*The object of this study is a large class of mathematical programming problems under conditions of uncertainty of initial data. The formulated object generates a subclass of problems of rational distribution of a limited resource under conditions of initial data described in terms of fuzzy mathematics. The conventional, standardly used method for solving such problems is based on optimization on average. To obtain such a solution, it is sufficient to replace all fuzzy initial data with their modal values in the analytical description of the mathematical model of the corresponding problem. To solve the resulting deterministic problem, one can use known methods of mathematical programming. However, the results of such a solution can be used in practice if the carriers of fuzzy parameters are specified compactly, that is, the intervals of possible values of fuzzy parameters of the problem are small. Otherwise, the implementation of this solution may lead to unpredictably large losses. Other alternative approaches are based on the use of insufficiently informative estimates of the best or worst possible values of fuzzy parameters of the problem. These circumstances make the statement of the problem and the objective of the study relevant: devising a method for solving the problem of rational distribution of a limited resource under conditions of fuzzy initial data. To solve the stated problem of rational distribution of a limited resource, a productive idea of constructing the proposed general optimization method under conditions of uncertainty of the initial data has been constructively implemented. In this case, the initial problem is reduced to a clear problem of optimizing a complex criterion constructed on the basis of the objective function of the initial problem and a set of membership functions of its fuzzy parameters. An example of solving the problem has been considered, leading to a solution that is better than that obtained on the basis of the modal values of the fuzzy parameters of the problem*

***Keywords:** distribution problem of mathematical programming, fuzzy initial data, method for solving the problem*

UDC 519.2
DOI: 10.15587/1729-4061.2024.310569

# METHOD FOR SOLVING DISTRIBUTIONAL PROBLEMS OF MATHEMATICAL PROGRAMMING UNDER CONDITIONS OF FUZZY INITIAL DATA

**Lev Raskin**
Doctor of Technical Sciences, Professor
Department of Software Engineering and Management Intelligent Technologies*

**Oksana Sira**
Doctor of Technical Sciences, Professor
Department of Computer Mathematics and Data Analysis*

**Artur Hatunov**
Department of Software Engineering and Management Intelligent Technologies*

**Roman Riabokon**
Department of Computer Mathematics and Data Analysis*

**Rodyslav Sinitsyn**
*Corresponding author*
Department of Computer Mathematics and Data Analysis*

*National Technical University «Kharkiv Polytechnic Institute»
Kyrpychova str., 2, Kharkiv, Ukraine, 61002

*Received date 05.06.2024*
*Accepted date 13.08.2024*
*Published date 30.08.2024*

***How to Cite:** Raskin, L., Sira, O., Hatunov, A., Riabokon, R., Sinitsyn, R. (2024). Method for solving distributional problems of mathematical programming under conditions of fuzzy initial data. Eastern-European Journal of Enterprise Technologies, 4 (4 (130)), 63–68. https://doi.org/10.15587/1729-4061.2024.310569*

## 1. Introduction

A typical problem of rational resource allocation in a standard statement is formulated as follows from [1, 2]. A limited resource equal to $b$ is distributed among $n$ consumers. The problem requires finding a plan $x=(x_1, x_2,..., x_n)$, which specifies the resource allocation that maximizes the total expected income, additively determined by a set of production functions $\varphi_j(x_j)$, $j=1,2,...,4$. The mathematical model of the problem takes the form: find a vector that delivers the maximum of the objective function:

$$L(x)=\sum_{j=1}^{n}\varphi_j(x_j), \tag{1}$$

and satisfying the limitation:

$$\sum_{j=1}^{n}x_j=b. \tag{2}$$

To solve the problem, the method of undetermined Lagrange multipliers is used. In this case, the Lagrange function is formed:

$$\phi(x)=L(x)-\lambda\left(\sum_{j=1}^{n}x_j-b\right)=\sum_{j=1}^{n}\varphi_j(x_j)-\lambda\left(\sum_{j=1}^{n}x_j-b\right).$$

Then:

$$\frac{d\phi(x)}{dx_j}=\frac{d\varphi_j(x)}{dx_j}-\lambda=0,\ j=1,2,...,n.$$

The resulting equations are independently solved with respect to $x_j$. To find the unknown uncertain multiplier $\lambda$, relation (2) is used.

The simple example below allows us to correctly state the problem that arises when solving optimization problems using known standard methods under conditions of fuzzy initial data.

Let $\varphi_j\left(x_j\right)=\alpha_j x_j^{1/2}, j=1,2,...,n$. In this case:

$$\phi(x)=\sum_{j=1}^{n} a_j x_j^{\frac{1}{2}}-\lambda\left(\sum_{j=1}^{n} x_j-b\right).$$

Then:

$$\frac{d\phi(x)}{dx_j}=\frac{1}{2}d_j x_j^{-\frac{1}{2}}-\lambda=0.$$

Hence:

$$x_j^{-\frac{1}{2}}=\frac{2\lambda}{a_j},\ x_j=\left(\frac{2\lambda}{a_j}\right)^{-2}=\frac{a_j^2}{(2\lambda)^2}\ j=1,2,...,n. \qquad (3)$$

Next:

$$\sum_{j=1}^{n} x_j=\frac{1}{(2\lambda)^2}\sum_{j=1}^{n} a_j^2=b,$$

whence:

$$\frac{1}{(2\lambda)^2}=\frac{b}{\sum_{j=1}^{n} a_j^2}.$$

Substituting the obtained ratio in (3), we finally have:

$$x_j=\frac{a_j^2}{\sum_{j=1}^{n} a_j^2},\ j=1,2,...n.$$

Let us consider another simple example of solving the resource allocation problem.

Let an enterprise manufacture two types of products. To this end, we introduce: $a_j$ – costs of manufacturing a unit of product of the $j$-th type, $j=1.2$; $b$ – total resource allocated for manufacturing the product; $c_j$ – average income received from selling a unit of product of the $j$-th type, $j=1.2$; $x_j$ – number of units of product of the $j$-th type planned for manufacturing, $j=1.2$.

Then $L(x_1, x_2)=c_1x_1+c_2x_2$ is the total average income received from implementing the plan $x=(x_1, x_2)$.

Now let us find the plan $x=(x_1, x_2)$, which maximizes the total average income from its implementation, satisfying the constraint on the total resource:

$$a_1x_1+a_2x_2=b.$$

This problem is easily solved as follows.

Enter $V_j=a_j \cdot x_j, j=1,2$. In this case:

$$x_j=\frac{V_j}{a_j}, j=1,2.$$

Then:

$$L=\frac{c_1}{a_1}V_1+\frac{c_2}{a_2}V_2, \qquad (4)$$

$$V_1+V_2=b. \qquad (5)$$

Now, the original problem is: find a plan $V=(V_1,V_2)$ that maximizes (4) and satisfies (5). The solution to this problem is obvious: the optimal plan has one non-zero component $V_j^*=b$, where:

$$j^*=\max_j\left\{\frac{C_1}{a_1},\frac{C_2}{a_2}\right\},$$

hence:

$$X_j^*=\frac{V_j^*}{a_j^*}=\frac{b}{a_j^*},\ L=C_j^*\cdot X_j^*=C_j^*\cdot\frac{V_j^*}{a_j^*}=\frac{C_j^*}{a_j^*}\cdot b. \qquad (6)$$

The result can be a solution to this problem for a specific set of source data.

Let:

$$a_1=2.5,\ a_2=2.4,\ b=100,\ c_1=\langle 10;1;1\rangle,\ c_2=\langle 9.5;1.4\rangle.$$

Then:

$$X_1=\frac{b}{a_1}=\frac{100}{2.5}=40,\ L\left(X_1\right)=\mu_1\cdot\frac{b_1}{a_1}=10\cdot 40=400,$$

$$X_2=\frac{b}{a_2}=\frac{100}{2,4}=41,\ L\left(X_2\right)=\mu_2\cdot\frac{b_1}{a_1}=9.5\cdot 41=389.5.$$

It is clear that similarly to what has been described, pessimistic and optimistic solutions to the problem can be obtained.

The problem is radically complicated if the initial data are not precise but are specified, for example, in terms of fuzzy mathematics [3, 4] by their modal values and definition interval. It is clear that the conventional procedure described above can be used in this case to calculate the average, as well as the pessimistic and optimistic values of the objective function. Of course, the results obtained in this case could be used in practical tasks only if the fuzzy numbers are specified on compact intervals. If this is not the case, then the resulting solution may turn out to be unpredictably erroneous. Thus, the known methods for solving optimization problems that use incomplete, local information about the uncertainty of the initial data are unreliable. This circumstance, firstly, significantly motivates the conduct of research in the field of solving optimization problems under uncertainty, and, secondly, poses the urgent task of devising a method for solving mathematical programming problems under fuzzy initial data. The solution to this problem significantly complements the toolset of widely used methods for solving optimization problems.

## 2. Literature review and problem statement

A possible approach to obtaining solutions of practical interest to this problem is to reduce the original fuzzy problem to a clear one [5]. In this case, all fuzzy parameters of the problem are successively assigned the minimum possible, then the maximum possible, and finally the modal values. The resulting values of the objective function determine only the pessimistic, optimistic, or average solutions, respectively. These solutions can be reasonably used in practice if the expected value is significantly greater than the length of the uncertainty interval of the fuzzy parameters of the problem. Otherwise, the resulting solution may be arbitrarily far from optimal.

In [6], the same idea is implemented for the case when the parameters of the problem are fuzzy numbers of the $(L–R)$-type. They are specified by sets $\alpha_j=\langle m_j, \alpha_j, \beta_j\rangle$, where $m_j$ are the modal values of these fuzzy numbers, and the values $\alpha_j$ and $\beta_j$, respectively, specify the left and right fuzziness coefficients. The accuracy of the solution obtained in this case is unpredictable.

In work [7], the same technique is used to find a solution to the problem that is optimal on average. It is clear that the solutions obtained by implementing this approach can be used in practice only if the numerical values of the parameters $\alpha_j$ and $\beta_j$ are small compared to $m_j$. The same unpromising idea is implemented by the procedure for solving the problem in the case where the fuzzy parameters are specified in intervals [8]. Here it is also necessary that the length of the interval for determining these fuzzy numbers be less than their modal value.

In numerous studies, a local method is built for solving a specific fuzzy problem specifically for this task. For example, in [9], the uncertainty is described in terms of probability theory. In [10], the same approach is implemented using a normal distribution. In [11], a solution is obtained for a specific linear objective function, and in [12], for the case of a quadratic constraint. However, these particular results do not resolve the problem of finding a general method for solving optimization problems under conditions of fuzzy initial data.

In real practice, solving a large number of problems of state optimization and system control requires solving exactly such problems. Known methods for solving these problems use only point information regarding the nature, type, and depth level of uncertainty. They do not take into account the information contained in the analytical descriptions of the membership functions of fuzzy parameters of the problem, which leads to a significant loss of accuracy. In this regard, research into the development of the designated method is advisable.

## 3. The aim and objectives of the study

The aim of our study is to devise a general method for solving distribution problems of mathematical programming under conditions when the parameters of the objective function of the problem are not clearly defined. Such an advancement significantly expands the toolset of actively used computational schemes in mathematical programming.

To achieve the goal, the following two tasks were set:

– to work out principles underlying the method for solving optimization problems under uncertainty;

– to devise a computational procedure for the numerical solution of distribution problems in mathematical programming under conditions of fuzzy initial data.

## 4. The study materials and methods

Application of fundamental principles from general systems theory to devise a universal optimization method under conditions of fuzzy initial data.

The object of our study is distribution optimization problems in which the parameters of the objective functions are specified fuzzy. The task is set to devise a general method for solving a conditional optimization problem for a case where the hypothesis is accepted that the parameters of the objective function are specified fuzzy. This stated problem is of a systemic nature since we are talking about devising a special method for obtaining a general method for solving a class of optimization problems that is important for practice.

The systemic nature of the problem necessitates the use of a set of fundamental principles of general systems theory for its solution, formulated by Bertalanffy [13] and forming the following list:

– purpose;

– structure;

– organization;

– properties (requirements).

## 5. Results of the study on devising a universal method for solving optimization problems under uncertainty

### 5. 1. Development of system principles for constructing the method and organizing the computational process

In accordance with the above list of fundamental principles for solving system problems, the design characteristics of the method and the computational procedure for its implementation were determined taking into account the specific features of the problem under consideration. The devised universal method for solving fuzzy problems of mathematical programming meets the requirements listed below.

Structure – a two-module computing system implementing an iterative procedure, at each step of which the next solution is formed, adjusted if necessary.

Organization – an iterative step-by-step procedure for obtaining a solution is implemented. The first module – as a result of solving the stated optimization problem, the next solution is determined. The second module – checking the optimality of the solution and adjusting the plan in the case of its non-optimality.

Properties (requirements):

– independence from the dimensionally of the optimization problem being solved;

– independence of the procedure implementation technology on the nature and type of the objective function and limitations of the original optimization problem;

– independence on the type and technique of describing the fuzzy parameters of the system, defined by its membership functions.

### 5. 2. Devising a computational procedure for a numerical solution implementing the proposed method

The essence and features of the implementation of the proposed method are presented using the example of solving a specific problem.

Let the income from the sale of manufactured products in a real problem of rational resource allocation be defined in terms of fuzzy mathematics by $(L–R)$-type functions.

$C_1$ and $C_2$ are specified – fuzzy numbers of $(L–R)$-type with a Gaussian membership function. $C_1=\langle m_1, \alpha_1, \beta_1\rangle$, $C_2=\langle m_2, \alpha_2, \beta_2\rangle$, and:

$$\mu(C_1)=\left\{\begin{array}{l}\exp\left\{-\dfrac{(m_1-C_1)^2}{2\alpha_1^2}\right\}, C_1\le m_1\\ \exp\left\{-\dfrac{(C_1-m_1)^2}{2\beta_1^2}\right\}, C_1> m_1\end{array}\right\}, \quad (7)$$

$$\mu(C_2)=\begin{Bmatrix}\exp\left\{-\frac{(m_2-C_2)^2}{2\alpha_2^2}\right\}, C_2\le m_2\\ \exp\left\{-\frac{(C_2-m_2)^2}{2\beta_2^2}\right\}, C_2> m_2\end{Bmatrix}. \quad (8)$$

A simple approximate solution to any fuzzy problem can be obtained if the values of the fuzzy parameters of this problem are set equal to their modal values, that is, in this case, assume $C_1=m_1$, $C_2=m_2$. In this case, relations (6) determine the desired solution. This solution can be implemented in practice if the value of the fuzziness coefficients ($\alpha_1$, $\beta_1$; $\alpha_2$, $\beta_2$) of the fuzzy numbers $C_1$ and $C_2$ are much less than the corresponding modal values ($m_1$, $m_2$). Otherwise, this solution may turn out to be unpredictably far from optimal, which is fraught with possible large losses. In this regard, the problem of finding an adequate optimization method that takes into account all the information regarding the uncertainty of the initial data remains relevant.

The above requirements are satisfied by the computational procedure implemented as follows. A two-step computational scheme for solving the problem is proposed. In the first step, the initial problem is solved using a suitable mathematical programming method under the assumption that the fuzzy parameters are fixed and set equal, for example, to their modal values. The result of this step is the obtained analytical relationships for calculating the sought variables of the problem. In the second step, an iterative procedure for optimizing the values of the fuzzy parameters is performed. To implement this, a technique from [4] can be used in combination with any zero-order optimization method.

Let $L(X, C)$ be the objective function of the problem, depending on the components of the vector $X=(X_1, X_2,..., X_n)$ and the set of parameters $C=(C_1, C_2,..., C_n)$, the analytical description of which is performed in terms of fuzzy mathematics by membership functions of the $(L-\lambda)$-type, having the form $(\mu_1(C_1), \mu_2(C_2),..., \mu_n(C_n))$. Then the solution to the original problem of maximizing the objective function $L(X, C)$ on the set of variables $X$ satisfying the constraints $G(X)=0$, and the set $C=(C_1, C_2,..., C_n)$ is obtained by optimizing the criterial function:

$$F(X,A)=L(X,A)\cdot\prod_{i=1}^{n}\mu_i(C_i), \quad (9)$$

under the same constraints. It is important that to implement this method it is sufficient to have the ability to calculate the numerical value of function (9) on any sets of variables $X$, $C$. This means that any zero-order optimization method, for example, the Nelder-Mead method, can be used to solve the problem. A similar idea for solving this problem without detailing was stated in [14].

A special property of the specific problem (1), (2) under consideration arises due to the separability and linearity of the objective function (1), as well as the linearity of the only constraint (2). In this case, the sought-after vector of the solution to the problem contains only one non-zero component. Taking this feature into account, the proposed general method can be simplified. The solution to the problem is obtained by sequential optimization for each of the variables, comparing the results, and choosing the best of them. In this case, in accordance with (6), the number of units of the product of the selected type for production and sale is determined by the ratio $X_i=\frac{b_i}{a_i}$, and the corresponding income is calculated from the formula $L=C_i\frac{b_i}{a_i}$. Then, taking into account (9), the $i$-th component of the criterial function takes the form:

$$F(C_i)=\frac{b_i}{a_i}C_i\mu_i(C_i)=\frac{b_i}{a_i}C_i\exp\left\{\frac{(C_i-m_i)^2}{2\beta_i^2}\right\}. \quad (10)$$

Here, for obvious reasons, the right branch of the membership function of the fuzzy parameter $C_i$ is chosen to describe the expected income.

As a result of differentiating (10) with respect to $C$ and equating the corresponding expression to zero, the equation is obtained:

$$\frac{\alpha F(C_i)}{\alpha C_i}=\frac{b_i}{a_i}\begin{bmatrix}\exp\left\{-\frac{(C_i-m_i)^2}{2\beta_i^2}\right\}-\\ -C_i\exp\left\{-\frac{(C_i-m_i)^2}{2\beta_i^2}\right\}\frac{(C_i-m_i)}{\beta_i^2}\end{bmatrix}=0.$$

Whence:

$$1-C_i\left(\frac{C_i-m_i}{\beta_i^2}\right)=0,$$

$$C_i^2-m_iC_i-\beta_i^2=0.$$

The positive root of this equation is:

$$\hat{C}_i=\frac{m_i+\sqrt{m_i^2+4\beta_i^2}}{2}. \quad (11)$$

At the same time, for the set of initial data selected above:

$$\hat{C}_1=\frac{10+\sqrt{100+4\cdot 1}}{2}=$$

$$=\frac{10+\sqrt{104}}{2}=\frac{10+10.2}{2}=10.1,$$

$$\hat{C}_2=\frac{9.5+\sqrt{9.5^2+4\cdot 4^2}}{2}=\frac{9.5+\sqrt{90.25+64}}{2}=$$

$$=\frac{9.5+\sqrt{154.25}}{2}=\frac{9.5+12.43}{2}=\frac{21.93}{2}=10.965.$$

The expected values of income from the sale of products of the first or second type, respectively, are equal to:

$$L(X_1)=\hat{C}_1\frac{b}{a_1}=10.2\cdot 40=408,$$

$$L(X_2)=\hat{C}_2\frac{b}{a_2}=10.96\cdot 40=449.$$

The obtained value of the expected income is significantly higher than that obtained earlier during optimization on average.

Another possibility of using the real uncertainty of the initial data in this problem is as follows. The probability is calculated that the expected income from the sale of products of the second type will exceed the average possible income from the sale of products of the first type. In order to impart a probability-theoretical nature to the fuzzy numbers $C_2$, the membership function $\mu_2(C_2)$ is transformed into a distribution density [14]. To this end, the area under the right branch of the curve describing the membership function of the fuzzy number $C_2$ is determined:

$$S=\int_{\mu_2}^{\infty}\exp\left\{-\frac{\left(C_2-m_2\right)^2}{2\beta_2^2}\right\}\alpha c_2 \underset{\frac{C_2-m_2}{\beta_2}=t}{=} \beta_2\int_0^{\infty}e^{-\frac{t^2}{2}}\alpha t=$$

$$=\sqrt{2\pi}\beta_2\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\alpha t=\frac{\sqrt{2\pi}\beta}{2}.$$

Next, the following function is introduced:

$$\varphi\left(C_2\right)=\frac{\mu\left(C_2\right)}{S}=\frac{2}{\sqrt{2\pi}\beta_2}\exp\left\{-\frac{\left(C_2-m_2\right)^2}{2\beta_2^2}\right\}. \qquad (12)$$

The obtained function (12) has the canonical properties of the distribution densities of random variables [10]. It is non-negative and its integral on the interval [9.5; ∞] is equal to one. Now the boundaries of the interval of $C_2$ values can be found for which the expected income from the sale of products of the second type exceeds the average income from the sale of products of the first type. To this end, the equation is solved:

$$C_2\exp\left\{-\frac{\left(C_2-m_2\right)^2}{2\beta_2^2}\right\}=10.$$

This equation has two roots: $C_2^{(1)}=10.1$; $C_2^{(2)}=14.3$. Now the probability that a random value $C_2$ with distribution density (12) will be in the interval [10.1;14.3] is determined:

$$P\left(10.1\le C_2\le 14.3\right)=$$

$$=\int_{10.1}^{14.3}\frac{2}{\sqrt{2\pi}\beta_2}\exp\left\{-\frac{\left(C_2-m_2\right)^2}{2\beta_2^2}\right\}\alpha c_2 \underset{\frac{C_2-m_2}{\beta_2}=t}{=}$$

$$\underset{\frac{C_2-m_2}{\beta_2}=t}{=}\frac{2}{\sqrt{2\pi}}\int_{\frac{10.1-2.5}{4}}^{\frac{14.3-9.5}{4}}\frac{1}{\sqrt{2\pi}\beta_2}e^{-\frac{t^2}{2}}=$$

$$=\varphi\left(1.2\right)-\varphi\left(0.15\right)=0.77-0.12=0.65.$$

Thus, with a probability of 0.65, the random $C_2$ value will be within the interval for which the value of income from the sale of products of the second type will exceed the average income received from the sale of products of the first type.

## 6. Discussion of results related to devising a universal method for solving optimization problems under uncertainty

The fundamental feature and important advantage of our result is explained by the constructive way of forming the objective function of problem (9), which provides the ability to take into account all available information regarding the uncertainty of the initial data. This feature radically distinguishes the proposed method from the known ones, which use an insufficiently informative (two or three-point) description of the membership functions of fuzzy parameters of the problem.

The multiplicative nature of the structure of the computational scheme for implementing the method provides the ability to implement it using a two-step iterative procedure. This feature of the technology for implementing the method provides a simple way to obtain the desired result.The proposed method could be used to solve any problem in mathematical programming and for any form of describing the uncertainty of the initial data. This, in fact, resolves the task of solving optimization problems under uncertainty of the initial data. It is clear that the method cannot be used if the original problem has no solution even in a clear statement.

The area of further research is to extend the method to the case where the uncertainty of the initial data is hierarchical, that is, the parameters of the membership functions of the fuzzy initial data are themselves not clearly defined.

## 7. Conclusions

1. System principles for the construction of a method and the computational procedure implementing it have been devised. These principles (goal, structure, organization, properties) clearly define and designate the direction and nature of our study.

2. A computational scheme for the method has been developed and implemented, ensuring its independence on the type and nature of the objective function and membership functions of the fuzzy parameters of the problem. It has been shown that correct consideration of the uncertainty of the initial data can radically improve the solution obtained by known methods.

## Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

## Funding

The study was conducted without financial support.

## Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

## References

1. Gill, P. E., Murray, W., Wright, M. H. (1981). Practical optimization. Academic Press Inc.
2. Taha, H. A. (2007). Operations Research: An Introduction. Pearson. Available at: https://dl.icdst.org/pdfs/files3/7e932ab65f9aa3de7122b4cea3587377.pdf

3. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8 (3), 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x
4. Liao, T. W., Egbelu, P. J., Sarker, B. R., Leu, S. S. (2011). Metaheuristics for project and construction management – A state-of-the-art review. Automation in Construction, 20 (5), 491–505. https://doi.org/10.1016/j.autcon.2010.12.006
5. Mandi, J., Demirovi, E., Stuckey, P. J., Guns, T. (2020). Smart Predict-and-Optimize for Hard Combinatorial Optimization Problems. Proceedings of the AAAI Conference on Artificial Intelligence, 34 (02), 1603–1610. https://doi.org/10.1609/aaai.v34i02.5521
6. İnce, N. (2021). Generalized Fuzzy Entropy Optimization Methods for Fuzzy Data Analysis. Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation, 453–460. https://doi.org/10.1007/978-3-030-85626-7_53
7. Li, F.-C., Jin, C.-X. (2008). Study on fuzzy optimization methods based on principal operation and inequity degree. Computers & Mathematics with Applications, 56 (6), 1545–1555. https://doi.org/10.1016/j.camwa.2008.02.042
8. Mai, D. S., Dang, T. H., Ngo, L. T. (2020). Optimization of interval type-2 fuzzy system using the PSO technique for predictive problems. Journal of Information and Telecommunication, 5 (2), 197–213. https://doi.org/10.1080/24751839.2020.1833141
9. Ekel, P. Ya. (1999). Approach to decision making in fuzzy environment. Computers & Mathematics with Applications, 37 (4-5), 59–71. https://doi.org/10.1016/s0898-1221(99)00059-0
10. Liu, G., Yu, J. (2007). Gray correlation analysis and prediction models of living refuse generation in Shanghai city. Waste Management, 27 (3), 345–351. https://doi.org/10.1016/j.wasman.2006.03.010
11. Halilović, D., Gligorić, M., Gligorić, Z., Pamučar, D. (2023). An Underground Mine Ore Pass System Optimization via Fuzzy 0–1 Linear Programming with Novel Torricelli-Simpson Ranking Function. Mathematics, 11 (13), 2914. https://doi.org/10.3390/math11132914
12. Kicsiny, R., Hufnagel, L., Varga, Z. (2022). Allocation of limited resources under quadratic constraints. Annals of Operations Research, 322 (2), 793–817. https://doi.org/10.1007/s10479-022-05114-3
13. Bertalanfi, L. fon. (1973). Istoriya i status obschey teorii sistem. V kn.: Sistemnye issledovaniya. Metodologicheskie problemy. Ezhegodnik. Moscow: «Nauka», 20–37.
14. Raskin, L., Sira, O. (2016). Method of solving fuzzy problems of mathematical programming. Eastern-European Journal of Enterprise Technologies, 5 (4 (83)), 23–28. https://doi.org/10.15587/1729-4061.2016.81292