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CONSTRUCTION OF A BALLISTIC MODEL OF THE MOTION OF UNCONTROLLED CARGO DURING ITS AUTONOMOUS HIGH-PRECISION DROP FROM A FIXED-WING UNMANNED AERIAL VEHICLE

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The object of this study is the process related to the fall of an uncontrolled cargo from a height of 400–600 meters relative to sea level in an air environment under the influence of gravity, air drag, and wind in the presence of an initial air speed of about 20 m/s. The study solves the task to build a ballistic model of the movement of an unguided cargo during autonomous high-precision dropping from an unmanned aerial vehicle of aircraft type. A system of equations was derived that explicitly describes the dependence of the cargo's travel speed and coordinates on time and takes into account the effect of gravity, air drag, and the influence of wind. The scope of application of the equations corresponds to drop heights of up to 400 m relative to the surface of the earth and the initial horizontal speed of the cargo up to 20 m/s. The resulting equations were analyzed using an example of a spherical load weighing 10 kg and the largest cross-sectional area of 7.1·10^{-2} m^2 when it falls from heights of 200, 300, and 400 m relative to the earth's surface. In the absence of wind, the horizontal component of the load's speed at the moment of falling is ≈(13–15) m/s, and the vertical component is ≈(50–60) m/s. At the same time, the horizontal displacement of the load under the conditions of a weak crosswind can reach ≈(150–220) m. With a vertical wind speed profile, the equivalent constant wind speed can be determined, resulting in the same effect on the load as a variable speed wind. An algorithm for determining the point of unloading the cargo has been proposed. The cargo delivery error has been evaluated. The most important parameters are the flight time of the cargo and the drop height. In order to achieve a hit accuracy of ±5 m, an error in determining the time of the fall of the load is not more than ≈0.16 s, and an error in determining the height of the drop is not more than ±8 m

Keywords: ballistic model of motion, cargo drop, aircraft-type unmanned aerial vehicle

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1. Introduction

Unmanned aerial vehicles (UAVs) are widely used for inspection and survey of territories and objects, terrain mapping, in agriculture, for search and rescue missions, for the delivery of cargo for various purposes, as well as for solving military tasks [1]. Moreover, all of these UAVs can operate autonomously without the need for human control in real time due to the presence of an autopilot, for example, with intelligent or adaptive control [2]. We note that the advantages of aircraft-type UAVs are low operating costs, simplicity of design, they are able to perform tasks under adverse or dangerous conditions, and their flight range is one of the largest [3]. Given this, aircraft-type UAVs are used in almost all of the areas listed above, including for cargo delivery. However, the delivery of cargo by such UAVs is associated with the difficulty of precision hitting the target area. Unlike copter drones, aircraft-type UAVs are not capable of hovering over the drop-off point, as well as vertically taking off – landing on almost any site. Therefore, when dropping from an aircraft-type UAV, the cargo will have a non-zero initial speed and move along a ballistic trajectory in the air environment, experiencing the effect of air drag. The impact of the peculiarities of meteorological conditions and the need to determine the aerodynamic characteristics for each type of falling body leads to even greater complications in the accurate delivery of cargo in this way.

Separately, worth considering is the height of the cargo drop. From the point of view of ensuring high accuracy of delivery and minimizing the probability of destruction of uncontrolled cargo when it hits the ground, the drop height should be minimal. On the other hand, when flying at an altitude of several tens of meters, there is a high probability that the UAV will collide with uneven terrain, trees, power lines, buildings, etc. The average height above sea level of the flat part of Ukraine is 175 m [4], and the highest buildings are of the Kyiv type TV towers reach a height of 385 m [5]. From this it follows that for carrying out UAV delivery missions in the flat part of Ukraine, the height of 400–600 m relative to sea level is optimal.

The speed of UAV at the moment of release is also essential from the point of view of accurate cargo delivery. It is obvious that in order to increase the accuracy of the delivery of the cargo with the maximum horizontal approach to the target, the indicated speed should be minimal. It should be added that the latter minimizes the influence of uncontrolled factors such as a gusty wind of variable direction during the movement of the cargo. However, the minimum speed of an aircraft-type UAV is limited by the stall speed. Thus, for one of the most popular UAVs of the aircraft type in Ukraine with a payload of up to 11 kg, the speed of release is 65 km/h (up to 18 m/s) [6]. So, focusing on a UAV with a payload of up to 11 kg, it can be assumed that the speed of the aircraft at the time of dropping the load will be close to 20 m/s.

All of the above predetermines the importance of building a ballistic model of cargo movement under the specified conditions for determining the drop point and precision hitting the target area. Thus, the construction of a ballistic model of the movement of unguided cargo during autonomous high-precision dropping from an aircraft-type UAV is a relevant task.

2. Literature review and problem statement

As evidenced by the review of scientific literature, considerable attention of researchers around the world focuses on the design of autonomous UAVs and high-precision cargo dropping systems [7–14]. For example, in [7], an autonomous aircraft-type UAV control system was developed and tested, which uses machine vision to identify targets, calculates the geographic coordinates of the target location, and plans a path to the drop point. In the paper, the ballistic model of cargo movement is a system of differential equations taking into account air drag and wind speed. At the same time, the numerical solution to the specified equations is performed by the on-board computer of UAV. But the issues of obtaining an analytical solution to the differential equations of cargo movement, which can simplify and speed up calculations during unloading, remained unsolved. The reason for this may be objective difficulties associated with taking into account the non-linear dependence of the air drag coefficient on the speed of the cargo. In addition, when dropping from high altitudes, the air density changes significantly, which also adds to the difficulties in finding the specified analytical solution. And, finally, the numerical solution is relatively accessible for implementation in the sense of the available hardware and software capabilities of the UAV on-board computer.

In [8], a mathematical model of a quadcopter was built to perform an air drop mission in a windy environment. The mathematical model of the free movement of cargo in the work takes into account wind speed and air drag. However, this model describes the unloading of cargo with zero initial horizontal speed and is also, as in [7], a system of differential equations. The analytical solution to this system of differential equations was also not obtained. The reasons for this are obviously the same as in [7].

Paper [9] reports a ballistic model of the free movement of cargo from high altitudes, but on a parachute, and also takes into account the influence of wind using Gaussian regression of the process. Within the framework of this ballistic model, the equations of coordinates and velocity of the load are recursive, which is explained by the need to obtain a numerical solution and the difficulty of determining analytical ratios. The ballistic model for accurate cargo drop from a UAV, which is described in [10], is also a system of differential equations, the solution to which is performed numerically in Python.

An option for overcoming the above difficulties in obtaining an analytical solution to differential equations is to consider the movement of the load under conditions that simplify the solution. This is the approach implemented in works [11, 12], in which the movement of cargo when dropped from UAV is described by analytical equations. However, these equations do not take into account the force of air drag and the effect of wind. Despite this, work [11] shows that if the load is dropped from a height of up to 10 m, in the worst case, the deviation from the target will be no more than 5 mm.

In [13], the system of differential equations of cargo motion is solved numerically by the Euler method. Therefore, in the work, as well as in [7–10], analytical expressions for the coordinates and speed of the load depending on time were not obtained. The reason for this has already been given above.

Work [14] considers a methodical approach to solving the aiming problem for dropping free-falling loads from UAV. The work shows the differential equations of motion of a freely falling load but the solutions to these equations are not given. The reason for this is that the solution to the differential equations of the movement of the cargo is beyond the scope of [14].

Thus, the ballistic model of the movement of unguided cargo during autonomous high-precision dropping from an aircraft-type UAV, which is a system of equations for the movement of the cargo, is clearly absent in the literature. The main reason for this is the difficulty of taking into account the drag force for a wide range of cargo speeds and the significant change in air density when dropping from high altitudes. Therefore, the differential equations of motion of an unguided load today are usually solved by numerical methods with the help of the on-board computer of UAV. However, when the load falls from a height of up to ≈400 m relative to the surface of the earth, its speed will not exceed 100 m/s. Estimates of the change in air density for such heights give a value of ≈5 %.

All this allows us to state that it is appropriate to conduct a study on the construction of a ballistic model of the movement of an unguided cargo during autonomous high-precision dropping from an aircraft-type UAV under the above conditions.

3. The aim and objectives of the study

The purpose of our study is to build a ballistic model of the movement of an unguided cargo during autonomous high-precision parachute-free dropping from an aircraft-type UAV, which is a system of equations of the movement of the cargo in an explicit form. This will make it possible to design a high-precision cargo drop system for aircraft-type UAVs.

To achieve the goal, the following tasks were set:

– to construct a mathematical model of the movement of uncontrolled cargo in the form of differential equations;

– to solve differential equations and obtain the equation of motion of an uncontrolled load in an explicit form;

– to perform an analysis of the resulting equations of motion of uncontrolled cargo;

– to consider the possibility of applying the resulting equations of motion of an uncontrolled load under the condition of wind of a constant direction, the speed of which changes with height according to the exponential law;

– to develop an algorithm for determining the drop point for high-precision cargo delivery using an aircraft-type UAV and estimate the cargo delivery error according to the resulting equations of motion.

4. The study materials and methods

The object of research is the process of uncontrolled cargo falling from heights of 400–600 meters relative to sea level in the air environment under the influence of gravity, air drag, and wind in the presence of an initial air speed of about 20 m/s.

The research hypothesis assumes that an adequate ballistic model of the movement of unguided cargo from heights up to 600 m relative to sea level could be obtained by analytically solving the corresponding differential equations.

The following assumptions and simplifications are adopted in our work:

– uncontrolled cargo is considered to be an absolutely solid body of the proper shape;

– during an uncontrolled fall, the load does not perform a rotational movement relative to the main axes of inertia;

– in the case of an uncontrolled fall of the load, the direction of the wind and its speed do not change both with time and with height;

– air drag coefficient for cargo is constant in the range of speeds of 0–100 m/s;

– air density does not change in the height range of 0–400 m relative to the earth's surface;

– the resulting movement of the load is the sum of the component of its movement in the absence of wind and the component of its movement caused by the action of the wind;

– the result of the influence of the wind is giving the cargo a velocity component of the same direction and magnitude as the wind velocity.

Research methods: analytical method for solving differential equations, mathematical modeling with the use of specialized mathematically oriented software for scientific and engineering calculations Mathcad 14.0 (USA).

The advantage of the analytical method for solving differential equations is that, as a result of its application, it is possible to derive the sought functions in an explicit form. Mathematical modeling in Mathcad 14.0 is used for their verification, visualization, and comparison.

When modeling, we consider a spherical load, the parameters of which are indicated in Table 1. The choice is due to the fact that this shape has a relatively small coefficient of air drag. In addition, the air drag coefficient and the cross-sectional area of the load of this shape do not depend on the direction of movement.

Table 1

Parameters of cargo in the form of a sphere that UAV drops

Parameter	CI-based parameter value
Cargo weight m	10 kg
Air drag coefficient ξ	0.48
The area of the medial section S	$7.1\cdot10^{-2}$ m^2
The initial airspeed of the cargo in the horizontal direction υ_{0x}	20 m/s
Time of falling from a height of 200 m	6.39 s
Time of falling from a height of 300 m	7.82 s
Time of falling from a height of 400 m	9.03 s

Table 1 also gives the time of the cargo falling from a height of 200–400 m relative to the ground surface without taking into account the force of air drag. The weight of the cargo is optimal for many types of air missions.

5. Results of research into the process of building a ballistic model of the movement of unguided cargo

5. 1. Mathematical model of uncontrolled load movement in the form of differential equations

The movement of cargo when dropped from an aircraft-type UAV will be considered relative to the horizontal non-rotating rectangular coordinate system Oηζλ and the horizontal rotational course rectangular coordinate system OXZY, similar to work [14]. As shown in Fig. 1, the course ψ of an aircraft-type UAV originates from the beginning of the orthodromic course η in the inertial frame of reference, ζOη lies in the horizon plane, axis ζ is directed along the orthodromic meridian and axis η – along the orthodromic parallel, axis λ indicates the vertical direction in this position. The location of XOZ corresponds to the horizontal plane of symmetry of UAV, while Y is directed vertically upwards along the geocentric or local vertical. The origin of coordinates O corresponds to the position of the center of mass of UAV.

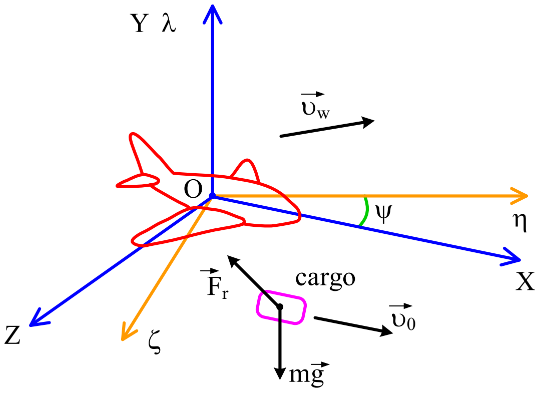

Fig. 1. Aircraft-type UAV cargo drop in rectangular coordinate systems [14]

The cargo at the moment of dropping has an initial speed $\vec{\upsilon}_0$ that coincides with the air speed of UAV at the same time point (Fig. 1). The uncontrolled movement of the cargo in the air environment occurs under the influence of gravity $m\vec{g}$ (m is the mass of the cargo), air drag $\vec{F}_r$ and wind speed $\vec{\upsilon}_w$. It was assumed that the gravitational force is directed along the Y and λ axes, and the transition from the OXZY coordinate system to $O\eta\zeta\lambda$ can be performed using the following formulas:

$$\begin{aligned} x &= \eta\cos\psi + \zeta\sin\psi, \\ y &= \lambda, \\ z &= \eta\sin\psi + \zeta\cos\psi. \end{aligned} \tag{1}$$

It is assumed that at the moment of dropping the cargo, the *X*-axis and the target point lie in the plane of UAV's path. Air drag, gravity, and wind speed were represented in the *OXYZ* coordinate system as follows:

$$\begin{aligned} \vec{F}_r &= F_{rx}\cdot\vec{i} + F_{ry}\cdot\vec{j} + F_{rz}\cdot\vec{k}, \\ m\vec{g} &= -mg_y\cdot\vec{j}, \\ \vec{\upsilon}_w &= \upsilon_{wx}\cdot\vec{i} + \upsilon_{wy}\cdot\vec{j} + \upsilon_{wz}\cdot\vec{k}, \end{aligned} \tag{2}$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ – ortho vectors of the rectangular coordinate system *OXYZ*; *x, y*, and *z* indices denote the projections of the specified vectors onto the *X, Y*, and *Z* axes, respectively. Note that the signs of the vector projections in (2) are determined by their direction relative to the axes of the *OXYZ* coordinate system for a specific case of cargo movement.

For low velocities of a body falling (less than ≈200 m/s [14]) in an air environment, the following expression for the value of air drag is valid:

$$F_r = \frac{1}{2}\xi S\rho\upsilon^2, \tag{3}$$

where ξ is the air drag coefficient; in general, it may take different values for the directions of movement relative to the *X, Y, Z* axes; *S* is the cross-sectional area of the cargo in the direction of the resistance force; in general, it may take different values for the directions of movement relative to the *X, Y, Z* axes; υ is the cargo speed relative to air flow; ρ is air density.

In the absence of wind, the road speed of the cargo, as well as its air speed, is equal to the speed υ. In the presence of wind, the ground speed of the cargo will be equal to the sum of the air speed of the cargo $\vec{\upsilon}_a$ and the wind speed $\vec{\upsilon}_w$. The air speed of the cargo before dropping is equal to the air speed of UAV. After dropping, the airspeed of the cargo is understood as its natural speed relative to the air in the absence of wind. Therefore, the following ratio can be written for the road speed of the cargo:

$$\vec{\upsilon}_s = \vec{\upsilon}_a + \vec{\upsilon}_w. \tag{4}$$

With the application of Newton's second law to the movement of the cargo after dropping from UAV, the following system of differential equations was built:

$$\begin{cases} ma_x = -\dfrac{1}{2}\cdot\xi_x\rho S_x\,\upsilon_{ax}^2, \\ ma_y = -mg + \dfrac{1}{2}\cdot\xi_y\rho S_y\,\upsilon_{ay}^2, \\ ma_z = -\dfrac{1}{2}\cdot\xi_z\rho S_z\,\upsilon_{az}^2, \end{cases} \tag{5}$$

where *m* is the mass of the cargo; a_x, a_y, a_z – cargo acceleration components; υ_{ax}, υ_{ay}, υ_{az} – projections of air speed of the cargo on the *X, Y, Z* axis.

5. 2. Analytical solution to the differential equations of motion of uncontrolled cargo

Equation (5) can be solved in general form by integration to determine the components of airspeed and payload coordinates as functions of time. During the integration, only the acceleration and airspeed of the cargo were considered to change over time. It is obvious that the solution to the first and third equations of system (5) will be similar. Therefore, solutions are given only for the first equation. After separating the variables in the first equation of system (5), we obtained:

$$\int\frac{d\upsilon_{ax}}{\upsilon_{ax}^{\ 2}} = -\int K_x\,\mathrm{d}t, \tag{6}$$

where $K_x = \dfrac{1}{2m}\cdot\xi_x\,\rho\,S_x$. For the solution to (6), the initial conditions were used, which imply that at the time of cargo unloading t=0 s and υ_{ax}=υ_{0x}. As a result, the following solution for the airspeed of the cargo along the *X* axis was obtained:

$$\upsilon_{ax}(t) = \frac{\upsilon_{0x}}{1 + K_x\,\upsilon_{0x}\,t}. \tag{7}$$

In a similar way, the time equation was derived for the air velocity of the cargo along the *Z* axis:

$$\upsilon_{az}(t) = \frac{\upsilon_{0z}}{1 + K_z\upsilon_{0z}\,t}, \tag{8}$$

where $K_z = \dfrac{1}{2m}\cdot\xi_z\,\rho\,S_z$. At the same time, the initial conditions used to obtain (8) imply that at the time of cargo unloading t=0 c and υ_{az}=υ_{0z}.

After separating the variables in the second equation of system (5), it was written:

$$-\frac{1}{K_y}\int\frac{\mathrm{d}\upsilon_{ay}}{\dfrac{g}{K_y} - \upsilon_{ay}^{\ 2}} = \int\mathrm{d}t, \tag{9}$$

where $K_y = \dfrac{1}{2m}\cdot\xi_y\,\rho\,S_y$. The solution to (9) is also based on the fact that when t=0 s, the air speed of the cargo was equal to υ_{0y}. At the same time, the following time dependence of the vertical component of the air velocity of the cargo was established:

$$\upsilon_{ay}(t) = \left(-\sqrt{\frac{g}{K_y}}\right)\cdot\left(\frac{C + e^{-2\sqrt{gK_y}\,t}}{C - e^{-2\sqrt{gK_y}\,t}}\right), \tag{10}$$

where:

$$C = \frac{\sqrt{\dfrac{g}{K_y}} - \upsilon_{0y}}{\sqrt{\dfrac{g}{K_y}} + \upsilon_{0y}}.$$

Thus, equations (7), (8), and (10) describe the change in airspeed of the cargo over time, when the countdown starts at the moment of drop. As noted above, the road speed of the cargo will be equal to the sum of the air speed and the wind speed:

$$\upsilon_{sx}(t)=\frac{\upsilon_{0x}}{1+K_x\,\upsilon_{0x}\,t}+\upsilon_{wx},$$

$$\upsilon_{sy}(t)=\left(-\sqrt{\frac{g}{K_y}}\right)\cdot\left(\frac{C+e^{-2\sqrt{gK_y}\,t}}{C-e^{-2\sqrt{gK_y}\,t}}\right)+\upsilon_{wy}, \tag{11}$$

$$\upsilon_{sz}(t)=\frac{\upsilon_{0z}}{1+K_z\upsilon_{0z}\,t}+\upsilon_{wz}.$$

As a result of integrating the equations for the components of the path speed, the dependence of the cargo coordinates on time in the $OXYZ$ system was found:

$$\begin{cases} x(t)=\dfrac{1}{K_x}\cdot\ln\left(1+K_x\upsilon_{0x}\cdot t\right)+\upsilon_{wx}\,t, \\ y(t)=\upsilon_{wy}\,t-\sqrt{\dfrac{g}{K_y}}\cdot t+\dfrac{1}{K_y}\cdot\ln\left(\dfrac{C+1}{C+e^{-2\sqrt{g\,K_y}\,t}}\right), \\ z(t)=\dfrac{1}{K_z}\cdot\ln\left(1+K_z\upsilon_{0z}\cdot t\right)+\upsilon_{wz}\,t. \end{cases} \tag{12}$$

It is clear that the path taken by the cargo along the X, Y, and Z axes can be obtained as the absolute value of the corresponding expression from (12).

5. 3. Analysis of the derived equations of motion of uncontrolled cargo

For the analysis of resulting equations (11) and (12), the traveling speed of the cargo in the time interval of falling up to 10–11 s was first considered. From Fig. 2, it can be seen that the component X of the cargo's traveling speed nonlinearly decreases with time, provided that the initial air speed of the cargo is 20 m/s. The vertical dashed lines on the plot indicate the time points when the cargo falls from heights of 200, 300, and 400 m, provided there is no wind along the X axis. Negative values of υ_{wx} in Fig. 2 correspond to the situation when the X component of the wind speed is directed opposite to the X component of the air speed of the cargo. It was established that the value of the X component of the cargo's path speed at the moment of fall for the specified drop heights, provided there is no wind along the X axis, is in the range of ≈(13–15) m/s.

The time dependences of the vertical component of the cargo speed in the absence of wind along the Y axis are shown in Fig. 3. It should be noted that the vertical component of the wind speed is much smaller than its horizontal component and plays a significant role only during strong convection or in the presence of orographic obstacles, when the air is forced to rise or flow down the slopes of elevations [15]. In addition, with increasing height relative to the earth's surface, the vertical component of wind speed decreases [16]. Thus, the vertical component of the wind speed can be neglected under the conditions of the fall of a cargo of a fairly significant mass from heights up to 600 m relative to sea level. Fig. 3 shows that at υ_{0y}=0 the absolute value of the vertical component of the cargo's path speed at the moment of fall is in the range of ≈(50–60) m/s.

The dependence of the Z component of the cargo's road speed on time is shown in Fig. 4, where the initial air speed of the cargo is 2 m/s, and the wind speed in the specified direction is 0, 5, and –5 m/s. One can see that under the conditions of insignificant values of the lateral air speed and weak wind, the traveling speed of the cargo along the Z axis in the investigated time interval changes slightly.

The change in x, y, and z coordinates of the cargo over time is shown in Fig. 5. The resulting dependences correspond to the wind speed (5; 0; –2) m/s and the initial air speed of the cargo (20; 0; 2) m/s. The displacement of the cargo along the X axis when falling from a height of 200 m is 154.178 m. The same displacements when falling from heights of 300 and 400 m are 190.791 m and 223.226 m, respectively. It is this distance under the specified conditions along the X axis between the UAV at the moment of drop and the target area that will ensure accurate cargo delivery. At the same time, the lateral offset of the cargo along the Z axis is – 0.203, – 0.325, and – 0.463 m for the drop height of 200, 300, and 400 m, respectively. If you need to deliver cargo within a radius of up to 5 meters from the center of the target zone, these are obviously satisfactory values. However, under the same conditions as shown in Fig. 5, but already at zero initial air speed of the cargo along the Z axis, the lateral displacement when falling from a height of 200–400 m relative to the surface of the earth already reaches 20–30 m. This requires correction of the course of UAV before dropping the cargo. Such a correction implies that the UAV needs to be shifted parallel to the X axis in the opposite direction from the direction of lateral cargo offset by a distance of 20–30 m.

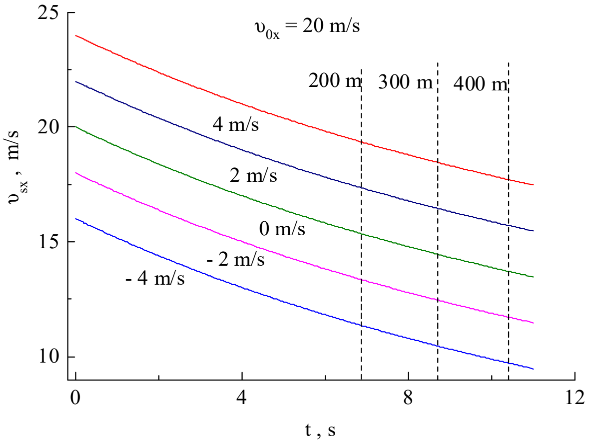

Fig. 2. Dependence of the X component of road speed of the cargo on time for different values of the wind speed along the X axis

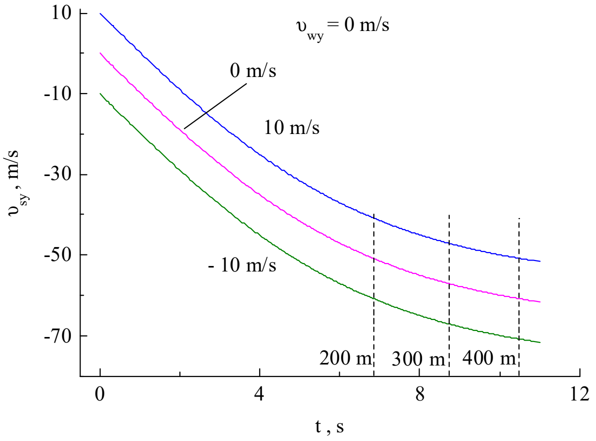

Fig. 3. Dependence of the Y component of the cargo's traveling speed on time for different values of its initial air speed along the Y axis

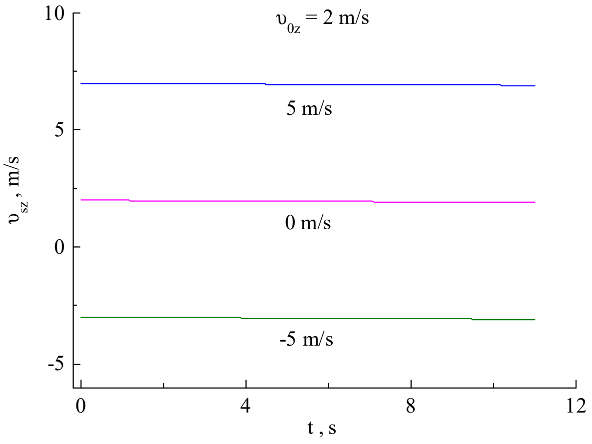

Fig. 4. Dependence of the Z component of the cargo traveling speed on time for different values of the wind speed along the Z axis

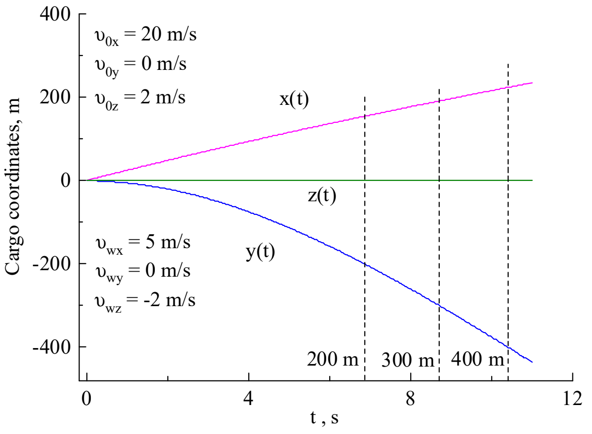

Fig. 5. Dependence of *x, y, z* coordinates of the cargo on time

Consequently, the initial conditions of the fall of the cargo significantly affect the accuracy of its delivery.

5. 4. Application of the derived equations of cargo movement under the condition of wind with exponential distribution of speed by height

Until now, it was believed that the UAV flies in the presence of wind, the speed and direction of which do not change. However, as evidenced by [17], the vertical profile of the wind speed is described by the law of power *p*:

$$\frac{\upsilon_{w1}}{\upsilon_{w2}}=\left(\frac{h_1}{h_2}\right)^p, \qquad (13)$$

where υ_{w1} is the wind speed at height h_1, and υ_{w2} is the wind speed at height h_2. The value of parameter *p* depends on the stability class of the atmosphere according to Pasquil [17]. The stability class of the atmosphere, in turn, is determined by the vertical gradient of air temperature. For example, the atmosphere is considered very stable when the vertical temperature gradient is greater than 4.0 °C per 100 m and $p\approx(0.4–0.6)$. When such a temperature gradient is less than –1.9 °C per 100 m, the atmosphere is considered very unstable and $p\approx0.1$. Accordingly, determining the value of *p* for a given state of the atmosphere requires measurements of the vertical temperature gradient.

The assessment of the lateral offset of the cargo was performed in the presence of a wind component along the *Z* axis with a speed of 5 m/s at a height of 400 m relative to the earth's surface. At the same time, the initial air speed of the cargo along the *Z* axis was assumed to be zero. It was considered that the state of the atmosphere is stable according to Pasquil and $p\approx0.5$. Then, for this case, the following dependence of wind speed on height was derived:

$$\upsilon_{wz}(h)=5\cdot\left(\frac{h}{400}\right)^{0.5}. \qquad (14)$$

According to (12), the dependence of altitude on time can be written as:

$$h(t)=400-\sqrt{\frac{g}{K_y}}\cdot t+\frac{1}{K_y}\cdot\ln\left(\frac{C+1}{C+e^{-2\sqrt{g\,K_y}\,t}}\right). \qquad (15)$$

In (15) it is assumed that the vertical component of the wind speed and the vertical component of the cargo's air speed are equal to zero. Then the dependence of the wind speed in the *Z* direction on time corresponds to the ratio:

$$\upsilon_{wz}(t)= \\ =5\cdot\left(\frac{400-\sqrt{\frac{g}{K_y}}\cdot t+\frac{1}{K_y}\cdot\ln\left(\frac{C+1}{C+e^{-2\sqrt{g\,K_y}\,t}}\right)}{400}\right)^{0.5}. \qquad (16)$$

When integrating (16) in the time interval from 0 to 10.40 s, which corresponds to the fall of the cargo from a height of 400 m, the lateral wear of the cargo was determined:

$$S_z=\int_0^{10.40}\upsilon_{wz}(t)\,\mathrm{d}t=60.466\text{ m}. \qquad (17)$$

As a result of dividing the obtained number by the time the cargo fell, the equivalent constant wind speed along the *Z* direction was determined:

$$\upsilon_{ew\,z}=\frac{S_z}{t}. \qquad (18)$$

By substituting in (18) the value of S_z from (17) and the time of 10.40 s, the value of the equivalent constant wind speed of 5.814 m/s was obtained.

Thus, knowing the vertical profile of the wind speed, one can get the equivalent value of the constant wind speed, which affects the movement of the cargo in the same way as the wind, the speed of which changes with height. Applying the value of the equivalent wind speed in equations (11) and (12), it is possible to determine the speed and coordinates of the cargo even under conditions of variable wind speed.

5. 5. Algorithm for determining the drop point and evaluating the cargo delivery error when using UAV

Based on the proposed ballistic model, it is possible to propose the following algorithm for determining the drop point for high-precision cargo delivery using an aircraft-type UAV (Fig. 6).

The horizontal flight range of the cargo after dropping from the indicated heights can be up to 200 m. To collect and process information from the UAV meters, a certain time is required, which will be the longer the lower the speed of the UAV. Therefore, it can be recommended to reduce the air speed to the minimum possible, which is about 20 m/s when the UAV approaches the target area at a horizontal distance of 2,000 3,000 m. At the same distance, the UAV autopilot directs its true flight course to the target area, and also changes the UAV flight height to the reset height (stage 1 in Fig. 6). After that moment, the UAV's airspeed, true course, and flight altitude do not change.

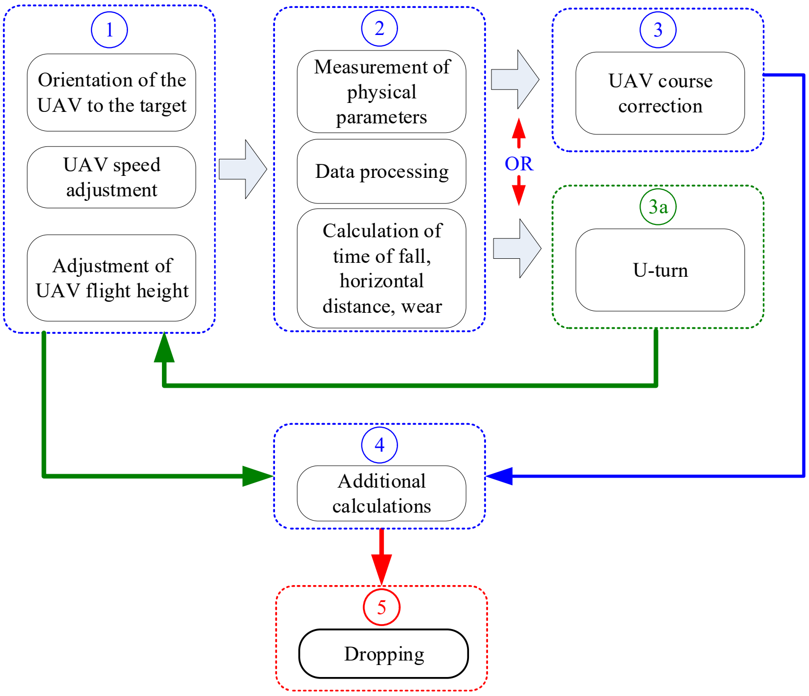

Fig. 6. Block diagram of the algorithm for determining the drop point for high-precision cargo delivery using an aircraft-type UAV

At the indicated speed, approximately 90–140 seconds remain for the acquisition and processing of data from the meters at UAV. Of course, the actual duration of this time will depend on the presence and speed of the wind. During this time, the UAV receives data on the current height of the flight relative to the ground, wind speed, and direction, air speed, atmospheric pressure, air temperature. Using the cargo motion equations (12), the UAV calculates the time the cargo falls from a given height, the horizontal flight distance of the cargo, and the distance of the cargo's lateral movement (stage 2 in Fig. 6). If the distance of the lateral movement of the cargo is more than 5 m, then the actual course is changed to one that is parallel to the previous one but shifted from it in the corresponding direction by the amount of the lateral movement of the cargo (stage 3 in Fig. 6). After that, the horizontal distance from the UAV to the target area is compared with the horizontal distance of the cargo flight (stage 4 in Fig. 6). When the horizontal distance from the UAV to the target area becomes greater than the horizontal flight distance of the cargo by the amount of the technical correction, reset occurs (stage 5 in Fig. 6). The technical correction includes the time for carrying out the last calculation operation before reset and the time from the submission of the command to the release of the cargo.

If there is such an opportunity, it is recommended that the UAV flies over the target area, collects all the data, and processes it, and after turning around, returns to the same course and performs the cargo drop. In this case, the following sequence of stages is performed: 1→2→3a→1→4→5 (Fig. 6).

The accuracy of delivery was evaluated within the framework of the constructed ballistic model of the movement of unguided cargo. One of the most important parameters that significantly affects the accuracy of cargo delivery is the time of fall and the height of the drop relative to the ground. These two parameters are interrelated, the accuracy of the calculation of horizontal along the X axis and lateral along the Z axis of the cargo depends on the accuracy of their determination. Therefore, it is necessary to assess how the inaccuracy of determining the time and height of the fall will affect the result of dropping the cargo.

It was believed that as a result of measurements and calculations, the UAV received the value of the time of the fall of the cargo t_m, and the real time of the fall of the cargo is t_r. For a cargo in the shape of a sphere, we shall estimate the deviation of horizontal and lateral offset for different deviations of t_m from t_r (Table 2). One can see from the data in Table 2 that under the condition of an error in measuring the height of the fall of up to 5–6 m and, accordingly, a fall time of 0.1 s, the error of hitting the cargo on the target along the X coordinate will be about 1.5 m. At the same time, the error of hitting the cargo on the target along the Z coordinate will be only 0.5 m.

Table 2

Deviation of the horizontal $|\Delta x|$ and lateral $|\Delta z|$ offset due to the error in determining the time $|\Delta t|$ and drop height $|\Delta y|$ of the cargo

Calculation conditions	$\lvert\Delta t\rvert=\lvert t_m-t_r\rvert$, s	$\lvert\Delta x\rvert$, m	$\lvert\Delta y\rvert$, m	$\lvert\Delta z\rvert$, m
$\upsilon_{0x}=20$ m/s, $\upsilon_{0y}=\upsilon_{0z}=0$ m/s, $\upsilon_{wx}=\upsilon_{wy}=0$ m/s, $\upsilon_{wz}=5$ m/s	Drop height relative to the ground surface ≈200 m			
	0.01	0.154	0.510	0.050
	0.1	1.538	5.085	0.5
	1	15.103	52.943	5
	Drop height relative to the ground surface ≈300 m			
	0.01	0.145	0.571	0.050
	0.1	1.448	5.700	0.5
	1	14.693	55.709	5
	Drop height relative to the ground surface ≈400 m			
	0.01	0.137	0.607	0.050
	0.1	1.374	6.065	0.5
	1	13.926	59.817	5

The accuracy of hitting the cargo at the target point, which is ±5 m along the X and Z coordinates, was considered acceptable. The data in Table 2 show that, under the specified conditions, the error of hitting the Z coordinate will be 5 m with a time determination error of 1 s for all cargo drop heights. At the same time, to determine the permissible error of the time of unloading the cargo and the permissible error of the X coordinates, the data from Table 2 was represented in Fig. 7. From Fig. 7, it can be concluded that for all considered drop heights, the error of hitting the target point along the X coordinate will be up to 5 m, when the error of the time of falling of the cargo will be no more than ≈0.16 s. This error in the time of the fall of the cargo corresponds to the error in determining the height of the cargo drop ≈(8–10) m.

Under the given conditions of dropping the cargo, the error of delivery to the specified point will be ±5 m, provided that the error in determining the height is no more than ±8 m or the error in determining the time of the cargo falling no more than ≈0.16 s.

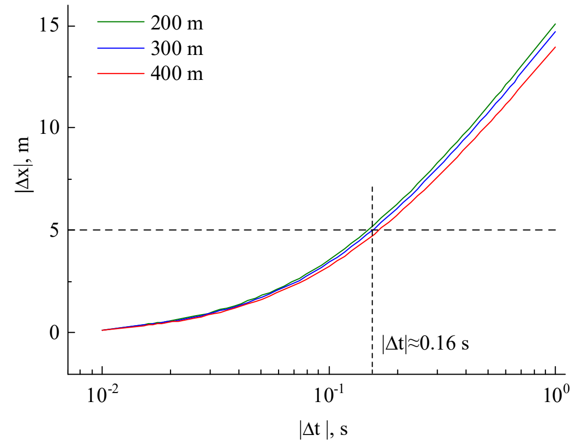

Fig. 7. Dependence of the absolute value of the horizontal offset deviation during cargo delivery on the absolute value of the fall time determination error

Therefore, it is necessary to adhere to the indicated intervals of determining the time and height of the drop in order to achieve the indicated accuracy of delivery.

6. Discussion of results related to the construction of the ballistic model of the movement of unguided cargo

The equations of motion of uncontrolled cargo (11) and (12) derived in this study, in contrast to works [11, 12], combine the influence of gravity, air drag, steady wind, and initial conditions of unloading. In addition, the scope of application of the cargo motion equations in works [11, 12] is limited by the drop height of only a few tens of meters. At the same time, it was established in [12] that the average error of cargo delivery from a height of 40 m is ±4 m. In the same study, the principal possibility of achieving a cargo delivery error from a height of up to 400 m relative to the earth's surface of ±5 m was shown.

Application of equations (11) and (12) to the fall of the cargo, the parameters of which are indicated in Table 1, give the time dependences of the speed components (Fig. 2–4) and coordinates (Fig. 5) of the cargo, which do not contradict the physics of its movement in the air environment under conditions of low speeds and flight altitude.

In contrast to [7–14], our study shows the possibility of applying the equivalent wind speed. Thus, equations (16) to (18) confirm that when the vertical wind speed profile is known, it is possible to correctly apply equations (11), (12) with the involvement of the value of the equivalent wind speed (18).

The proposed algorithm for determining the drop point for high-precision cargo delivery using an aircraft-type UAV (Fig. 6) is the result of the analysis of equations (11), (12). The most critical parameter for accurate cargo delivery, as evidenced by the data in Table 2 and Fig. 7, is the horizontal offset of the cargo along the *X*-axis because, as expected, it is in this direction that the cargo has the highest speed in the horizontal plane. At the same time, it is important to accurately determine the time and height of the fall of the cargo relative to the surface of the earth.

The advantage of this study is that, unlike works [7–10, 13, 14], the equation of motion (11), (12) of an uncontrolled cargo was derived in an explicit form. In addition, in contrast to [7–10, 13, 14], in this paper, the movement of the cargo is considered as the sum of the component of its movement in the absence of wind and the component of its movement caused by the action of the wind. The resulting equations are applicable at an initial velocity of about 20 m/s and a drop height of up to 600 m relative to sea level. In addition, these equations take into account the effect of gravity, air drag, and wind.

It is expected that the application of the research results could simplify and speed up the UAV control systems related to managing the cargo drop with a delivery accuracy of ±5 m.

The limitations of our study are that the constructed ballistic model of uncontrolled cargo movement is valid for low heights and speeds of dropping cargo weighing up to 10 kg. At higher speeds, it is necessary to take into account the dependence of the air drag coefficient on the speed of the cargo. In addition, at significantly higher drop heights, the change in air density with height will have a significant impact on the movement of the cargo.

The shortcomings of this study include the fact that the resulting equations of uncontrolled movement of the cargo do not take into account possible fluctuations relative to the center of mass and rotation of the cargo relative to its own axes of inertia. The role of the Magnus effect has also not been assessed. In addition, this theoretical study needs experimental verification.

The continuation of this study involves the development of software based on our equations for managing the dropping of cargo from UAV. This will provide an opportunity to conduct a series of test resets to determine the experimental cargo delivery error. It is likely that during experiments it will be found that the cargo is prone to rotation relative to its center of mass. In the presence of wind due to the Magnus effect, this could lead to additional deviations from the calculated trajectory and would require correction of the model built.

7. Conclusions

1. A mathematical model of the movement of uncontrolled cargo was constructed in the form of differential equations that include the action of gravity and air drag. The model differs from similar ones in that the influence of wind of constant direction and speed is taken into account already at the stage of solving differential equations.

2. The equation of the movement of the cargo after unloading was derived, which, unlike similar works, clearly represents the speed and coordinates of the cargo as a function of time.

3. The resulting equations for a spherical cargo with a mass of 10 kg and an area of the largest cross-section of the cargo of $7.1 \cdot 10^{-2}$ m^2 from heights of 200, 300, and 400 m relative to the earth's surface were analyzed. Our analysis reveals that in the absence of wind, the horizontal component of the cargo's speed at the moment of fall is ≈(13–15) m/s, and the vertical component is ≈(50–60) m/s. The horizontal displacement of the cargo under conditions of a weak crosswind can reach ≈(150–220) m.

4. It was established that in the presence of a vertical wind speed profile of an exponential power type, it is possible to determine the equivalent speed of a constant wind, which leads to the same effect on the cargo as the wind of variable speed.

5. An algorithm for determining the cargo drop point has been proposed, which includes the stage of possible

course correction, if the lateral offset of the cargo according to the calculations exceeds the permissible value of ±5 m. In addition, this algorithm takes into account two possible scenarios: UAV reaching the drop point after passing over the target and U-turn for more accurate estimates and a single run to the reset point. The evaluation of cargo delivery error shows that the most important parameters are cargo flight time and drop height. The amount of horizontal offset of the cargo depends most strongly on them. In order to achieve a hit accuracy of ±5 m, an error in determining the time of the fall of the cargo is not more than ≈0.16 s, and an error in determining the height of the drop is not more than ±8 m.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Darvishpoor, S., Roshanian, J., Raissi, A., Hassanalian, M. (2020). Configurations, flight mechanisms, and applications of unmanned aerial systems: A review. Progress in Aerospace Sciences, 121, 100694. https://doi.org/10.1016/j.paerosci.2020.100694
2. Telli, K., Kraa, O., Himeur, Y., Ouamane, A., Boumehraz, M., Atalla, S., Mansoor, W. (2023). A Comprehensive Review of Recent Research Trends on Unmanned Aerial Vehicles (UAVs). Systems, 11 (8), 400. https://doi.org/10.3390/systems11080400
3. Hassanalian, M., Abdelkefi, A. (2017). Classifications, applications, and design challenges of drones: A review. Progress in Aerospace Sciences, 91, 99–131. https://doi.org/10.1016/j.paerosci.2017.04.003
4. Petruk, V. H., Bondarchuk, O. V., Trach, I. A., Panasiuk, O. Yu., Tsaruk, Yu. I., Vakoliuk, V. D. (2016). Ekoheohrafiia ta ekoturyzm. Vinnytsia: TOV «Nilan-LTD», 178. Available at: https://files.znu.edu.ua/files/Bibliobooks/Inshi74/0054541.pdf
5. Shimanovsky, A. V. (2016). Creation of unique long-span constructions with the use of modern methods of mechanics. Visnik Nacional'noi' Akademii' Nauk Ukrai'ni, 01, 86–93. https://doi.org/10.15407/visn2016.01.086
6. Tekhnichni kharakterystyky BpAK PD-2. Available at: https://ukrspecsystems.com/uk/drones/pd-2-uas
7. Mathisen, S. G., Leira, F. S., Helgesen, H. H., Gryte, K., Johansen, T. A. (2020). Autonomous ballistic airdrop of objects from a small fixed-wing unmanned aerial vehicle. Autonomous Robots, 44 (5), 859–875. https://doi.org/10.1007/s10514-020-09902-3
8. Liuswanto, C., Jenie, Y. I. (2021). Modelling and design of flight control for quadcopter in ballistic airdrop mission under wind perturbation. IOP Conference Series: Materials Science and Engineering, 1173 (1), 012064. https://doi.org/10.1088/1757-899x/1173/1/012064
9. Yang, S., Jeon, S. (2019). Recursive Path Planning and Wind Field Estimation for Precision Airdrop. Journal of Guidance, Control, and Dynamics, 42 (6), 1429–1437. https://doi.org/10.2514/1.g003944
10. Mitridis, D., Mathioudakis, N., Panagiotou, P., Yakinthos, K. (2021). Development of a Cargo Airdrop Modeling Method for a Tactical Blended-Wing-Body UAV. IOP Conference Series: Materials Science and Engineering, 1024 (1), 012044. https://doi.org/10.1088/1757-899x/1024/1/012044
11. Ivanovic, A., Orsag, M. (2022). Parabolic Airdrop Trajectory Planning for Multirotor Unmanned Aerial Vehicles. IEEE Access, 10, 36907–36923. https://doi.org/10.1109/access.2022.3164434
12. Mardiyanto, R., Pujiantara, M., Suryoatmojo, H., Dikairono, R., Irfansyah, A. N. (2019). Development of Unmanned Aerial Vehicle (UAV) for Dropping Object Accurately Based on Global Positioning System. 2019 International Seminar on Intelligent Technology and Its Applications (ISITIA), 86–90. https://doi.org/10.1109/isitia.2019.8937269
13. Mathisen, S. H., Grindheim, V., Johansen, T. A. (2017). Approach Methods for Autonomous Precision Aerial Drop from a Small Unmanned Aerial Vehicle. IFAC-PapersOnLine, 50 (1), 3566–3573. https://doi.org/10.1016/j.ifacol.2017.08.624
14. Petrov, V., Shalyhin, A., Kudriavtsev, A. (2020). Methodical approach to the solution of the objective of the aim for discharge of freely falling goods from unmanned aircraft. Science and Technology of the Air Force of Ukraine, 1 (38), 84–90. https://doi.org/10.30748/nitps.2020.38.10
15. Netrobchuk, I. M. (2019). Meteorolohiya ta klimatolohiya. Lutsk: Vezha-Druk, 108.
16. Chellali, F., Khellaf, A., Belouchrani, A. (2012). Identification and analysis of wind speed patterns extracted from multi-sensors measurements. Stochastic Environmental Research and Risk Assessment, 27 (1), 1–9. https://doi.org/10.1007/s00477-012-0558-0
17. Touma, J. S. (1977). Dependence of the Wind Profile Power Law on Stability for Various Locations. Journal of the Air Pollution Control Association, 27 (9), 863–866. https://doi.org/10.1080/00022470.1977.10470503