UDC 621.389
DOI: 10.15587/1729-4061.2024.312762

DEVISING AN ANALYTICAL METHOD FOR ESTIMATING AIRCRAFT POSITIONING ACCURACY BY AN INFOCOMMUNICATION NETWORK OF OPTOELECTRONIC STATIONS

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The object of this study is the monitoring processes of airspace in the visible and infrared frequency ranges using an infocommunication network of optoelectronic tracking stations.

The problem addressed is the assessment of accuracy in positioning aerial vehicles in airspace, considering random and systematic errors in video surveillance.

The proposed method allows for the analytical evaluation of variance in the coordinates of aerial vehicles in airspace at any given moment, depending on the variance in errors from all components of the video surveillance process.

The following results are reported:

– mathematical models for both open and covert video surveillance by the infocommunication network of optoelectronic stations based on the trajectories of aerial vehicles, which enable the estimation of their coordinates for each moment of video surveillance;

– analytical relationships between the variance of geolocation of aerial vehicles and the variance of errors in all components of the video surveillance process.

A key feature of the method is its practical independence on the type and size of aerial vehicles. The method requires the availability of metrological characteristics of the optoelectronic station instruments and synchronized measurements of azimuth, elevation angle (for covert), and slant range (for open video surveillance).

The numerical values of the root mean square deviations of the positioning errors of aerial vehicles under various video surveillance conditions range from 0.1 to 0.35 meters, confirming the effectiveness of using the infocommunication network of optoelectronic stations as an instrumental tool for high-precision trajectory measurements

Keywords: optoelectronic station, infocommunication network, airspace monitoring, aerial vehicles

Received date 15.07.2024
Accepted date 23.09.2024
Published date 23.10.2024

How to Cite: *Tevjashev, A., Zemlyaniy, O., Shostko, I., Kostaryev, D., Paramonov, A. (2024). Devising an analytical method for estimating aircraft positioning accuracy by an infocommunication network of optoelectronic stations. Eastern-European Journal of Enterprise Technologies, 5 (9 (131)), 36–48. https://doi.org/10.15587/1729-4061.2024.312762*

1. Introduction

Modern optical-electronic stations (OESs) with video cameras in the visible and infrared frequency range and laser rangefinders (LRs) have significantly increased efficiency and expanded the possibilities of continuous airspace monitoring.

The use of OES allows for the detection, recognition, and high-precision tracking of small-sized highly dynamic aerial vehicles (HDAVs), manufactured, in particular, using stealth technology.

In addition, OESs, compared to radar stations, provide a significant increase in the accuracy of determining the parameters of trajectories of all types of aircraft.

During open observation, OESs provides direct measurements of three parameters characterizing the location of the aircraft in the airspace in the spherical coordinate system: azimuth, angle of elevation and inclination, range.

When using a group of radars connected to each other by an information communication network (ICN), there is a problem related to estimating the limits of positioning accuracy of stationary or moving sources of radio radiation by difference-far-range and angle-measuring methods. In terms of radar theory, this problem belongs to the field of multi-position passive radar systems and, despite the large body of research on its solution, it remains relevant.

In the case of the use of a group of OES, combined with each other by ICN, the possibility of not only open but also covert video surveillance of the trajectory of the aircraft movement also appears. It is believed that in such a case UAV is not irradiated by LR and does not have the opportunity to establish the fact of its detection and accompaniment.

This is possible in the case when the detected UAV is accompanied by at least two OES ICN.

In the case of using OES ICN, the problem of high-precision trajectory measurements is solved for video surveillance of the trajectory of the aircraft but the assessment of actual accuracy of UAV positioning in a real time scale for each time point remains extremely relevant.

2. Literature review and problem statement

A large body of research addressed the tasks related to using OES for monitoring air, land, and sea space. In works [1, 2], separate studies on OES monitoring are reported but the issue of accuracy of their application in various situations is not considered. Paper [3] raises the question of improving accuracy methods. However, the problem of high-precision trajectory measurements and accuracy of aircraft positioning for each time point remains extremely relevant.

The problem of high-precision trajectory measurements and positioning accuracy of aircraft for each time point is studied in [4, 5] but partial cases of measurements and accuracy are also considered.

The mathematical model and method of optimal placement of optical-electronic systems for trajectory measurements of aerial objects during testing are presented in works [6, 7] but there is no emphasis on the principles of optimal placement of objects.

In [8, 9], an information and measurement system based on a wireless sensor information communication network for polygonal tests of guided and unguided rockets are investigated but there are unsolved issues related to measurement accuracy when using a wireless network.

The solution to this problem requires a detailed analysis of the causes of occurrence, degree of influence, and methods of suppression (neutralization) of all types of systematic and random errors of video surveillance of OES ICN of aircraft movement trajectories, which is addressed in work [10] but the results are not given.

In [11, 12], it is not the assessment of the positioning accuracy of radio radiation sources (RRS) that is considered but the limits of accuracy using the difference-distance measuring and/or angle measuring method of positioning. The methods are used both during the positioning of stationary RRSs, for example, base stations of mobile communication networks [11], and mobile RRSs, for example, subscriber stations [12]. The researched task of DRI positioning in terms of radar theory belongs to the field of multi-position passive radar systems. Therefore, it becomes necessary to consider a general approach to the assessment of the limits of accuracy in the positioning of radio radiation sources (RRS) by other approaches and methods.

Works [13, 14] show that modern OES for trajectory measurements are implemented in a modular design and contain the following modules:

1. A rotary support platform (RSP) with a two-axis gimbal suspension, on which an optical-electronic module with a system of global positioning, leveling of the platform, and its gyro stabilization is placed.
2. Optical-electronic module – with visible and infrared video cameras and a laser rangefinder.

The OES provides, under the LOS (Line of Sight) conditions, a circular survey of the airspace in the optical and infrared frequency ranges, detection, identification, recognition, and automatic support of aircraft. However, approaches to other frequency ranges are not considered.

In [15, 16], a description and individual characteristics of ground-based optical, optical-electronic, quantum-optical, laser-television means and systems of high-precision trajectory measurements, which are used at scientific research test sites, cosmodromes, laboratory-testing bases, and test sites, are given. Therefore, for a more accurate representation of the airspace survey, it is necessary to unravel other LOS visibility conditions, which are possible by modern OES.

Since each OES has a limited viewing area, when building an effective system of video surveillance of the airspace over a large area, it is necessary to use OES ICN. The systematic solution to the problem of building OES ICN [17, 18] includes the solution of an ordered set of interrelated tasks. One of the primary tasks is to determine the minimum number and optimal placement of OESs in such a way as to minimize the total volume of dead zones in the field of airspace video control and increase the accuracy of aircraft positioning for each time point t of video surveillance of the OES ICN aircraft. Therefore, ICN of OES must be spatially distributed, echeloned, highly reliable, and viable throughout the entire period of its operation

In addition, OES ICN should be considered as an instrumental means of video surveillance and high-precision trajectory measurements.

And this means that the metrological characteristics of OES ICN trajectory measurements are estimates of mathematical expectations of the coordinates of the location of aircraft for each time point t of video surveillance and estimates of variances in their errors for each of the coordinates. The problem of estimating the positioning accuracy of OES ICN aircraft is currently practically unsolved, and its solution is one of the urgent tasks of high-precision trajectory measurements in the visible and infrared frequency ranges. Moreover, the methods for estimating the accuracy of aircraft positioning should work in real-time systems with built-in analytics and minimal costs of computing resources. Therefore, the construction of mathematical models of video surveillance and an analytical method for evaluating the metrological characteristics of OES ICN of trajectory measurements for each time point t is a relevant task that is solved in this study.

3. The aim and objectives of the study

The purpose of our study is to devise an analytical method for estimating the variance in aircraft positioning errors for each fixed moment of video surveillance of OES ICN along its trajectory. This will make it possible to increase the effectiveness of both the results of the study of the dynamic characteristics of all types of aircraft during their range tests, as well as the effectiveness of all types of means for defeating air targets under combat conditions.

To achieve the goal, the following tasks were solved:

– to build mathematical models of open and hidden video surveillance of OES ICN along the trajectories of aircraft movement and use them to estimate mathematical expectations for the coordinates of aircraft location in air space for each moment of video surveillance time;

– to derive analytical dependences for calculating the variance in aircraft positioning errors for each time point depending on the variance in all types of random video surveillance errors of OES ICN along the trajectory of the aircraft movement;

– to conduct a numerical modeling study of resulting dependences for their consistency and reliability.

4. The study materials and methods

The object of our research is the process of devising and researching an analytical method for assessing the accuracy of positioning of aircraft in the air space. As a measure of accuracy of aircraft positioning, the estimates of variance in errors are used for each of the coordinates of aircraft location in the air space at each time point, depending on the error variances of all component video surveillance processes.

The research hypothesis assumes that the errors of all components of video surveillance processes are random variables with a normal distribution law and known parameters.

The following assumptions were accepted. To determine the coordinates of aircraft location in the airspace relative to the location of OES, two instrumental coordinate systems of OES video cameras are used: the instrumental spherical coordinate system (ISCS) and the instrumental Cartesian coordinate system (ICCS). The global geodetic coordinate system WGS-84 is used to quantitatively describe the position and movement of aircraft in near-Earth space. The estimates of coordinates of aircraft location were obtained as a result of the formation and solution of the NAU mathematical model of open video surveillance by the analytical method and numerical MNC of the redefined LAU systems of the mathematical model of covert video surveillance. The variances in the estimates of coordinates of aircraft location from the variances in the errors of all the component processes of video surveillance were calculated according to analytical dependences, which were derived by the method of statistical linearization of nonlinear functions of random arguments.

The following simplifications were adopted. Linearization of nonlinear functions of random arguments by expanding them into a Taylor series was carried out with accuracy up to linear terms. To increase the accuracy of variance estimates, expansion into a Taylor series can be carried out with accuracy up to the quadratic terms (covariance matrices).

The research was carried out analytically using the methods of probability theory, mathematical statistics, linear algebra, and included the following:

– selection and justification of coordinate systems, mathematical models of open and hidden video surveillance, and methods for researching the results of video surveillance;

– identification of the sources of systematic errors associated with both the design features of the practical implementation of OES, as well as with errors in the horizontalization of OES RSP and the binding of the instrumental coordinate systems of OES to the local meridian;

– assessment of the degree of influence of systematic errors on the effectiveness of estimates of the coordinates of aircraft location.

The principal research implied obtaining, by the method of statistical linearization, analytical dependences of variance in the estimates of coordinates of aircraft location on variances in the random errors of all components of video surveillance processes. Verification of the resulting dependences was carried out by a numerical method.

The equipment used in the research was a test sample of OES with its metrological characteristics and proprietary specialized software [4].

5. Results of investigating the analytical method for assessing the accuracy aircraft positioning in the air space

5. 1. Construction of mathematical models of open and covert video surveillance by the information communication network of optoelectronic stations based on the trajectories of aircraft movements

To use OES as an instrumental tool when conducting external trajectory measurements, the video camera is installed on RSP. This provides the ability to rotate the video camera in two planes – a horizontal plane (in azimuth) to an angle of $n\times360$ and a vertical plane (in position) to an angle of 0–90 degrees.

Visible and infrared video cameras are used as tools. Therefore, to determine the coordinates of aircraft location in relation to the coordinates of OES location, two instrumental coordinate systems of the video cameras of OES are used: the instrumental spherical coordinate system (ISCS) and the instrumental Cartesian coordinate system (ICCS).

It is assumed that the beginning of the instrumental systems of video cameras coincides with the location point of video camera matrices of the visible (*Oc*) and infrared (Op) frequency range through which the optical axes of the video cameras pass. Theoretically, the coordinate axes ISCS and ICCS of OES video cameras have the following directions: the *Y* axis is directed vertically upwards, the *X* axis is directed to the north, the *Z* axis so that the coordinate system is right. Since the video cameras are installed on RSP, the spatial position of which is an adjustable value, the direction of the coordinate axes of the instrumental coordinate systems of OES video cameras, as a rule, does not coincide with the theoretical directions.

To compensate for this discrepancy, two instrumental coordinate systems OES ICS and OES ICCS are also used. They have a common origin that coincides with the point of intersection of the axes of the gimbal suspension O (Fig. 1), and the direction of the axes coincides with the theoretical direction of the axes of the video cameras.

The use of OES ISCS and ICCS makes it possible to determine the estimates of aircraft location in relation to the location of a specific OES. The WGS-84 geodetic coordinate system is used to conduct external trajectory measurements along the entire trajectory of the aircraft.

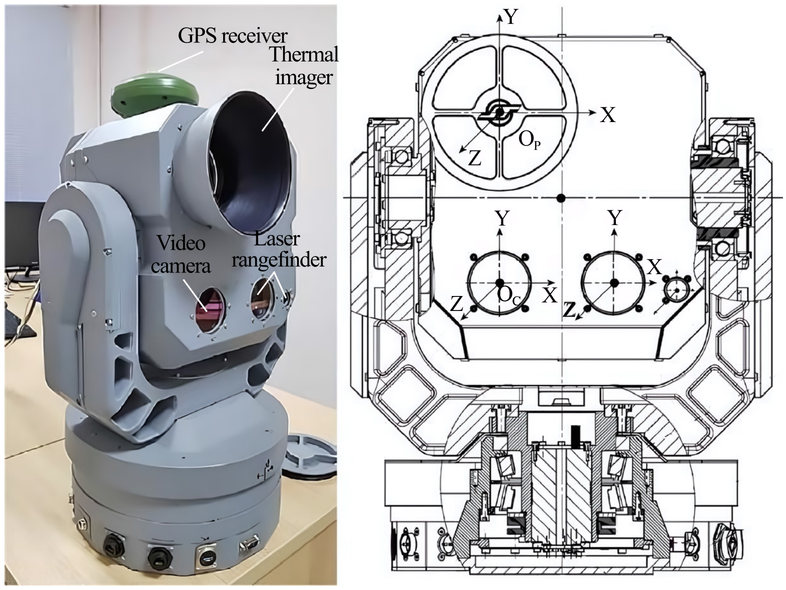
Thus, estimates of aircraft location in relation to the position of a specific OES were obtained.

The World Geodetic Coordinate System WGS-84 is intended for the quantitative description of the position and movement of objects located on the Earth's surface and in space around the Earth. The position of aircraft in the WGS-84 system can be found in the form of spatial rectangular coordinates $(X, Y, Z)^T$, where X, Y, Z are the coordinates of aircraft [km] in the Greenwich coordinate system (GCS) or its coordinates (B, L,H)T in the geodetic coordinate system (GDCS).

Coordinate B – geodetic latitude – the angle between the normal from the location point of aircraft and the plane of the equator [rad].

Coordinate L – geodetic longitude – the angle between the plane of the meridian of aircraft location point and the plane of the initial meridian [rad].

Coordinate H – geodetic height – segment of the normal from aircraft location to the surface of the ellipsoid [m].



a *b*

Fig. 1. Instrumental tool for conducting external trajectory measurements: *a* — appearance of the design implementation of the optical-electronic station; *b* — direction of the axes of coordinate system of the video cameras and the laser range finder in an opto-electronic station

Geodetic coordinates refer to the global ellipsoid, the dimensions and shape of which are determined by the values of the semi-major axis and compression. The center of the ellipsoid coincides with the origin of the WGS-84 coordinate system, the axis of rotation of the ellipsoid coincides with the *Z* axis, and the plane of the initial meridian – with the *XOZ* plane.

Instrumental coordinate systems of OES are strictly related to the design features of OES implementation, its spatial location on the surface of the earth, and the direction of the *OX* axis, relative to which the azimuth is counted (measured).

The main source of instrumental (systematic) errors in measuring the azimuth and angle of aircraft location relative to the location of OES are:

1) the actual deviation of the origin of ISCS and ICCS of video cameras from the origin of ISCS and ICCS of OES (crossing points of the axes of the gimbal suspension of OES RSP);

2) errors between the directions of ISCS and ICCS axes of OES video cameras and the directions of ISCS and ICCS axes of OES.

Fig. 1, *a* shows the appearance of the design implementation of OES. The actual deviation of the beginning of the action of video cameras of the visible Oc, the infrared Op of the frequency range, and the laser rangefinder OD from the point of intersection of the axes of the gimbal suspension OES RSP (*b*).

These deviations are one of the sources of constant instrumental errors in measuring the azimuth and elevation angle of aircraft.

They can theoretically be easily eliminated by moving the origins of the Oc, Op and O_D coordinate systems to the point of intersection of the RSP gimbal suspension axes.

The value of the systematic error in the estimation of the coordinates of aircraft location $_i$ depending on the actual deviation of the start of ISC of the video cameras and the laser rangefinder from the point *O* is calculated according to the formula:

$$\rho_i=\sqrt{x_i^2+y_i^2+z_i^2}\ ,\ \ i=1,2,3, \tag{1}$$

where $(x_i, y_i, z_i)^T$ are the location coordinates of the video camera matrices of the visible range i=1, the infrared frequency range i=2, and the laser rangefinder i=3. For small-sized OES, they are insignificant and are practically neglected.

Each OES for each time point under the open video surveillance mode provides measurement of three parameters – azimuth α_i, elevation angle β_i, and slant range D_i from OES to the aircraft, which determine the location (α_i, β_i, D_i) of aircraft in the IR of cameras. Next, these coordinates are transformed into the coordinates of aircraft location $(X^/,Y^/,Z^/)^T$ in the ICCS of the video cameras. Under actual conditions, adjustment technologies are used to minimize the discrepancy between the directions of ICCS axes of the video cameras and the axes of ICCS of OES (horizontalization of RSP and binding of the *X* axis of ICCS of the video cameras to the local meridian). Since the further processing of the results of trajectory measurements is carried out in the ICCS of OES, the coordinates of the location of aircraft in the ICCS of video cameras of the visible and infrared frequency range $(X^/,Y^/,Z^/)^T$ must be translated into the coordinates $(X,Y,Z)^T$ of the ICCS of OES. If the IDBC of the video cameras completely coincides with the IDBC of OES, such a translation is carried out by equating the corresponding coordinates, i.e., $X=X^/$, $Y=Y^/$, $Z=Z^/$.

It is assumed that the ICCS of video cameras and the ICCS of OES have a common origin, but usually differ by corresponding angles in the direction of each coordinate axis. To compensate for this discrepancy, a matrix of rotations of the video cameras' ICCS with respect to OES's ICCS is used. Each of the turning angles $(\alpha_x, \alpha_z, \alpha_y)$ due to the adjustment of OES RSP can turn out to be both positive and negative. Therefore, there are $n=2^3=8$ rotation matrices. Without violating commonality, the rotation matrix for positive angles α_x, α_z, α_y was considered:

$$P_{xzy}\left(\alpha_x,\alpha_z,\alpha_y\right)=P_x\left(\alpha_x\right)\cdot P_z\left(\alpha_z\right)\cdot P_y\left(\alpha_y\right)=\begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix}, \tag{2}$$

where elements of the matrix $P_{xzy}(\alpha_x, \alpha_z, \alpha_y)$ take the following form:

$P_{11}=\cos\alpha_z\cos\alpha_y$; $P_{12}=-\sin\alpha_z$; $P_{13}=\cos\alpha_z\sin\alpha_y$;

$P_{21}=\cos\alpha_x\sin\alpha_z\cos\alpha_y+\sin\alpha_x\sin\alpha_y$; $P_{22}=\cos\alpha_x\cos\alpha_z$;

$P_{23}=\cos\alpha_x\sin\alpha_z$;

$P_{31}=\sin\alpha_x\sin\alpha_z\cos\alpha_y-\cos\alpha_x\sin\alpha_y$; $P_{32}=\sin\alpha_x\cos\alpha_z$;

$$P_{33}=\sin\alpha_x\sin\alpha_z\sin\alpha_y+\cos\alpha_x\cos\alpha_y. \tag{3}$$

In this case, the estimates of the coordinates of aircraft location in the ICCS of OES are equal to:

$$\acute{\mathbf{x}}_p(\boldsymbol{\omega}) = P_{xzy}(\alpha_x, \alpha_z, \alpha_y) \cdot \acute{\mathbf{x}}'_p(\boldsymbol{\omega}), \tag{4}$$

where

$$\acute{\mathbf{x}}'_p(\boldsymbol{\omega}) = \left(\acute{x}'_p(\omega), \acute{y}_p(\omega), \acute{z}'_p(\omega)\right)^T,$$

$$\acute{\mathbf{x}}_p(\boldsymbol{\omega}) = \left(\acute{x}'_p(\omega), \acute{y}_p(\omega), \acute{z}'_p(\omega)\right)^T$$

are the estimated coordinates of the point in the ICCS of video cameras and in the ICCS of OES.

The value of the systematic error of estimating the coordinates of HDAV location depending on the adjustment errors of OES RSP was calculated according to the formula:

$$\acute{\rho}(\omega) = \\ = \sqrt{\left(x(\omega) - \acute{x}'(\omega)\right)^2 + \left(y(\omega) - \acute{y}'(\omega)\right)^2 \left(z(\omega) - \acute{z}'(\omega)\right)^2}. \tag{5}$$

Various technologies and equipment, including laser, optical, and navigation, are used to minimize errors between the directions of the axes of ICCS of video cameras and the ICCS of OES [2]. An analysis of instrumental errors of aircraft location estimates related to random errors of RSP adjustment was carried out using the method of simulation modeling. The model of normal distribution of random errors with non-zero positive values of mathematical expectations of turning angles (α_x, α_z, α_y) and ($\sigma_{\alpha x}$, $\sigma_{\alpha z}$, $\sigma_{\alpha y}$) – variances in the residual errors of turning angles of OES RSP after its adjustment is used as a model of RSP adjustment.

Fig. 2, *a*, *b* shows the dependence of change in the values of the systematic error $\acute{\rho}(\omega)$ in the estimation of coordinates of aircraft location on change in the azimuth and the elevation angle at the given values of the mathematical expectations and variances in OES platform adjustment errors.

Estimates of mathematical expectations and variances in the systematic error $\acute{\rho}(\omega)$ for different values of the slant range to aircraft are given in Tables 1, 2.

Analysis of the dependence of OES measurement instrumental error in the coordinates of aircraft location on the studied parameters allows us to draw the following conclusions:

Table 1

Estimates of mathematical expectations and variances for different values of the slant range from OES to aircraft

D=1000 m					
A	0°	15°	30°	45°	60°
$\bar{\rho}$, m	0.2274	0.1934	0.1596	0.1388	0.1596
σ_ρ, m	0.0262	0.0471	0.0711	0.0895	0.0711
D=5000 m					
A	0°	15°	30°	45°	60°
$\bar{\rho}$, m	1.1372	0.9668	0.7982	0.6942	0.7981
σ_ρ, m	0.1311	0.2355	0.3554	0.4473	0.3554
D=10000 m					
A	0°	15°	30°	45°	60°
$\bar{\rho}$, m	2.2745	1.9336	1.5964	1.3885	1.5963
σ_ρ, m	0.2621	0.4711	0.7108	0.8947	0.7109
D=15000 m					
A	0°	15°	30°	45°	60°
$\bar{\rho}$, m	3.4117	2.9004	2.3946	2.0827	2.3944
σ_ρ, m	0.3932	0.7066	1.0662	1.3420	1.0663

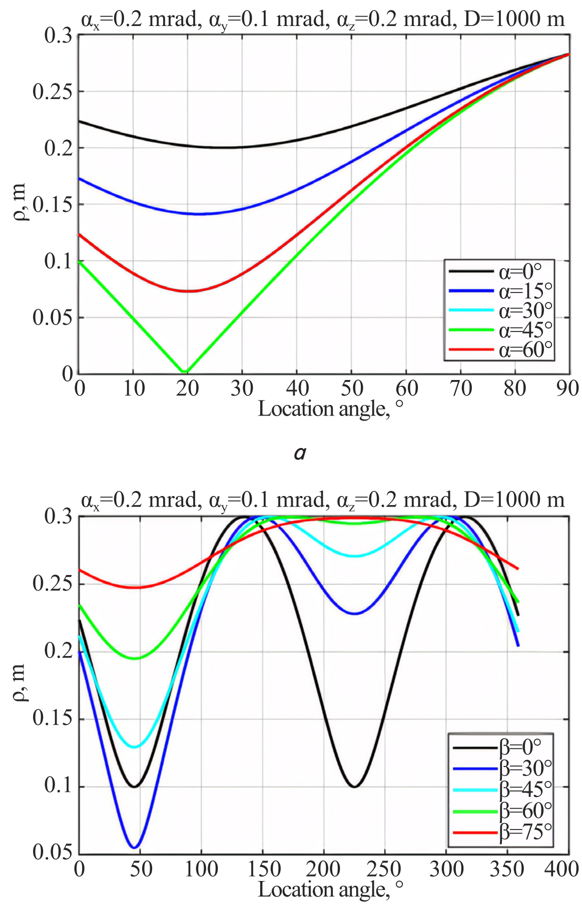


Fig. 2. Dependence of the instrumental error: *a* – on the change in azimuth; *b* – on a change in the location angle

Table 2

Estimates of mathematical expectations and variances for different values of the slant range to aircraft

D=1000 m					
B	0°	30°	45°	60°	75°
$\bar{\rho}$, m	0.2127	0.2289	0.2489	0.2667	0.2787
σ_ρ, m	0.0691	0.0714	0.0552	0.0371	0.0185
D=5000 m					
β	0°	30°	45°	60°	75°
$\bar{\rho}$, m	1.0635	1.1446	1.2445	1.3335	1.3933
σ_ρ, m	0.3453	0.3572	0.2762	0.1854	0.0927
D=10000 m					
β	0°	30°	45°	60°	75°
$\bar{\rho}$, м	2.1270	2.2893	2.4891	2.6670	2.7865
σ_ρ, м	0.6906	0.7144	0.5525	0.3708	0.1854
D=15000 m					
B	0°	30°	45°	60°	75°
$\bar{\rho}$, м	3.1905	3.4339	3.7336	4.0005	4.1798
σ_ρ, м	1.0358	1.0716	0.8287	0.5562	0.2782

1. The dependence of the numerical value of instrumental error $\acute{\rho}(\omega)$ (Systematic error) in determining aircraft coordinates is non-linear on the studied parameters.

2. In order to minimize the impact of instrumental error, the technology for adjusting the OES RSP should be devised and implemented in such a way that the maximum adjustment error does not exceed ±0.2 mrad along each axis.

3. To reduce the dependence of instrumental error on the influence of external factors (environmental temperature), it is necessary to use a gyro-stabilizing OES RSP based on laser gyroscopes.

4. These methods make it possible to reduce the instrumental errors of OES measurement of azimuth and elevation angle, but it is practically impossible to completely eliminate them.

5. An effective way to minimize systematic errors is the use of spatial redundancy of video surveillance of aircraft trajectory at each time point by several spatially separated OES in the ICN of OES [12].

During the analysis of the sources of influence of random errors of direct measurements of azimuth, elevation angle, and slant range on the statistical properties of estimates of parameters of aircraft trajectories, the following assumptions were adopted:

1) all measurements of model parameters yield random errors;

2) the metrological characteristics of measuring instruments are known a priori, that is, the statistical properties of measurement errors are known a priori (the normal distribution law of measurement errors with zero mathematical expectation and known variance);

3) the results of calculating the parameters of aircraft trajectories for each time point are also random variables, the statistical properties of which are determined by the type of nonlinear equations of the mathematical model and the statistical properties of errors in the measured parameters.

The "EPS-Dream-10Hz" receiver is used for direct measurements of the coordinates of OES location in the GDCS and in GCS at each OES of ICN. It is designed for automatic continuous evaluation of current coordinates.

Simultaneously with solving the main navigation task, the receiver "EPS-Dream-10Hz" enables reception and consideration of differential corrections (when working under differential mode (DGPS)) and exchange of information with external systems and devices. The accuracy of determining the coordinates of OES location – root mean square error (RMS): autonomously – 1.5 m; SBAS – 0.9 m; DGPS – 0.5 m.

A 1PPS signal at the output of the "EPS-Dream-10Hz" receiver was used to synchronize the timer of the personal computer and PCM, which ensures the accuracy of synchronization with SCP.

When carrying out tractor measurements by the method of open observation, for each time point t, OES measures three parameters: the azimuth $\alpha_t(\tilde{\omega})$, of elevation angle $\beta_t(\tilde{\omega})$ and the slant range $D_t(\tilde{\omega})$ from OES to aircraft. The vector of measured parameters $\left(\alpha_t(\tilde{\omega}),\beta_t(\tilde{\omega}),D_t(\tilde{\omega})\right)^T$ determines aircraft location in ISCS at the time point t_k. In this case, the components of the vector of estimates of the coordinates of aircraft location $\left(\acute{\bar{x}}_{it_k}(\omega),\acute{\bar{y}}_{it_k}(\omega),\acute{\bar{z}}_{it_k}(\omega)\right)^T$ in the ICCS of OES are determined by the following expressions [18, 19]:

$$\acute{\bar{x}}_{it_k}(\omega)=\acute{\bar{D}}_{it_k}(\omega)\cos\acute{\bar{\beta}}_{it_k}(\omega)\cos\acute{\bar{\alpha}}_{it_k}(\omega), \quad (6)$$

$$\acute{\bar{y}}_{it_k}(\omega)=\acute{\bar{D}}_{it_k}(\omega)\cos\acute{\bar{\beta}}_{it_k}(\omega), \quad (7)$$

$$\acute{\bar{z}}_{it_k}(\omega)=\acute{\bar{D}}_{it_k}(\omega)\cos\acute{\bar{\beta}}_{it_k}(\omega)\sin\acute{\bar{\alpha}}_{it_k}(\omega). \quad (8)$$

We shall consider vector $\left(\acute{\bar{x}}_{it_k}(\omega),\acute{\bar{y}}_{it_k}(\omega),\acute{\bar{z}}_{it_k}(\omega)\right)^T$ as estimates of the mathematical expectations for the coordinates of aircraft location in the ICCS of the i-th OES at the time point t_k. The model for recalculating the vector of coordinates of aircraft location from ICCS – t_k-th OES into the vector of estimates of the coordinates $\left(x^*_{it_k}(\omega),y^*_{it_k}(\omega),z^*_{it_k}(\omega)\right)^T$ of aircraft location in GCS takes the form of an affine transformation [18, 19]:

$$\left\|\begin{matrix}x^*_{it_k}(\omega)\\ y^*_{it_k}(\omega)\\ z^*_{it_k}(\omega)\end{matrix}\right\|=\boldsymbol{A_{iG}}(\boldsymbol{\omega})\cdot\left\|\begin{matrix}\acute{\bar{x}}_{it_k}(\omega)\\ \acute{\bar{y}}_{it_k}(\omega)\\ \acute{\bar{z}}_{it_k}(\omega)\end{matrix}\right\|+\left\|\begin{matrix}\acute{\bar{x}}_{0Gi}(\omega)\\ \acute{\bar{y}}_{0Gi}(\omega)\\ \acute{\bar{z}}_{0Gi}(\omega)\end{matrix}\right\|. \quad (9)$$

The transformation matrix $A_{iG}(\omega)$ takes the following form:

$$\boldsymbol{A_{iG}}(\boldsymbol{\omega})=\left\|\begin{matrix}-\cos L_i(\omega)\cdot\sin B_i(\omega) & \cos L_i(\omega)\cdot\cos B_i(\omega) & -\sin L_i(\omega)\\ -\sin L_i(\omega)\cdot\sin B_i(\omega) & \sin L_i(\omega)\cdot\cos B_i(\omega) & \cos L_i(\omega)\\ \cos B_i(\omega) & \sin B_i(\omega) & 0\end{matrix}\right\|, \quad (10)$$

where $(B_i(\omega),L_i(\omega),H_i(\omega))^T$ is the vector of coordinates of the location of the 1st OES in WGS-84; $\left(\acute{\bar{x}}_{0Gi}(\omega),\acute{\bar{y}}_{0Gi}(\omega),\acute{\bar{z}}_{0Gi}(\omega)\right)^T$ is a vector of estimates of the mathematical expectations of the coordinates of the location of the 1st OES in GCS. Since the aircraft at the time point t_k is at different distances from each i-th OES, the estimates $\left(x^*_{it_k}(\omega),y^*_{it_k}(\omega),z^*_{it_k}(\omega)\right)^T$, i=1, 2, ..., n, must be considered as the results of indirect non-equipoint measurements. In this case, estimates of the mathematical expectation for the coordinates of aircraft location in GCS at the time point t_k are equal to [17, 18]:

$$\acute{\bar{X}}_{Gt_k}(\omega)=\sum_{i=1}^{n}W_{xi}x^*_{it_k}(\omega)\Big/\sum_{i=1}^{n}W_{xi}, \quad (11)$$

$$\acute{\bar{Y}}_{Gt_k}(\omega)=\sum_{i=1}^{n}W_{yi}y^*_{it_k}(\omega)\Big/\sum_{i=1}^{n}W_{yi}, \quad (12)$$

$$\acute{\bar{Z}}_{Gt_k}(\omega)=\sum_{i=1}^{n}W_{zi}z^*_{it_k}(\omega)\Big/\sum_{i=1}^{n}W_{zi}, \quad (13)$$

where $W_{xi}=1\big/\sigma^2_{iX_Gt_k}$, $W_{yi}=1\big/\sigma^2_{iY_Gt_k}$, $W_{zi}=1\big/\sigma^2_{iZ_Gt_k}$, i=1,2,...,n.

Estimates of variances in the mathematical expectation for the coordinates of HDAV location in GCS at the time point t_k are equal to:

$$\sigma^2_{iX_Gt_k}=1\Big/\sum_{i=1}^{n}W_{Xit_k},\quad \sigma^2_{iY_Gt_k}=1\Big/\sum_{i=1}^{n}W_{Yit_k},$$

$$\sigma^2_{iZ_Gt_k}=1\Big/\sum_{i=1}^{n}W_{Zit_k}. \quad (14)$$

Estimates of the mathematical expectations for the coordinates of aircraft location in GCS, calculated according to (11) to (13), take into account the spatial redundancy of the measurements of coordinates of aircraft location in the ICS of OES. They are less biased and more efficient compared to the estimates obtained from each i-th OES.

When carrying out trajectory measurements by the method of covert observation of aircraft location, for each time point t of each i-th OES, two parameters are measured: the azimuth $\alpha_{it}(\tilde{\omega})$ and the angle of the site $\beta_{it}(\tilde{\omega})$. In this case, to determine the coordinates of aircraft location, additional information about the results of the measurement of the azimuth $\alpha_{jt}(\tilde{\omega})$ and the place angle $\beta_{jt}(\tilde{\omega})$ is required at the same aircraft at the same time point t from other j-th OESs in ICS j=1,2,...,n.

The calculation of estimates of the directional coefficients of the line of sight from OES to the aircraft at the time point t_k is performed sequentially for each OES, the number of which is in the list I={i, j, ..., l} of OESs in PCM, which conduct video surveillance of this aircraft at the time t_k. Estimates of the mathematical expectations for the directional coefficients $\acute{\bar{\boldsymbol{l}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega})$, $\acute{\bar{\boldsymbol{m}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega})$, $\acute{\bar{\boldsymbol{n}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega})$ of the line of sight from the i-th OES on aircraft at the time point t_k in ICCS are calculated as follows:

$$\acute{\bar{\boldsymbol{l}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}) = \cos\acute{\bar{\beta}}_{it_k}(\omega)\cos\acute{\bar{\alpha}}_{it_k}(\omega), \tag{15}$$

$$\acute{\bar{\boldsymbol{m}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}) = \sin\acute{\bar{\beta}}_{it_k}(\omega), \tag{16}$$

$$\acute{\bar{\boldsymbol{n}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}) = \cos\acute{\bar{\beta}}_{it_k}(\omega)\sin\acute{\bar{\alpha}}_{it_k}(\omega), \tag{17}$$

where $\acute{\bar{\alpha}}_{it_k}(\omega)$, $\acute{\bar{\beta}}_{it_k}(\omega)$ are estimates of the mathematical expectations of the azimuth and elevation angle from the i-th OES on aircraft in ISCS. The vector $\left(\acute{\bar{l}}_{it_k}(\omega), \acute{\bar{m}}_{it_k}(\omega), \acute{\bar{n}}_{it_k}(\omega)\right)^T$ of mathematical expectations for the directional coefficients of the line of sight from the i-th OES on aircraft at the time point t_k in GCS is equal to:

$$\begin{Vmatrix} \acute{\bar{l}}_{it_k}(\omega) \\ \acute{\bar{m}}_{it_k}(\omega) \\ \acute{\bar{n}}_{it_k}(\omega) \end{Vmatrix} = \boldsymbol{A_{iG}} \cdot \begin{Vmatrix} \acute{\bar{\boldsymbol{l}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}) \\ \acute{\bar{\boldsymbol{m}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}) \\ \acute{\bar{\boldsymbol{n}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}) \end{Vmatrix}, \tag{18}$$

where $\left(\acute{\bar{\boldsymbol{l}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}), \acute{\bar{\boldsymbol{m}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega}), \acute{\bar{\boldsymbol{n}}}_{\boldsymbol{it_k}}(\boldsymbol{\omega})\right)^T$ is the vector of estimates of mathematical expectations for the directional coefficients of the line of sight from the i-th OES on aircraft at the time point t_k in ICCS; $\boldsymbol{A_{iG}}$ is the matrix of transformation of ICCS of the i-th OES into the GCS of the form (10). Then, the equations of the sighting lines of the i-th OES on aircraft at the time point t_k in ICCS take the following form:

$$\frac{x_{t_k}(\omega) - \acute{\bar{x}}_{0Gi}(\omega)}{\acute{\bar{l}}_{it_k}(\omega)} = \frac{y_{t_k}(\omega) - \acute{\bar{y}}_{0Gi}(\omega)}{\acute{\bar{m}}_{it_k}(\omega)} = \frac{z_{t_k}(\omega) - \acute{\bar{z}}_{0Gi}(\omega)}{\acute{\bar{n}}_{it_k}(\omega)}, \tag{19}$$

where $\left(\acute{\bar{x}}_{0Gi}(\omega), \acute{\bar{y}}_{0Gi}(\omega), \acute{\bar{z}}_{0Gi}(\omega)\right)^T$ is the vector of estimates of mathematical expectations for the coordinates of location of the ith OES in GCS.

From (19), we derive a system of two linear equations, which specifies the position of the sighting lines from each i-th OES on aircraft in GCS at the time point t_k:

$$\begin{cases} \acute{\bar{m}}_{it_k}(\omega)\left(x_{t_k}(\omega) - \acute{\bar{x}}_{0Gi}(\omega)\right) = \acute{\bar{l}}_{it_k}(\omega)\left(y_{t_k}(\omega) - \acute{\bar{y}}_{0Gi}(\omega)\right); \\ \acute{\bar{n}}_{it_k}(\omega)\left(y_{t_k}(\omega) - \acute{\bar{y}}_{0Gi}(\omega)\right) = \acute{\bar{m}}_{it_k}(\omega)\left(z_{t_k}(\omega) - \acute{\bar{z}}_{0Gi}(\omega)\right), \end{cases} \quad i=1, 2..., n. \tag{20}$$

The system of line of sight equations (20) is a redefined LAU system because it contains 2*n equations and three unknown variables. To represent the LAU subsystem (20) in a standard form at the time point t_k, the following notations were introduced:

$$\bar{m}_i = \acute{\bar{m}}_{it_k}(\omega); \ \bar{n}_i = \acute{\bar{n}}_{it_k}(\omega); \ \bar{l}_i = \acute{\bar{l}}_{it_k}(\omega);$$

$$x = x_{t_k}(\omega); \ \bar{x}_i = \acute{\bar{x}}_{0Gi}(\omega);$$

$$y = y_{t_k}(\omega); \ \bar{y}_i = \acute{\bar{y}}_{0Gi}(\omega);$$

$$z = z_{t_k}(\omega); \ \bar{z}_i = \acute{\bar{z}}_{0Gi}(\omega).$$

In this case, the LAU system (20) takes the form:

$$\bar{m}_i(x - \bar{x}_i) = \bar{l}_i(y - \bar{y}_i),$$

$$\bar{n}_i(y - \bar{y}_i) = \bar{m}_i(z - \bar{z}_i), \ i=1, 2, ..., n. \tag{21}$$

After transforming system (21) to the standard form, the following is obtained:

$$\bar{A}x = \bar{B}, \tag{22}$$

where $(x, y, z)^T$ is the vector of the estimated coordinates of aircraft location in GCS at the time point t_k; matrix $\bar{A}$ and vector $\bar{B}$ of free coefficients take the following form:

$$\bar{A} = \begin{bmatrix} \bar{m}_1 & -\bar{l}_1 & 0 \\ 0 & \bar{n}_1 & -\bar{m}_1 \\ ... & ... & ... \\ ... & ... & ... \\ \bar{m}_i & -\bar{l}_i & 0 \\ 0 & \bar{n}_i & -\bar{m}_i \\ ... & ... & ... \\ ... & ... & ... \\ \bar{m}_n & -\bar{l}_n & 0 \\ 0 & \bar{n}_n & -\bar{m}_n \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \bar{m}_1\bar{x}_1 & -\bar{l}_1\bar{y}_1 \\ \bar{n}_1\bar{y}_1 & -\bar{m}_1\bar{z}_1 \\ ... & ... \\ ... & ... \\ \bar{m}_i\bar{x}_i & -\bar{l}_i\bar{y}_i \\ \bar{m}_i\bar{y}_i & -\bar{m}_i\bar{z}_1 \\ ... & ... \\ ... & ... \\ \bar{m}_n\bar{x}_n & -\bar{l}_n\bar{y}_n \\ \bar{m}_n\bar{y}_n & -\bar{m}_n\bar{z}_n \end{bmatrix}. \tag{23}$$

All equations in subsystem (23) are linearly independent, such a system is overdetermined, and instead of an exact solution, a vector that minimizes the norm of the incoherence of the system of equations should be sought. The vector $(x^*, y^*, z^*)^T$ of the solution to LAU subsystem (23) determines the estimates of the mathematical expectation for the coordinates of aircraft location at the time point t_k. The solution to subsystem (23) is derived by standard LSM [9], which would make it possible to calculate the global minimum of the norm of the discrepancy $\left\|\bar{A} \cdot x - \bar{B}\right\|$.

5. 2. Deriving analytical dependences for calculating the variance in aircraft positioning errors

Mathematical models of parameters of aircraft trajectories in the considered coordinate systems are nonlinear functions of random arguments. This leads to the fact that nonlinear transformations of random variables can lead not only to a change in the parameters of the function of random arguments but also to a change in the form of its distribution function. Therefore, a priori determination of the statistical properties of the dependent variables of the nonlinear stochastic model from the statistical properties of the independent variables is possible only approximately using the statistical linearization method.

Estimates of the mathematical expectations for the components of the coordinate vector of aircraft location in the ICCS of OES can be found based on the results of direct measured values of the components of the vector of coordinates of aircraft in the ISCS of OES. The frequency of video surveillance cameras is usually 30 frames per second, and the frequency of measuring (estimating) coordinates is once per second, i.e., there is temporal redundancy in video measurement. This allows viewing video measurements as mathematical expectation estimates obtained by averaging every 30 video observations received per second. To simplify the record, we omit the index t; estimates of mathematical expectations for random variables will be denoted by: $\acute{\bar{\alpha}}_t(\omega)=\alpha(\omega)$, $\acute{\bar{\beta}}_t(\omega)=\beta(\omega)$, $\bar{D}_t(\omega)=D(\omega)$, $\bar{X}_t(\omega)=X_M$, $\bar{Y}_t(\omega)=Y_M$, $\bar{Z}_t(\omega)=Z_M$. In this case, the vectors $(\alpha(\omega),\beta(\omega), D(\omega))^T$ and $(X_M,Y_M,Z_M)^T$ determine the estimates of the mathematical expectation for the components of aircraft location vectors in ISCS and ICCS. The relationship between the components of these vectors is determined by dependences of the form (6) to (8), which take the form:

$$X_M(\omega)=D(\omega)\cdot\cos\alpha(\omega)\cdot\cos\beta(\omega), \tag{24}$$

$$Y_M(\omega)=D(\omega)\cdot\sin\beta(\omega), \tag{25}$$

$$Z_M(\omega)=D(\omega)\cdot\sin\alpha(\omega)\cdot\sin\beta(\omega). \tag{26}$$

Each of the expressions (24) to (26) will be considered as a nonlinear function of random arguments $(\alpha(\omega), \beta(\omega), D(\omega))^T$. Assuming that $(\alpha(\omega), \beta(\omega), D(\omega))^T$ are not correlated with each other, the variance estimates $\sigma^2_{X_M}$, $\sigma^2_{Y_M}$, $\sigma^2_{Z_M}$ are calculated according to the expression:

$$\sigma^2_{X_M}=\left(\frac{\partial X_M}{\partial D}\right)^2\Bigg|_m\cdot\sigma^2_D+\left(\frac{\partial X_M}{\partial \alpha}\right)^2\Bigg|_m\cdot\sigma^2_\alpha+\left(\frac{\partial X_M}{\partial \beta}\right)^2\Bigg|_m\cdot\sigma^2_\beta, \tag{27}$$

$$\sigma^2_{Y_M}=\left(\frac{\partial Y_M}{\partial D}\right)^2\Bigg|_m\cdot\sigma^2_D+\left(\frac{\partial Y_M}{\partial \alpha}\right)^2\Bigg|_m\cdot\sigma^2_\alpha+\left(\frac{\partial Y_M}{\partial \beta}\right)^2\Bigg|_m\cdot\sigma^2_\beta, \tag{28}$$

$$\sigma^2_{Z_M}=\left(\frac{\partial Z_M}{\partial D}\right)^2\Bigg|_m\cdot\sigma^2_D+\left(\frac{\partial Z_M}{\partial \alpha}\right)^2\Bigg|_m\cdot\sigma^2_\alpha+\left(\frac{\partial Z_M}{\partial \beta}\right)^2\Bigg|_m\cdot\sigma^2_\beta. \tag{29}$$

Partial derivatives included in expressions for $\sigma^2_{X_M}$, $\sigma^2_{Y_M}$, $\sigma^2_{Z_M}$:

$$\frac{\partial X_M}{\partial D}=\cos\alpha(\omega)\cos\beta(\omega),$$

$$\frac{\partial X_M}{\partial \alpha}=-D(\omega)\sin\alpha(\omega)\cos\beta(\omega),$$

$$\frac{\partial X_M}{\partial \beta}=-D\cos\alpha(\omega)\sin\beta(\omega),$$

$$\frac{\partial Y_M}{\partial D}=\sin\beta(\omega),\quad \frac{\partial Y_M}{\partial \alpha}=0,$$

$$\frac{\partial Y_M}{\partial \beta}=D(\omega)\cos\beta(\omega),$$

$$\frac{\partial Z_M}{\partial D}=\sin\alpha(\omega)\cos\beta(\omega),$$

$$\frac{\partial Z_M}{\partial \alpha}=D(\omega)\cos\alpha(\omega)\cos\beta(\omega),$$

$$\frac{\partial Z_M}{\partial \beta}=-D(\omega)\sin\alpha(\omega)\sin\beta(\omega). \tag{30}$$

To calculate the estimates of the mathematical expectations for coordinates $(X_M,Y_M,Z_M)^T$ of aircraft location in GCS based on the estimates of the coordinates of its location in ICCS $(X_M,Y_M,Z_M)^T$ and the estimates of coordinates of OES location in GCS $(X_{0Г},Y_{0Г},Z_{0Г})^T$, we use the expression:

$$\begin{Vmatrix}X_G\\Y_G\\Z_G\end{Vmatrix}=A_{GM}\cdot\begin{Vmatrix}X_M\\Y_M\\Z_M\end{Vmatrix}+\begin{Vmatrix}X_{0G}\\Y_{0G}\\Z_{0G}\end{Vmatrix}. \tag{31}$$

The transition matrix from ICCS to GCS takes the following form [19]:

$$A_{GM}(\omega)=$$
$$=\begin{Vmatrix}-\sin L\cdot\sin A_m-\cos L\cdot\sin B\cdot\cos A_m & \cos L\cdot\cos B & -\sin L\cdot\cos A_m+\cos L\cdot\sin B\cdot\sin A_m\\ \cos L\cdot\sin A_m-\sin L\cdot\sin B\cdot\cos A_m & \sin L\cdot\cos B & \cos L\cdot\cos A_m+\sin L\cdot\sin B\cdot\sin A_m\\ \cos B\cdot\cos A_m & \sin B & -\cos B\cdot\sin A_m\end{Vmatrix}, \tag{32}$$

where $(B_i(\omega),L_i(\omega), H_i(\omega))^T$ is the vector of coordinates of the location of the i-th OES in WGS-84; $\left(\acute{\bar{x}}_{0Gi}(\omega),\acute{\bar{y}}_{0Gi}(\omega),\acute{\bar{z}}_{0Gi}(\omega)\right)^T$ – vector of estimates of mathe matical expectations for the coordinates of OES location in GCS; A_m is an estimate of the mathematical expectation for an error in linking the direction of the X axis of OES ICCS to the direction of the local meridian. We ignore horizontalization errors. All elements of matrix (32) are random variables. Thus, estimates of the mathematical expectations for the components of aircraft coordinate vector in GCS take the following form:

$$\begin{aligned}X_Г=&-X_M\cdot(\sin L\cdot\sin A_m+\cos L\cdot\sin B\cdot\cos A_m)+\\&+Y_M\cdot\cos L\cdot\cos B+\\&+Z_M\cdot\begin{pmatrix}-\sin L\cdot\cos A_m+\\+\cos L\cdot\sin B\cdot\sin A_m\end{pmatrix}+X_0,\end{aligned} \tag{33}$$

$$\begin{aligned}Y_Г=&-X_M\cdot(\cos L\cdot\sin A_m-\sin L\cdot\sin B\cdot\cos A_m)+\\&+Y_M\cdot\sin L\cdot\cos B+\\&+Z_M\cdot\begin{pmatrix}\cos L\cdot\cos A_m+\\+\sin L\cdot\sin B\cdot\sin A_m\end{pmatrix}+Y_0,\end{aligned} \tag{34}$$

$$\begin{aligned}Z_Г=&X_M\cdot\cos B\cdot\cos A_m+\\&+Y_M\cdot\sin B-Z_M\cdot\cos B\cdot\sin A_m+Z_0.\end{aligned} \tag{35}$$

Estimates of variance in the components of mathematical expectations for the aircraft coordinate vector in GCS take the following form:

$$\sigma^2_{X_G}=\left(\frac{\partial X_G}{\partial X_M}\right)^2\bigg|_m\cdot\sigma^2_{X_M}+\left(\frac{\partial X_G}{\partial Y_M}\right)^2\bigg|_m\cdot\sigma^2_{Y_M}+$$
$$+\left(\frac{\partial X_G}{\partial Z_M}\right)^2\bigg|_m\cdot\sigma^2_{Z_M}+\left(\frac{\partial X_G}{\partial A_m}\right)^2\bigg|_m\cdot\sigma^2_{A_m}+\left(\frac{\partial X_G}{\partial L}\right)^2\bigg|_m\cdot\sigma^2_L+$$
$$\left(\frac{\partial X_G}{\partial B}\right)^2\bigg|_m\cdot\sigma^2_B+\left(\frac{\partial X_G}{\partial X_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{X_{0G}}+$$
$$+\left(\frac{\partial X_G}{\partial Y_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{Y_{0G}}+\left(\frac{\partial X_G}{\partial Z_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{Z_{0G}}, \qquad (36)$$

$$\sigma^2_{Y_G}=\left(\frac{\partial Y_G}{\partial X_M}\right)^2\bigg|_m\cdot\sigma^2_{X_M}+\left(\frac{\partial Y_G}{\partial Y_M}\right)^2\bigg|_m\cdot\sigma^2_{Y_M}+$$
$$+\left(\frac{\partial Y_G}{\partial Z_M}\right)^2\bigg|_m\cdot\sigma^2_{Z_M}+\left(\frac{\partial Y_G}{\partial A_m}\right)^2\bigg|_m\cdot\sigma^2_{A_m}+\left(\frac{\partial Y_G}{\partial L}\right)^2\bigg|_m\cdot\sigma^2_L+$$
$$\left(\frac{\partial Y_G}{\partial B}\right)^2\bigg|_m\cdot\sigma^2_B+\left(\frac{\partial Y_G}{\partial X_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{X_{0G}}+$$
$$+\left(\frac{\partial Y_G}{\partial Y_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{Y_{0G}}+\left(\frac{\partial Y_G}{\partial Z_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{Z_{0G}}, \qquad (37)$$

$$\sigma^2_{Z_G}=\left(\frac{\partial Z_G}{\partial X_M}\right)^2\bigg|_m\cdot\sigma^2_{X_M}+\left(\frac{\partial Z_G}{\partial Y_M}\right)^2\bigg|_m\cdot\sigma^2_{Y_M}+$$
$$+\left(\frac{\partial Z_G}{\partial Z_M}\right)^2\bigg|_m\cdot\sigma^2_{Z_M}+\left(\frac{\partial Z_G}{\partial A_m}\right)^2\bigg|_m\cdot\sigma^2_{A_m}+\left(\frac{\partial Z_G}{\partial L}\right)^2\bigg|_m\cdot\sigma^2_L+$$
$$\left(\frac{\partial Z_G}{\partial B}\right)^2\bigg|_m\cdot\sigma^2_B+\left(\frac{\partial Z_G}{\partial X_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{X_{0G}}+$$
$$+\left(\frac{\partial Z_G}{\partial Y_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{Y_{0G}}+\left(\frac{\partial Z_G}{\partial Z_{0G}}\right)^2\bigg|_m\cdot\sigma^2_{Z_{0G}}. \qquad (38)$$

The partial derivatives included in expressions for $\sigma^2_{X_\Gamma}$, $\sigma^2_{Y_\Gamma}$, $\sigma^2_{Z_\Gamma}$ take the following form:

$$\frac{\partial X_G}{\partial X_M}=-\sin L\cdot\sin A_m-\cos L\cdot\sin B\cdot\cos A_m,$$

$$\frac{\partial X_G}{\partial Y_M}=\cos L\cdot\sin B,$$

$$\frac{\partial X_G}{\partial Z_M}=-\sin L\cdot\cos A_m+\cos L\cdot\sin B\cdot\sin A_m,$$

$$\frac{\partial X_G}{\partial A_m}=-X_M\left(\sin L\cdot\cos A_m-\cos L\cdot\sin B\cdot\sin A_m\right)+$$
$$+Z_M\left(\sin L\cdot\sin A_m+\cos L\cdot\sin B\cdot\cos A_m\right),$$

$$\frac{\partial X_G}{\partial L}=-X_M\left(\cos L\cdot\sin A_m-\sin L\cdot\sin B\cdot\cos A_m\right)-$$
$$-Y_M\cdot\sin L\cdot\cos B+Z_M\left(-\cos L\cdot\cos A_m-\sin L\cdot\sin B\cdot\sin A_m\right),$$

$$\frac{\partial X_G}{\partial B}=-X_M\cdot\cos L\cdot\cos B\cdot\cos A_m-Y_M\cdot\cos L\cdot\sin B+$$
$$+Z_M\cdot\cos L\cdot\cos B\cdot\sin A_m,$$

$$\frac{\partial X_G}{\partial X_{0G}}=1,\quad \frac{\partial X_G}{\partial Y_{0G}}=0,\quad \frac{\partial X_G}{\partial Z_{0G}}=0,$$

$$\frac{\partial Y_G}{\partial X_M}=\cos L\cdot\sin A_m-\sin L\cdot\sin B\cdot\cos A_m,$$

$$\frac{\partial Y_G}{\partial Y_M}=\sin L\cdot\cos B,$$

$$\frac{\partial Y_G}{\partial Z_M}=\cos L\cdot\cos A_m+\sin L\cdot\sin B\cdot\sin A_m,$$

$$\frac{\partial Y_G}{\partial A_m}=X_M\left(\cos L\cdot\cos A_m+\sin L\cdot\sin B\cdot\sin A_m\right)-$$
$$-Z_M\left(\cos L\cdot\sin A_m-\sin L\cdot\sin B\cdot\cos A_m\right),$$

$$\frac{\partial Y_G}{\partial L}=-X_M\left(\sin L\cdot\sin A_m+\cos L\cdot\sin B\cdot\cos A_m\right)+$$
$$+Y_M\cdot\cos L\cdot\cos B-Z_M\left(\sin L\cdot\cos A_m-\cos L\cdot\sin B\cdot\sin A_m\right),$$

$$\frac{\partial Y_G}{\partial B}=-X_M\cdot\sin L\cdot\cos B\cdot\cos A_m-Y_M\cdot\sin L\cdot\sin B+$$
$$+Z_M\cdot\sin L\cdot\cos B\cdot\sin A_m,$$

$$\frac{\partial Y_G}{\partial X_{0G}}=0,\quad \frac{\partial Y_G}{\partial Y_{0G}}=1,\quad \frac{\partial Y_G}{\partial Z_{0G}}=0,$$

$$\frac{\partial Z_G}{\partial X_M}=\cos B\cdot\cos A_m,\quad \frac{\partial Z_G}{\partial Y_M}=\sin B,\quad \frac{\partial Z_G}{\partial Z_M}=-\cos B\cdot\sin A_m,$$

$$\frac{\partial Z_G}{\partial A_m}=-X_M\cdot\cos B\cdot\sin A_m-Z_M\cdot\cos B\cdot\cos A_m,$$

$$\frac{\partial Z_G}{\partial L}=0,$$

$$\frac{\partial Z_G}{\partial B}=-X_M\cdot\sin B\cdot\cos A_m+Y_M\cdot\cos B+$$
$$+Z_M\cdot\sin B\cdot\sin A_m,$$

$$\frac{\partial Z_G}{\partial X_{0G}}=0,\quad \frac{\partial Z_G}{\partial Y_{0G}}=0,\quad \frac{\partial Z_G}{\partial Z_{0G}}=1.$$

Expressions (36) to (38), taking into account the corresponding time derivatives, are analytical dependences of variances in the errors of aircraft location for each of the coordinates in GCS.

Let us construct the confidence limits of scattering ellipsoid, in which, with a given probability, the aircraft is located in ICCS i of OES at the time point t_k.

The resulting interval estimates of variance in the aircraft coordinates in ICCS of i^{th} OES at the time point t_k,, which allows us to construct the confidence limits of scattering ellipsoid, in which, with a given probability, the aircraft is located. The equation of the scattering ellipsoid centered at a point $\left(\acute{\bar{x}}_{it_k}(\omega),\acute{\bar{y}}_{it_k}(\omega),\acute{\bar{z}}_{it_k}(\omega)\right)^T=\left(X_M,Y_M,Z_M\right)^T$ takes the following form:

$$\frac{\left(X-X_M\right)^2}{a^2}+\frac{\left(Y-Y_M\right)^2}{b^2}+\frac{\left(Z-Z_M\right)^2}{c^2}=1, \tag{39}$$

where $(X_M, Y_M, Z_M)^T$ is the vector of estimates of the mathematical expectation for the coordinates of aircraft location in the ICCS of i^{th} OES at the time point t_k; a, b, c are the semi-axes of the ellipsoid.

The dimensions of semi-axes of the ellipsoid are determined by the probability with which the actual aircraft coordinates will be in the considered interval in relation to the mathematical expectation along each of the axes.

Dimensions of the confidence limits of the ellipsoid are given in Table 3.

Table 3

Dimensions of confidence limits of the scattering ellipsoid

Probability P	a	b	c
0,8664	$1.5\sigma_{X_M}$	$1.5\sigma_{Z_M}$	$1.5\sigma_{Z_M}$
0,9544	$2\sigma_{X_M}$	$2\sigma_{Z_M}$	$2\sigma_{Z_M}$
0,0876	$2.5\sigma_{X_M}$	$2.5\sigma_{Z_M}$	$2.5\sigma_{Z_M}$
0,9974	$3\sigma_{X_M}$	$3\sigma_{Z_M}$	$3\sigma_{Z_M}$

Fig. 3 shows the scattering ellipsoid of aircraft location in the ICCS of OES at P=0.9974 and the dimensions of the semi-axes of the aircraft $a=3\sigma_{X_M}$, $b=3\sigma_{Y_M}$, $c=3\sigma_{Z_M}$.

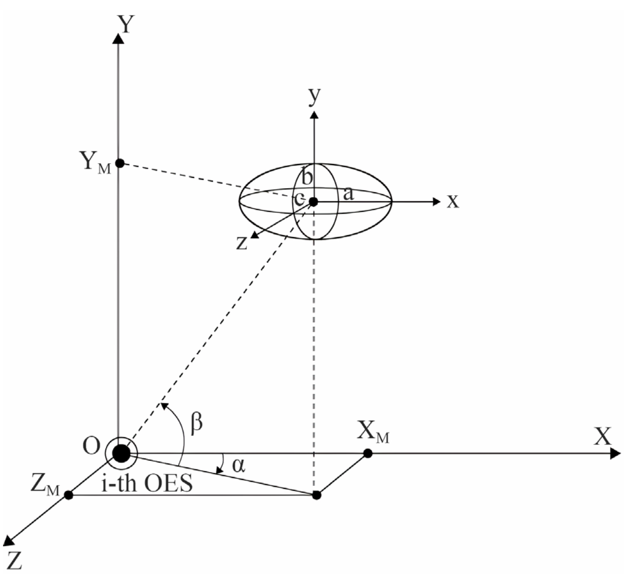


Fig. 3. Scattering ellipsoid of aircraft location in the instrumental Cartesian coordinate system of the optical-electronic station

Thus, the confidence limits of the scattering ellipsoid, in which, with a given probability, the aircraft is located in the ICCS of the i-th OES at the time point t_k are constructed.

5. 3. Investigating the derived dependences by a numerical modeling method

The results of the analytical dependence of variances in the estimates of components of the vector of estimates of the mathematical expectation for the coordinates of aircraft location in the ICCS of the i-th OES and in GCS at the time t_k were verified by a numerical method. The study of the dependence of estimates of variances (SQV) of aircraft location in GCS on the estimates of mathematical expectations and variances of all measured values was carried out using a numerical method.

As input, we used:

– numerical values of mathematical expectations for the coordinates of OES location in GCS;

– sets of results from direct measurements of azimuth, elevation angle, and slant range;

– SQV values of random measurement errors (30 words)

Based on the results of direct measurements using the rover RTK|GHCC complex of the Dream-RTK receiver, we obtain numerical values of mathematical expectations for the coordinates of OES location vector in GCS and their variances. The numerical value of mathematical expectations for the coordinates of OES location vector in GCS were equal to:

$$\acute{\bar{L}}(\omega)=36.005725000000005\cdot 2\cdot\pi/360 \text{ rad};$$

$$\acute{\bar{B}}(\omega)=49.865086998432581\cdot 2\cdot\pi/360 \text{ rad};$$

$$\acute{\bar{H}}(\omega)=1.960063724255 \text{ m}; \ \acute{\bar{A}}_m(\omega)=0 \text{ rad}.$$

The corresponding numerical values of the mathematical expectations for coordinates:

$$\acute{\bar{X}}_{0G}(\omega)=3332380.619421877 \text{ m};$$

$$\acute{\bar{Y}}_{0G}(\omega)=2421625.012094292 \text{ m};$$

$$\acute{\bar{Z}}_{0G}(\omega)=4853129.795749833 \text{ m}.$$

The values of real metrological characteristics of random errors in OES measurements, reported in work [4], were used as initial data for calculating variance estimates for the coordinates of aircraft location:

$$\bar{\sigma}_\alpha=(0.2e-3)/3 \text{ rad}; \ \bar{\sigma}_\beta=(0.2e-3)/3 \text{ rad};$$

$$\bar{\sigma}_D=0.2 \text{ м}; \ \bar{\sigma}_L=0.000001\cdot 2\cdot\pi/360 \text{ rad};$$

$$\bar{\sigma}_B=0.000001\cdot 2\cdot\pi/360 \text{ rad}; \ \bar{\sigma}_H=0.1 \text{ m};$$

$$\bar{\sigma}_\rho=(0.2e-3)/3 \text{ rad}; \ \bar{\sigma}_{A_m}=\sqrt{\bar{\sigma}_L^2+\bar{\sigma}_\rho^2} \text{ rad};$$

$$\bar{\sigma}_{X_{0G}}=0.1 \text{ m}; \ \bar{\sigma}_{Y_{0G}}=0.1 \text{ m}; \ \bar{\sigma}_{Z_{0G}}=0.1 \text{ m}.$$

The procedure for processing the source data:

– for known values of variances in random measurement errors $\left(\sigma_L^2,\sigma_B^2,\sigma_H^2\right)^T$, estimates of variances $\left(\sigma_{X_{0G}}^2,\sigma_{Y_{0G}}^2,\sigma_{Z_{0G}}^2\right)^T$ in the coordinates of OES location in GCS were calculated;

– in accordance with (11) to (13), the calculation of estimates of the mathematical expectations for the coordinates of aircraft location in the ICCS of OES was performed, and as a result of solving the LAU system in GCS – for different values of azimuth and elevation angle;

– in accordance with (27) to (29), the SQV calculation of the coordinates of aircraft location was performed in the ICCS of OES, and, in accordance with (33) to (35), in GCS for different values of azimuth and elevation angle.

The results of calculations of the estimates of mathematical expectations and SQV coordinates of aircraft location in the ICCS OES and in GCS are given in Tables 4–6 for different values of the mathematical expectations for the azimuth $\bar{\alpha}=m_{\alpha}$, the location angle $\bar{\beta}=m_{\beta}$, and the slant range:

$$\bar{D}=m_D,\ \acute{\bar{X}}_M(\omega)=m_{X_M},\ \acute{\bar{Y}}_M(\omega)=m_{Y_M},\ \acute{\bar{Z}}_M(\omega)=m_{Z_M},$$

$$\acute{\sigma}_{X_M}(\omega)=\sigma_{X_M},\ \acute{\sigma}_{Y_M}(\omega)=\sigma_{Y_M},\acute{\sigma}_{Z_M}(\omega)=\sigma_{Z_M},$$

$$\acute{\bar{X}}_G(\omega)=m_{X_G},\ \acute{\bar{Y}}_G(\omega)=m_{Y_G},\ \acute{\bar{Z}}_G(\omega)=m_{Z_G},$$

$$\acute{\sigma}_{X_G}(\omega)=\sigma_{X_G},\acute{\sigma}_{Y_G}(\omega)=\sigma_{Y_G},\acute{\sigma}_{Z_G}(\omega)=\sigma_{Z_G}.$$

Table 4

Results of calculating the estimates of mathematical expectations and SQV in the coordinates of aircraft location in the ICCS of OES and in GCS for $m_{\alpha}=\pi/4$, $m_{\beta}=\pi/4$

m_D, m	m_{X_M}, m	m_{Y_M}, m	m_{Z_M}, m	σ_{X_M}, m	σ_{Y_M}, m	σ_{Z_M}, m
100	50.0000	70.7107	50.0000	0.1001	0.1415	0.1001
500	250.0000	353.5534	250.0000	0.1027	0.1434	0.1027
1000	500.0000	707.1068	500.0000	0.1106	0.1491	0.1106
3000	1.5000e+03	2.1213e+03	1.5000e+03	0.1732	0.2000	0.1732
5000	2.5000e+03	3.5355e+03	2.5000e+03	0.2560	0.2749	0.2560
1000	3332146.1685	2422072.7166	4853992.6940	0.1579	0.1594	0.1689
3000	3331677.2667	2422968.1256	4855718.4906	0.2067	0.2393	0.2236
5000	3331208.3649	2423863.5347	4857444.2871	0.2798	0.3478	0.3049

Table 5

Results of calculating the estimates of mathematical expectations and SQV in the coordinates of aircraft location in the ICCS of OES and in GCS for $m_{\alpha}=\pi/6$, $m_{\beta}=\pi/4$

m_D, m	m_{X_M}, m	m_{Y_M}, m	m_{Z_M}, m	σ_{X_M}, m	σ_{Y_M}, m	σ_{Z_M}, m
100	61.2372	70.7107	35.3553	0.1226	0.1415	0.0709
500	306.1862	353.5534	176.7767	0.1247	0.1434	0.0745
1000	612.3724	707.1068	353.5534	0.1312	0.1491	0.0850
3000	1.8371e+03	2.1213e+03	1.0607e+03	0.1871	0.2000	0.1581
5000	3.0619e+03	3.5355e+03	1.7678e+03	0.2656	0.2749	0.2461
1000	3332162.7603	2421903.7426	4854065.1281	0.1588	0.1526	0.1743
3000	3331727.0421	2422461.2038	4855935.7929	0.2091	0.2373	0.2236
5000	3331291.3239	2423018.6650	485780.64576	0.2840	0.3497	0.2987

Table 6

Results of calculating the estimates of mathematical expectations and SQV in the coordinates of aircraft location in the ICCS of OES and in GCS for $m_{\alpha}=\pi/3$, $m_{\beta}=\pi/3$

m_D, m	m_{X_M}, m	m_{Y_M}, m	m_{Z_M}, m	σ_{X_M}, m	σ_{Y_M}, m	σ_{Z_M}, m
100	25.0000	86.6025	43.3013	0.0502	0.1732	0.0868
500	125.0000	433.0127	216.5064	0.0540	0.1740	0.0905
1000	250.0000	866.0254	433.0127	0.0645	0.1764	0.1014
3000	750.0000	2.5981e+03	1.2990e+03	0.1323	0.2000	0.1803
5000	1.2500e+03	4.3301e+03	2.1651e+03	0.2102	0.2404	0.2774
1000	3332423.0335	2422191.1062	4853953.0445	0.1538	0.1508	0.1739
3000	3332507.8618	2423323.2945	4855599.5419	0.1985	0.2164	0.2092
5000	3332592.6899	2424455.4827	4857246.0394	0.2661	0.3083	0.2660

Our analysis of results from calculating the estimates of mathematical expectations and SQV in the coordinates of aircraft location in the ICCS of OES and in GCS revealed their significant dependence on both the mathematical expectations of the azimuth, the elevation angle, and the slant range. At the same time, the dependence of SQV on the slant range is close to quadratic. The resulting estimates are consistent while the degree of their adequacy can be assessed as a result of specially designed, conducted, and processed natural (polygon) experiments.

6. Discussion of results related to devising an analytical method for assessing the accuracy of aircraft positioning in the air space

In the analytical method for obtaining estimates of mathematical expectations for the coordinates of aircraft location in the ICCS of OES and in GCS, mathematical models of open and hidden video surveillance of the ICN of OES were used along the trajectories of the movement of the aircraft. Analytical dependences of variances in the geo-positioning errors of aircraft depending on variances in the errors of all component video surveillance processes were derived by the method of statistical linearization of nonlinear functions of random arguments as a result of their expansion into a Taylor series with accuracy up to linear terms. Such linearization makes it possible to effectively use analytical dependences in real-time systems. To increase the accuracy of variance estimates, Taylor series expansion can be performed with accuracy not only for linear but also for quadratic terms (covariance matrices). The use of covariance matrices to calculate variance in the aircraft positioning errors in real-time systems turned out to be impractical both due to a significant increase in the complexity of calculations and a slight increase in the efficiency of variance estimates.

Expressions (27) to (29) and (36) to (38), taking into account the corresponding partial derivatives, are the analytical dependences of variances in the aircraft location errors according to each coordinate in the ICCS of the i-th OES and in GCS.

The main limitations of the analytical method include:

– high requirements for time synchronization accuracy of OES PCM hardware;

– for covert surveillance, the need for simultaneous monitoring of aircraft trajectories by two or more OES ICNs.

If these restrictions are met, the devised method is resistant to changes in the laws of distribution of random errors of video surveillance and makes it possible to obtain quite effective estimates of variances in the errors of coordinates of aircraft location in the air space.

Our method makes it possible to obtain fairly effective estimates of error variances in the coordinates of aircraft location in the airspace. The estimate is termed effective if its variance is minimal among other estimates. Reasonably efficient estimates are those estimates whose variance approaches the minimum but is not the minimum.

The scattering ellipsoid, which is currently the main form of the area for evaluating the metrological characteristics of radio-technical and opto-electronic systems, is a rather rough approximation of the actual area in which the aircraft is with a given probability for each fixed time point of video surveillance. This is due to the fact that the law of distribution of errors in the coordinates of aircraft location as a result of nonlinear transformations of normally distributed random variables will, as a rule, be asymmetric and different from the normal one. The study of the actual laws of distribution of errors in the estimation of coordinates of aircraft location and the construction of such areas is a possible way to advance future studies.

As a drawback, one should not the use of a least-squares method (LSM) for estimating mathematical expectations for coordinates of aircraft location for each time point, which is the most effective for the normal law of the distribution of video surveillance errors. Nonlinear transformations of random errors in direct measurements used in the devised method lead to changes not only in the parameters of the normal distribution law but also in the distribution law itself. In this case, the use of LSM can lead to biased and ineffective estimates of coordinates of aircraft location and, therefore, to a shift and loss of effectiveness of the analytical estimates of variance in the coordinates. Identification of the actual laws of distribution of random errors in the video observations of OES ICN along the trajectories of aircraft movement and the selection of optimal algorithms for processing their results require further theoretical and field studies. It is necessary to further verify our results by the method of mathematical modeling with representative samples of up to tens of thousands of observations and by planning, conducting, and processing the results of large-scale polygon tests.

7. Conclusions

1. We have constructed mathematical models of open and covert video surveillance of OES ICN along aircraft movement trajectories. Estimates of the mathematical expectations for the coordinates of aircraft location were derived as a result of solving the mathematical model of open video surveillance by the analytical method of NAU system and the mathematical model of covert video surveillance redefined by the LAU system using numerical LSM.

2. We have built analytical expressions of dependence for calculating the variance in aircraft positioning errors for each time point depending on variance in all types of random video surveillance errors of OES ICN along aircraft trajectory movement. The results of our study make it possible to obtain real-time estimates of SQV by each of the coordinates of aircraft location in the air space for each fixed moment of aircraft video surveillance by OES ICN.

3. Using the method of numerical modeling, the resulting dependences were studied for their consistency and reliability. Numerical values of SQV estimates of aircraft location coordinate errors for various video surveillance conditions are in the range from 0.1 to 0.35 m. This confirms the effectiveness of using OES ICN as an instrumental tool for high-precision trajectory measurements. The reported analytical method for estimating the aircraft positioning accuracy by OES ICN is workable. The obtained estimates of variances in the errors of coordinates of aircraft location in the air space could be used in real-time systems. To verify the method, we envisage the planning, conducting, and processing the results of large-scale polygon tests of the experimental model of OES ICN with different types of aircraft, for different weather conditions and arbitrary trajectories of aircraft movement.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Funding

The source of funding for producing the OES prototype for detecting and tracking moving objects in the air environment is the R&D work “Devising technology for high-precision targeting of aerial objects” under the contract dated 09.03.2021, No. DZ/105–2021 and the R&D work “Devising technology for detection and high-precision escort of aerial objects” according to the contract dated 05.11.2018, No. DZ/60-2018.

Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Acknowledgments

The authors of the paper express their gratitude to the team at the research plant of the Kharkiv National University of Radio Electronics, and personally to engineers A. Cherkashin, M. Yeletin, and V. Krylov.

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