig. 4. Block diagram of the 'f\_interv' subprogram algorithm for determining the boundaries of a fuzzy interval that estimates the measured quantity

#### 4. 5. Software for estimating underaccounting and actual electricity consumption

Procedures for estimating the uncertainty of electricity measurement for specific conditions, corresponding to subsystem II of the general algorithm, Fig. 2, implemented as EMRL software, Fig. 5.

The EMRL software provides, based on the data received from the digital meter at certain time intervals, an estimate of the most likely underaccounting and the actual consumption of electricity. It is developed in Microsoft Visual Studio Professional 2022 Version 17.10.0 (Microsoft Corporation, USA). The data necessary for the program to work are contained in files of the \*.xml type. The electricity metering unit characteristics file (metering\_unitObj1.xml) includes the values of the meter parameters, current measuring transformers and dependencies characterizing the metering unit in reduced load mode. The file with meter readings (dataObj1.xml) is generated during the reduced load mode. After reading the input data, the main window of the program displays: duration of reduced load; consumption according to readings; estimation of real consumption; the most possible underaccounting. Such data are summarized for a day and throughout the studied time period.

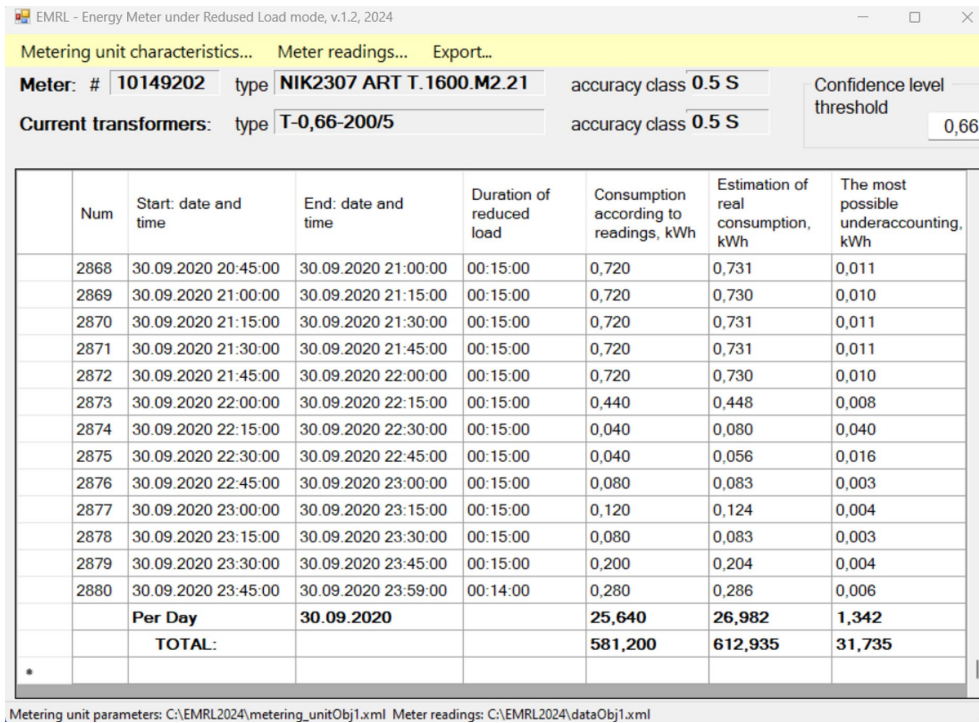


Fig. 5. EMRL software

4. 6. Experimental data

The accuracy of electricity metering is assessed using the results of the PJSC ‘Rivneoblenergo’ experiment [27] for consumer No. 1. The experiment was conducted for a private enterprise (Rivne), whose electricity metering unit is equipped with a PI1 meter No. 10149202 type NIK2307 ART T.1600.M2.21 (accuracy class 0.5 S, manufactured by ‘NIK’ LLC, Ukraine). The meter was connected to the line using three measuring current transformers of type T-0.66-200/5 (accuracy class 0.5 S, manufactured by PJSC ‘Uman Zavod ‘Megommeter’, Ukraine). In addition, the metering unit was equipped with a PI2 directly connected meter No. 10011723, type NIK2307 ARP3 T.1600.M2.21 (accuracy class 0.5 S, manufactured by ‘NIK’ LLC, Ukraine). The duration of the experiment is from September 1 to 30, 2020. Every 15 minutes, the following data were automatically recorded for each of the meters: positive active energy A+ (OBIS 1.8.0, kW·h); negative active energy A- (OBIS 2.8.0, kW·h); positive reactive energy R+ (OBIS 3.8.0, kVAr·h); negative reactive energy R- (OBIS 4.8.0, kVAr·h). According to the calculations in [27], the load power factor was 0.997, which confirms the correctness of the assumption about the active nature of the load.

4. 7. Procedure for assessing the accuracy and checking the adequacy of the results

To assess the accuracy of determining the amount of electricity consumed during the reduced load mode, it is suggested to use the following indicator:

$$\gamma = \frac{|W_{PI2} - W_{EMRL}|}{W_{PI2}} \cdot 100\%, \tag{17}$$

where  $W_{EMRL}$  – the amount of electricity consumed during the period of reduced load, estimated using the EMRL, kW·h;  $W_{PI2}$  – valid amount of energy for the same period, measured by a direct-on meter, kW·h.

To estimate the value of the indicator  $\gamma$  and, accordingly, the accuracy of the EMRL software, the modified Harrington scale can be used, which is given by the equation (Table 1):

$$d(\gamma) = \exp\{-\exp[\gamma - 4]\}. \tag{18}$$

The adequacy of the  $W_{EMRL}$  estimates to the actual  $W_{PI2}$  energy consumption during reduced load is supposed to be checked by examining the absolute residuals:

$$\varepsilon = W_{PI2} - W_{EMRL}. \tag{19}$$

The results obtained using the EMRL are considered adequate to the actual energy consumption if the residuals  $\varepsilon$  satisfy the following requirements [28]:

- 1) are independent normally distributed random variables;
- 2) are characterized by zero mean;
- 3) there is no autocorrelation of the first order between sample residual values.

Table 1

Evaluating the EMRL accuracy on a desirability scale

EMRL accuracy	Desirability value $d$
Very good	1.00...0.80
Good	0.80...0.63
Satisfactory	0.63...0.37
Bad	0.37...0.20
Very bad	0.20...0.00

The Kolmogorov-Smirnov test is supposed to be used to test the sample of residuals  $\varepsilon$  for normality of distribution. Checking the mean for a zero value is carried out according to Student’s t-test. The absence of autocorrelation of the first order between the residuals  $\varepsilon$  can be detected using the Durbin-Watson statistic.

**5. Results of the study of accuracy of electrical energy metering at reduced load**

**5.1. Identification of characteristics of measuring devices at reduced load**

As a result of the characteristics identification of current transformers T-0.66-200/5, estimates of the parameter values of the statistical model (4) were obtained, namely:  $\hat{\mu} = -1.369 \cdot 10^{-4}$  p.u.;  $\hat{\beta} = 9.932 \cdot 10^{-1}$  p.u.

For a set of confidence levels  $\lambda = 0.80, 0.75, \dots, 0.20$ , using the developed GS software, the characteristics of the counter PI1 No. 10149202 type NIK2307 ART T.1600.M2.21 were identified in the reduced load mode. The results are presented in the form of estimations of the approximation coefficients of parameters sets of the fuzzy function (5) boundaries, Table 2. The sum of exponents (16) was used for approximation.

The received characteristics of the measuring equipment are recorded in the metering\_unitObj1.xml file. Parameter values from such a file are read by the EMRL when clicking on the «Metering unit characteristics...» button. Based

on the test measurements, the marginal confidence level  $\lambda_b^* = 0.66$  was established, which is entered in the «Confident level threshold» field, Fig. 5.

**5.2. Estimation of electricity consumption and the most possible underaccounting based on the array of experimental data**

The experimental data used to evaluate the accuracy of electricity metering contain 2,880 time intervals. Such a data array was saved in the dataObj1.xml file and uploaded to the EMRL program using the «Meter readings...» button. The results of calculations of real consumption and the most possible underaccounting for each of the time intervals, each day and monthly indicators were displayed in the program window, Fig. 5. The results summarized for every 5 days are shown in the Table 3.

The results of the comparison of the absolute underaccounting values  $\Delta W_{EMRL}$ , estimated by the EMRL, and  $\Delta W_{PI2}$ , which are calculated based on the readings of the PI2 directly connected meter, are illustrated by the diagram in Fig. 6. Such a comparison is made for each day of the month.

Table 2

Estimates of approximation coefficients of parameter sets for fuzzy function boundaries (5) by the sum of exponents (16) for selected confidence levels

A set of parameters	Phase	Confidence level number $j$ (confidence level $\lambda$ )	Value of approximation coefficients				
			$K^{(1)}$	$K^{(2)}$	$K^{(3)}$	$K^{(4)}$	$K^{(5)}$
{L}	A	1 (0.80)	$-8.41 \cdot 10^2$	1.58	$5.82 \cdot 10^{-2}$	2.24	-2.48
		...	...	...	...	...	...
		13 (0.20)	$-3.35 \cdot 10^2$	8.09	$1.05 \cdot 10^{-1}$	$3.42 \cdot 10^{-1}$	-3.03
	B	1 (0.80)	$-1.78 \cdot 10^4$	$-1.00 \cdot 10^2$	$2.49 \cdot 10^{-2}$	$9.73 \cdot 10^3$	$9.87 \cdot 10^1$
		...	...	...	...	...	...
		13 (0.20)	$-5.17 \cdot 10^2$	$-3.53 \cdot 10^1$	$6.65 \cdot 10^{-2}$	$3.06 \cdot 10^{-1}$	-2.48
	C	1 (0.80)	$-3.92 \cdot 10^4$	$-2.08 \cdot 10^3$	$2.29 \cdot 10^{-2}$	$6.76 \cdot 10^4$	$2.08 \cdot 10^3$
		...	...	...	...	...	...
		13 (0.20)	$-6.28 \cdot 10^2$	$-3.09 \cdot 10^7$	$7.35 \cdot 10^{-2}$	$2.96 \cdot 10^8$	$3.09 \cdot 10^7$
{R}	A	1 (0.80)	$-6.61 \cdot 10^3$	7.61	$2.97 \cdot 10^{-2}$	$1.58 \cdot 10^1$	-7.37
		...	...	...	...	...	...
		13 (0.20)	$-2.31 \cdot 10^3$	$5.57 \cdot 10^1$	$3.99 \cdot 10^{-2}$	$8.62 \cdot 10^1$	$-5.28 \cdot 10^1$
	B	1 (0.80)	$-2.56 \cdot 10^4$	$-9.84 \cdot 10^1$	$2.33 \cdot 10^{-2}$	$-1.09 \cdot 10^4$	$9.91 \cdot 10^1$
		...	...	...	...	...	...
		13 (0.20)	$-2.60 \cdot 10^4$	$1.01 \cdot 10^2$	$2.33 \cdot 10^{-2}$	$-6.02 \cdot 10^3$	$-9.88 \cdot 10^1$
	C	1 (0.80)	$-4.91 \cdot 10^4$	$-3.90 \cdot 10^2$	$2.21 \cdot 10^{-2}$	$-2.92 \cdot 10^4$	$-3.90 \cdot 10^2$
		...	...	...	...	...	...
		13 (0.20)	$-6.02 \cdot 10^4$	$-2.29 \cdot 10^2$	$2.16 \cdot 10^{-2}$	$-2.49 \cdot 10^4$	$2.34 \cdot 10^2$
{Y}	A	×	$-1.19 \cdot 10^4$	$-8.52 \cdot 10^2$	$2.59 \cdot 10^{-2}$	$6.03 \cdot 10^3$	$8.51 \cdot 10^2$
	B	×	$-2.34 \cdot 10^4$	$-1.01 \cdot 10^2$	$2.37 \cdot 10^{-2}$	$4.67 \cdot 10^4$	$1.01 \cdot 10^3$
	C	×	$-5.87 \cdot 10^2$	$-2.57 \cdot 10^3$	$2.15 \cdot 10^{-2}$	$3.71 \cdot 10^4$	$2.57 \cdot 10^3$

Table 3

EMRL estimates of actual consumption and the most likely underaccount for five-day time intervals

Time interval	Energy consumption by readings of the metering unit $W_{PI1}$ , kW·h	Estimating energy consumption by the EMRL software $W_{EMRL}$ , kW·h	Energy consumption by readings of the directly connected meter $W_{PI2}$ , kW·h	Evaluation of the most possible underaccount by the EMRL software	
				$\Delta W_{EMRL}$ , kW·h	$\delta W_{EMRL}$ , %
01–05.09.2020	123.040	126.117	125.706	3.077	2.4
06–10.09.2020	108.000	112.029	111.381	4.029	3.6
11–15.09.2020	91.160	96.895	95.725	5.735	5.9
16–20.09.2020	88.160	92.957	94.664	4.797	5.2
21–25.09.2020	87.800	94.679	94.216	6.879	7.3
26–30.09.2020	83.040	90.258	90.007	7.218	8.0
Total	581.200	612.935	611.699	31.735	5.2

The comparison results of the relative values of underaccounting  $\delta W_{EMRL}$ , estimated by the EMRL software, and  $\delta W_{PI2}$ , calculated from the readings of the PI2 meter, are shown in Fig. 7. Moreover, such a comparison is made depending on the actual electricity consumption of  $W_{PI2}$ , which is determined by the directly connected meter.

Underaccounting values, which are plotted on the graph, Fig. 7, calculated relative to the real levels of electricity consumption for each day during the month under research.

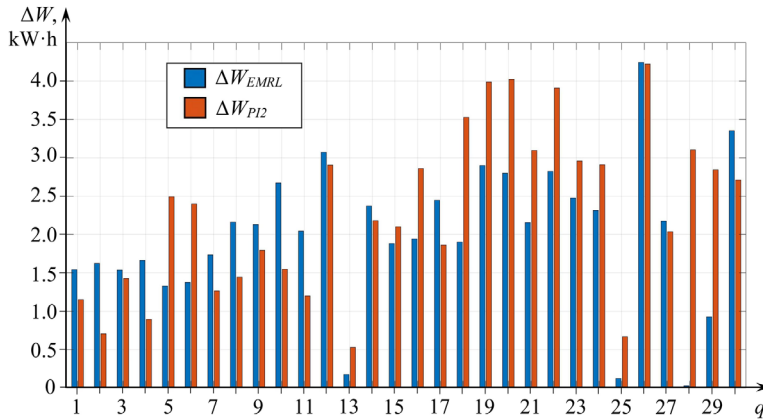


Fig. 6. Comparison of the daily absolute values of the underaccounting  $\Delta W_{EMRL}$ , kW·h, estimated by the EMRL software, and  $\Delta W_{PI2}$ , kW·h, which are calculated from the readings of the directly connected meter, on  $q$  days during September 2020

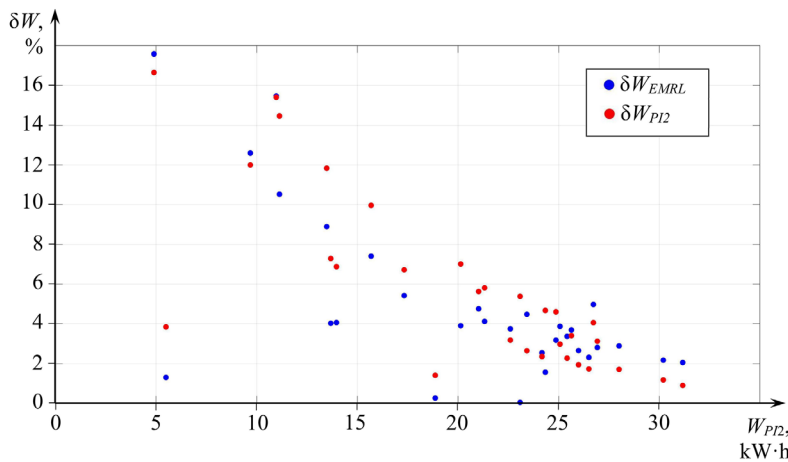


Fig. 7. Daily relative values of underaccounting  $\delta W_{EMRL}$ , %, estimated by the EMRL, and  $\delta W_{PI2}$ , %, calculated from the readings of the directly connected meter depending on the actual consumption of electricity  $W_{PI2}$ , kW·h

### 5. 3. Evaluation of accuracy and adequacy verification of the results of the consumed electricity amount assessment at a reduced load

For the daily amount of electricity, the value of the indicator  $\gamma$  was calculated according to (17), as a result of which a histogram was plotted, Fig. 8.

The number of bins was determined by Sturges's rule:

$$n = 1 + \lceil \log_2 N \rceil = 1 + \lceil \log_2 30 \rceil = 5. \tag{20}$$

The results of evaluating the EMRL software accuracy using the modified desirability function (18) are shown in Fig. 9. The  $d(\gamma)$  values were estimated for a sample of  $\gamma$  estimated by daily amount of energy during the month

under study. The probability of an accuracy rating of «very good» is  $7/10=0.700$ , «good» –  $4/30=0.133$ , «satisfactory» –  $3/30=0.100$ , «bad» –  $1/30=0.033$ , «very bad» –  $1/30=0.033$ .

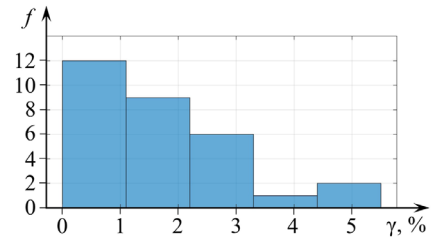


Fig. 8. Histogram of indicator  $\gamma$ , %, sample values frequencies  $f$  hits to the corresponding bin

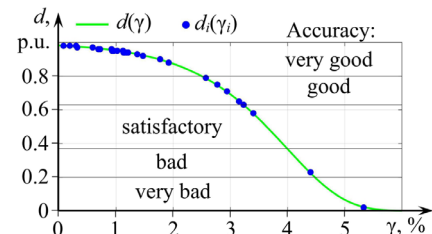


Fig. 9. Experimental points  $\gamma$ , %, on the curve of the modified desirability function  $d(\gamma)$ , p.u.

The results of adequacy verification of the consumed electricity amount assessment are as follows. Regarding the absolute residuals  $\epsilon$  (Fig. 10), calculated according to dependence (19) for each day of the studied month (sample size  $N=30$ ), statistical hypotheses are put forward. The null hypothesis assumes that the sample conforms to the normal distribution law, the alternative hypothesis does not conform. The experimental value of the Kolmogorov-Smirnov statistic is  $K_0=0.1285$ . The critical value of the criterion equals  $K_c=0.2417$  at the significance level  $\alpha=0.05$ . The fulfillment of the inequality  $K_0 < K_c$  does not give grounds to reject the null hypothesis, which indicates a normal distribution of the absolute residuals  $\epsilon$ .

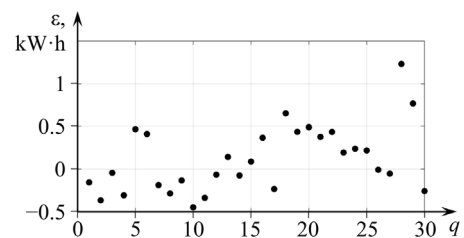


Fig. 10. Residues  $\epsilon$ , kW·h, for each day  $q$  during the studied month

The second condition of adequacy is checked according to the Student's t-test. The following hypotheses are put forward.  $H_0: m[\epsilon]=0$ .  $H_1: m[\epsilon] \neq 0$ . The empirical value of the criterion  $t_0=1.6537$ . The critical value  $t_c=t(1-\alpha; N-1) = t(0.95; 29)=1.70$  at  $1-\alpha=0.95$ . This does not give grounds to reject the null hypothesis and indicates the zero value of the residuals mean.



The Durbin-Watson test was used to check the third adequacy condition. The null hypothesis indicates the independence of the random deviations of the residuals  $\varepsilon$ , Fig. 11. An alternative hypothesis lies in autocorrelation between sample values of  $\varepsilon$ .

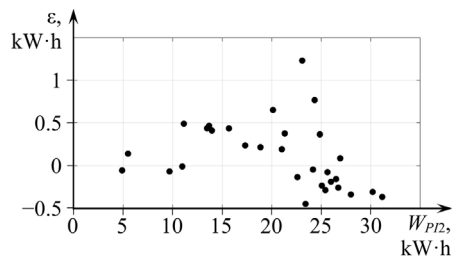


Fig. 11. Residues  $\varepsilon$ , kW·h, depending on actual energy consumption  $W_{PI2}$ , kW·h

The empirical value of the criterion  $D_0=1.5526$ . The critical value belongs to the interval  $[d_L; d_U]=[1.352; 1.489]$  at  $\alpha=0.05$ . Since the condition  $D > d_U$  is fulfilled, there are no grounds for rejecting the null hypothesis. This indicates the statistical insignificance of the first-order autocorrelation between the residuals  $\varepsilon$ .

## 6. Discussion of the results of the accuracy assessment of electrical energy metering at reduced load

The received estimates of parameters  $\hat{\mu}'$ ,  $\hat{\beta}$  of static characteristics (4), which describes the current transformers, are explained by the deviation of the transformation coefficient from the rated value in the area of low primary currents. The rated transformation ratio when measuring currents in relative units is equal to 1. At the same time, this ratio with the obtained parameter estimates in the area of low loads is 1.007, which explains the deviation of the actual secondary current from the expected one. The selection for the boundaries (6) and (7) approximation of the sum of two exponents (16), the estimates of whose parameters  $K^{(1)}, \dots, K^{(5)}$  for each phase and confidence level are obtained in the Table 2, is explained by the significant nonlinearity of the  $\delta W(I)$  dependence. Such nonlinearity takes place in the area of reduced loads, when the means of measuring equipment in the metering unit function in the non-class-standardized accuracy of the current area. Using the sum of exponents (16) with coefficient estimates (Table 2) makes it possible to use one function to describe the  $\delta W(I)$  dependence without using the piecewise linear approximation method.

For five-day time intervals, the most possible underaccounting, estimated by the EMRL software, ranges from 2.4 % to 8.0 %, Table 3. In absolute terms – from 3.077 kW·h to 7.218 kW·h. The monthly level of underaccounting is 5.2 %, which corresponds to 31.735 kW·h. For every 5 days, the deviation of the  $W_{EMRL}$  estimate of the energy consumption obtained using the software from the  $W_{PI2}$  consumption according to the readings of the PI2 meter did not exceed 1.7 kW·h, which corresponds to 1.8 %. The monthly estimate of consumption by the EMRL differs from the actual level by 1.2 kW·h (0.2 %). Similar deviations of daily consumption volumes, Fig. 6, are in the range from 0.009 kW·h to 1.231 kW·h. Analyzing the change in the relative values of underaccounts  $\delta W_{EMRL}$  and  $\delta W_{PI2}$  when the  $W_{PI2}$  level of electricity consumption increases, it is possible to establish

a trend of their significant decrease, Fig. 7. With a daily consumption of up to 10 kW·h, the level of underaccounting can reach 18 %, with consumption from 10 kW·h to 20 kW·h – from 4 % to 16 %. If the daily consumption exceeds 20 kW·h, the underaccounting is not more than 6 %. This is explained by the significantly non-linear nature of the dependence (12). The average monthly level of underaccounting according to the readings of the direct connection meter is 4.5 %, the estimate calculated by the EMRL is 5.2 %.

The absolute value of the relative deviation  $\gamma$ , the dependence (17), of the estimated amount of electricity consumption from the actual with the highest probability  $12/30=0.4$  lies in the range from 0 % to 1 %, Fig. 8. The probability of finding such an indicator in the range from 1 % to 2 % is  $9/30=0.3$ . Deviation  $\gamma > 2$  % with a probability of 0.3. That is, with a probability of 0.7, the accuracy of estimating by the EMRL the amount of electricity consumed can be estimated with a relative error not exceeding 2 %. When applying the psychophysical scale of desirability (18), the probability of accuracy ratings of «very good» and «good» is at least 0.833, Fig. 9. The probability of «bad» and «very bad» ratings does not exceed 0.066. The analysis of the adequacy verification results in accordance with the three requirements formulated in section 4.7 allowed to establish the following. The conclusion made according to the Kolmogorov-Smirnov criterion about not rejecting the hypothesis about normal distribution of residuals  $\varepsilon$ , Fig. 10, confirms the fulfillment of the first adequacy requirement. Not rejecting the hypothesis about the zero value of the residuals mean, for which the Student's criterion was used, allows to talk about the fulfillment of the second requirement of adequacy. Since, according to the Durbin-Watson criterion, the hypothesis about the independence of random deviations  $\varepsilon$ , Fig. 11, was not rejected, then there are grounds to talk about the fulfillment of the third requirement of adequacy. Since all three requirements are satisfied at the significance level of 0.05, it is possible to talk about the adequacy of the consumed electricity estimation results at reduced load using the EMRL software with experimental data. This confirms the main hypothesis of the research about the possibility of electricity measurement at a reduced load of the metering unit.

The use of a fuzzy function, the boundaries of which are described by deterministic dependencies, to describe the uncertainty of electricity measurement in the reduced load mode, compared to the machine learning algorithm [13], has the following advantage. The proposed approach can be applied in existing systems of commercial electricity accounting without updating the hardware. Also, unlike the method [20], which involves equipping the measuring current transformer with an additional winding, the developed method uses the indications of existing measuring equipment. This reduces the costs of implementing the proposed solution. In contrast to the method [12], which is characterized by significant errors with unbalanced data arrays, the proposed approach is suitable for estimating underaccounting with asymmetric phase currents of the load. This advantage is explained by the fact that the fuzzy function (8) underlying the operation of the EMRL depends on each of the phase currents. The advantages should also include the simplification of the assessment of real consumption and the level of underaccounting of electricity due to the use of the continuous function (16) with coefficient estimates (Table 2). In contrast to the method of piecewise linear approximation [29], with this approach, there are no

difficulties in combining the approximating lines at the discontinuity points. This reduces the volume of calculations, accordingly – the software algorithm for estimating the level of underaccounting of electricity is simplified.

The capabilities of the EMRL software allow to solve the existing problem, namely, to increase the accuracy of electric energy accounting in distribution power grids with a reduced load current of the metering unit. The solution to the problem is achieved by the following. The software can be used as part of automated systems of commercial electricity metering or advanced metering infrastructure to determine the most possible underaccounting due to the operation of metering units at a reduced load. Also, EMRL is suitable for control verification of the electricity metering units of industrial consumers for the significance of the duration of the reduced load regime. The use of such software will give energy supply organizations the opportunity to assess the operating conditions of a specific non-household consumer's metering unit and formulate recommendations for improving electricity metering and, accordingly, clarifying financial calculations. For example, based on the data obtained from the EMRL, a significant, from the point of view of the energy supply company, level of underaccounting of electric energy was revealed. Then recommendations can be made to the consumer regarding the reconstruction or technical reequipment of the metering unit, taking into account the actual level of currents when choosing measuring equipment. Technical reequipment of the metering unit should provide for the selection of measuring current transformers in accordance with the actual power of the consumer. Also, current transformers can be selected of a higher accuracy class. The use of the software provides a reasonable assessment of the level of underaccounting of electricity based on the actual operating modes of the metering unit. Also, EMRL can be used by consumers for technical accounting of electricity at the enterprise and self-monitoring of the operation mode of the metering unit. The scope of EMRL can be extended to renewable sources of electricity, in particular to solar power plants.

Limitations of the proposed approach to increasing the accuracy of electricity metering include the possibility of application only for metering units equipped with electromagnetic current transformers.

The disadvantages of the proposed EMRL software include the lack of direct integration with existing commercial electricity accounting systems, in particular: NovaSyS, ASKUE.net etc. In addition, the evaluation of the characteristics of measuring equipment of a specific type in the reduced load mode requires special experiments. This complicates the implementation of the proposed approach.

In the course of further research, it is planned to simplify the procedure for identifying the characteristics of the metering unit. This can be achieved by avoiding conducting special experiments to estimate the values of the fuzzy function (8) parameters. Instead, the values of the currents flowing through the metering unit in the operating mode can be used. This will reduce the time of implementation of the proposed solution for various types of measuring equipment.

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## 7. Conclusions

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1. As a result of the identification of the measuring equipment characteristics of the specific electricity metering unit at a reduced load, the possibility of representing the dependence of the relative measurement error on the current values by a fuzzy function has been confirmed. The boundaries of such a characteristic are approximated with an error of no more than 3 % by the sum of two exponents, which is explained by their significant nonlinearity in the area of reduced current.

2. Based on the array of experimental data, reflecting the monthly consumption of electricity by an industrial consumer at 15-minute intervals, the following estimates were obtained using the proposed EMRL software. The most possible underaccounting of electricity is from 2.4 % to 8.0 %. The average monthly level of underaccounting is 5.2 %. The monthly estimate of consumption by the EMRL software differs from the actual value by 0.2 %. The trend of a significant decrease in the relative value of underaccounting with an increase in the level of electricity consumption was revealed. With a daily consumption of up to 10 kW·h, the underaccounting can reach 18 %, and with a consumption of more than 20 kW·h, the underaccounting does not exceed 6 %. This trend is explained by the non-linearity of the metrological characteristics of measuring equipment in the area of low currents.

3. The accuracy of estimating by the EMRL software the consumed amount of electricity with a probability of 0.7 can be estimated with a relative error not exceeding 2 %. The probability of psychophysical accuracy assessments of EMRL «very good» and «good» is at least 0.833. The adequacy of the results of estimating the amount of consumed electricity at reduced load using the EMRL software was confirmed by experimental data at a significance level of 0.05.

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## Conflict of interest

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The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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## Financing

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The study was conducted without financial support.

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## Availability of data

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The manuscript has no associated data.

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## Use of artificial intelligence

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The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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