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DETERMINING THE EFFECT OF LONGITUDINAL STIFFENERS ON THE DEFORMATION OF MULTILAYER ELLIPSOIDAL SHELLS UNDER NON-STATIONARY LOADS

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The object of this study is the deformation of multilayer ellipsoidal shells under non-stationary distributed load. It is proposed to reinforce the shell with longitudinal stiffeners to enhance the strength of it. The application of Timoshenko's theory of shells and cores enabled the investigation of the influence of longitudinal stiffeners on the deformation of multilayer ellipsoidal shells under non-stationary loads with the discrete positioning of the stiffeners. A mathematical model of shell oscillations under various types of short-term non-stationary loads was based on the Hamilton-Ostrogradsky variational principle. The numerical algorithm based on the application of the integral-interpolation approach to the construction of finite difference schemes in spatial coordinates, combined with an explicit finite-difference scheme for the time coordinate, was used to solve the task set.

It has been determined that reinforcing stiffeners significantly affect the deformed state of the multilayer ellipsoidal shell. The "shell-stiffeners" relation was taken into account as a base for research. The maximum deformation ε_{11} of the smooth three-layer ellipsoidal shell was on average 1.4 times greater than the deformation ε_{11} of the reinforced three-layer ellipsoidal shell throughout the entire studied time interval. It was determined that over time the presence of reinforcing stiffeners has a more significant impact on the deformed state of the reinforced ellipsoidal shell.

A distinctive feature of this research is the consideration of the discrete placement of reinforcing stiffeners which made it possible to investigate the impact of longitudinal stiffeners on the deformation of multilayer ellipsoidal shells under non-stationary loads.

The results could be used for the investigation of applied problems in research organizations and design bureaus when designing shell structures

Keywords: three-layer shells, forced oscillations, non-stationary load, reinforcing stiffeners, numerical algorithm

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1. Introduction

Shells and shell structures are widely used in modern industries such as manufacturing, shipbuilding, aerospace, etc. During the operation of shells and shell structures, they are inevitably exposed to various types of dynamic loads. This requires a detailed analysis of the stability and dynamic properties of both individual elements and nodes, as well as the structure as a whole. Such an analysis could be carried out with the help of physical field experiments, but the experimental method is sometimes an expensive and not always safe process. At the current stage of development of science and technology, computer experiments using modern information systems and technologies are becoming more and more common. Conducting computer experiments makes it possible to carry out a significant number of virtual tests with various physical, mechanical, and geometric parameters of shells and shell structures, which makes it possible to choose the optimal solution for the given problem.

That is why the construction of new mathematical models of shells and shell structures, the development of methods for calculating structures, taking into account various design parameters and load options, are relevant for modern scientists across the world.

2. Literature review and problem statement

In [1], a mathematical model of geometrically nonlinear oscillations of three-layer sandwich shells was built, which describes the oscillations of the structure with amplitudes comparable to its thickness. The high-order shift theory was used to derive the model. However, the model proposed in the work does not allow its use to detect deformations of three-layer ellipsoidal shells.

Nonlinear forced vibration of an axially moving cylindrical shell made of functionally gradient materials was studied in [2]. Donnell's thin shell theory was used to derive the equations of motion of cylindrical shells. The Galerkin method and the multiscale method were used for the analysis. With the help of parametric studies, the influence of the transverse stress and the amplitude of the external excitation on the nonlinear forced vibration was clarified. The method reported in the work is limited to the use of cylindrical shells, which makes it impossible to use it for calculating deformations of ellipsoidal shells.

An analysis of the behavior of multilayer axisymmetric shells under the influence of various internal pressure conditions was carried out [3]. Critical parameters of the design, including material properties, geometric parameters, etc., were determined. However, the studies in the cited work are limited to the analysis of the internal pressure on the shell.

In [4], the dynamic behavior of load-bearing components of composite structures was investigated. The high-order Galerkin method for transient processes and oscillations of multilayer plates and shells has been numerically confirmed. The starting point of the formulation was a generalized theory for multi-layered shells of arbitrary curvature, based on the distribution of the covariant displacement components over the shell thickness. But the use of such a research method does not make it possible to consider shells reinforced by stiffeners.

In works [5, 6], the results of studies of graduated multilayer cylindrical shells reinforced with graphene plates are reported. The influence of various factors, including geometric parameters, boundary conditions, and material properties, on the behavior of multilayer cylindrical shells after deflection and during torsion was considered. However, the authors of this procedure did not choose ellipsoidal reinforced shells for research.

A study of the behavior after bending of multilayer functionally graded graphene lamellar-reinforced composite cylindrical and spherical panels of the shell, resting on elastic foundations, subjected to central pinching and pressure forces was carried out [7]. The theory of higher-order shear deformation and nonlinear von K rm n strain-displacement relationships was applied. The method proposed by the authors requires complex computational calculations in the case of its application for the study of ellipsoidal shells.

In [8], the results of studies of the dynamic behavior of a multilayer ring nanocomposite plate, reinforced with graphene, under the action of thermomechanical loads are reported. The author's approach consisted in replacing the reinforced shell with a structurally orthotropic shell. The peculiarity of this procedure provided a mostly qualitative characterization of the deformation process but the numerical interpretation of the deformation process for discretely reinforced shells remained problematic.

The results of numerical studies using the theory of shear deformation of the first order and the finite element method were considered in work [9]. This method was applied to the study of sandwich composite panels with a cellular core under the influence of transverse mechanical loads. Static and dynamic analyses of sandwich panels were considered and compared with the available literature to confirm the findings. However, the use of structural orthotropy did not make it possible to take into account the features of the force interaction between the stiffeners and cladding.

The theories of the equivalent single-layer shell of high order, based on the thickness expansion of the covariant components of the displacement field, are presented for the analysis of the linear bending of multilayer shells in [10]. Examples of bending loads of plates and shells modeled by different theories and characterized by different materials, geometries, boundary conditions and cutouts are considered. The disadvantage of this procedure is the use of a constructive-orthotropic model, in which it was allowed to determine only averaged deformations, which in turn could lead to significant errors, even with frequent reinforcement of stiffeners.

The vibration characteristics of a lattice multilayer truncated conical shell with composite stiffeners are considered in [11]. The equations of motion together with the boundary conditions of the shells were derived using the classic theory of Donnell shells and the fuzzy stiffness technique. The equations were solved using the Galerkin method. ABAQUS simulations were presented to study the vibration behavior of conical shells with one and three shells. However, within the framework of this theory, it is impossible to determine a number of important regularities associated with the presence of discretely placed stiffeners.

In [12], a three-dimensional solution for the bending of doubly curved shells subjected to mechanical loading was investigated. Partial differential equations were solved using Navier closed-form summation, which is valid only for shells with constant radii of curvature of the middle surface and with simple support boundary conditions at the stiffeners of the shell. The equation was solved by discretizing the thickness profile using a Legendre grid distribution. Layer-by-layer capabilities of the method were guaranteed by imposing interlaminar continuity out of plane stresses, displacements, and load thickness profile. The results are given for cylindrical, spherical panels, and rectangular plates. The methodology proposed in the work requires the use of complex computational algorithms, which prevents its widespread use in modeling shell deformations.

In [13], a general 3D exact shell solution for thermomechanical analysis of a heterogeneous group of single- and multilayer isotropic, composite, and sandwich structures is shown. Plates, cylinders, cylindrical and spherical shells were investigated using orthogonal mixed curvilinear coordinates. The elastic part of the proposed 3D model is based on a consolidated layer-by-layer exact solution that uses the exponential matrix method to solve the differential equations of equilibrium in the thickness direction. The closed-form solution was obtained assuming boundary conditions with a simple support. However, the 3D model proposed by the authors cannot be used for the study of ellipsoidal reinforced shells due to geometric and structural features.

On the basis of the Kirchhoff-Love theory and von Kármán geometric nonlinearity, a nonlinear vibration analysis of multilayer sandwich plates is reported [14]. The rectangular plates were considered to be laminated from orthotropic composite layers that are functionally graded, anisograting gratings, or metal foam materials. An impermeability condition was developed, which takes into account a moderately large deflection of the contact surface to correct the pressure distribution of the fluid flow. With the help of Galerkin methods and many large-scale perturbations, a closed solution in terms of the maximum amplitude of oscillations was obtained. The resulting equations of shell oscillations belonged to parabolic type equations. But for calculations of wave processes under short-term loads, equations of the hyperbolic type should be used. This inconsistency leads to the impossibility of applying the theory proposed in the paper to study deformations under short-term non-stationary loads.

Paper [15] considered the problem of forced oscillations of three-layer ellipsoidal shells reinforced with longitudinal stiffeners under the action of a non-stationary distributed load. The research of the stiffener shell involved taking into account the discrete placement of the stiffeners. The effect of the stiffeners on the amount of deflection of the reinforced ellipsoidal shell was analyzed. However, the cited paper did not consider the issue of the effect of stiffeners on the amount of deformation of the reinforced ellipsoidal shell.

From our review of the literature, it could be concluded that the question of determining the effect of longitudinal stiffeners on the deformation of multilayer ellipsoidal shells under non-stationary loads remains unresolved.

3. The aim and objectives of the stud

The purpose of our research is to identify the deformations of multilayer ellipsoidal shells reinforced by longitudinal stiffeners under non-stationary loads, taking into account the discrete placement of the stiffeners. This will make it possible to choose the optimal options for shell structures, that is, to reduce the mass of the shell due to a lighter filler and to reduce its deformations under non-stationary loading.

To achieve the goal, the following tasks are set:

– statement of the problem of deformation of multilayer ellipsoidal shells taking into account the discrete placement of stiffeners;

– derivation of equations of oscillations of multilayer ellipsoidal shells supported by longitudinal stiffeners and natural boundary conditions;

– application of a numerical method for solving problems of non-stationary oscillations of multilayer ellipsoidal shells supported by longitudinal stiffeners;

– comparative analysis of deformations of smooth and reinforced with longitudinal stiffeners of multilayer ellipsoidal shells.

4. The study materials and methods

The object of our study is the deformation of multilayer ellipsoidal shells supported by longitudinal stiffeners under the action of a non-stationary distributed load.

In the study, it was hypothesized that the presence of reinforcing stiffeners would reduce the maximum deformation of a reinforced three-layer ellipsoidal shell compared to a smooth three-layer ellipsoidal shell. The shell layers and discrete reinforcing stiffeners were assumed to be rigidly connected. The following research methods were used in this work to accomplish our goal, to solve the problems, and to confirm the proposed hypothesis.

At the stage of setting the problem of deformation of multilayer ellipsoidal shells discretely supported by longitudinal stiffeners, the method of imaginary construction of the object under study was used. The imaginary experiment method was chosen because it enabled the detection of the influence of reinforcing stiffeners on the deformation of multilayer ellipsoidal shells under non-stationary loading without conducting expensive empirical studies.

The equation of oscillations of multilayer ellipsoidal shells supported by longitudinal stiffeners and natural boundary conditions under non-stationary loading was obtained through the use of such research methods as imaginary experiment and simulation. The chosen methods have shown their effectiveness for detecting the influence of various factors on the deformations of multilayer ellipsoidal shells reinforced by stiffeners.

During dynamic loading of multilayer reinforced ellipsoidal shells, local disturbances in the area of changes in the physical and mechanical parameters of the stiffeners led to a significant redistribution of the parameters of the stress-strain state in the entire studied area. The complexity of the processes that arose at the same time necessitated the use of a modern integral-interpolation numerical method for solving problems of the behavior of reinforced ellipsoidal shell structures taking into account the discrete placement of stiffeners. This method was based on the construction of finite-difference schemes by spatial coordinates and an explicit finite-difference scheme of the “cross” type by time coordinate. The choice of this approach has made it possible to solve the system of partial differential equations in the presence of spatial gaps, which ensured that the discreteness of the reinforcing stiffeners was taken into account. To implement the developed numerical algorithm, a computer method of search design was used, which was based on the use of modern computer and information technologies. The computerization of the research object was performed by using the programming language FortranPowerStation (USA).

In order to solve the given task of comparative analysis of the deformations of smooth and reinforced by longitudinal stiffeners, the following research methods were used: graphic method, analysis, comparison. The graphical method was implemented using the MATLAB (USA) application software package for numerical analysis and programming. The graphic method made it possible to visualize the results of the influence of reinforcing stiffeners on the deformation of multilayer ellipsoidal shells reinforced by longitudinal stiffeners when they are subjected to non-stationary load, and this made it possible to see the deformation process of multilayer ellipsoidal shells reinforced by stiffeners in dynamics. According to the results of this method, an analysis of the dynamic phenomenon of deformation was carried out and the deformations of smooth and reinforced with longitudinal stiffeners of multilayer ellipsoidal shells were compared to confirm the research hypothesis.

5. Results of investigating the effect of reinforcing stiffeners on the deformation of multilayer ellipsoidal shells

5. 1. Stating the problem of deformation of multilayer ellipsoidal shells taking into account the discrete placement of stiffeners

The geometry of the middle surface of the ellipsoidal shell was given by the following relations [16]:

$$x = R\sin\alpha_1 \sin\alpha_2, \quad y = R\sin\alpha_1 \cos\alpha_2, \quad z = kR\cos\alpha_1,$$

where α_1, α_2 are Gaussian curvilinear coordinates on the shell surface, α_1 corresponds to the meridional direction, and α_2 corresponds to the circumferential direction; $k=b/a$ – ellipticity parameter; a, b are the semi-axes of the ellipse.

The ratio of the components of the metric and the shape of the middle surface of the shell:

$$a_{11}=R^2\left(\cos^2\alpha_1+k^2\sin^2\alpha_1\right),\quad a_2=R^2\sin^2\alpha_1,$$

$$b_{11}=kR\left(\cos^2\alpha_1+k^2\sin^2\alpha_1\right)^{-\frac{1}{2}},$$

$$b_{22}=kR\sin^2\alpha_1\left(\cos^2\alpha_1+k^2\sin^2\alpha_1\right)^{-\frac{1}{2}}. \qquad (1)$$

According to formulas (1), the coefficients of the first quadratic form and the curvature of the middle surface of the ellipsoidal shell:

$$A_1=a\left(\cos^2\alpha_1+k^2\sin^2\alpha_1\right)^{\frac{1}{2}},\quad A_2=a\sin\alpha_1,$$

$$k_1=-\frac{b}{a^2}\left(\cos^2\alpha_1+k^2\sin^2\alpha_1\right)^{-\frac{3}{2}},$$

$$k_2=-\frac{b}{a^2}\left(\cos^2\alpha_1+k^2\sin^2\alpha_1\right)^{-\frac{1}{2}}.$$

To build a mathematical model of the processes of forced oscillations of a three-layer ellipsoidal shell supported by longitudinal stiffeners, a hyperbolic system of nonlinear differential equations of the Tymoshenko-type shell theory was considered [17]. It was assumed that the law of change of displacements along the thickness of the ellipsoidal shell in the coordinate system (s_1, s_2, z) had the form from [18].

The deformed state of the i-th reinforcing element directed along the α_1 axis was determined through the components of the generalized vector of displacements of the centers of gravity of the stiffener cross sections according to [15]. Contact conditions between the components of the generalized vector of movements of the centers of gravity of the cross sections of the ith stiffener and the components of the generalized vector of movements of the initial median surface of the smooth ellipsoidal shell:

$$u_{1i}(s_1)=u_1(s_1,s_{2i})+h_{ci}\varphi_1(s_1,s_{2i}), \qquad (2)$$

$$u_{2i}(s_1)=u_2(s_1,s_{2i})+h_{ci}\varphi_2(s_1,s_{2i}),$$

$$u_{3i}(s_1)=u_3(s_1,s_{2i}),$$

$$\varphi_{1i}(s_1)=\varphi_1(s_1,s_{2i}),$$

$$\varphi_{2i}(s_1)=\varphi_2(s_1,s_{2i}),$$

where h_{ci}=0.5(h+h_i) is the distance from the middle surface of the smooth ellipsoidal shell to the line of the center of gravity of the cross section of the ith stiffener; h_i is the height of the reinforcing i-th stiffener directed along the axis α_1, s_{2i} is the coordinate of the line projecting the centers of gravity of the cross-sections of the i-th stiffener onto the coordinate median surface of the cladding.

5. 2. Results of the derivation of vibration equations of multilayer ellipsoidal shells reinforced by stiffeners

Vibration equations for a reinforced ellipsoidal shell and boundary conditions were derived using the Hamilton-Ostrogradsky variational principle.

Variational equation of the shell with longitudinal discretely reinforced stiffeners in the α_1 and α_2 directions:

$$\int_{t_1}^{t_2}\left[\delta(P-K)+\delta A\right]\mathrm{d}t=0,$$

where:

$$P=P_0+\sum_{i=1}^{n_1}P_i,\quad K=K_0+\sum_{i=1}^{n_1}K_i, \qquad (3)$$

P_0, K_0 – potential and kinetic energy of the cladding; P_i, K_i – potential and kinetic energies, respectively, of the ith reinforcing stiffener directed along the α_1 axis; A is the work of external forces. Expressions for δP and δK values are given in [19].

The stressed-strained state of the elastic structure was described using a geometrically nonlinear version of the shell theory in the quadratic approximation [20]. At the same time, the deformation ratios for a smooth three-layer shell are:

$$\varepsilon_{11}=\frac{\partial u_1}{\partial s_1}+k_1u_3+\frac{1}{2}\theta_1^2,$$

$$\varepsilon_{22}=\frac{\partial u_2}{\partial s_2}+\frac{1}{A_2}\frac{\partial A_2}{\partial s_1}u_1+k_2u_3+\frac{1}{2}\theta_1^2,$$

$$\varepsilon_{12}=\omega+\theta_1\theta_2,\quad \varepsilon_{13}=\varphi_1+\theta_1,\quad \varepsilon_{23}=\varphi_2+\theta_2,$$

$$\omega=\omega_1+\omega_2,\quad \omega_1=\frac{\partial u_2}{\partial s_1},$$

$$\omega_2=\frac{\partial u_1}{\partial s_2}-\frac{1}{A_2}\frac{\partial A_2}{\partial s_1}u_2,\quad \theta_1=\frac{\partial u_3}{\partial s_1}-k_1u_1,$$

$$\theta_2=\frac{\partial u_3}{\partial s_2}-k_2u_2,\quad \chi_{11}=\frac{\partial\varphi_1}{\partial s_1},\quad \chi_{22}=\frac{\partial\varphi_2}{\partial s_2}+\frac{1}{A_2}\frac{\partial A_2}{\partial s_1}\varphi_1,$$

$$\chi_{12}=\tau_1+\tau_2+k_1\omega_1+k_2\omega_2,$$

$$\tau_1=\frac{\partial\varphi_2}{\partial s_1},\quad \tau_2=\frac{\partial\varphi_1}{\partial s_2}-\frac{1}{A_2}\frac{\partial A_2}{\partial s_1}\varphi_2.$$

The values of efforts and moments were expressed through the values of deformations [15].

Deformation ratios for the ith stiffener:

$$\varepsilon_{11i}=\frac{\partial u_1}{\partial s_1}\pm h_{ci}\frac{\partial\varphi_1}{\partial s_1}+k_{1i}u_3+\frac{1}{2}\theta_{1j}^2+\frac{1}{2}\theta_{2j}^2,$$

$$\varepsilon_{12i}=\theta_{2i},\quad \varepsilon_{13i}=\varphi_1\theta_{1i},$$

$$\theta_{1i}=\frac{\partial u_3}{\partial s_1}-k_{1i}\left(u_1\pm h_{ci}\varphi_1\right),\quad \theta_{2i}=\frac{\partial u_2}{\partial s_1}\pm h_{ci}\frac{\partial\varphi_2}{\partial s_1},$$

$$\chi_{11i}=\frac{\partial\varphi_1}{\partial s_1},\quad \chi_{12i}=\frac{\partial\varphi_2}{\partial s_1}.$$

Expressions of moment forces for the i-th stiffener are given in [15].

Equation of oscillations of a nonuniform shell in differential form:

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}\left(A_2T_{11}\right)-\frac{\partial A_2}{\partial s_1}T_{22}\right]+k_1\overline{T}_{13}+$$

$$+\frac{1}{A_1}\frac{\partial}{\partial s_2}\left(A_1T_{21}\right)=I_1\cdot\frac{\partial^2u_1}{\partial t^2}+I_2\cdot\frac{\partial^2\varphi_1}{\partial t^2},$$

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}\left(A_2T_{12}\right)-\frac{\partial A_2}{\partial s_1}T_{21}\right]+k_2\bar{T}_{23}+$$
$$+\frac{1}{A_1}\frac{\partial}{\partial s_2}\left(A_1T_{22}\right)=I_1\cdot\frac{\partial^2 u_2}{\partial t^2}+I_2\cdot\frac{\partial^2\varphi_2}{\partial t^2},$$

$$\frac{1}{A_2}\frac{\partial}{\partial s_1}\left(A_2\bar{T}_{13}\right)-k_1T_{11}-k_2T_{22}+$$
$$+\frac{1}{A_1}\frac{\partial}{\partial s_2}\left(A_1\bar{T}_{23}\right)+P_3=I_1\cdot\frac{\partial^2 u_3}{\partial t^2}, \tag{4}$$

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}\left(A_2M_{11}\right)-\frac{\partial A_2}{\partial s_1}M_{22}\right]-$$
$$-T_{13}+\frac{1}{A_1}\frac{\partial}{\partial s_2}\left(A_1M_1\right)=I_2\cdot\frac{\partial^2 u_1}{\partial t^2}+I_3\cdot\frac{\partial^2\varphi_1}{\partial t^2},$$

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}\left(A_2M_{12}\right)+\frac{\partial A_2}{\partial s_1}M_{21}\right]+$$
$$+\frac{1}{A_1}\frac{\partial}{\partial s_2}\left(A_1M_{22}\right)-T_{23}=I_2\cdot\frac{\partial^2 u_2}{\partial t^2}+I_3\cdot\frac{\partial^2\varphi_2}{\partial t^2},$$

$$I_1=\sum_{r=1}^{R}\rho_r h_r,\quad I_2=\sum_{r=1}^{R}\rho_r\frac{z_{r+1}^2-z_r^2}{2},\quad I_3=\sum_{r=1}^{R}\rho_r\frac{z_{r+1}^3-z_r^3}{3}.$$

Equation of oscillations of the i-th reinforcing stiffener directed along the α_1 axis:

$$\frac{\partial T_{11i}}{\partial s_1}+k_{1i}T_{13i}+\left[S\right]_i=\rho_iF_i\left(\frac{\partial^2 u_1}{\partial t^2}\pm h_{ci}\frac{\partial^2\varphi_1}{\partial t^2}\right),$$

$$\frac{\partial T_{12i}}{\partial s_1}+\left[T_{22}\right]_i=\rho_iF_i\left(\frac{\partial^2 u_2}{\partial t^2}\pm h_{ci}\frac{\partial^2\varphi_2}{\partial t^2}\right),$$

$$\frac{\partial T_{13i}}{\partial s_1}-k_{1i}T_{11i}+\left[T_{23}\right]_i=\rho_iF_i\frac{\partial^2 u_3}{\partial t^2}, \tag{5}$$

$$\frac{\partial M_{11i}}{\partial s_1}-T_{13i}\pm h_{ci}\left(\frac{\partial T_{11i}}{\partial s_1}+k_{1i}T_{13i}\right)+\left[H\right]_i=$$
$$=\rho_iF_i\left(\pm h_{ci}\frac{\partial^2 u_1}{\partial t^2}+\left(h_{ci}^2+\frac{I_{1i}}{F_i}\right)\frac{\partial^2\varphi_1}{\partial t^2}\right),$$

$$\frac{\partial M_{12i}}{\partial s_1}\pm h_{ci}\frac{\partial T_{12i}}{\partial s_1}+\left[M_{22}\right]_i=$$
$$=\rho_iF_i\left(\pm h_{ci}\frac{\partial^2 u_2}{\partial t^2}+\left(h_{ci}^2+\frac{I_{cri}}{F_i}\right)\frac{\partial^2\varphi_2}{\partial t^2}\right).$$

Boundary conditions:

$$\bar{U}\left(\alpha_{10},\alpha_2\right)=\bar{U}\left(\alpha_{1N},\alpha_2\right)=0,$$
$$\bar{U}\left(\alpha_1,\alpha_{20}\right)=\bar{U}\left(\alpha_1,\alpha_{2N}\right)=0. \tag{6}$$

Zero initial conditions were assumed for all components of the generalized displacement vector at t=0 [15].

Systems of differential equations (4), (5) are systems of nonlinear partial differential equations with respect to spatial coordinates s_1, s_2 and time coordinate t. In connection with this, there was a need to apply numerical methods for solving the initial boundary value problems from the theory of discretely supported ellipsoidal shells.

5. 3. Algorithm for the application of a numerical method for studying the vibrations of multi-layered ellipsoidal shells reinforced by stiffeners

The construction of the numerical algorithm was based on the use of the integral-interpolation method of construction of difference schemes in the spatial coordinates s_1, s_2 and the explicit finite-difference scheme of integration in the time coordinate t [16]. The operation of integrating equations (4) was performed using the explicit approximation along the time coordinate, as a result, the difference equations for the smooth shell were built. After performing the integration operation of equations (5) using explicit approximation along the time coordinate, the difference equations for the i-th reinforcing stiffener were derived.

Ratios for $\Omega_2=\left\{s_{1l-1}\le s_1\le s_{1l};s_{2m-1/2}\le s_2\le s_{2m+1/2}\right\}$:

$$\varepsilon^n_{11l-1/2,m}=\frac{u^n_{1l,m}-u^n_{1l-1,m}}{\Delta s_1}+k_{1l-1/2}u^n_{3l-1/2,m}+\frac{1}{2}\left(\theta_{1l-1/2,m}\right)^2,$$

$$\varepsilon^n_{22l-1/2,m}=\frac{u^n_{2l-1/2,m+1/2}-u^n_{2l-1/2,m-1/2}}{\Delta s_2}+$$
$$+\psi_{2l-1/2}u^n_{1l-1/2,m}+k_{2l-1/2}u^n_{3l-1/2,m}+\frac{1}{2}\left(\theta_{1l-1/2,m}\right)^2,$$

$$\theta^n_{11l-1/2,m}=\frac{u^n_{3l,m}-u^n_{3l-1,m}}{\Delta s_1}-k_{1l-1/2}u^n_{1l-1/2,m},$$

$$\omega^n_{1l-1/2,m}=\frac{u^n_{2l,m}-u^n_{2l-1,m}}{\Delta s_1},$$

$$\omega^n_{2l-1/2,m}=\frac{u^n_{1l-1/2,m+1/2}-u^n_{1l-1/2,m-1/2}}{\Delta s_2}-\psi_{2l-1/2}u^n_{2l-1/2,m},$$

$$\omega^n_{l-1/2,m}=\omega^n_{1l-1/2,m}+\omega^n_{2l-1/2,m},$$

$$\varepsilon^n_{13l-1/2,m}=\varphi^n_{1l-1/2,m}+\theta^n_{1l-1/2,m},$$

$$\chi^n_{12l-1/2,m}=\frac{\varphi^n_{2l,m}-\varphi^n_{2l-1,m}}{\Delta s_1}+\frac{\varphi^n_{1l-1/2,m+1/2}-\varphi^n_{1l-1/2,m-1/2}}{\Delta s_2}-$$
$$-\psi_{2l-1/2}\varphi^n_{2l-1/2,m}+k_{1l-1/2}\omega^n_{1l-1/2,m}+k_{2l-1/2}\omega^n_{2l-1/2,m},$$

$$\theta^n_{2l-1/2,m}=\frac{u^n_{3l-1/2,m+1/2}-u^n_{3l-1/2,m-1/2}}{\Delta s_2}-k_{2l-1/2}u^n_{2l-1/2,m},$$

$$\chi^n_{22l-1/2,m}=\frac{\varphi^n_{2l-1/2,m+1/2}-\varphi^n_{2l-1/2,m-1/2}}{\Delta s_2}-\psi_{2l-1/2}\varphi^n_{1l-1/2,m}.$$

Similar deformation ratios were recorded for other areas.

5. 4. Numerical analysis of deformations of smooth and stiffener-reinforced multilayer ellipsoidal shells and comparison of calculation results

In this work, the dynamic deformation of a three-layer ellipsoidal shell reinforced by longitudinal stiffeners with rigidly clamped edges (6) was studied in the region $D=\left\{\alpha_{10}\le\alpha_1\le\alpha_{1N},\ \alpha_{20}\le\alpha_2\le\alpha_{2N}\right\}$ under the action of a distributed normal load $P_3(\alpha_1, \alpha_2, t)$:

$$P_3(\alpha_1, \alpha_2, t) =$$
$$= A \cdot \sin\frac{\pi t}{T}\left[\eta(t) - \eta(t - T)\right],$$

where A is the load amplitude; T is the duration of the load. In the calculations, it was assumed that $A=10^6$ Pa; $T=50\cdot10^{-6}$ s. The problem was solved for $k=\frac{a}{b}=1.5$.

Geometric and physical-mechanical parameters of a smooth three-layer shell:

$$\alpha_{10} = \frac{\pi}{12}, \quad \alpha_{1N} = \pi - \frac{\pi}{12},$$

$$\alpha_{20} = -\frac{\pi}{2}, \quad \alpha_{2N} = \frac{\pi}{2}, \quad \frac{a}{h} = 30,$$

$$h = h_1 + h_2 + h_3, \quad h_1 = h_3 = 10^{-2},$$

$$h_2 = 3\cdot10^{-2},$$

$$E_1^1 = 7\cdot10^{10}, \quad E_2^1 = E_1^1,$$

$$E_1^2 = \frac{E_1^1}{1{,}000}, \quad E_2^2 = E_1^2,$$

$$\nu_{12}^1 = \nu_{21}^1 = \nu_{12}^2 = \nu_{21}^2 =$$
$$= \nu_{12}^3 = \nu_{21}^3 = 0.33,$$

$$\rho_1 = \rho_3 = 2.7\cdot10^3 \ \mathrm{kg}/\mathrm{m}^3,$$

$$\rho_2 = 3\cdot10^2 \ \mathrm{kg}/\mathrm{m}^3.$$

Longitudinal reinforcing elements were placed along the α_1 coordinate in the sections $\alpha_{2i} = -\frac{\pi}{4} + \frac{\pi}{4}i, \ i = 0,1,2.$

Geometric and physical-mechanical parameters of reinforcing stiffeners:

$$h_i = 4\cdot h, \quad F_i = 4\cdot h^2, \quad E_i = E_1^1, \quad G_i = G_{12}^1, \quad \rho_i = \rho_1.$$

Fig. 1–4 show the results of calculations for the time interval t=80T. During the analysis, the absolute values of the deformation value ε_{11} were selected.

Fig. 1 shows the dependence of the deformation value ε_{11} on time at the point $\left(\alpha_1 = \frac{\pi}{2}; \alpha_2 = 0\right)$, at which ε_{11}, in the case of a smooth three-layer ellipsoidal shell, reached its maximum value at the time interval t=80T. The curve with index 1 is the case of a smooth three-layer ellipsoidal shell, and the curve with index 2 is the case of a reinforced three-layer ellipsoidal shell.

The illustration in Fig. 1 demonstrates that the maximum value of the deformation value ε_{11} for the case of a smooth three-layer ellipsoidal shell at the time $t=7T$ took a value $\left.\varepsilon_{11}\left(\frac{\pi}{2};0\right)\right|_{t=7T} = 1.31\cdot10^{-3}$ m, which is 1.5 times greater than the value of the deformation ε_{11} for the case of a reinforced three-layer ellipsoidal shell at the time $t=7T$ – $\left.\varepsilon_{11}\left(\frac{\pi}{2};0\right)\right|_{t=7T} = 8.86\cdot10^{-4}$ m.

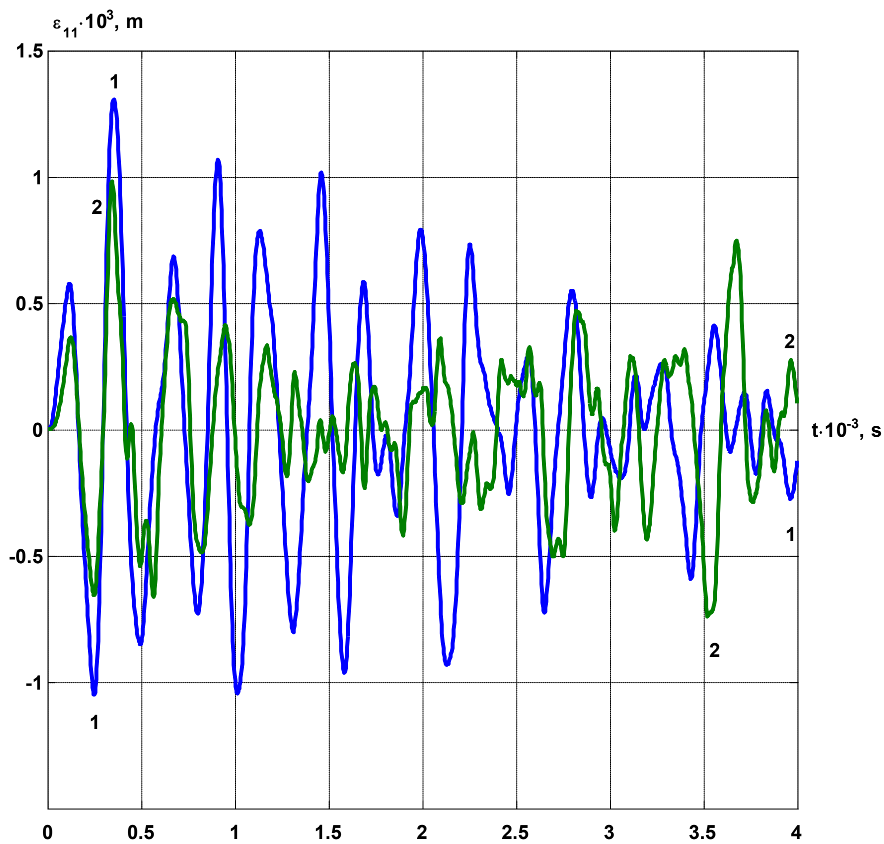

Fig. 1. Dependence of the amount of deformation ε_{11} on time at point $\left(\frac{\pi}{2};0\right)$: curve 1 – case of a smooth three-layer ellipsoidal shell; curve 2 – the case of a reinforced three-layer ellipsoidal shell

Fig. 2 shows the dependence of the deformation value ε_{11} on time at the point $\left(\alpha_1=\frac{\pi}{2};\alpha_2=\frac{\pi}{20}\right)$, at which the deformation value ε_{11}, in the case of a reinforced three-layer ellipsoidal shell, reached its maximum value at the time interval t=80T.

Analysis of the graphic material in Fig. 2 revealed that the maximum absolute value of the deformation value ε_{11} for the case of a smooth three-layer ellipsoidal shell was observed at the time t=7T – $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{\pi}{20}\right)\right|_{t=7T}=1.26\cdot10^{-3}$ m – 1.3 times greater than the value of the deformation ε_{11} for the case of a reinforced three-layer ellipsoidal shell at the time t=7T – $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{\pi}{20}\right)\right|_{t=7T}=9.81\cdot10^{-4}$ m.

Fig. 3 shows the dependence of the deformation value ε_{11} on the spatial coordinate α_2 in the cross-section $\alpha_1=\frac{\pi}{2}$ for the case of a smooth ellipsoidal shell at different time points t: t_1=1T, t_2=7T, t_3=80T. The plot shows how the amount of deformation ε_{11} behaves over time.

The largest value of the deformation value ε_{11} for a smooth shell at the time t=80T – $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{\pi}{6}\right)\right|_{t=80T}=6.44\cdot10^{-4}$ m – is 2 times less than the largest value of the deformation value (at the time t=7T): – $\left.\varepsilon_{11}\left(\frac{\pi}{2};0\right)\right|_{t=7T}=1.31\cdot10^{-3}$ m.

The largest value of the deformation value of the smooth shell at the time t=7T – $\left.\varepsilon_{11}\left(\frac{\pi}{2};0\right)\right|_{t=7T}=1.31\cdot10^{-3}$ m – is 9 times greater than the largest value of the deformation value at the time t=80T – $\left.\varepsilon_{11}\left(\frac{\pi}{2};0\right)\right|_{t=80T}=1.46\cdot10^{-4}$ m.

Fig. 4 shows the dependence of the deformation value ε_{11} on the spatial coordinate α_2 in the cross-section $\alpha_1=\frac{\pi}{2}$ for the case of a reinforced ellipsoidal shell at different time points t: t_1=1T, t_2=7T, t_3=80T. The plot shows how the amount of deformation ε_{11} behaves over time, as well as the location of reinforcing stiffeners.

The largest value of the deformation ε_{11} for the case of the reinforced shell at the time t=80T is equal to $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{5\pi}{20}\right)\right|_{t=80T}=3.71\cdot10^{-4}$ m, 3 times less that the largest value of the deformation $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{5\pi}{20}\right)\right|_{t=7T}=9.81\cdot10^{-4}$ m (at the time t=7T).

The largest value of the deformation value for the case of the reinforced shell $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{\pi}{20}\right)\right|_{t=7T}=9.81\cdot10^{-4}$ m at the time t=7T is 8 times greater than the largest value of the deformation value $\left.\varepsilon_{11}\left(\frac{\pi}{2};\frac{\pi}{20}\right)\right|_{t=80T}=1.28\cdot10^{-4}$ m at the time t=80T.

Therefore, at time point t=7T, the maximum strain ε_{11} for a smooth three-layer ellipsoidal shell was almost 32 % greater than the maximum strain ε_{11} in a reinforced three-layer ellipsoidal shell. At time point t=80T, the maximum strain ε_{11} for a smooth three-layer ellipsoidal shell was almost 42 % greater than the maximum strain ε_{11} in a reinforced three-layer ellipsoidal shell.

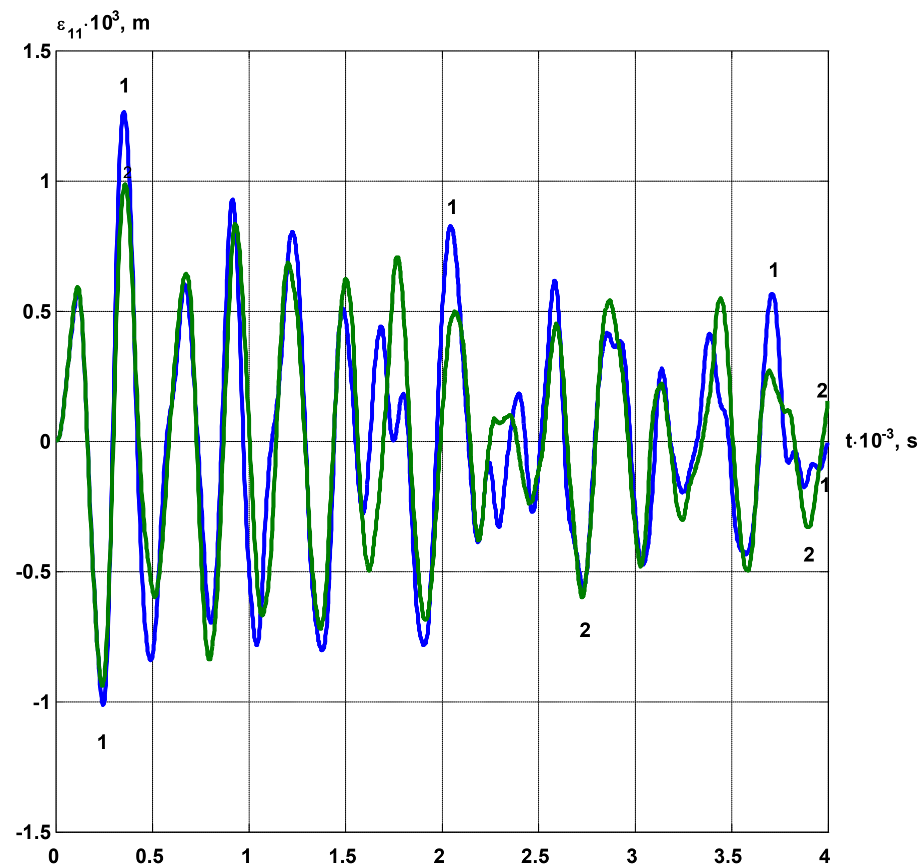

Fig. 2. Dependence of the amount of deformation ε_{11} on time at a point $\left(\frac{\pi}{2};\frac{\pi}{20}\right)$: curve 1 – case of a smooth three-layer ellipsoidal shell; curve 2 – the case of a reinforced three-layer ellipsoidal shell

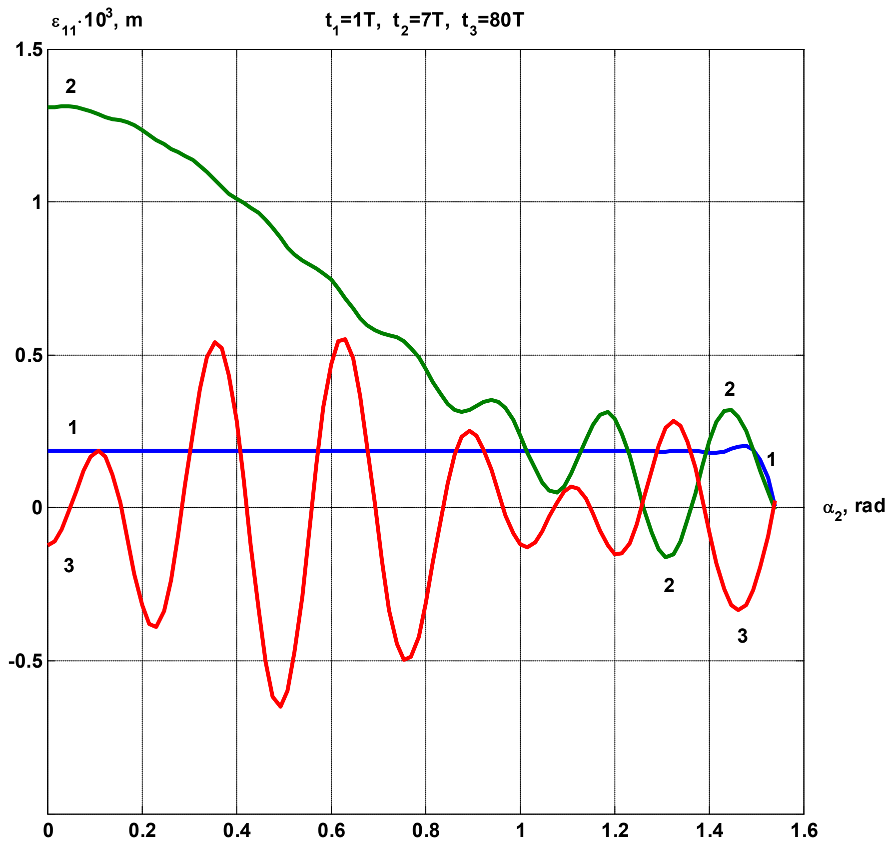

Fig. 3. Dependence of the amount of deformation ε_{11} on the spatial coordinate α_2 in the cross-section $\alpha_1 = \frac{\pi}{2}$ for the case of a smooth shell: 1 – t_1=1T; 2 – t_1=7T; 3 – t_1=80T

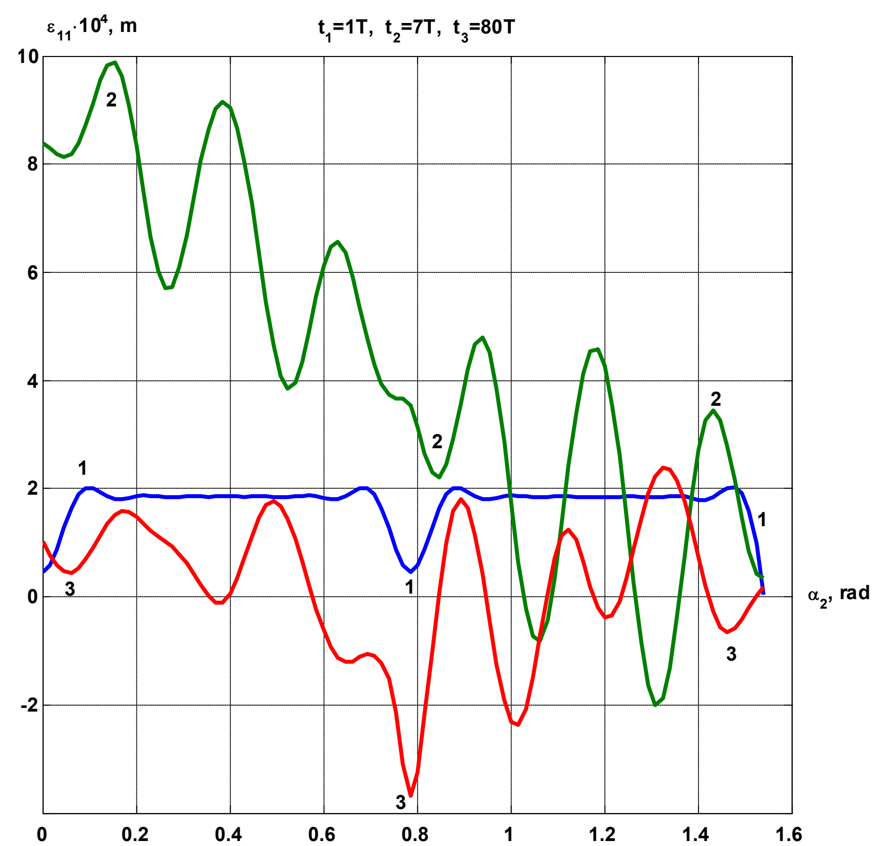

Fig. 4. Dependence of the amount of deformation ε_{11} on the spatial coordinate α_2 in the cross-section $\alpha_1 = \frac{\pi}{2}$ for the case of a reinforced shell: 1 – t_1=1T; 2 – t_1=7T; 3 – t_1=80T

The practical convergence of our results was checked. A comparison of the calculation results was performed depending on the values of the discrete steps in the spatial coordinates α_1 and α_2. Since the equation of oscillations in the smooth region (4) and on the *i*th discontinuity line (5) was derived as a system of linear differential equations in partial derivatives with respect to the variables s_1, s_2, t in the presence of spatial discontinuities along the coordinate s_1, then on coarse grids a satisfactory solution is difficult. Calculations were performed for cases N=50, N=100, N=200, N=400. Table 1 gives the values of the deformation values ε_{11i} at time point $t=7T$ in the section $\alpha_1=\frac{\pi}{2}$ and, respectively, at the point $\alpha_2=0$ for the case of a reinforced three-layer ellipsoidal shell.

Table 1

Practical convergence of the value ε_{11}

Grid dimensionality	$\varepsilon\cdot^{4-}10$, m
N=50	7.47
N=100	8.36
N=200	8.81
N=400	8.86

Analysis of the numerical results reveled that the difference for the amount of deformation ε_{11} in the section $\alpha_2=0$: for the cases n=50, n=100 reaches 11 %, for n=100, n=200 – 5 %, and for n=200, n=400 – 1 %.

Our constructed numerical algorithms for solving problems in the theory of discretely reinforced multilayer ellipsoidal shells under the action of non-stationary load were checked on test calculations. A comparative analysis with axisymmetric oscillations of the spherical shell was carried out. The results of calculations of this problem were compared with the results reported in [12]. The calculations, according to the numerical method devised in this work, satisfactorily agreed with the analytical solutions [12], which confirmed the reliability of our results.

6. Discussion of results of investigating the influence of reinforcing stiffeners on the deformation of multilayer ellipsoidal shells

In this work, a comparative analysis of the deformations of smooth and reinforced with longitudinal stiffeners of multilayer ellipsoidal shells was carried out using the graphical method of research. The graphic material in Fig. 1–4 demonstrate that the absolute maximum value of the deformation value ε_{11} for the case of a smooth three-layer ellipsoidal shell turned out to be greater than the absolute maximum value of the deformation value ε_{11} for the case of a reinforced three-layer ellipsoidal shell over the entire investigated time interval $t=80T$. Our results are explained by the fact that when the multi-layered ellipsoidal shells were reinforced, there was a redistribution of the parameters of the stress-strain state in the entire studied inhomogeneous structure. Based on physical considerations, reinforcement of the three-layer shell with stiffeners reduced the amount of deformation ε_{11}.

The advantages of our study in comparison with similar well-known ones are as follows. Unlike the studies reported in [1, 3, 10, 11], the problem of deformation of multilayer shells was solved on the basis of the finite difference method. It was this method that made it possible to study non-stationary oscillations of discretely supported shells taking into account spatial gaps. Solving the problem using the finite difference method showed that the proposed method, in contrast to study [4], contributed to the dynamic analysis of the deformation of axisymmetric shells for different types of internal and external loads.

The problem has been solved, which was confirmed by the convergence of the numerical implementation of the given problem with the use of the proposed method. We have analyzed the data, which are given in Table 1. It turned out that a reasonable accuracy of the obtained data was achieved at N=200.

Our solutions were compared with the solutions by other authors. The comparative analysis of the data obtained in [12] gave satisfactory results, which confirmed the reliability of our results. Also given in [15], the conclusions that the absolute value of the maximum deflection u_3 for a smooth ellipsoidal shell is greater than the absolute maximum value of the deflection u_3 in a reinforced ellipsoidal shell are consistent with the conclusions of this study.

Owing to the research carried out in the work, it was possible to report the results of the study on deformations of a three-layer ellipsoidal shell reinforced with longitudinal stiffeners under the influence of a non-stationary, normally distributed load. Numerical implementation of the problem of deformation of reinforced ellipsoidal shells during consideration of the action on the shell of a non-stationary load became especially difficult. Objective difficulties arose due to the fact that the process of deformation of discretely supported ellipsoidal shells was described by a system of nonlinear partial differential equations with respect to spatial coordinates s_1, s_2 and time coordinate t. The difficulty of solving such problems was the presence of discontinuous coefficients in the equations of oscillations along the s_1 coordinate. The research described in this paper suggested a way to overcome these difficulties. The numerical algorithm for solving problems of the theory of discretely reinforced multilayer ellipsoidal shells was constructed as follows: solutions were sought in the smooth region of the ellipsoidal shell (4) and separately on the lines of spatial discontinuities (5). The solutions found were "glued" on the lines of discontinuities with the help of kinematic conjugation conditions. This method has made it possible to obtain robust results for discretely reinforced multilayer ellipsoidal shells with different boundary conditions and under different loads.

The limitations in this research are within the applicability of the finite difference method, for example, solving the problem of dynamic stability. The disadvantage of this study was that increasing the accuracy of the solution led to computational difficulties.

The development of our research may deal with the field of strength of reinforced multilayer ellipsoidal shells with different openings, in contact with an elastic base, etc. In these cases, the difficulty will be to correctly account for these conditions.

7. Conclusions

1. The problem of deformation of discretely reinforced longitudinal stiffeners of multilayer ellipsoidal shells under the action of a non-stationary load has been stated. Owing

to the application of the geometrically linear version of the theory of elastic shells and rods of the Tymoshenko type, and the proposed contact conditions, it was possible to take into account the discreteness of the placement of longitudinal stiffeners on a multilayer ellipsoidal shell.

2. The use of the Hamilton-Ostrogradsky variational principle has made it possible to build a mathematical model of the vibrations of multilayer ellipsoidal shells supported by longitudinal stiffeners under various types of loads. The resulting equations of the hyperbolic type helped investigate the deformations of stiffener-reinforced multilayer ellipsoidal shells under short-term non-stationary loads.

3. The equation of oscillations in the smooth region and for the ith stiffener has been derived as a system of partial differential equations with respect to the variables s_1, s_2, t in the presence of spatial discontinuities along the s_1 coordinate. Such a system could not be solved by known analytical methods. Numerical methods proposed in other literature sources either could not be applied to the study of reinforced ellipsoidal shells, or they required complex computational algorithms. A numerical algorithm based on the calculations of the integral-interpolation method of constructing finite-difference schemes by spatial coordinates and an explicit finite-difference scheme of the "cross" type by time coordinate made it possible to derive a solution to the given problem.

4. A comparative analysis of the deformations of a smooth and reinforced with longitudinal stiffeners of a multilayer ellipsoidal shell was carried out. The analysis of our results revealed that the reinforcing stiffeners significantly affect the deformed state of the multilayer ellipsoidal shell. It was established that the maximum strain ε_{11} of a smooth three-layer ellipsoidal shell is on average 1.4 times greater than the strain ε_{11} of a reinforced three-layer ellipsoidal shell over the entire investigated time interval. It was determined that over time the presence of reinforcing stiffeners has a greater influence on the deformed state of the reinforced ellipsoidal shell. In this way, the hypothesis put forward in the work was confirmed, that the presence of reinforcing stiffeners would reduce the maximum deformation of a reinforced three-layer ellipsoidal shell in comparison with a smooth three-layer ellipsoidal shell.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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